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## Abstract

*Radiative corrections to pion decays are calculated. The spectra of real photons emitted in the $\pi\mu$ -decay and $\pi e$ -decay are different in form. Therefore, the ratio of decay probabilities for the same value of the cut-off of the photon spectrum in both channels depends essentially upon the cut-off. At the same time the radiation effects give the contribution to the total probability of decay (the decay with photon of any energy) which is equal to 3.93 per cent. The principal contribution to the correction comes from the difference in the probabilities of radiation of photons by an electron and $\mu$ -meson. The formula is obtained for lepton and photon spectra in pion decays.*

The radiative corrections in the weak interactions have been investigated in a number of papers /1–9/. It has been found in these papers that the corrections have different features in various processes. In the $\beta$ -decay where in the final state there arise two charged particles of the same helicity, their interaction (together with the influence of their masses) yields integrals which diverge at the upper limit. In the $\mu$ -decay processes where the charged particles have the same helicity as well but one of them vanishes in the initial state and another particle appears in the final one, similar integrals cancel* and the corrections can be calculated. Unlike the first case where the divergence is connected apparently with general difficulties of the four-fermion interaction, in the second case a main part of the effect should be expected to be connected with the real radiation of photons possessing comparatively low energies and a correction value should be determined mainly by the probability of such a decay. In this case it is natural that the radiation of a quantum will have a great influence upon the angular and energy distribution of the reaction products but it will influence less strongly the total probability value. For instance, in the case of the $\mu-e$ -decay the Michel parameter determining the electron spectrum changes by 5%, the radiative corrections being taken into account, while the $\mu$ -meson life time changes only by 0.5%. A similar role should be played by the radiative corrections to the ratio of two possible types of the pion decay.

Recent investigations of the pion decay have showed /10/ /11/ that the experimental value of the ratio of pion $e$— and $\mu$—decays is close to the value predicted by the Feynman—Gell—Mann theory. Namely, without the account of radiative corrections the theory yields /12/ :

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* If we plot the graphs so that the decay were described as a transformation of one charged particle into another ($\mu B e$ , $p B e$ ) then a divergent integral appears when the charged particle helicity changes in the transition.

$$R_0 = \frac{(W_{e\nu})_0}{(W_{\mu\nu})_0} = \frac{(m_\pi^2 - m_e^2)^2}{(m_\pi^2 - m_\mu^2)^2} \cdot \frac{m_e^2}{m_\mu^2} = 1{,}282 \cdot 10^{-4}, \qquad /1/$$

where $(W_{e\nu})_0$ and $(W_{\mu\nu})_0$ are uncorrected probabilities of the pion $e$ — and $\mu$ —decays. Radiative corrections to both modes of decay have been calculated by Berman/6/, Kinoshita/8/ who has shown that the value of the correction to the ratio /1/ is surprisingly large and amounts to 14%.

Berman calculates as usual the sum of 'radiative corrections' and 'probability of real soft quantum radiation'. This value depends strongly upon the value of the quantum energy cut-off, or what is the same, upon the experimental loss of the electron energy $\Delta E$ (the above numerical result corresponds to $\Delta E =$ 0.25 Mev)

It is obvious that in this case the correction can be large if the photon spectrum shape is different in both decays. In this case the cut-off will single out a different part of the lepton spectrum. As the calculations show, this is, in fact, just the principle cause of the large value of the correction. Kinoshita/8/ has mentioned this in his paper, however he has not give all the formulae.

All the calculations are repeated in the present paper, the formula for the correction to the decay probability being somewhat different from that of Kinoshita, (although the numerical difference with the Kinoshita's value is negligible). Besides, the formulae for the photon spectra are given. The lepton spectrum obtained by us coincides with that calculated in/14/, /15/, /16/.

The calculations have been made by a standard method. In doing so the radiation from nucleon loops was neglected, since they contain the nucleon mass in the denominator (comp. for example/17/).

If we choose a system of coordinates where the pion is at rest, then the radiative decay probability equals/8/:

$$\frac{W_{e\nu\gamma}}{(W_{e\nu})_0} = \frac{\alpha}{\pi}\left\{ b(\mu)\left[\ln\frac{\lambda}{m_\pi} - \ln(1-\mu^2) - \frac{1}{2}\ln\mu + \frac{3}{4}\right] - \frac{\mu^2(10-7\mu^2)}{2(1-\mu^2)^2}\cdot\ln\mu \right.$$
$$\left. + \frac{2(1+\mu^2)}{(1-\mu^2)}\cdot L(1-\mu^2) + \frac{(15-21\mu^2)}{8(1-\mu^2)} \right\}. \qquad /2/$$

In the formula $\lambda$ is the infra-red cut-off, $\mu$ is the ratio of the lepton mass to that of pion,

$$L(x) = \int_0^x \frac{\ln(1-t)}{t}dt = -\sum_{k=1}^{\infty}\frac{x^k}{k^2}\ (|x|\leq 1), \qquad b(\mu) = 2\left(\frac{1+\mu^2}{1-\mu^2}\ln\mu + 1\right). \qquad /3/$$

The photon and lepton spectra are given by the formulae: $\mathcal{E}_\gamma = E_\gamma/m_\pi$, $\mathcal{E}_e = E_e/m_\pi$

$$dW_{e\nu\gamma}(\mathcal{E}_\gamma) = (W_{e\nu})_0\frac{\alpha}{\pi}\left\{ -\frac{2}{\mathcal{E}_\gamma} + \frac{(4-5\mu^2-2\mathcal{E}_\gamma)}{(1-\mu^2)^2} + \frac{\mu^2}{(1-2\mathcal{E}_\gamma)(1-\mu^2)^2} \right.$$
$$\left. + \left[\frac{(1+\mu^2)}{(1-\mu^2)}\cdot\frac{1}{\mathcal{E}_\gamma} - \frac{2(1-\mu^2-2\mathcal{E}_\gamma)}{(1-\mu^2)^2}\right]\cdot\ln\frac{1-2\mathcal{E}_\gamma}{\mu^2} \right\} d\mathcal{E}_\gamma, \qquad /4/$$

$$dW_{e\nu\gamma}(\varepsilon_e)=(W_{e\nu})_0\frac{\alpha}{\pi}\left\{\frac{4}{(1-\mu^2)(1+\mu^2-2\varepsilon_e)}\left[\varepsilon_e\ln\frac{\varepsilon_e+\sqrt{\varepsilon_e^2-\mu^2}}{\varepsilon_e-\sqrt{\varepsilon_e^2-\mu^2}}-2\sqrt{\varepsilon_e^2-\mu^2}\right]\right.$$

$$\left.+\frac{(1+\mu^2-2\varepsilon_e)}{(1-\mu^2)^2}\cdot\ln\frac{\varepsilon_e+\sqrt{\varepsilon_e^2\mu^2-\mu^2}}{\varepsilon_e-\sqrt{\varepsilon_e^2\mu^2-\mu^2}}\right\}d\varepsilon_e\ . \qquad /5/$$

the formula /5/ coincides with that of paper /16/.

Taking into account all the Feynman graphs with the virtual photon emission up to $e^2$, we obtain the probability of a nonradiative $\pi e$ -decay (or $\pi\mu$ -decay):

$$W_{e\nu}=(W_{e\nu})_0\left\{1+\frac{\alpha}{\pi}\left[-\frac{3}{2}\ln\frac{L}{m_\pi}-b(\mu)\left(\ln\frac{\lambda}{m_\pi}-\frac{1}{2}\ln\mu+\frac{3}{4}\right)+\frac{(3-2\mu^2)}{(1-\mu^2)}\ln\mu-\frac{3}{8}\right]\right\}, \qquad /6/$$

where $L$ and $\lambda$ are the upper and lower limit of the virtual photon energy respectively. The formula of the paper /8/ differs from the above one in that instead of the term $-3/8$ there is the term $-1/4$ (after making correction for the mass).

By summing up /2/ and /6/ we find the total (radiative + nonradiative) pion decay probability:

$$\frac{W_{e\nu}+W_{e\nu\gamma}}{(W_{e\nu})_0}=1+\frac{\alpha}{\pi}\left\{-\frac{3}{2}\ln\frac{L}{m_\pi}-b(\mu)\cdot\ln(1-\mu^2)+\frac{(6-20\mu^2+11\mu^4)}{2(1-\mu^2)^2}\ln\mu\right.$$

$$\left.+\frac{2(1+\mu^2)}{(1-\mu^2)}L(1-\mu^2)+\frac{(6-9\mu^2)}{4(1-\mu^2)}\right\}. \qquad /7/$$

In /8/ the last term was $(13-19\mu^2)/8(1-\mu^2)$ (after making correction for the mass). By using the formula /7/ we can calculate radiative correction to /1/.

For the probability of the emission of a lepton with the energy less than $E_{max}-\Delta E$ we get:

$$\frac{W_{e\nu\gamma}(E<E_{max}-\Delta E)}{(W_{e\nu})_0}=\frac{\alpha}{\pi}\left\{-b(\mu)\left(\ln\frac{m_\pi}{2\Delta E}+2\ln(1-\mu^2)-\frac{3}{4}\right)+\frac{2(1+\mu^2)}{(1-\mu^2)}\left[L(1-\mu^2)-L\left(\frac{2}{1-\mu^2}\frac{\Delta E}{m_\pi}\right)\right]\right.$$

$$\left.-\left[\frac{\mu^2(10-7\mu^2)}{2(1-\mu^2)^2}+\frac{4(1-3\mu^2)}{(1-\mu^2)^3}\frac{\Delta E}{m_\pi}\right]\ln\mu+\left[\frac{(15-21\mu^2)}{8(1-\mu^2)}-\frac{4(1+3\mu^2)}{(1-\mu^2)^2}\frac{\Delta E}{m_\pi}\right]\right\}. \qquad /8/$$

If the linear terms in $\Delta E$ are neglected the formula turns into the Kinoshita formula /8/. Now we can write down the formula for the correction to the ratio of $e$ – and $\mu$ -decay probabilities for which leptons have an energy which differs from the maximum one less than by $\Delta E$. By substituting the values of constants we obtain:

$$R(\Delta E)=R_0\left\{1-(4{,}647\cdot10^{-3})\cdot\left[30{,}12-4{,}611\left(\ln\frac{2\Delta E_e}{m_e}-\frac{2\Delta E_e}{m_e}\right)-L\left(\frac{2\Delta E_e}{m_e}\right)\right]\right\}. \qquad /9/$$

Numerical values and the comparison with the results of Berman[6] and Kinoshita[8] are tabulated :

| | $\Delta E_e$ (in units of $m_e$) | 0.5 | 10 | 20 | 30 | total energy region |
|---|---|---|---|---|---|---|
| $\left(\frac{R_0-R}{R_0}\right)\cdot 10^2$ | Berman | 13.9 | 7.6 | 6.1 | 5.3 | 3.9[8] |
| | The present paper | 14.0 | 7.8 | 6.5 | 5.8 | 3.93 |

For the sake of illustration we give the curve of the dependence of the ratio on the cut-off $\Delta E_e$ . Note, that for the value of $\Delta E_e = 10$ Mev, which has been taking in the experiments[11], $R = 1.198 \cdot 10^{-4}$ (the experimental value yields $R = (1{,}18 \pm 0{,}08) \cdot 10^{-4}$).

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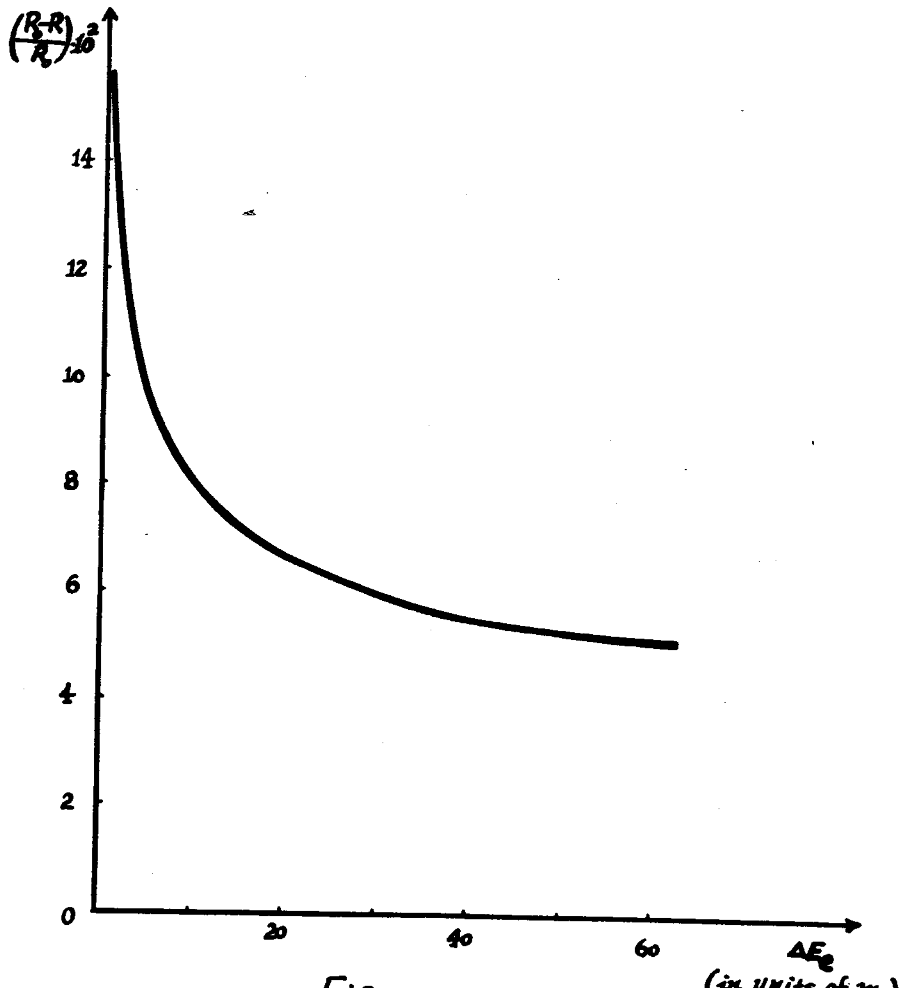


Fig.

Dependence of the ratio $\frac{R_0-R}{R_0}$ upon the interval of energy $\Delta E_e$ of electron.

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