ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

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D-685

$K\bar{K}$ –PAIR PRODUCTION IN THE $\pi\pi$ –COLLISIONS

Дубна 1961

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Submitted to JETP

Abstract

The equations for the partial waves of the $K\bar{K}$-pair production in the $\pi\pi$-collisions have been obtained by means of the Mandelstam representations. The solutions of these equations are given in the general form. It is shown that the presence of the resonance in the P-phase shift of the $\pi\pi$-scattering does not contradict the requirements of the existence and uniqueness of the solutions obtained.

1. Introduction

In investigating the scattering of K-mesons by nucleons by means of the Mandelstam representations it is necessary to know the amplitude of the $K\bar{K}$-pair production process in the $\pi\pi$-collisions.

Up to the present the reaction $\pi+\pi \longrightarrow K+\bar{K}$ was considered in numerous papers /1,2,3/. In ref./1/ one has obtained the S-wave of the process $\pi+\pi \to K+\bar{K}$ in rough approximation, and in ref./3/ one has obtained a general form of the solution for the ρ-wave. In ref./2/ one has used the pole ideology which is not being considered in the present paper.

In this paper the integral equations for the partial waves of the process $\pi+\pi \to K+\bar{K}$ are derived with the aid of the double Mandelstam representations. These equations contain the dependence on the π-π-scattering phase shifts and the π-K-scattering partial waves. The requirement of the existence and uniqueness of solutions we are considering leads to certain requirements impossed on the $\pi\pi$-scattering phase shifts and the πK-scattering partial waves.

It is impossible for the time being to mean to obtain good quantitative results. Depending on various behaviours of the $\pi\pi$-scattering phase shifts at infinity we can obtain various solutions for the $K\bar{K}$-pair production partial waves. In addition no use can be made for the present of the π-K-scattering amplitudes ($T^{1/2} = T^{3/2}$) obtained in ref./4,5/, because the solutions of $T^{1/2} = T^{3/2}$ are roughly approximate. In the given paper we restrict ourselves, therefore, to obtaining solutions for the lowest order partial waves in a general form.

The further progress in the investigation of the $\pi\pi$-scattering process will enable us to obtain solution of the integral equations for the πK-scattering partial waves as well as for the partial waves of the process under consideration.

II. Kinematics and the analytical properties of the $\pi+\pi \to K+\bar{K}$ process amplitude

We introduce the following invariant variables:

$$S_1 = -(p_1 + q_1)^2$$
$$S_2 = -(p_1 + q_2)^2$$
$$S_3 = -(p_1 + p_2)^2$$

where p_1 and p_2 are the four-momenta of the K and $\bar{K}$ -mesons, respectively, and q_1 and q_2 are the four-momenta of π -mesons. In the c.m.s. of the reaction $\pi + \pi \to K + \bar{K}$ (in the following it will be denoted as reaction III) the variables S_1, S_2 and S_3 are of the form

$$\begin{aligned} S_1 &= M^2 - \mu^2 - 2q^2 + 2z\sqrt{q^2(q^2+\mu^2-M^2)} \\ S_2 &= M^2 - \mu^2 - 2q^2 - 2z\sqrt{q^2(q^2+\mu^2-M^2)} \\ S_3 &= 4(\mu^2+q^2) = 4(M^2+p^2) \end{aligned} \qquad /1/$$

where M and μ are the masses of the K and π mesons, respectively, $\vec{p}$ and $\vec{q}$ are moments of the K and π -mesons, respectively

$$z = \cos\theta = \frac{\vec{p}\cdot\vec{q}}{|\vec{p}|\cdot|\vec{q}|}$$

The isotopic structure of the process $\pi + \pi \to K + \bar{K}$ has the form

$$T_{\alpha\beta} = A(s_1, s_2, s_3)\,\delta_{\alpha\beta} + \frac{1}{2}[\tau_\alpha, \tau_\beta]\, B(s_1, s_2, s_3)$$

The amplitudes of the T^0 and T^1-states with the isotopic spin zero and unit, correspondingly are connected with the functions $A(s_1, s_2, s_3)$ and $B(s_1, s_2, s_3)$ by the simple relations:

$$T^0 = \sqrt{6}\,A\,; \qquad T^1 = 2B \qquad /2/$$

The functions $A(s_1, s_2, s_3)$ and $B(s_1, s_2, s_3)$ satisfy the following conditions of the crossing symmetry:

$$A(s_1, s_2, s_3) = A(s_2, s_1, s_3)\,; \qquad B(s_1, s_2, s_3) = -B(s_2, s_1, s_3) \qquad /3/$$

We shall consider the functions $A(s_1, s_2, s_3)$ and $B(s_1, s_2, s_3)$ in the c.m.s. of the reaction III as functions of the variable q^2 for fixed value of $z = \text{Const}$. We assume that the functions $A(s_1, s_2, s_3)$ and $B(s_1, s_2, s_3)$ comply with the Mandelstam representations. Then we obtain the following cuts in the plane q^2:

1. From the reaction I ($\pi + K \to \pi' + K'$) the cut is in the interval $[-\infty, x_{1m}]$
2. From the reaction II ($\pi' + K \to \pi + K'$) the cut is in the interval $[-\infty, x_{2m}]$
3. From the reaction III – the cut is in the interval $[0, \infty]$

$$x_{1m} = -\frac{M+\mu}{2(1-z^2)}\left[M+\mu+(1-z^2)(\mu-M) - z\sqrt{(M+\mu)^2-(1-z^2)(M-\mu)^2}\right]$$
$$x_{2m} = -\frac{M+\mu}{2(1-z^2)}\left[M+\mu+(1-z^2)(\mu-M) + z\sqrt{(M+\mu)^2-(1-z^2)(M-\mu)^2}\right]$$

Besides there is one more cut – the kinematic one, which lies in the interval $0 \le q^2 \le M^2 - \mu^2$. We remove this cut by the method suggested in ref./6/. For this purpose we shall consider the symmetrical and antisymmetrical in root $k(q^2) = \sqrt{q^2(q^2+\mu^2-M^2)}$ combinations of the functions $A(s_1, s_2, s_3)$ and $B(s_1, s_2, s_3)$

$$\Phi_s(q^2, z) = \frac{\Phi(q^2, z + k(q^2)) + \Phi(q^2, z, -k(q^2))}{2}$$
$$\Phi_a(q^2, z) = \frac{\Phi(q^2, z, +k(q^2)) - \Phi(q^2, z, -k(q^2))}{2k(q^2)} \qquad /4/$$

where Φ denotes the functions A and B. From the conditions /3/ and relations /4/ it follows

$$\Phi_s(q^2, z) = A(q^2, z)$$
$$\Phi_a(q^2, z) = \frac{B(q^2, z)}{k(q^2)} \qquad /5/$$

By writting now the Cauchy theorem for the functions $A(q^2, z)$ and $\frac{B(q^2, z)}{k(q^2)}$, we obtain the following relations

$$Re\, A(q^2, z) = \frac{1}{\pi} P \int_0^\infty \frac{Im\, A(x, z)}{x - q^2} dx + \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{Im\, A(x, z)}{x - q^2} dx + \frac{1}{\pi} P \int_{-\infty}^{x_{2m}} \frac{Im\, A(x, z)}{x - q^2} dx$$

$$Re\, B(q^2, z) = \frac{P}{\pi} \int_0^\infty \frac{Im\, B(x, z)}{x - q^2} \cdot \frac{k(q^2)}{k(x)} dx + \frac{P}{\pi} \int_{-\infty}^{x_{1m}} \frac{Im\, B(x, z)}{x - q^2} \frac{k(q^2)}{k(x)} dx + \frac{P}{\pi} \int_{-\infty}^{x_{2m}} \frac{Im\, B(x, z)}{x - q^2} \frac{k(q^2)}{k(x)} dx \qquad /6/$$

In Eqs. /6/ the amplitudes $Im\, A(x, z)$ and $Im\, B(x, z)$ are continued analytically throughout the region $0 \le q^2 \le M^2 - \mu^2$.

III. Integral equations for the partial waves and their solution

We use further the connection of the coefficients $A(s_1, s_2, s_3)$ and $B(s_1, s_2, s_3)$ with the amplitudes $T^{1/2}(s_1, s_2, s_3)$ and $T^{3/2}(s_1, s_2, s_3)$ (for the first and second channels):

$$A = \frac{2T^{3/2} + T^{1/2}}{3}$$
$$B = \frac{T^{1/2} - T^{3/2}}{3} \qquad /7/$$

and we restrict ourselves in the cosideration to the small values of the orbital momentum ℓ (0 or 1).

By making use of the relations /2/ and /7/ we write the equations /6/ in the following form:

$$\mathrm{Re}\, T^0(q^2,z) = \frac{1}{\pi} P\int_0^\infty \frac{\mathrm{Im}\, T^0(x,z)}{x-q^2}dx + \frac{1}{\pi}\sqrt{\frac{2}{3}}\, P\int_0^\infty \left\{ \frac{\mathrm{Im}\left[2T^{3/2}(x,z_1)+T^{1/2}(x,z_1)\right]}{f_1(x,z_1)-q^2}\cdot\frac{\partial f_1(x,z_1)}{\partial x}dx \right. \qquad /8a/$$

$$\left. + \frac{\mathrm{Im}\left[2T^{3/2}(x,z_2)+T^{1/2}(x,z_2)\right]}{f_2(x,z_2)-q^2}\cdot\frac{\partial f_2(x,z_2)}{\partial x}dx \right\}$$

$$\mathrm{Re}\, T^1(q^2,z) = \frac{1}{\pi} P\int_0^\infty \frac{\mathrm{Im}\, T^1(x,z)}{x-q^2}\cdot\frac{K(q^2)}{K(x)}dx + \frac{P}{6\pi}\int_0^\infty \left\{ \frac{\mathrm{Im}\left[T^{1/2}(x,z_1)-T^{3/2}(x,z_1)\right]}{f_1(x,z_1)-q^2}\cdot\frac{K(q^2)}{K[f_1(x,z_1)]}\cdot \right. \qquad /8b/$$

$$\left. \cdot\frac{\partial f_1(x,z_1)}{\partial x}dx + \frac{\mathrm{Im}\left[T^{1/2}(x,z_2)-T^{3/2}(x,z_2)\right]}{f_2(x,z_2)-q^2}\cdot\frac{K(q^2)}{K[f_2(x,z_2)]}\cdot\frac{\partial f_2(x,z_2)}{\partial x}dx \right\}$$

where $f_i(x,z_i) = -\mu^2 - \frac{x}{2}(1-z_i)$, $(i = 1, 2)$ and z_1 and z_2 are the cosines of angles between π -mesons in the reactions I and II respectively.

By using the unitarity condition in the form:

$$\mathrm{Im}\, T_\ell^i(q^2) = \frac{|\vec{q}|}{8\pi W(q^2)}\, T_\ell^i(q^2)\, \Pi_\ell^{*i}(q^2) \qquad /9/$$

where $W(q^2) = \sqrt{S_3}$, and Π_ℓ^i is the π-π -scattering amplitude, we get from the Eqs. /8/ the following equation for partial waves of the process we are considering

$$\mathrm{Re}\, T_0(q^2) = \frac{P}{8\pi^2}\int_0^\infty \frac{\sqrt{x}}{W(x)}\cdot\frac{T_0(x)\cdot\Pi_0^*(x)}{x-q^2}dx + F_0(q^2) \qquad /10a/$$

$$\mathrm{Re}\, T_1(q^2) = \frac{P}{8\pi^2}\int_0^\infty \frac{\sqrt{x}}{W(x)}\cdot\frac{T_1(x)\cdot\Pi_1^*(x)}{x-q^2}\cdot\frac{K(q^2)}{K(x)}dx + F_1(q^2) \qquad /10b/$$

where

$$F_0(q^2) = \frac{1}{\pi}\sqrt{\frac{2}{3}}\int_{-1}^{+1}dz\cdot P\int_0^\infty\left\{ \frac{\mathrm{Im}\left[2T^{3/2}(x,z_1)+T^{1/2}(x,z_1)\right]}{f_1(x,z_1)-q^2}\cdot\frac{\partial f_1(x,z_1)}{\partial x}dx + \right.$$

$$\left. + \frac{\mathrm{Im}\left[2T^{3/2}(x,z_2)+T^{1/2}(x,z_2)\right]}{f_2(x,z_2)-q^2}\cdot\frac{\partial f_2(x,z_2)}{\partial x}dx \right\}$$

$$F_1(q^2) = \frac{1}{6\pi}\int_{-1}^{+1}dz\, P\int_0^\infty\left\{ \frac{\mathrm{Im}\left[T^{1/2}(x,z_1)-T^{3/2}(x,z_1)\right]}{f_1(x,z_1)-q^2}\cdot\frac{\partial f_1(x,z_1)}{\partial x}\cdot\frac{K(q^2)}{K[f_1(x,z_1)]}dx + \right.$$

$$\left. + \frac{\mathrm{Im}\left[T^{1/2}(x,z_2)-T^{3/2}(x,z_2)\right]}{f_2(x,z_2)-q^2}\cdot\frac{K(q^2)}{K[f_2(x,z_2)]}\cdot\frac{\partial f_2(x,z_2)}{\partial x}dx \right\}$$

One of the most essential approximations in obtaining Eqs. /10/ is the fact that we restrict ourselves only to two π -meson intermediate state in the unitarity condition /9/.

Eqs. /10a/ and /10b/ together with Eqs. /25/, /26/ from/4/ form an approximate system of equations for the π- K interaction (for the set $\ell = 0.1$). We suggest to carry out one subtraction in the Eq. /10a/. The presence of the factor $\frac{K(q^2)}{K(x)}$ provides the convergence of the integrals in the Eq. /10b/.

We shall seek first the solution of the equation /10b/.

If we assume that in the low energy region the solution for the first and second processes

$T^{3/2}(q^2) = T^{1/2}(q^2)$ is written as a strict equality then the unique solution for the state $T_1^1(q^2)$ is zero (identically) and the $K\bar{K}$ -pair production amplitude does not depend on the angle ϑ , i.e. we have the isotropy in the $K\bar{K}$ -distribution (in the c.m.s.).

However, the requirement that the solutions $T^{3/2}$ and $T^{1/2}$ be equal strictly (in the low-energy region) is too strong. It is more natural to assume that $T^{3/2} \approx T^{1/2}$. In this case the quantity $F_1(q^2)$ will be different from zero, although it might appear rather small (in comparison with $Re\, T_1(q^2)$ and the integral term).

If $F_1(q^2)$ is a small quantity, then, by omitting it, we get the Eq. /10b/ in the form

$$Re\, T_1(q^2) = \frac{1}{8\pi^2} P \int_0^\infty \frac{\sqrt{x}}{W(x)} \cdot \frac{T_1(x)\, \Pi_1^*(x)}{x - q^2} \cdot \frac{k(q^2)}{k(x)}\, dx \qquad /11/$$

We shall find first the solution of Eq. /11/, and then take into account the case $F_1(q^2) \neq 0$

Let us introduce the notation $\Psi(q^2) = \frac{T_1(q^2)}{k(q^2)}$. Since $\Pi_1^*(x) = \frac{8\pi W(x)}{\sqrt{x}} e^{-i\delta_1(x)} \sin\delta_1(x)$ then the equation /11/ takes the form

$$\Psi(q^2) = \frac{1}{\pi} \int_0^\infty \frac{e^{-i\delta_1(x)} \sin\delta_1(x)}{x - q^2} \Psi(x)\, dx \qquad /12/$$

Note that $\Psi(q^2)$ decreases $\sim \frac{1}{q^2}$ at infinity. Let us introduce the function of the complex variable ξ as follows

$$\Phi_1(\xi) = \frac{1}{2\pi i} \int_0^\infty \frac{e^{-i\delta_1(x)} \sin\delta_1(x)}{x - \xi} \Psi(x)\, dx \qquad /13/$$

From where one can see that

$$\Psi(q^2) = 2i\, \Phi_1^+(q^2) \qquad /14/$$

where Φ_1^+ denotes the limiting value of the function $\Phi_1^+(\xi)$ on the upper edge of the cut. The problem of finding the function $\Phi_1(\xi)$ is solved by reducing it to the adge Riemann problem/7/: from /13/ it follows that on the contour $[0, \infty]$ there is the relation:

$$\Phi_1^+(q^2) = e^{2i\delta_1(q^2)} \Phi_1^-(q^2) \qquad /15/$$

We supplement the integration contour up to the total real axis $[-\infty, +\infty]$, by determining the phase shift $\delta_1(q^2)$ on the negative semi-axis $[0, -\infty]$ so that $\delta_1(q^2) = 0$ for $q^2 \leq 0$.

Once the index of the problem is equal to zero, i.e. if

$$\frac{1}{2\pi i} \int_{-\infty}^{+\infty} d\ln\left(e^{2i\delta_1(x)}\right) = 0 \qquad /16/$$

(it is obvious that in this case the $\pi\pi$ -scattering phase shift equals zero at infinity), then there exists only the trivial zero solution of the problem. However, if we shall demand that there exist the unique solution of the problem within the accuracy up to the constant multiplier (but not a trivial one), then it is necessary to demand, that the index of the problem /16/ be unit. Then the $\pi\pi$ -scattering phase shift at infinity must equal π and ρ -phase shift must have at least one maximum. In this case the solution is of the form .

$$\Phi_1^+(q) = a \cdot \frac{X_1^+(q)}{q + i\mu^2} \qquad /17/$$

where a is a certain constant defined from the comparison with the experiment

$$X_1^+(q) = e^{\Gamma_1^+(q)} \qquad /18/$$

$$\Gamma_1(q) = \frac{1}{2\pi i}\int_{-\infty}^{+\infty} \ln\left[\frac{x+i\mu^2}{x-i\mu^2}\cdot e^{2i\delta_1(x)}\right]\frac{dx}{x-q} \qquad /19/$$

From /14/, /17/–/19/ we find that

$$\Psi(q^2) = 2ia \exp\left[i\delta_1(q^2) + \frac{\rho}{\pi}\int_{-\infty}^{+\infty}\frac{\delta_1(x)}{x-q^2}dx\right] \qquad /20/$$

Since $\delta_1(\infty) = \pi$ then in order that the integral convergence in /20/ it is necessary to make one subtraction.

The account of the negative cut (as well as the highest states in the unitarity condition) leads to the appearence of the inhomogeneous term $F_1(q^2)$. In this case the relation /14- will take the form

$$\Psi(q^2) = 2i\Phi^+(q^2) + \frac{F_1(q^2)}{K(q^2)} \qquad /21/$$

and the relation /15/ will be of the form:

$$\Phi_1^+(q^2) = \exp\left(2i\delta_1(q^2)\right)\Phi_1^- + \frac{F_1(q^2)}{K(q^2)}\exp\left(i\delta_1(q^2)\right)\sin\delta_1(q^2) \qquad /22/$$

We assume that in the relation /22/ the free term of the Riemann problem $\frac{F_1(q^2)}{K(q^2)}\exp(i\delta_1(q^2))\sin\delta_1(q^2)$ satisfies the Hölder condition. If, as before, the index of the Riemann problem is believed to be equal to (1), then the general solution of Eq. /10b/ has the form:

$$\Phi_1^+(q) = X_1^+(q)\left[A^+(q) + \frac{a}{(q+i\mu^2)^+}\right] \qquad /23/$$

where

$$A^+(q) = \frac{1}{2\pi i}\int_{-\infty}^{+\infty}\frac{F_1(x)\exp(i\delta_1(x))\sin\delta_1(x)}{K(x)\cdot X_1^+(q)\cdot(x-q)}dx$$

However, if the inhomogeneous term is present the requirement of the existence of a non-zero solution is also satsfied in the case when the index of the problem is zero. Then $\Gamma_1(q) = \frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{\delta_1(x)}{x-q}dx$, the quantity "a" should be assumed to be equal to zero, and the unique solution of Eq. /10b/ is written in the form (see formulae /8/, /21/, /23/)

$$\Psi(q^2) = 2i\,X^+(q^2)\,A^+(q^2) + \frac{F_1(q^2)}{K(q^2)} \qquad /24/$$

Before to proceed to the solution of Eq. /10a/ it is necessary to make one subtraction. After this subtraction has been done at the point q_0^2 the equation /10a/ is written in the form

$$\mathrm{Re}\,T_0(q^2) = \frac{q^2 - q_0^2}{8\pi^2}\,P\int_0^\infty \frac{\sqrt{x}}{W(x)} \cdot \frac{T_0(x)\,n_0^*(x)}{(x-q_0^2)(x-q^2)}\,dx + F_0(q^2, q_0^2) \qquad /25/$$

where

$$F_0(q^2, q_0^2) = \mathrm{Re}\,T_0(q_0^2) + F_0(q^2) - F_0(q_0^2)$$

The Eq. /25/ coincides with Eq. /10b/ up to the change

$$\frac{T_1(x)}{K(x)} \to \frac{T_0(x)}{x - q_0^2}\,; \qquad \frac{F_1(x)}{K(x)} \to \frac{F_0(x, q_0^2)}{x - q_0^2}$$

$$\delta_1(x) \to \delta_0(x) \qquad /26/$$

By supposing as earlier that the free term of the Riemann problem $\frac{F_0(x,q_0^2)}{x-q_0^2}\,e^{i\delta_0(x)}\sin\delta_0(x)$ satisfies the Hölder condition, and the index of the problem is equal to (+1), then analogically to the foredoing solution of the problem for the p -wave, we obtain a general solution of the Riemann problem in the form

$$\Phi_0^+(q) = X_0^+(q)\left[B^+(q) + \frac{b}{(q + i\mu^2)_+}\right] \qquad /27/$$

where

$$X_0^+(q) = \exp\left[\Gamma_0^+(q)\right]$$

$$\Gamma_0(q) = \frac{1}{2\pi i}\int_{-\infty}^{+\infty} \ln\left[\frac{x + i\mu^2}{x - i\mu^2}\exp\left(2i\delta_0(x)\right)\right]\frac{dx}{x - q}$$

$$B^+(q) = \frac{1}{2\pi i}\int_{-\infty}^{+\infty} \frac{F_0(x, q_0^2)}{x - q_0^2} \cdot \frac{\exp\left(i\delta_0(x)\right)\cdot\sin\delta_0(x)}{X_0^+(x)\cdot(x - q)}\,dx$$

The constant "b" is determined from the comparison with the experimental data.

If the index of the Riemann problem is equal to zero, then in /28/ we have to assume

$$\Gamma_0(q) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{\delta_0(x)}{x - q}\,dx$$

and the quantity "b" is assumed to be equal to zero. Then the unique solution of Eq. /25/ will be of the form

$$\Phi_0^+(q) = X_0^+(q)\,B^+(q) \qquad /29/$$

Let us note that even if the solutions $T^{1/2}$ and $T^{3/2}$ are equal strictly, the free term of Eq. /25/ turns out to be different from zero. This leads to the fact that we obtain the unique solution, non-trivial for the S-wave too in the case when the index of the Riemann problem equals zero.

Conclusion

Eq. /10a/ and /10b/ contain the following essential approximations:

a) the consideration of the problem is restricted only to the lowest waves (s and p)*

b) the unitarity condition /11/ does not take into account the contribution of highest states. Strictly speaking, the unitarity condition /11/ can be used only in the region $0 \leq q^2 \leq 3\mu^2$.

In the low-energy region under consideration the approximation b) will not introduce, apparently, large errors. The approximation a) is more rough (see footnote*). The solutions /20/ and /24/ for the amplitude T_1^1 and analogically /27/ and /29/ for the amplitude T_0^0 are very different, which is connected with the behaviour of the $\pi\pi$ -scattering phase shift at infinity. The choice of the index of the problem was related to the requirementof the existence and uniqueness of the solution. Since the inhomogeneous term in Eq. /10/ exists, in principle, it is not obligatory to require that the index of the problem be equal to (+1).

However, in the case, when the index of the problem is equal to unit the $\pi\pi$ -scattering phase shift has at least one resonance,and the solution has one indefinite constant; when the index is zero, there are no direct indications that the resonance exist, although the possibility of its existence is not denied.

At present there are indications that the resonance in the p -phase shift of the $\pi\pi$ -scattering exists/9/. As it follows from the consideration we have been undertaken the presence of such a resonance does not contradict the conditions of the existence and uniqueness of our problem.

We can also obtain other solutions depending on various behaviour of the $\pi\pi$ -scattering phase shift at infinity. These solutions will contain a larger number of indefinite constants when the index of the problem is positive, or require additional conditions to be imposed when the index is negative. However, we have no reason to choose the values of the index different from zero.

In conclusion we express our deep gratitude to professor H.T.Tzu, A.V.Efremov and L.D.Soloviev for useful discussions.

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* The method for obtaining integral equations for partial waves by using the ortogonality properties of the Legendre polynomials leads to large errors in the unphysical angle region ($|\cos\theta| > 1$).Therefore, to obtain more strict quantitative results it is advisable to use the amplitude of the $\pi + \pi \to k + \bar{k}$ process at the point $\cos\theta = -1$/8/.

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Received by Publishing Department on April 3, 1961.