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ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

Лаборатория ядерных проблем
Лаборатория теоретической физики

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D – 682

LOW ENERGY LIMIT FOR γn SCATTERING AMPLITUDE AND CROSSING SYMMETRY

Дубна 1961

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Submitted to JETP

Abstract

The low energy limit for γN scattering amplitude has been obtained with the aid of the single nucleon terms of the invariant amplitudes. The account of the crossing symmetry requirements permits to get the next terms by ν for $Q^2 \to 0$, as well as an expression for the limiting value of the first derivative with respect to Q^2 for $Q^2 \to 0$.

I

Low, Gell-Mann and Goldberger have shown[1], that the requirements of the relativistic and gauge invariance make it possible to express the limiting value for the low energy amplitudes of γ- rays scattering on spin ½ particles and the limiting value of the derivative of the frequency amplitude for $\nu \to 0$ in terms of the charge and the magnetic moment of the particle.

Later on, this result was generalized[2] to the case of elastic scattering of γ rays by particles with other spins, as well as to the bremsstrahlung[3].

This result for elastic scattering is also valid when only C.P invariance is assumed.

One is led to the low-energy theorem if the single nucleon terms in the dispersion relations are considered for γN -scattering[4,5,6]. (An analogous result holds for the bremstrahlung[7]).

This note is concerned with the limiting theorem for γN -scattering basing upon the one-nucleon terms.

The requirement for the invariant functions $T_i(\nu, Q^2)$ $(i = 1, \dots, 6)$ to be crossing symmetrical allows to get additional components to the limiting values of the functions $R_i(\nu, 0)$, which are characteristic of the γ-N scattering matrix in the c.m.s., as well as of the limiting values of the derivatives of the amplitudes by Q^2 for $\nu \to 0$. (The quantities T_i and R_i are determined, e.g., in[6]).

II

The invariant functions $T_i(\nu, Q^2)$ are related to the scalar functions $R_i(\nu, Q^2)$

$$T_1 - T_3 = \frac{8MW^2}{(W^2-M^2)^2}\left[\nu - \frac{W-M}{W+M}\,\frac{Q^2}{M}\right](R_3+R_4) - \frac{4W}{W+M}\left[1 - \frac{4Q^2W^2}{(W^2-M^2)^2}\right](R_1+R_2)$$

$$T_2 - T_4 = \frac{8MW^2}{(W^2-M^2)^2}\left[1 + \frac{2W}{M}\,\frac{Q^2}{(W+M)^2}\right](R_3+R_4) + \frac{4W}{(W+M)^2}\left[1 - \frac{4Q^2W^2}{(W^2-M^2)^2}\right](R_1+R_2)$$

$$T_1 + T_3 = \frac{8MW^2}{(W^2-M^2)^2}\left[\nu - \frac{W-M}{W+M}\,\frac{Q^2}{M}\right](R_3-R_4) + \frac{16W^3Q^2}{(W+M)(W^2-M^2)^2}(R_1-R_2)$$

$$T_2 + T_4 = \frac{8MW^2}{(W^2-M^2)^2}\left[1 + \frac{2W}{M}\,\frac{Q^2}{(W+M)^2}\right](R_3-R_4) - \frac{16W^3Q^2}{(W+M)^2(W^2-M^2)^2}(R_1-R_2)$$

$$\frac{M\nu + Q^2}{W^2}\,T_5 = \frac{8W^2Q^2}{(W^2-Q^2)^2}(R_5-R_6) - (R_3-R_4) \tag{1}$$

$$\frac{M\nu + Q^2}{W}\,T_6 = \left(2 - \frac{8W^2Q^2}{(W^2-M^2)^2}\right)(R_5+R_6) + (R_3+R_4)$$

where W is the total energy in the c.m.s., while ν and Q^2 are two invariants which characterize the kinematics of the process $W^2 - M^2 = 2M\nu + 2Q^2$.

The pole terms for $T_i(\nu, Q^2)$ are of the form

$$T_1^\circ = \frac{2e^2}{M}\,\frac{M^2Q^2}{Q^4 - M^2\nu^2}\;;\quad T_2^\circ = \frac{e^2}{M}\,\frac{M^2\nu}{Q^4 - M^2\nu^2}\;;\quad T_3^\circ = 0 \tag{2}$$

$$T_4^\circ = -\frac{e^2(1+\lambda)^2}{M}\,\frac{M^2\nu}{Q^4 - M^2\nu^2}\;;\quad T_5^\circ = M\,T_6^\circ = \frac{e^2(1+\lambda)}{M}\,\frac{M^2Q^2}{Q^4 - M^2\nu^2}\,.$$

where the units in which $\hbar = c = 1$ are adopted and the magnetic moment is $\mu = \frac{e(1+\lambda)}{2M}$.

For $Q^2 = 0$, it follows from (1)

$$
\begin{aligned}
T_1 - T_3)_0 &= \frac{2W^2}{M\nu}(R_3+R_4)_0 - \frac{4W}{W+M}(R_1+R_2)_0 \\
T_2 - T_4)_0 &= \frac{2W^2}{M\nu^2}(R_3+R_4)_0 + \frac{4W}{(W+M)^2}(R_1+R_2)_0 \\
T_1 + T_3)_0 &= \frac{2W^2}{M\nu}(R_3-R_4)_0 \\
T_2 + T_4)_0 &= \frac{2W^2}{M\nu^2}(R_3-R_4)_0 \\
T_5)_0 &= -\frac{W^2}{M\nu}(R_3-R_4)_0 \\
T_6)_0 &= \frac{W}{M\nu}\left[2(R_5+R_6)+R_3+R_4\right].
\end{aligned}
\tag{3}
$$

By differentiating the relations in (1) with respect to Q^2 in the limit $Q^2 \to 0$ we obtain

$$
(T_1-T_3)'_0 \equiv \lim_{Q^2\to 0}\frac{d}{dQ^2}(T_1-T_3) = \frac{2W^2}{M\nu}(R_3+R_4)'_0 - \frac{4W}{(W+M)}(R_1+R_2)'_0 -
$$

$$
- \frac{2(2W^3+M^2W+M^3)}{M^2\nu^2(W+M)}(R_3+R_4)_0 + \frac{4(R_1+R_2)_0}{W(W+M)}\left[\frac{W^4}{M^2\nu^2} - \frac{M}{W+M}\right];
$$

$$
(T_2-T_4)'_0 = \frac{2W^2}{M\nu^2}(R_3+R_4)'_0 + \frac{4W}{(W+M)^2}(R_1+R_2)'_0 + \frac{4}{M^2\nu^2}\left[\frac{W^3}{(W+M)^2} - \right.
$$

$$
\left. - M - \frac{M^2}{\nu}\right](R_3+R_4)_0 - \frac{4}{(W+M)^2}\left[\frac{W^3}{M^2\nu^2} - \frac{1}{W} + \frac{2}{W+M}\right](R_1+R_2)_0
$$

$$
(T_1+T_3)'_0 = \frac{2W^2}{M\nu}(R_3-R_4)'_0 - \frac{2(2W^3+WM^2+M^3)}{M^2\nu^2(W+M)}(R_3-R_4)_0 + \frac{4W^3}{(W+M)M^2\nu^2}(R_1-R_2)_0
$$

$$(T_2+T_4)'_0 = \frac{2W^2}{M\nu^2}(R_3-R_4)'_0 + \frac{4(R_3-R_4)_0}{M^2\nu^2}\left[\frac{W^3}{(W+M)^2} - M - \frac{M^2}{\nu}\right] - \frac{4W^3}{(W+M)^2}\frac{(R_1-R_2)_0}{M^2\nu^2}$$

$$T_5)'_0 = \frac{W^2}{M\nu}\left[\frac{2W^2}{M^2\nu^2}(R_5-R_6)_0 - (R_3-R_4)'_0 + \frac{M}{W^2\nu}(R_3-R_4)_0\right]$$

$$T_6)'_0 = \frac{W}{M\nu}\left[-\frac{2W^2}{M^2\nu^2}(R_5+R_6)_0 + (2R_5+2R_6+R_3+R_4)'_0 - \right. \tag{4}$$
$$\left. - \frac{M+\nu}{W^2\nu}(R_3+R_4+2R_5+2R_6)_0\right].$$

One can see from (2), that $T_1 - T_3$ and T_2+T_4 do not contain a pole at $Q^2=0$. Then it follows from (1), that R_1+R_2 and $\frac{R_3 \pm R_4}{\nu}$ are finite at $\nu \to 0$.

As far as the functions $T_2 \pm T_4$ have the singularity of the form

$$\frac{e^2}{M}\left[-1 \pm (1+\lambda)^2\right]\frac{1}{\nu}$$

it follows from (1) that

$$\left.\frac{R_3 \pm R_4}{\nu}\right)_0 = -\frac{e^2}{2M^2}\left[1 \pm (1+\lambda)^2\right] \tag{5}$$

for $\nu \to 0$, what is in consistence with the low energy theorem.

Owing to the fact that T_5 and T_6 do not contain the pole at $Q^2=0$, $\left.\frac{R_5+R_6}{\nu}\right)_0$ has to remain constant for $\nu \to 0$. By analogy, from the condition that $(T_1 \pm T_3)'_0$ do contain the second order pole

$$(T_1 \pm T_3)'_0 = -\frac{2e^2}{M\nu^2}$$

and $\nu(T_2-T_4)'_0$ does not contain any, it follows that

$$(R_1 \pm R_2)_0 = -\frac{e^2}{M} \tag{6}$$

while $(R_3 \pm R_4)'_0 \to Const$ and $\nu(R_1 \pm R_2)'_0 \to Const$ for $\nu \to 0.$
From

$$T_5)'_{0\,p} = M\,T_6)'_{0\,p} = -\frac{e^2(1+\lambda)}{2M^2}$$

one can draw a conclusion that

$$(R_5 \pm R_6)_0 = \pm \frac{e^2(1+\lambda)}{2M^2}\,\nu \qquad (7)$$

and $(R_3 + R_4 + 2R_5 + 2R_6)'_0 \to Const$ for $\nu \to 0.$

We see that formulae (5)-(7), obtained from the consideration of pole terms (2), involve the results of the limiting theorem for $Q^2 = 0.$

III

It should be noted that with the aid of the crossing symmetry conditions it is possible to get additional information about the limit of low energies. It follows from the crossing symmetry that, e.g., the quantity $T_1 - T_3$ must be an even function ν.

If we substitute into the first relation from (2)

$$(R_1 + R_2)_0 = -\frac{e^2}{M} + \alpha_1 \nu + \cdots$$

$$(R_3 + R_4)_0 = -\frac{e^2}{2M^2}\left[1 + (1+\lambda)^2\right]\nu + \alpha_3 \nu + \cdots \qquad (8)$$

and take into account that for small ν $W = (M^2 + 2M\nu)^{1/2} \cong M + \nu$ then we can obtain the relation

$$\alpha_3 M - \alpha_1 = \frac{e^2}{M}\left[1/2 + (1+\lambda)^2\right] \qquad (9)$$

from the condition that there is no linear dependence on ν.

The crossing symmetry requirements lead to the fact that the quantity $\nu(T_2 - T_4)$ must be an even function of ν. The absence of the terms linear over ν leads to

$$\alpha_3 M = \frac{e^2}{M}\left[3/2 + (1+\lambda)^2\right]. \qquad (10)$$

From (8) - (10) we have

$$\left(R_1+R_2\right)_0 = -\frac{e^2}{M}\left(1-\frac{\nu}{M}\right) + O(\nu^2) \qquad (11)$$

$$\left(R_3+R_4\right)_0 = -\frac{e^2}{2M^2}\left(1-\frac{3\nu}{M}\right)\nu - \frac{e^2}{2M^2}(1+\lambda)^2\left(1-\frac{2\nu}{M}\right)\nu + O(\nu^3).$$

The functions T_1+T_3, T_5, $\nu(T_2+T_4)$ and T_6 must be even functions of ν. An analogous consideration leads to

$$\left(R_3-R_4\right)_0 = -\frac{e^2}{2M^2}\left[1-(1+\lambda)^2\right]\left(1-\frac{2\nu}{M}\right)\nu + O(\nu^3) \qquad (12)$$

whereas

$$\left(R_3+R_4+2R_5+2R_6\right)_0 = -\frac{e^2\lambda^2}{2M^2}\left(1-\frac{\nu}{M}\right)\nu + O(\nu^3) \qquad (13)$$

and

$$\left(R_5+R_6\right)_0 = \frac{e^2(1+\lambda)}{2M^2}\nu + \frac{e^2}{4M^3}\left[\lambda^2-3-2(1+\lambda)^2\right]\nu^2 + O(\nu^3). \qquad (13')$$

The function $(T_1-T_3)'_0$ is an even function of ν. Substituting into (4)

$$\left(R_3+R_4\right)'_0 = \alpha'_3 + \cdots \quad ; \quad \left(R_1+R_2\right)'_0 = \frac{\alpha'_1}{\nu} + \cdots$$

from the condition of the absence of the component proportional to ν^{-1} we get

$$2M\alpha'_3 - 2\alpha'_1 = \frac{e^2}{M}\left[1-2(1+\lambda)^2\right].$$

An analogous condition on the even function $\nu(T_2-T_4)'_0$ leads to

$$2M\alpha'_3 = -\frac{e^2}{M}\left[3+2(1+\lambda)^2\right].$$

Then

$$\alpha'_1 = -2\frac{e^2}{M^2}.$$

Hence

$$\left(R_1+R_2\right)'_0 = -2\frac{e^2}{M^2}\frac{1}{\nu} + O(1) \; ; \; \left(R_3+R_4\right)'_0 = -\frac{e^2}{2M^3}\left[3+2(1+\lambda)^2\right] + O(\nu). \qquad (14)$$

The conditions for the first order poles to disappear in the even functions $(T_1+T_3)'_0$ and $\nu(T_2+T_4)_0$ lead to

$$(R_1-R_2)_0 = -\frac{e^2}{M}\left(1-\frac{3\nu}{M}\right)+O(\nu^2)$$

$$(R_3-R_4)'_0 = \frac{e^2}{2M^3}\left[-3+2(1+\lambda)^2\right]+O(\nu). \qquad (15)$$

Analogous conditions for the functions $T_5)'_0$ and $T_6)'_0$ require that

$$(R_5-R_6)_0 = -\frac{e^2(1+\lambda)}{2M^2}\nu+\frac{e^2}{4M^3}\left[-2+8(1+\lambda)+(1+\lambda)^2\right]\nu^2+O(\nu^3) \qquad (16)$$

$$(R_3+R_4+2R_5+2R_6)'_0 = -\frac{e^2}{2M^3}(2\lambda^2-2\lambda-1)$$

Let us emphasize that the low energy dependence we have got is valid for the amplitudes in the c.m.s. The results obtained may prove useful in the dispersion analysis of γ quanta scattering by nucleons

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Received by Publishing Department on February 23, 1961.