701

ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

ЛАБОРАТОРИЯ ЯДЕРНЫХ ПРОБЛЕМ

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Abstract

Using the scintillation counter method measurement is made of the values, average with respect to two states of the hyperfine structure, of the $\bar{a}_0$ assymetry coefficients of $(\mu - e)$-decay electrons in mesic atoms of silver, red and black phosphorus, and of τ the life time of mesons in these modifications of phosphorus. The quatities $\bar{a}_0$ for red (insulator) and black (conductor) phosphorus directly indicate that the time of the relaxation of meson spins in mesic atoms decreases owing to the presence of conductivity electrons. On the basis of $\bar{a}_0$ and τ are found the n_1 and n_0 level populations in the states $F=1$ and $F=0$ in phosphorus as well as the λ_1 and λ_0 probabilities of the capture of mesons by the nucleus in these states. The values found directly indicate that $\lambda_1 \neq \lambda_0$ (weak interaction is spin-dependent) and $\lambda_0 > \lambda_1$ (the capture from the state $F=0$ occurs more rapidly than from the state $F=1$). The lower limit of the magnitude $\frac{\Delta\lambda}{\lambda} = \frac{\lambda_1 - \lambda_0}{\frac{3}{4}\lambda_1 + \frac{1}{4}\lambda_0}$ obtained is direct evidence in favour of the $(A - xV)$-interaction.

Introduction

It was mentioned in ref. /1/ that the measurement of the probability of the capture of muons by nuclei in two states of the hyperfine structure can furnish information on the spin dependence of weak interaction in the process $\mu^- + p \to n + \nu$. The idea of the proposition reduces to this: if probability of the capture in two states is different, the time dependence of the number of electrons in the $(\mu - e)$-decay in isolated mesic atoms is not described by a simple exponent and the logarythm of the decay curve must have a positive curvature. Yet an experimental check of the theory is possible only for the production of mesic atoms in a medium. The presence of a medium can lead to muon transition between hyperfine structure levels and consequently to a more complex pattern. It was shown in ref. /2/ that the existence of the transitions and the spin dependence on the interaction proves useful in the experimental investigation of the type of the interaction. Thus, if the probability of the transition R is known the measurement of the sign and magnitude of the curvature K of the logarythm of the decay curve uniquely determines the type of the interaction. Yet the problem of determining the quantity R is very complex /3-5/ and has not yet been solved experimentally /6/. Therefore no final conclusions can be drawn on the variant of interaction if one has nothing to go by but the experimental data on the curvature K /6/.

From refs. /5,7/ it follows that unique conclusions on the type of interaction can be drawn also by measuring the quantities n_1 and n_0, the level populations in the state $F=1$ and $F=0$, and $\Delta\lambda$, the difference of the probabilities of the capture of mesons by the nucleus in these states. The convenient object for experiment is phosphorus with nuclear spin $I=\frac{1}{2}$. Phosphorus, indeed, forms several allotropic modifications. If it is taken into account that conductivity electrons have a kind of catalytic effect, decreasing the time of nuclear spin relaxation, red (insulator) and black (conductor) phosphorus are of the greatest interest. It may be expected that due to the conversion on conduction electrons[2,5,7] the difference in the quantities n_1 and n_0 for these modifications at the moment the mesons are captured by nuclei will be sufficient for the measurement of their values and the quantity $\Delta\lambda$ to be possible.

In the present paper the quantities n_1 , n_0 and $\Delta\lambda$ were determined by measuring the asymmetry in the electron angular distribution of ($\mu-e$) -decay electrons and lifetime of μ^--mesons in red and black phosphorus.

Experiment

a) Measurement of Asymmetry in the Angular Distribution of ($\mu-e$) -Decay Electrons.

If the method offered in ref.[8] is used for measuring the asymmetry, the precession curve observed in experiment for substances with nuclear spin $I\neq 0$ will be determined by the superposition of the curves of the precession of μ -mesons decaying from both states of the hyperfine structure. Consequently, the quantity of the $\bar{a}_0$ asymmetry coefficient in the electron angular distribution $1+a\cos\theta$ integrated over energy will be one average over two states. The transitions will be evidenced in that $\bar{a}_0$ will depend on time[5,6]. The relative measurements[9,10] of the a_0 asymmetry coefficients averaged over the entire spectrum for substances with $I=0$ show that a_0 does not depend on time and the atomic number Z . If a_0 does not depend on Z the level populations $F=1$ and $F=0$ at the moment the mesons are captured by phosphorus nuclei can be assessed through comparing the quantities $\bar{a}_0$ for silver and phosphorus. Indeed, in silver mesic atoms having $I=\frac{1}{2}$ and nuclear magnetic moment $\mu_N<0$, the transitions will occur from the state $F=0$ to the state $F=1$. Since mesons are depolarized in the state $F=0$, and the spin direction 'lingers' in the state $F=1$, the presence of transitions will not affect the quantity $\bar{a}_0$: $\bar{a}_0$ will be constant in time and equal to $\bar{a}_0=\frac{1}{2}a_0$ [11]. In the case of phosphorus mesic atoms, having $I=\frac{1}{2}$ and $\mu_N>0$ the state $F=0$ is the lowest and the existence of transitions will lead to a decrease of $\bar{a}_0$ with time. At the initial moment $t=0$ $\bar{a}_0=\bar{a}_{0g}=\frac{1}{2}a_0$ for phosphorus. Then the average effective (for the time interval from 0 to t) level populations $\bar{n}_1$ and $\bar{n}_0$ can be found from the following relation if it is taken into account that $\bar{n}_1+\bar{n}_0=1$:

$$\frac{\bar{a}_P}{\bar{a}_{Ag}} = \frac{\bar{a}_P}{\bar{a}_0} = \frac{\bar{n}_1 a_1 + \bar{n}_0 a_0}{\frac{3}{4} a_1 + \frac{1}{4} a_0} \qquad /1/$$

where $\bar{a}_P$ and $\bar{a}_{Ag}$ are the quantities $\bar{a}_0$ for phosphorous and silver respectively measured for the time from 0 to t and a_1 and a_0 are the asymmetry coefficients in the states $F=1$ and $F=0$ depending on the nucleus spin alone[11]. Sulphur and cadmium can be used out of the substances with $I=0$ and Z close to Z of phosphorus and silver to check that a_0 does not depend on Z. Indeed, a_0 should not depend on Z, according to the theoretical predictions made in ref.[12] for $Z \geq 15$ when the absolute quantities of the quantum yields of $K-$ and $L-$ series in mesic atoms are constant[13] and the effect of the Auger transitions is insignificant. The quantities a_0 and $\bar{a}_0$ can be found by measuring the number of electrons N_{max} and N_{min} for two values of the strength of the magnetic field $\pm H$ (which contains the target[14]), corresponding to the calculation by the formula.

$$t_1 + \Delta t = \frac{T}{2} = \frac{\pi mc}{eH},$$

where t_1, is the delay time, Δt the width of the 'gate' and T the precession period of a 'free' meson spin (sulphur, cadmium) or a mesic nuclear spin (phosphorus, silver).

The experimental conditions and set-up are the same as in the investigations published before[9,14]. Fig. 1 represents the block diagram of the set-up. The negative μ-mesons stopped in target 6 were detected by circuit 10 of anticoincidences 1+2−3. The pulses from scheme 10 stopped for 0.1 mcsec opened passage circuit ('gate') 12 for 1.2 mcsec. Through the 'gate' passed the pulses from the ($\mu - e$) -decay electrons from circuit 11 of coincidences 4+5 to be detected by a recalculation device. The targets were 15 × 15 cm; and 8 gr/cm^2 thick. In the experiments with sulphur and phosphorus paraffin filter 9 between counters 4 and 5 was 7 gr/cm^2 thick. In the experiments with silver and cadmium paraffin filter 9 was replaced by an aluminium one, also 7 gr/cm^2 thick. Owing to the use of the aluminium filter the efficiency of the detection of γ-rays of energy less than 10 MeV emitted from the target as a result of meson absorption was made less than 10^{-3}. The thickness of the targets and paraffin filter was considerable for the asymmetry to be measured at the end of the electron decay spectrum where asymmetry coefficient proves to be larger than the coefficient a_0.

The quantities $\xi = \frac{N_{max}}{N_{min}}$ obtained are given in the third column of table 1.

Table I

Substance	Nuclear Spin	$\xi = \frac{N_{max}}{N_{min}}$
Cadmium	0	1.10 ± 0.01
Silver	½	1.05 ± 0.01
Sulphur	0	1.10 ± 0.01
Phosphorus (red)	½	1.04 ± 0.01
Phosphorus (black)	½	1.00 ± 0.01

The corrections were introduced for the delay time, the width of the gate, τ the lifetime of mesons and the solid angle of the electron detector. For the corrections taking account of the decay and capture of mesons use was made of the values τ for red and black phosphorus obtained in the present paper (see below) and for sulphur, cadmium and silver those obtained in ref. /15/. The errors indicated are standard statistical deviations.

b) Measuring the Lifetime of μ^--Mesons in Phosphorus.

When the conventional method is used for measuring the lifetime, the μ^--meson decay curve observed in phosphorus is determined by the superimposition of the decay curves of μ^--mesons decaying from two states of the hyperfine structure. Then the measured quantity of the sum $S = \frac{\sum t_i n_i}{\sum n_i}$ (here n_i is a number of electrons at the moment t_i) is averaged over the states $F=1$ and $F=0$, since it can readily be shown that $S = \bar{n}_1 S_1 + \bar{n}_0 S_0$ (see appendix 1). It is evident that the values of the population of the levels $\bar{n}_1$ and $\bar{n}_0$ are known for red and black phosphorous, the measurement of the quantities S determines S_1 and S_0 uniquely, and hence τ_1 and τ_0 , the lifetimes of μ -mesons in the states $F=1$ and $F=0$.

The block diagram of the experimental set-up used in these experiments is represented in Fig. 1. Magnetizing coil 7 was used for compensating the scattering magnetic field of a synchrocyclotron. The electron equipment worked in the following manner. The pulses from circuit 10 of anticoincidences 1+2−3 started up trigger 13 which produced positive rectangular stable pulses of 5 microsec. Passage circuit ('gate') 14 was also opened for 5 microsec. Through the gate passed the pulses from circuit 11 of coincidences 4+5. Stopped in 12 for 0.2 mcsec., these pulses triggered pulse-forming device 16. Delay device 12 was introduced for detecting the zero time between the pulses on the screen of analyser 18. The pulses intended for triggering converter 17 came to its first entrance from circuit 16 and to the second one from pulse-forming device 15 started up by the trailing front of a pulse from trigger 13. At the converter exit the pulses the amplitude of which is proportional to the interval of time between two pulses entering its entrances were analyzed with a 128-canal amplitude AMA-3C analyzer 18 /16/. The linearity of the equipment was checked by a set of delay cables

PK3–401 with a delay of 0.52 microsec. The identity of the delay was checked by the resonance method accurately within 0.5%. Fig. 2 represents the dependence of the canal number n of the analyser on the delay t of the second pulse in the coverter with respect to the firs one. It is clear from Fig. 2 that nonlinearity did not exceed one per cent. The stability of the calibration of the equipment for 15 hours of work was better than 1 per cent. For determining the zero canal of the analyser scintillation counters 1,2,3,4,5 were situated along the 'axis' of a meson beam, with the anticoincidence canals switched off.

All conditions of the experiments with red and black phosphorus were identical. In the case of black phosphorus (red phosphorus) the number of electrons registered in the interval of time from 0 to 2.4 microsec which corresponded to 80 analyser canals was equal to 19272 (12088). The time interval from 2.4 to 3.3 microsec was used for calculating the background. At the moment $t=0$ the relation between the number of pulses from electrons and that from the background was equal to 23. The number of the gate openings amounted to 600 $^{1}/$sec., i.e., 6 for one pulse of the accelerator.

For calculating the quantities S the pulses were summated for 4 canals, i.e., for the time interval 0.127 microsec. The measured values of S were found to be as follows:

$S_{б}=(0.540 \pm 0.007)$ microsec for black phosphorus and

$S_{2}=(0.590 \pm 0.012)$ microsec for red phosphorus. The errors listed are standard statistical deviations.

Discussion of the Results Obtained

It is clear from table 1 that the quantities ξ for sulphur and cadmium proved to be identical within statistical errors. The values of a_0 obtained with the aid of ξ coincide, within experimental errors, with the values a_0 obtained in ref. [9]. The values $\bar{a}_0$ found through the quantities ξ proved also to be equal for silver and red phosphorus in the present paper and on the basis of measurement of a larger number of points on the precession curve for red phosphorus in ref. [17]. In the experiments with black phosphorus no dependence of the count rate of electrons on the current in the magnetic coil was detected. The maximum asymmetry of electrons observed in these experiments with silver and red phosphorus, with the frequency of a spin precession of the mesic nucleus being one half of that of a free μ -meson, once again [17] directly points to the fact that the spin of a negative μ -meson is equal to one half.

The constancy of the quantities a_0 measured for sulphur and cadmium is in accordance with the theoretical predictions made in ref. [12]. Consequently, it is possible to find, using the quantities ξ for silver and phosphorus, the average effective values $\bar{n}_1$ and $\bar{n}_0$ of the level population $F=1$ and $F=0$ in two modifications if use is made of the relation (1):

$$\frac{a_e}{a_{eq}} = \frac{\bar{n}_1 a_1 + \bar{n}_0 a_0}{\frac{3}{4} a_1 + \frac{1}{4} a_0}$$

where a_1 and a_0 are equal respectively[11] to $\frac{1}{3}\frac{2I+3}{2I+1}$ and $\frac{1}{3}\frac{2I-1}{2I+1}$. The time interval 0 to 1.3 microsec (which corresponds to two lifetimes τ of red phosphorus[15]) was used for measuring asymmetry. The values $\bar{n}_1$ and $\bar{n}_0$ in this time interval proved to be as follows: $\bar{n}_1 = 0$ and $\bar{n}_0 = 1$ for black phosphorus and $\bar{n}_1 = \frac{3}{5}$ and $\bar{n}_0 = \frac{2}{5}$ for red phosphorus. The quantities $\bar{n}_1$ and $\bar{n}_0$ favour the view that the probabilities of the transition of μ-mesons between the hyperfine structure levels in mesic atoms of the two modifications are essentially different. This fact is direct evidence that the time of the relaxation of meson spin in mesic atoms decreases owing to conduction electrons[2,5,7].

Let us now turn to the measurements of the quantities S for black and red phosphorus. The measurement of S was carried out in the time interval 0 to 2.4 microsec (which corresponds to '4τ' for red phosphorus) . If the fact that the number of ($\mu - e$) -decay electrons appearing in the intervals 2τ and 4τ are approximately the same, the expressions S for the two modifications can be written in the following way

$$S_r = \bar{n}_{1r} S_1 + \bar{n}_{0r} S_0$$

$$S_B = \bar{n}_{1B} S_1 + n_{0B} S_0 \tag{2}$$

If the quantities S_1 and S_0 are found from these relations, it is possible, using the tables of coefficients given in ref.[18], to obtain τ_1 and τ_0, the lifetimes of μ-mesons in the states $F=1$ and $F=0$, and consequently λ_1 and λ_0, the probabilities of the capture of mesons by the nucleus from these states.

The quantity S_B measured in the present paper proved to be less than the quantity S_r. The difference between them is quite appreciable, exceeding as it does three standard deviations. These fact, as well as the values $\bar{n}_1$ and $\bar{n}_0$ for red and black phosphorus as they follow from the relation (2), directly testify that $\lambda_1 \neq \lambda_0$ (weak interaction depends on spin) and $\lambda_0 > \lambda_1$ (capture from the state $F=0$ occurs more rapidly than from the state $F=1$). On the basis of the measured quantities S_B and S_r definite conclusions can be drawn on the variant of weak interaction in the process $\mu^- + p \to n + \nu$ if the lower limit of the quantity $\frac{\Delta\lambda}{\lambda} = \frac{\lambda_1 - \lambda_0}{\frac{3}{4}\lambda_1 + \frac{1}{4}\lambda_0}$ is found. The latter can be obtained from eq. (2) for the values $\bar{n}_1$ and $\bar{n}_0$ equal to $\bar{n}_1 = 3/4$ and $\bar{n}_0 = 1/4$ for red phosphorus and $\bar{n}_1 = 0$ and $\bar{n}_0 = 1$ for black phosphorus (see appendix II) . The wanted quantities for the present case proved to be as follows

$S_1 = (0.61 \pm 0.016).10^{-6}$ sec, $S_0 = (0.54 \pm 0.007).10^{-6}$sec,

upper limit $\Lambda_1 = \lambda_1 + \lambda_{dec} = (1.42 \pm 0.035).10^{6}$ sec^{-1}, lower limit $\Lambda_\bullet = \Lambda_0 + \Lambda_{dec} = (1.72 \pm 0.022).10^6\ \text{sec}^{-1}$,

lower limit $\left(\frac{\Delta\Lambda}{\Lambda}\right)_{low} \simeq -0.29 \pm 0.04$.

The value of the meson decay probability Λ decay was taken to be $4.505.10^5\ \text{sec}^{-1}$. The errors listed are statistical.

Fig. 3 represents (for hydrogen) the dependence $\frac{\Delta\Lambda}{\Lambda}$ on the quantity $x' = \frac{G_A^{\Lambda}}{G_V \Lambda}$ calculated for the interaction type $A + xV + P$ (with allowing for the weak magnetism effect) on the basis of the formulas given in ref./7/. In the case of the capture of mesons by the phosphorus nucleus the quantity $\frac{\Delta\Lambda}{\Lambda}$ was calculated in a similar way for $x' = -1.21$ on the basis of the Schmidt model as well as the Meyer-Yensen model in ref/5/. The calculated quantities $\frac{\Delta\Lambda}{\Lambda}$ proved to be equal to -0.25 and -0.45 respectively. If comparison is made between these values and that given in fig.3 it can be claimed that the quantity $\frac{\Delta\Lambda}{\Lambda}$ for phosphorus is 1/15 and 1/9 that for hydrogen. From Fig. 3 it follows that the quantity $\left(\frac{\Delta\Lambda}{\Lambda}\right)_{low}$ obtained taken with allowances for double statistical error, indicates that all values $x' > 0$ (Schmidt model) and the values $x' > 0$ in the interval $0 < x' < 5$ (Meyer-Yensen model) are excluded, since at $x' \to \infty$ $\frac{\Delta\Lambda}{\Lambda} \to 2.5$. Thus if we take into consideration that $/x'/ = 1.25$ for the β-decay it can be concluded that the results of the present experiments are direct evidence in favour of the ($A - xV$) -interaction. Using the average effective values $\bar{n}_1$ and $\bar{n}_\bullet$ and the values S the quantity $\frac{\Delta\Lambda}{\Lambda}$ can be obtained for phosphorus. The comparison of this quantity with the theoretical predictions/3,5,19/ makes it possible to obtain information on the probability of the absorption of mesons by the protons of the different nuclear shells. It was impossible, however, to draw this comparison, since the error in the quantity $\frac{\Delta\Lambda}{\Lambda}$ proved to be very large as a result of considerable indefiniteness in the quantities $\bar{n}_1$ and $\bar{n}_0$ obtained by the method used in the present paper.

In conclusion the authors express their gratitude to D.Chultem for the aid rendered in this investigation.

Appendix I

1. It will be shown that $S = \bar{n}_1 S_1 + \bar{n}_\bullet S_0$. Let the sums $S_1 = \frac{\sum n_i' t_i}{N_1}$ and $S_0 = \frac{\sum n_i^\circ t_i}{N_0}$ calculated in the time interval from 0 to t, correspond to the exponents $e^{-\frac{t}{\tau_1}}$ and $e^{-\frac{t}{\tau_0}}$. Let us introduce the notations $N_1 + N_0 = N$, $\frac{N_1}{N} = a$ and $\frac{N_0}{N} = 1 - a$.

After adding the sums S_1 and $S_\bullet$ we obtain

$$S = aS_1 + (1-a)S_0 = \frac{\sum (n_i' + n_i^\circ) t_i}{N}$$

where $n_i = n_i' + n_i^\circ$ designates the total number of decays at the moment t_i. Obviously, the quantities a and $1-a$ are equal to the level populations $\bar{n}_1$ and $\bar{n}_0$ respectively when τ_1 and τ_0 do not differ strongly.

II. The expressions S for the two phosphorus modifications are of the form

$$a_1 S_1 + (1-a_1) S_0 = S_I$$
$$a_2 S_1 + (1-a_2) S_0 = S_{II}$$

The difference $S_1 - S_0$ is equal to $S_1 - S_0 = \frac{S_I - S_{II}}{a_1 - a_2}$. Derivatives of the difference $S_1 - S_0$ over a_1 and a_2 will equal respectively

$$\frac{\partial (S_1 - S_0)}{\partial a_1} = - \frac{S_I - S_{II}}{(a_1 - a_2)^2}$$
$$\frac{\partial (S_1 - S_0)}{\partial a_2} = - \frac{S_I - S_{II}}{(a_1 - a_2)^2}$$

From these expressions it follows that the quantity $S_1 - S_0$ decreases when a_1 increases and a_2 decreases. Obviously, the value of $S_1 - S_0$ will be the smallest when $a_1 \sim \bar{n}_1 = \frac{3}{4}$ and $a_2 \sim \bar{n}_2 = 0$. If one takes into account that $|S_1 - S_0| \sim |\Lambda_1 - \Lambda_0|$ the value of $|\Lambda_1 - \Lambda_0|$ will also be the lowest at $\bar{n}_1 = ¾$ and $\bar{n}_2 = 0$.

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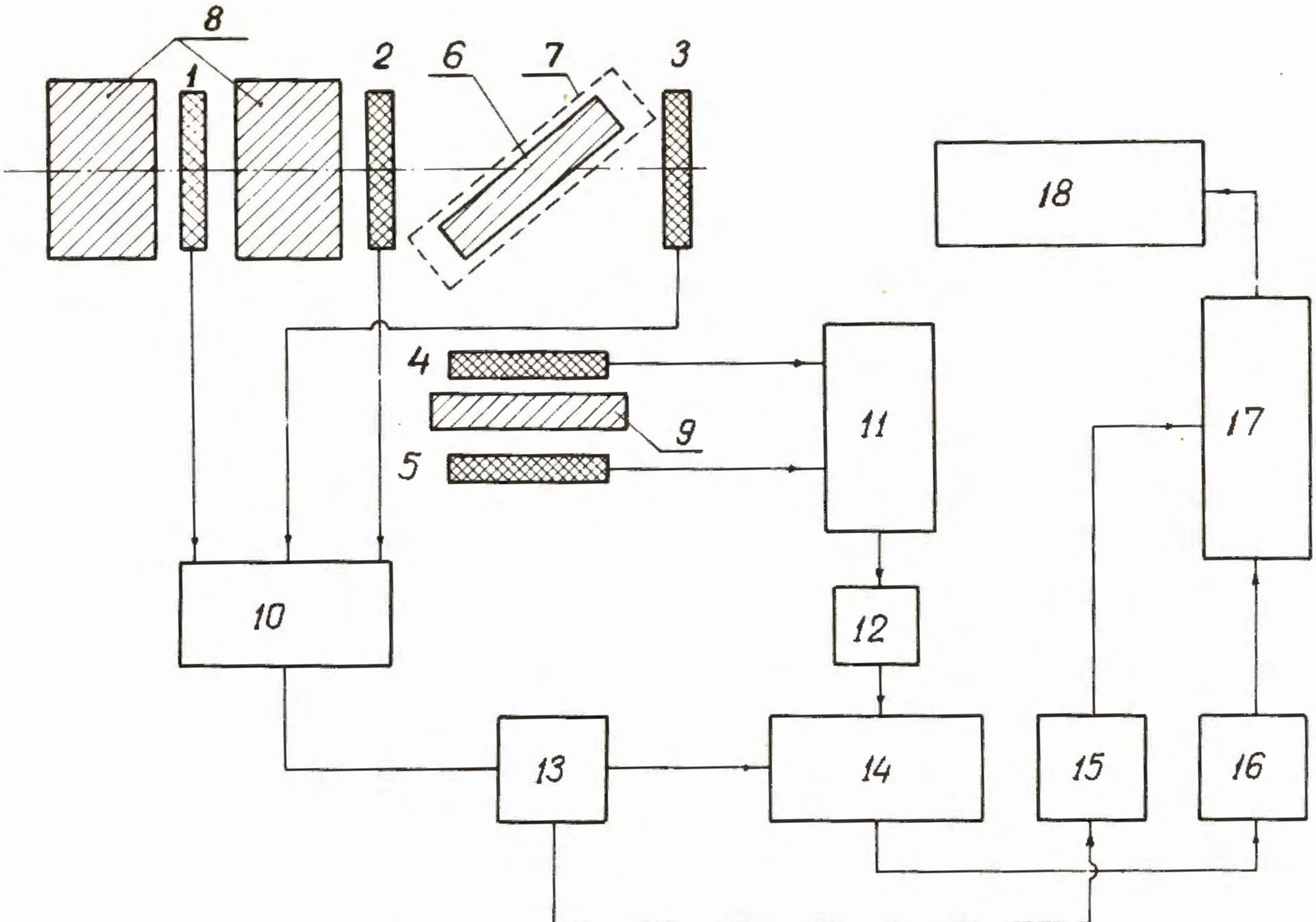

Fig.1

Block diagram of the set-up.

1,2,3,4 and 5 – scintillation counters, 6 – target, 7 – magnetizing coil, 8 – copper filters, 9 – paraffin filter, 10 – anticoincidence circuit, 11– coincidence circuit, 12 – delay device, 13 – gate interval trigger, 14–passage circuit (gate), 15 and 16 – forming devices, 17 – convertor and 18–amplitude analyzer.

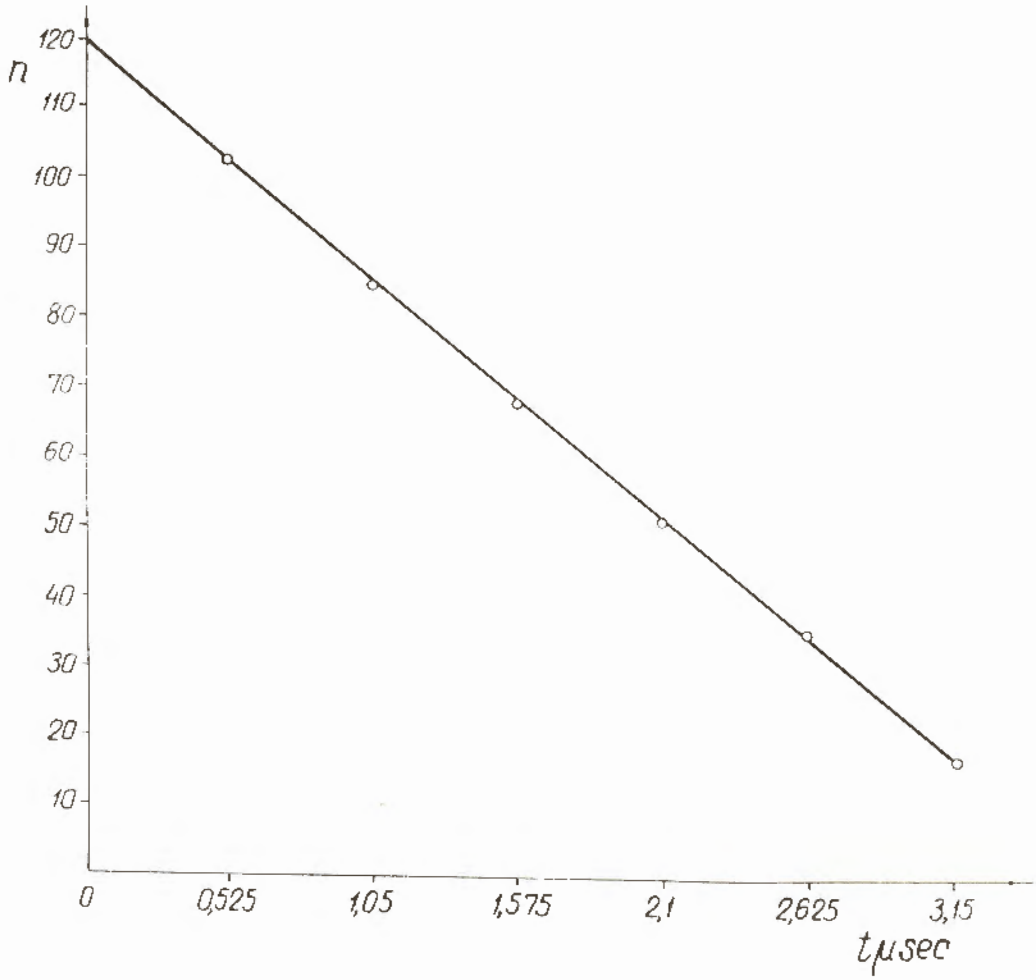

Fig. 2

Dependence of the number of the analyzer channel n on the delay t of the second pulse with respect to the first one in the convertor.

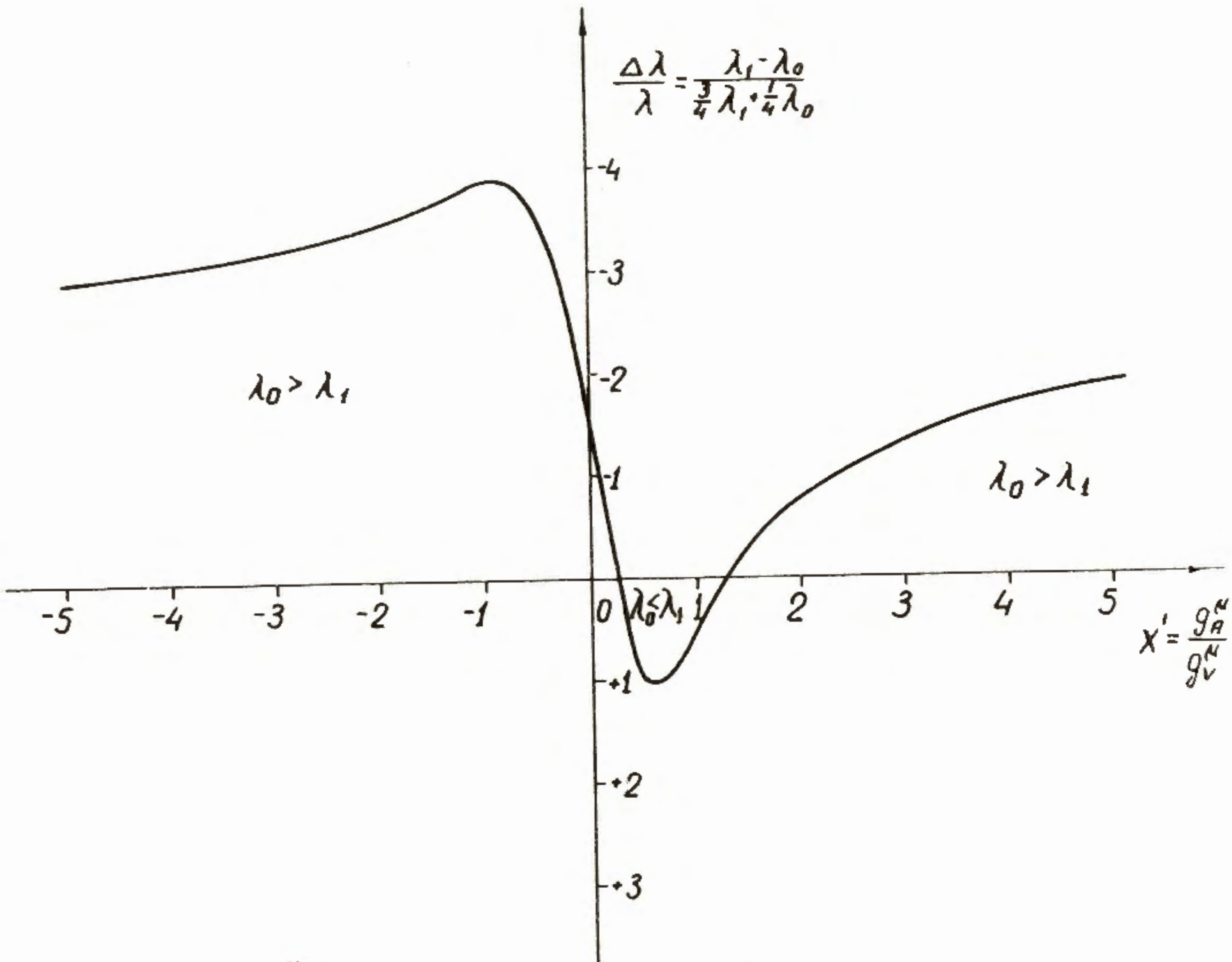

Fig. 3

Dependence of $\frac{\Delta\lambda}{\lambda} = \frac{\lambda_1 - \lambda_0}{\frac{3}{4}\lambda_1 + \frac{1}{4}\lambda_0}$ on $x' = \frac{G_A^N}{G_V^N}$ for hydrogen.