ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

Лаборатория теоретической физики

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ON THE FORM FACTOR OF THE π_0 MESON

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Now, it seems that the experiments of using head on electron-positron beam from the accelerator is possible in the near future. It is interesting to note that the form factor of the π^o meson can be measured in the reaction:

$$e^+ + e^- \longrightarrow \pi^o + \gamma \ . \tag{1}$$

Taking into consideration of the invariant property of the Hamiltonian under translation and rotation of space the effective Hamiltonian of a pseudoscalar π^o meson interacting with the electromagnetic fields has the following form:

$$\iiint d^4x\, d^4y\, d^4z\, \tilde{F}\left((x-z)^2, (y-z)^2, (x-y)^2\right) \varepsilon_{\alpha\beta\rho\sigma} \frac{\partial A_\alpha(x)}{\partial x_\beta} \frac{\partial A_\rho(y)}{\partial y_\sigma} \phi_o(z) , \tag{2}$$

where ϕ_o and A_α are the pseudoscalar π^o field and the electromagnetic field respectively, $\varepsilon_{\alpha\beta\rho\sigma}$ is the antisymmetrical forth-rank tensor.

Then the Feynman diagram of (1) corresponding to the electromagnetic interaction of the lowest order is shown in Fig. 1. The S matrix element for this diagram is:

$$\langle q, k | S | P_e \cdot P_n \rangle = \frac{-e}{(2\pi)^2} \frac{1}{\sqrt{q_o k_o}} \varepsilon_{\mu\beta\rho\sigma} \bar{v}(-\vec{P}_n) \gamma_\mu u(\vec{P}_e) \cdot e_\rho (P_n + P_e)_\beta \cdot$$
$$\cdot \frac{k_\sigma}{(P_n + P_e)^2} \cdot F\left((P_n+P_e)^2, 0, m_{\pi^o}^2\right) , \tag{3}$$

where P_e, P_n, k and q are the four-momenta of the electron, positron, photon and the π^o meson respectively, e_ρ the polarization vector of the photon, F is the Fourier component of $\tilde{F}$ in the momentum space and is by definition the form factor of the π^o meson.

The total cross section for the process (1) is:

$$\sigma(E) = \frac{e^2}{4\pi} \frac{1}{3} \cdot \frac{(1-x)^3 (1+2y)}{(1+x)(1-4y)^{1/2}} \cdot F^2(-E^2, 0, m_{\pi^o}^2) , \tag{4}$$

where E is the total energy in the center of mass system, $x = m_{\pi^o}^2 / E^2$, $y = m_e^2 / E^2$, m_{π^o} and m_e are the masses of π^o and electron respectively. As evident from the formula (4), the measurement of $\sigma(E)$ in the experiment will give some informations about the electromagnetic formfactor of π^o meson. In order to estimate the order of magnitude of $\sigma(E)$ instead of $F^2(-E^2, 0, m_{\pi^o}^2)$ we substitute into (4) $F^2(0, 0, m_{\pi^o}^2)$ which relates with τ, the life time of π^o, by the following expression:

$$F^2(0, 0, m_{\pi^o}^2) = \frac{8\pi}{m_{\pi^o}^3 \tau} \ . \tag{5}$$

If we use $\tau = (2.3 \pm 0.8) \cdot 10^{-16}\, \mathrm{sec}$, then from (4) we have:

$$\sigma(E)=f(E)\sigma\,, \tag{6}$$

where $\sigma=(28^{+14}_{-98})\cdot 10^{-35}cm^2$ $f(E)$ is plotted in Fig. 2 against the ratio E/m_{π^0}.

It is interesting to note also that such an experiment can give some informations about the contribution of intermediate 3π state to the form factor of π^0.

Using the typical technique of dispersion relations, one can show that (for instance, cf./2/):

$$\frac{1}{2\pi(\pi q_0)^{1/2}}\varepsilon_{\mu\beta\rho\sigma}\cdot\bar{v}\gamma_\mu u\cdot e_\rho K_\beta k_\sigma\cdot F(K^2,0,m_{\pi^0}^2)$$

$$=e_\rho\bar{v}\gamma_\mu u\int d^4z\, e^{i(k+K)\frac{z}{2}}\langle q|T(J_\mu(\tfrac{z}{2})J_\rho(-\tfrac{z}{2}))|0\rangle\,, \tag{7}$$

where $J_\rho(\frac{z}{2})$ the current of the strong interacting particles. Further, for simplicity, we define:

$$F(K^2,0,m_{\pi^0}^2)\equiv F(\nu)\,, \tag{8}$$

with $\nu=K^2$. It is easy to prove by using the equation (7), that $F(\nu)$ is an analytic function on the ν plane expect a branch cut on the real axis extending from $4m_{\pi^0}^2$ to ∞. Then the dispersion relation for $F(\nu)$ can be easily written down as following:

$$F(\nu)=F(0)+\frac{\nu}{\pi}\int_{4m_{\pi^0}^2}^{\infty}\frac{I_m F(\nu')}{\nu'(\nu'-\nu)}d\nu'\,, \tag{9}$$

where $I_m F(\nu)$ is given by:

$$\frac{1}{2\pi(\pi q_0)^{1/2}}\varepsilon_{\mu\beta\rho\sigma}\cdot\bar{v}\gamma_\mu u\cdot e_\rho K_\beta k_\sigma\cdot I_m(K^2,0,m_{\pi^0}^2)$$

$$=(2\pi)^4\cdot\bar{v}\gamma_\mu u\cdot\sum_n\langle q|J_\rho(0)|n\rangle\langle n|J_\mu(0)|0\rangle\,\delta^4(k-P_n)\,, \tag{10}$$

where $n=2\pi, 3\pi\ldots$ etc. Further, we shall confine ourself to the 3π intermediate state. The contribution of the 2π intermediate state can be calculated by using the knowledges of 1) the nucleon structure (the vertex function $\langle 2\pi|J_\mu(0)|0\rangle$) and 2) the photoproduction of pion on nucleon (the vertex function $\langle q|J_\rho(0)|2\pi\rangle$). Then, one may hope that subtracting from $F(\nu)$ the contribution of the 2π intermediate state, it will give an estimation to the contribution of the 3π intermediate state. But since problems 1) and 2) are up to now not solved well enough, we are only going to discuss in the following what conclusions can be made from the measurement of $F(\nu)$ under such a situation. From equation (9) we can see that for small ν $(\nu\ll 4m_{\pi^0}^2)$:

$$F(\nu)=F(0)+a\nu\,, \tag{11}$$

where a is a constant.

For larger $\nu(\nu<4m_\pi^2)$ if the 2π and 3π intermediate state contributions are both resonant, and have nearly the same resonant energies then we have:

$$F(\nu) \sim F(0) + \frac{b\nu}{(\nu_0-\nu)}, \tag{12}$$

where b is a constant and ν_0 is the square of resonant energy of the 2π state. If experiment shows that the result is of the form (12), then one can assert either the 3π intermediate state gives no important contribution to the form factor or it is important, but the resonant energy is nearly the same as that of the 2π state. On the contrary, if experiment shows that $F(\nu)$ is very different from the form (12), it means that the 3π intermediate state gives important contribution and is either of non-resonant type or of resonant type, but the resonant energies differs largely from ν_0.

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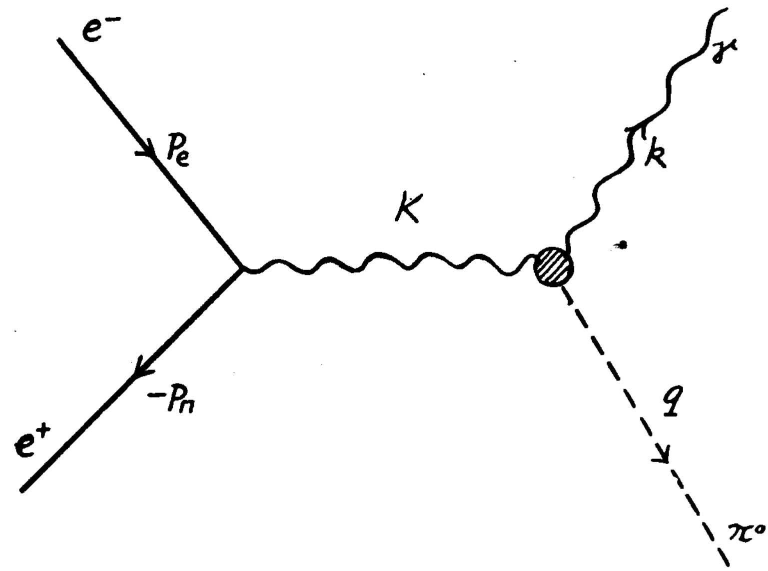

Fig. 1.

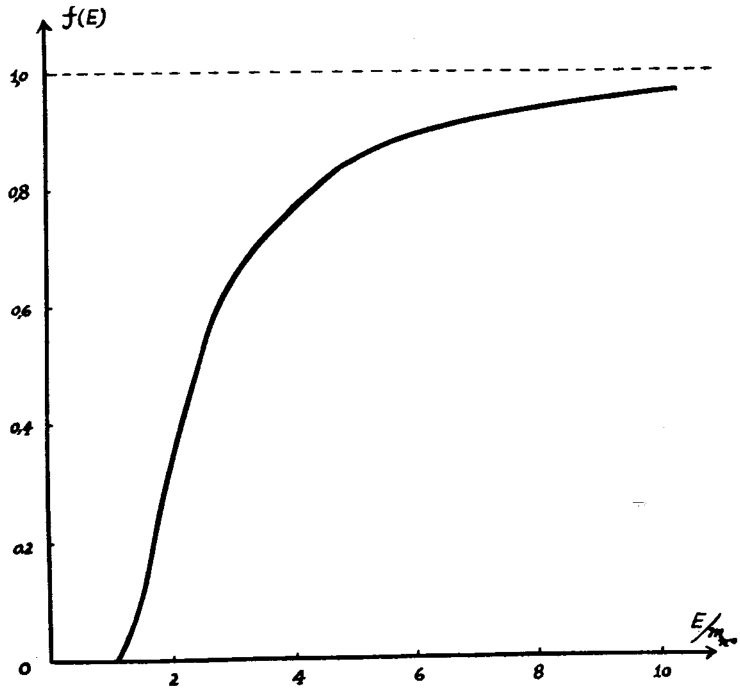

Fig. 2.

Dependence of the function $f(E)$ upon the energy of electron–positron pair.