ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

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THE NEUTRAL MODEL FOR THE INVESTIGATION OF THE PION-PION SCATTERING

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THE NEUTRAL MODEL FOR THE INVESTIGATION OF THE PION-PION SCATTERING

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Abstract

The equation describing the scattering of the neutral pseudoscalar mesons in the low energy region is investigated. The general solution obtained is similar to that of the Chew-Low equation found by Castillejo, Dalitz and Dyson/1/. The solution has too different possible asymptotic behaviours at high energy. In the first case, the scattering amplitude decreases as $(\ln E)^{-1}$ at high energy; it corresponds to the result of the renormalizable perturbation theory. In the seoond case, the amplitude decreases as E^{-4}, such solution has no correspondence with the result of the renormalizable perturbation theory. In certain sense, it is connected with the non-renormalizable Lagrangian $(\partial_\mu \varphi \partial_\mu \varphi)^2$. The second class of solutions has a number of interesting properties. In particular, it shows degeneracy in the limit of the switching off of the interaction.

1. Introduction

In the last few years, attempts have been made to build the theory of strong interaction in the low energy region based on the analytic property of the scattering amplitude as postulated by Mandelstam[2] and the unitarity condition. They all start from the assumption that the scattering amplitude in the low energy region is essentially determined by the nearest singularities/3/, so that a self-contained theory of the low energy phenomena can be formulated without the detailed knowledge of the high energy processes. It is therefore justified to approximate the unitarity condition by the contributions from the two particles intermediate states. Such approximate unitarity condition together with the spectral representation lead to a system of non-linear equations for the scattering amplitudes/2,4/.

These equations involve two continuous variables and are very complicated. Further approximation is made by using the first few terms of the Legendre expansion to represent the scattering amplitude. Using this method, Chew and Mandelstam/5/ obtained a closed system of non-linear integral equations for the lowest partial waves of the $\pi\pi$- scattering. Equations for a number of other processes have been derived in a similar approach afterwards/6,7/.

In these papers, the imaginary parts of the scattering amplitudes in the crossing integrals are obtained by analytic continuation with the help of the Legendre expansion into the region $|\cos\theta| > 1$. Large errors are introduced by keeping only the first few terms of the Legendre expansion/8,9,10/. The errors are particularly serious in the high energy region of the crossing processes. Integrals involving higher partial waves even diverge. The solutions of the equations are unstable with respect to small perturbations in the high energy region. In particular, the analytic continuation with the help of Legendre expansion leads to the inconsistency of the results on the parameters of the ρ -wave resonance of the $\pi\pi$- scattering derived from the investigation of the πN- scattering on the one hand and that of the nucleon structure on the other/11/. It also leads to difficulty in the search for a stable solution of the equation for the $\pi\pi$- scattering with a large ρ - wave.

The analytic continuation of the scattering amplitude into the region $|\cos\theta| > 1$ with the help of the Legendre expansion leads therefore finally to results in contradiction with the initial assumption on the self-containedness of the low energy processes. The above mentioned difficulties raise doubt on the possibility of building a self-contained theory of the strong interaction in the low energy region.

However, we believe, that such pessimism is not yet well founded. It is very probable, that the above mentioned difficulties can be overcamed by using a somewhat different approach to the derivation of the equations for the partial waves proposed in /9,10/. In this approach, no use is made of the analytic continuation with help of the Legendre expansion. The structure of the crossing integrals of the equations for the partial waves is quite different from that of the equations of the Chew-Mandelstam type. In particular, these integrals converge better in the high energy region. This new approach may lead to an explanation of the low energy phenomena free from internal contradiction. It is therefore interesting to obtain the numerical solutions of the equations for the partial waves of various processes, first of all, that of the $\pi\pi$-scattering. The equations for the $\pi\pi$ - scattering are derived in /8/ using the new approach. A simplified version of the equations is proposed in /9/, where it is assumed, that the f- and d- waves are small in comparison with the s- and p- waves and can thus be neglected. In fact, these equations were derived by only using the dispersion relation for the forward scattering, which has been proved rigorously.

Before embarking on the numerical solution of these equation, it is useful to have some indication on certain general properties of the equations of such type. However, the analytical investigation of the equations proposed in /9/ turns out to be rather complicated. Therefore, we study first the neutral version of the equations proposed in /9/. The investigation leads to a number of important conclusions, which have to be taken into account during the analytical investigation and the numerical solution of equations of such type.

II. The Integral Equation and the Behaviour of its Solution in the High Energy Region

In the case under consideration, there is only one invariant scattering amplitude A , which is a function of three usual invariant variables

$$s = 4(\nu+1)\,; \qquad u = -2\nu(1+c)\,; \quad t = -2\nu(1-c) \tag{2.1}$$

where $\nu = q^2/\mu^2$, $c =$ cos θ; q and θ are the momentum and the scattering angle in the center of mass system respectively. As a result of the crossing symmetry, A is symmetric with respect to all three variables in (2.1). Its Legendre expansion contains therefore only terms of even degree. In accordance with the approximation used in /9,10/, we approximate the forward scattering amplitude by the S -wave

$$A(\nu, c=1) \cong A_0(\nu) \equiv A(\nu)\,. \tag{2.2}$$

Taking into account, that $A(\nu) = \lim_{\varepsilon\to 0} A(\nu+i\varepsilon)$ and that A is a symmetrical function of S and U , we obtain

$$A(-1-\nu) = A^*(\nu). \tag{2.3}$$

The unitarity condition for the S -wave can be written in the form

$$\operatorname{Im} A(\nu) = K(\nu)|A(\nu)|^2; \quad K(\nu) = \sqrt{\frac{\nu}{\nu+1}}. \qquad (24)$$

This formula is exact only up to $\nu = 3$, where the first inelastic process begins to appear. We assume, however, that even if (2.4) is used for all $\nu \geq 0$, the solution in the region of small ν receives only unimportant modifications due to the error of (2.4) in the high energy region. From (2.4) follows, that $A(\nu)$ is finite, so that one subtraction is sufficient for the dispersion relation. We choose to make the subtraction at the symmetrical point $\nu = -½$ and obtain

$$A(\nu) = \lambda + \frac{\nu + ½}{\pi} \int_0^\infty d\nu' \, \frac{\operatorname{Im} A(\nu')}{\nu' + ½} \left\{ \frac{1}{\nu' - \nu} - \frac{1}{\nu' + \nu + 1} \right\}. \tag{2.5}$$

From (2.5) is obvious, that the assumption

$$\lim_{\nu \to \infty} \operatorname{Im} A(\nu) = c > 0$$

leads to logarithmical increase of $\operatorname{Re} A(\nu)$ as $\nu \to \infty$ in contradiction with (2.4). Thus, we have $A(\infty) = 0$. The equation for $A(\nu)$ can therefore be written without subtraction.

$$A(\nu) = \frac{1}{\pi} \int_0^\infty \operatorname{Im} A(\nu') \left\{ \frac{1}{\nu' - \nu} + \frac{1}{\nu' + \nu + 1} \right\} d\nu'. \tag{2.6}$$

From (2.6) follows

$$\lambda = \frac{2}{\pi} \int_0^\infty \frac{\operatorname{Im} A(\nu')}{\nu' + ½} \, d\nu' > 0. \tag{2.7}$$

Therefore, the equation without subtraction (2.6) is mathematically equivalent to the equation with subtraction (2.5). It follows therefore, that the existence of the manifold of the solutions connected with the parameter λ is not a consequence of the subtraction.

III. The Solution of the Equation

It is convenient to introduce the following new variable

$$\omega = (2\nu + 1)^2; \qquad B(\omega) \equiv A(\nu). \tag{3.1}$$

Equation (2.4) has then the form

$$\mathrm{Im}\, B(\omega) = k(\omega)|B(\omega)|^2; \quad k(\omega) = K(\nu); \quad \omega \geq 1 \tag{3.2}$$

while (2.6) becomes

$$B(\omega) = \frac{1}{\pi}\int_1^\infty \frac{\mathrm{Im}\, B(\omega')}{\omega' - \omega}\, d\omega'. \tag{3.3}$$

The equation (3.3) can be solved by the method of Castillejo, Dalitz and Dyson[1]. The function $B(z)$ as a function of the complex variable $z = \omega + iy$ has the following properties:

1. analytic in the complex plane z with a cut $[1, \infty)$ along which

$$\mathrm{Im}\, B(\omega + i0) = k(\omega)\, |B(\omega + i0)|^2.$$

It is evident from (3.3) that

$$B^*(z) = B(z^*).$$

2. is a generalized R - function

$$\mathrm{Im}\, B(z) = \lambda(z)\, \mathrm{Im}\, z, \qquad \lambda(z) = \frac{1}{\pi}\int_1^\infty \frac{k(\omega')\,|B(\omega')|^2}{|\omega' - z|^2}\, d\omega' > 0.$$

Thus, $B(z)$ has no zero except possibly on the real axis and at infinity.

3. $B(\omega)$ is real and larger than zero for ω real ana smaller than 1.

4. $B(\omega)$ may have any number of isolated zeros in the interval $(1, \infty)$.

Let us investigate the function

$$H(z) = \frac{1}{B(z)} \tag{3.4}$$

$H(z)$ has the following property:

1. $H(z)$ is analytic in the complex plane with the cut $[1, \infty)$, along which $\mathrm{Im}\, H(\omega + i0) = -k(\omega)$. Besides, there is $H(z^*) = H^*(z)$.

2. $H(z)$ is a generalized R - function, has no zero when $\mathrm{Im}\, z \neq 0$.

3. $H(z)$ has no pole except in the inter val $(1, \infty)$ where there may be any number of isolated poles of the first order. Poles of higher order have not the properties of the generalized R - function.

4. $H(z)$ has no zero on the real axis. It follows, that the general form of $H(z)$ is

$$H(z) = \frac{1}{\lambda} - \frac{z}{\pi}\int_1^{\infty} d\omega' \frac{k(\omega')}{\omega'(\omega'-z)} - cz - zR(z) \qquad (3.5)$$

where

$$R(z) = \sum_n \frac{R_n}{\omega_n(\omega_n - z)}, \qquad 1 < \omega_n < \infty. \qquad (3.6)$$

From (3.5) follows

$$\mathrm{Im}\, H(z) = -\mathrm{Im}\, z \left\{ \lambda'(z) + c + \sum \frac{R_n}{|\omega_n - z|^2} \right\}.$$

$$\lambda'(z) = \frac{1}{\pi}\int_1^{\infty} \frac{k(\omega')}{|\omega' - z|^2} d\omega'.$$

In order that $H(z)$ has the property specified in 2. there must be

$$R_n \geq 0, \quad c \geq 0 \qquad (3.7)$$

$H(z)$ decreases monotonously in the interval $(-\infty, 1]$. In order that $H(z)$ has the property specified in 4), so that it has no zero in this interval, it is necessary and sufficient, that

$$\frac{1}{\lambda} \geq \frac{1}{\pi} J(1) + c + R(1) \qquad (3.8)$$

where

$$J(\omega) = \omega \int_1^{\infty} d\omega' \frac{k(\omega')}{\omega'(\omega'-\omega)} = \pi - 2\sqrt{x}\, Q_0(\sqrt{x}) - \frac{2}{\sqrt{x}} Q_0\left(\frac{1}{\sqrt{x}}\right) \qquad (3.9)$$

Q_0 denotes the Legendre function of the second kind.

$Q_0(\xi) = -\frac{1}{2}\ln\frac{\xi-1}{\xi+1}$. x is defined as: $x = \frac{\sqrt{\omega}-1}{\sqrt{\omega}+1} = \frac{v}{v+1}$.

Summarizing, we have the following general solution of the equation (3.3)

$$B(\omega) = \frac{\lambda}{1 - \frac{\lambda}{\pi} J(\omega) - \lambda c\omega - \lambda\omega R(\omega)}. \qquad (3.10)$$

The function $R(\omega)$ is defined in (3.6), λ, c, R_n have to satisfy the conditions (3.7) and (3.8). The condition (2.7) is a consequence of the conditions (3.7) and (3.8).

IV . Comparison with the Result of the Perturbation Theory

It is interesting to see the correspondence between (3.10) and the result of the perturbation theory. Assuming that λ is small, we may develop the denominator of (3.10) and obtain

$$A(\nu) = \lambda + \frac{\lambda^2}{\pi} J((2\nu+1)^2) + \lambda^2 c\,(2\nu+1)^2 + \lambda^2 (2\nu+1)^2 R((2\nu+1)^2) + O(\lambda^3). \quad (4.1)$$

The first two terms in (4.1) correspond to the first and second order contributions in the perturbation theory based on the interaction Lagrangian

$$\mathcal{L}^{(0)}_{int} = \frac{4\pi}{3} \lambda \varphi^4 . \quad (4.2)$$

The fourth term corresponds to the pole contribution of the second order terms in the perturbation theory from the interaction Lagrangian

$$\mathcal{L}^{(1)}_{int} = \sum_n g_n \Phi_n \varphi^2 . \quad (4.3)$$

(See in this connection the paper of Dyson[13]). Φ_n describes here the unstable particle with the mass $m_n > 2$. The relation between g_n , λ , m_n , ω_n and R_n can be established easily in the theory of perturbation.

The third term is in a certain sense connected with the non-renormalizable interaction Lagrangian

$$\mathcal{L}^{(2)}_{int} = f \left[\left(\frac{\partial\varphi}{\partial x_\mu} \cdot \frac{\partial\varphi}{\partial x_\mu} \right)^2 - \frac{1}{3} \varphi^4 \right] \quad (4.4)$$

with $f = 2\pi\lambda^2 c$.

Of course, the connection between (4.4) and the third term in (4.1) is not clear cut, as there is yet no consistent theory of perturbation for such interaction Lagrangian. However, it is shown by one of the author[14], that similar case exists in the non-relativistic theory. We postpone the discussion about this interesting point to the last section and limit ourselves in the present section to the discussion of the correspondence between the first two terms of (4.1) and the interaction Lagrangian (4.2).

The first and second order contributions to the S -wave in the perturbation theory can be calculated easily.

We obtain
$$A_{p.th.} = \lambda_0 + \frac{\lambda_0^2}{\pi} A^{(2)}_{p.th.}(\nu) + \cdots \quad (4.5)$$

Here the amplitude is renormalized at the threshold. The suffix "$p.th.$" denotes here the results of the perturbation theory. $A^{(2)}_{p.th.}(\nu)$ is of the following form:

$$A^{(2)}_{p.th.}(\nu) = 6 - 2\sqrt{x}\, Q_0(\sqrt{x}) - \frac{4}{\sqrt{x}} Q_0\left(\frac{1}{\sqrt{x}}\right) - 2\,\frac{1-x}{x} Q_0^2\left(\frac{1}{\sqrt{x}}\right) \quad (4.6)$$

(4.5) and (4.6) should be compared with the first two terms of (4.1), which can be written in the form (4.5) by transfering the point of renormalization to the threshold. We have then

$$A^{(2)}_{i.eq.}(\nu) = 2 - 2\sqrt{x}\,Q_0(\sqrt{x}) - \frac{2}{\sqrt{x}}\,Q_0\left(\frac{1}{\sqrt{x}}\right). \qquad (4.7)$$

The suffix "$i.\,eq$" denotes here the results derived from the integral equation. The first order term in λ_0 in the perturbation theory agrees with that of the solution of the integral equation by definition. It is interesting to compare the second order terms in λ_0 given in (4.6) and (4.7) at the threshold.

a) In the perturbation theory

$$A^{(2)}_{p.th.} = -\frac{8}{3}x - \frac{52}{45}x^2 - \frac{248}{315}x^3 - \dots$$

b) In the solution of the integral equation

$$A^{(2)}_{i.eq.} = -\frac{8}{3}x - \frac{16}{15}x^2 - \frac{24}{35}x^3 \dots$$

At the threshold of the first inelastic process at $\nu = 3$

$$A^{(2)}_{p.th.}(3) = -3{,}521 \;; \qquad A^{(2)}_{i.eq.}(3) = -3{,}323.$$

We see therefore the solution of the integral equation agrees very well with the corresponding result of the perturbation theory in the low energy region. Error of the second order term is 6% at $\nu = 3$. This agreement supports the initial assumption on the possibility of building a self-contained theory of the strong interaction in the low energy region.

V. The Solution with Resonance

From (3.10) follows, that there are two different kinds of possible asymptotic behaviour for the solutions of the integral equation as $\nu \to \infty$

a)

$$A(\nu) \simeq \frac{\pi}{2\ln\nu} \qquad (5.1)$$

which corresponds to the absence of non-renormalizable interaction.

b)

$$A(\nu) \simeq -\frac{1}{c\,\nu^2} \qquad (5.2)$$

which corresponds to the non-renormalizable interaction Lagrangian (4.4). These asymptotic behaviours do not depend on the function $\mathcal{R}(\omega)$, which describes the influence of the unstable particles. For the sake of simplicity, we shall limit our discussion to the case, in which there is no unstable particle. This is equivalent to the case, in which the phase shift of the scattering never becomes zero in the interval $0 < \nu < \infty$. It is then obvious, that there is no resonance in the solution of the type a), while in the solution of the type b) there is a resonance at the point $\nu_r = \frac{1}{2}\left(\frac{1}{\sqrt{\lambda c}} - 1\right) = \frac{1}{2}\left(\sqrt{\frac{2\pi\lambda}{f}} - 1\right).$ (5.3)

If λ and f are magnitudes of the same order, the resonance occurs in the low energy region. The resonance solution for small λ can be written in the following form

$$A(\nu) = \frac{\lambda/2}{1-\frac{2\nu+1}{2\nu_\lambda+1} - i\frac{\lambda}{2}K(\nu)\theta(\nu)} + \frac{\lambda/2}{1+\frac{2\nu+1}{2\nu_\lambda+1} + i\frac{\lambda}{2}K(-1-\nu)\theta(-1-\nu)} . \tag{5.4}$$

In the limiting case of $\lambda \to 0$, the imaginary part of $A(\nu)$ can be approximated by the δ –functions

$$\Im m\, A(\nu) \simeq \frac{\pi\lambda}{2}(\nu_\lambda + \tfrac{1}{2})\left\{\delta(\nu_\lambda - \nu) - \delta(\nu_\lambda + \nu + 1)\right\}. \tag{5.5}$$

The real part can be represented by the pole terms

$$\mathcal{R}e\, A(\nu) \simeq \frac{\lambda}{2}(\nu_\lambda + \tfrac{1}{2})\left\{\frac{1}{\nu_\lambda - \nu} + \frac{1}{\nu_\lambda + \nu + 1}\right\}. \tag{5.6}$$

Therefore with ν_λ fixed, the width approaches zero with λ. In the case of $\lambda \to 0$, we obtain there-refore non-zero solution. At the point of resonance ν_λ, which can take arbitrary value, the phase shift jumps suddenly from 0 to π. Thus the solution shows degeneracy at $\lambda = 0$.

It is to be noticed, that in this case, the d-wave A_2 will be proportional to λ. Its main contribution comes from the crossing integral. It is always small due to the large denominator appearing in the integral. For example, in the interval $0 < \nu < 6$ and $\nu_\lambda = 3$ the numerical estimation shows

$$\frac{5 A_2(\nu)}{A(\nu)} \leq 6\% .$$

It is very likely, that the equations describing the scattering of the charged π - mesons have also different types of solutions similar to those given in (3.10). It can be shown, that the solutions of the equations for the scattering of the charged π - mesons tend to zero as $\nu \to \infty$. Beside the type of solutions with the asymptotic behaviour $A(\nu)\,|_{\nu\to\infty} \sim \frac{1}{\ln \nu}$ which corresponds to the result of the renormalizable perturbation theory, there may exists another type of solutions, which tends to zero quicker then $\frac{1}{\ln \nu}$ as $\nu \to \infty$. Such type of solutions will also exhibite degeneracy in the limit of the switching off of the interaction as described above.

We would like to add here a few remarks on the possibility of obtaining the solution (3.10) with the " $\frac{N}{D}$ " method proposed by Chew and Mandelstam/5/ . The form of the integral representation for the function D depends very much on the asymptotic behaviour or the phase shift as $\nu \to \infty$. This representation is determined only up to a polynomials of the n th degree, where

$$n = \frac{1}{\pi}\left\{\delta(\infty) - \delta(0)\right\}.$$

The form of the representation (V.12) proposed in/5/ corresponds to $n = 0$. Thus from the outset, Chew and Mandelstam excluded the possibility of the existence of an odd number of resonances of the partial waves. The equations of the form (V .11), (V.12) of/5/ cannot have solutions of the form (5.4). It needs a second subtraction of the equation (V.12) to make the existence of the resonance solution of the form

(5.4) possible. Taylor arrived at the same conclusion in his recent work[17].

It is to be pointed out, that equations of the type (V.11), (V.12) derived with the " $\frac{N}{D}$ " method are not convenient for ensuring the crossing symmetry of the real part of the scattering amplitude.

VI. Discussion

First of all, we would like to say a few words on the formal aspect of the results. Many features of the solution a) and b) have close correspondence with those of the expressions for the model Green functions in the renormalizable and the non-renormalizable theories proposed in[18,19].

The solution a) is very similar to the expression of the Green function for the photon. Its spectral representation can be written down without subtraction (Eq. (2.6)). However, if we develop the expression under the integral sign in power series of λ , then logarithmic divergences appear in each term of the power series after the integration. If one subtraction is made of the spectral representation, equation (2.5) is obtained. Then no divergence will appear.

The solution b) corresponds to the Green function of the non-renormalizable theory in certain sense. If the expression under the integral sign in (2.6) is developed into power series in λ and f , then a series of divergent integrals appear, the degree of divergence of which increases with the degree of f. Such divergences cannot be get rid of by any finite number of subtractions. Solutions of the type b) have therefore no correspondence whatsoever with the theory of perturbation.

However, there is no justification to throw away solutions of the type b) in favor of solutions of the type a) which is in fact nothing else than the analytic continuation of the result of the perturbation theory into the region of large λ .

As pointed out above, solutions of the type b) show degeneracy in the limit of the switching off of the interaction. It is pointed out by Bogolubov[20], that solutions of such type are of great interest in many problems of the statistical physics. Now it seems very likely, that such solutions will also be important in the problems of the theory of elementary particles. It is well known, that the 33-resonance in the π-N scattering is rather narrow. The preliminary estimation using the data of the nucleon structure shows, that the p - wave of the π-π scattering has very narrow resonance. However, theories, which explain the existence of the 33-resonance, have great difficulty in explaining why the resonance is so narrow[21,22]. Attempts to explain the narrow p -resonance in the π-π scattering encounter still greater difficulty[23,24]. However, solutions of the type b) lead to narrow resonance in a natural way.

We would like to make the following important remark based on the explicite form of the solution (3.10). Integral equations derived with the help of the dispersion relation, unitary condition and crossing relation do not give a unique description of the scattering processes. In order to fix the solution completely, it is necessary to specify a set (consists of infinite number) of parameters. This fact is not surprising. The dispersion relation only the consequence of the most general properties of the theory, such as causality and relativistic invariance. It does not specify the concrete mechanism of the interaction. In this sense, the relativistic dispersion theory corresponds completely to the non-relativistic models. (See for example/13,14/).

Therefore, in order to build a theory based on the integral equations derived from the dispersion relation, it is further necessary to specify a series of properties of the solutions of these integral equations. For example, in the case of the neutral model under investigation, it is sufficient to specify the value of the scattering amplitude at the threshold, the asymptotic behaviour at $\nu \to \infty$ and to demand that the phase shift never equal to zero for finite ν. The same aim can be achieved by fixing the subtraction constants. The first subtraction gives the threshold value of the amplitude. The second subtraction constant (Viz. the derivative of the amplitude at the threshold) fixes the asymptotic behaviour of the amplitude, if the zero of the amplitude is exclude. This method is convenient for fixing the solution during the numerical solution of the integral equation.

It is interesting to speculate on the physical meaning of the parameters defining the solution. The first possible interpretation is, that these parameters correspond in fact to the Lagrangians (4.2), (4.3) and (4.4), so that interactions, which is not renormalizable in the perturbation theory, play an important role in the pion physics (see in this connection/25/). In other words, the dispersion relation may offer a possibility of settling the question of the existence of non-renormalizable strong interaction by means of comparing the consequences of the theory with the experiments.

The second possible interpretation is, that these parameters take into account the influences of the inelastic processes on the elastic process in the low energy region. We have, therefore, here a possibility of taking into account the effect of inelastic processes phenomenologically in a scheme, in which the two particles approximation of the unitarity condition is used.

Further investigations show that the solutions of the integral equations describing the scattering of charged pions also possess the important properties of the solutions of the neutral model discussed in this section. Result of these investigations will be published in future communications.

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