ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

Лаборатория ядерных проблем

Лаборатория теоретической физики

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D-681

ON THE ROLE OF ONE-PION POLE DIAGRAM IN γ-RAYS SCATTERING BY PROTONS

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Submitted to JETP

It is shown that under right choice of the sign of the pole $\gamma - n$ scattering diagram due to a neutral point decay its contribution of the cross section of γ rays scattering by protons decreases considerably. To obtain the information on the neutral pion lifetime it is necessary to improve essentially the experimental accuracy.

1. Introduction

Some years ago Low drew his attention/1/ to the existence of the pole diagram which is due to the neutral pion decay in the amplitude for elastic scattering of γ quanta by protons. From the point of view of double dispersion relations for γ-N scattering the account of this diagram is equivalent to the consideration of the nearest singularity by Q^2. There are some interesting ideas concerning double dispersion relations for γ-N scattering in the paper by Nelipa and Filkov. Zhizhin/2/ considered the contribution of this amplitude in states with definite value of angular momenta. Recently, Hyman/3/and other authors, especially Jacob and J.Mathews/4/ pointed out, that the addition of the one-pion pole amplitude improves considerably the agreement between the theoretical and experimental results in the γ - quanta energy renge of 100-250 MeV. This problem is being treated in detail in the recently published paper by Bernardini, Yamagata et al/5/*.

It is well-known that the analysis which makes use of the dispersion relations/6,7/, leads to the values for the scattering cross sections exceeding the experimental values in this energy interval.

In this paper we wish to draw the attention of the readers to the sign of the pole amplitude which is of great importance since the interference terms play the main role. It follows from the results of Goldberger and Treiman/8/ for neutral pion decay and from the dispersion relations for the forward scattering we used earlier/7/, that (a relative) sign of the pole diagram differs from that used by Jacob and J.Mathews. Thus, the addition of the pole diagram does not improve the agreement between the theoretical and experimental results the difference between which has to be accounted for in another manner.

2. Scattering Amplitude

Let us denote the vectors of nucleon momenta in the initial and final states by p and p' respectively, and by q and q' the same for γ quanta. Since they satisfy the conservation law

$$q + p = q' + p' \tag{1}$$

it is convenient to introduce the following four orthogonal vectors

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* The authors of this paper are grateful to Nelipa, Filkov and to dr. Yamagata for sending the results of their papers prior the publication.

$$\mathcal{K}=\frac{1}{2}(q+q') \quad , \quad Q=\frac{1}{2}(q'-q)=\frac{1}{2}(p-p') \; ;$$

$$\mathcal{P}'=\mathcal{P}-\frac{(\mathcal{P}\cdot\mathcal{K})}{\mathcal{K}^2}\mathcal{K} \quad ; \quad N_\mu = i\,\varepsilon_{\mu\nu\sigma\rho}\,\mathcal{P}'_\nu\,\mathcal{K}_\sigma\,Q_\rho \tag{2}$$

where

$$\mathcal{P}=\frac{1}{2}(p+p').$$

It is possible to construct two independent scalar of these four vectors

$$Q^2 \quad \text{and} \quad M\nu=-(\mathcal{P}\cdot\mathcal{K}). \tag{3}$$

The lengths of the vectors introduced in (2) are connected with Q^2 and $M\nu$ by the following relations

$$\mathcal{K}^2=-Q^2 \quad ; \quad \mathcal{P}^2=-Q^2-M^2$$

$$\mathcal{P}'^2=\mathcal{P}^2-\frac{(\mathcal{P}\mathcal{K})^2}{\mathcal{K}^2}=\frac{1}{Q^2}\left[M^2\nu^2-Q^2(Q^2+M^2)\right] \tag{4}$$

$$N^2=-\mathcal{P}'^2\mathcal{K}^2Q^2=Q^2\left[M^2\nu^2-Q^2(Q^2+M^2)\right].$$

The S matrix element for γN scattering may be put as

$$\langle p'q'|S-1|pq\rangle=\frac{i}{2\pi}\,\delta\,(p'+q'-p-q)\,\frac{M}{(p_0p_0'q_0q_0')^{1/2}}\,N \tag{5}$$

where

$$N=\bar{u}\;\bar{u}(p')\,e'_\mu\,N_{\mu\nu}\,e_\nu\,u(p)=$$

$$=2\pi^2 i\left(\frac{p_0p_0'}{M^2}\right)^{1/2}\int d^4z\,e^{-i\mathcal{K}z}\langle p'|T\left(e'j\left(\tfrac{z}{2}\right)\right)\left(e\,j\left(-\tfrac{z}{2}\right)\right)|p\rangle. \tag{6}$$

In the c.m.s. the differential cross section is given by

$$\frac{d\sigma}{do}=\sum_{\text{spin}}\left|\frac{M}{W}N\right|^2 \tag{7}$$

where $W^2=-(\mathcal{P}+\mathcal{K})^2$ is the square of the total energy in the c.m.s.

The scattering amplitude N may be written as a sum of six invariant functions

$$e'_\mu N_{\mu\nu} e_\nu = \frac{(e'\mathcal{P}')(e\mathcal{P}')}{\mathcal{P}'^2}\left[T_1 + i\hat{K}T_2\right] + \frac{(e'N)(eN)}{N^2}\left[T_3 + i\hat{K}T_4\right]$$

$$- \frac{(e'\mathcal{P}')(eN) - (e'N)(e\mathcal{P}')}{(\mathcal{P}'^2 N^2)^{1/2}} i\gamma_5 T_5 + \qquad (8)$$

$$+ \frac{(e'\mathcal{P}')(eN) + (e'N)(e\mathcal{P}')}{(\mathcal{P}'^2 N^2)^{1/2}} \gamma_5 \hat{K} T_6 ; \qquad (\hat{K} = \gamma_\mu K_\mu)$$

In some cases it is also convenient to represent the amplitude as an operator in the spin space in terms of six noninvariant functions R_i

$$\frac{M}{W} e'_\mu N_{\mu\nu} e_\nu = R_1(\vec{e}'\vec{e}) + R_2(\vec{s}'\vec{s}) + iR_3(\vec{\sigma}[\vec{e}'\vec{e}]) + iR_4(\vec{\sigma}[\vec{s}'\vec{s}])$$

$$+ iR_5\left[(\vec{\sigma}\vec{k})(\vec{s}'\vec{e}) - (\vec{\sigma}\cdot\vec{k}')(\vec{s}\vec{e}')\right] \qquad (9)$$

$$+ iR_6\left[(\vec{\sigma}\cdot\vec{k}')(\vec{s}'\vec{e}) - (\vec{\sigma}\cdot\vec{k})(\vec{s}\vec{e}')\right]$$

where $\vec{s} = [\vec{k}\vec{e}]$, $\vec{s}' = [\vec{k}'\vec{e}']$ while $(\vec{e}, \vec{k})$ and $(\vec{e}', \vec{k}')$ are unit vectors of the photon polarization and momentum before and after scattering, respectively.

3. The Matrix Element of the Neutral Pion Decay

The S matrix for the neutral pion decay is of the form

$$\langle q'q|S|q_\pi\rangle = \frac{i}{(2\pi)^{3/2}}\frac{1}{\sqrt{2\omega_K}}(2\pi)^4\delta(q_\pi - q - q')\langle q'q|J(0)|0\rangle \qquad (10)$$

where q and q' are the photon momenta; q_π - is the four-momentum of a pion; $J(x)$ is the pion field current which is determined to be

$$J(x) = i\frac{\delta S}{\delta\varphi(x)}S^+ = ig_0\bar{\psi}(x)\gamma_5\tau_3\psi(x) \qquad (11)$$

where $\varphi(x)$ is the meson field operator; $\psi(x)$ is the nucleon field operator, and g_0 is the nonrenormalized constant of pion-nucleon interaction. The Heisenberg equation for the meson field may be written as

$$(-\Box^2+m_\pi^2)\,\varphi(x)=J(x). \tag{12}$$

In Goldberger and Treiman's notations/8/

$$M=(2\pi)^3\sqrt{4qq'}\,\langle q'q|J|0\rangle=-i\,\varepsilon_{\mu\nu\sigma\lambda}\,e'_\mu e_\nu q_\sigma q'_\lambda\,F\left[(q+q')^2\right] \tag{13}$$

where $F(q^2)$ is a form-factor. $F(-m_\pi^2)$ enters the expression for the S matrix..

The probability of the neutral pion decay is

$$w=\sum_{q'qe'e}\frac{|\langle q'q|S|q_\pi\rangle|^2}{\frac{1}{(2\pi)^3}V\cdot T}=\frac{1}{(2\pi)^2}\delta(q_\pi-q'-q)\,d^3q\,d^3q'$$
$$\frac{1}{8q_{\pi0}q'_0q_0}\,q^2q'^2\sum_{e'e}\left[(\vec{e}\vec{s}')+(\vec{e}'\vec{s})\right]^2|F|^2. \tag{14}$$

Summing over e and e' and integrating with respect to the angles, we get in the rest system of a pion

$$w=\frac{m_\pi^3}{64\pi}|F|^2. \tag{15}$$

The pion lifetime τ is equal to

$$\tau=\frac{64\pi}{m_\pi^3|F|^2}. \tag{16}$$

By using the dispersion relations technique, Goldberger and Treiman have shown that

$$F(0)=-\frac{ge^2}{4\pi^2m_\pi}(1+\mu_p)\frac{I_0+\rho I_1}{1+\frac{g^2}{4\pi}I_1}, \tag{17}$$

where

$$\rho=\frac{2\mu_p-(\mu_p^2-\mu_n^2)}{1+\mu_p} \tag{18}$$

and μ_p and μ_n are anomalous magnetic moments of a proton and a neutron, respectively; I_0 and I_1 are positive integrals. It follows from (17) that

$$F(0)g < 0. \tag{19}$$

This sign is important for what follows further.

4. One-Pion Diagram for γ Quanta Scattering by Protons

The S matrix element of the pole diagram is equal to

$$\langle p'q'|S-1|pq\rangle = i\,(2\pi)^4\,\delta(p'+q'-p-q)\,\langle p'|J_N(0)|p\rangle \cdot$$

$$\cdot\frac{1}{(p'-p)^2+m_\pi^2}\,\langle q'|J_\pi(0)|q\rangle = ig\,\frac{i}{(2\pi)^3}\,\bar{u}(p')\,\gamma_5\,u(p) \tag{21}$$

$$\delta(p'+q'-p-q)\,(2\pi)^4\,\frac{1}{(p'-p)^2+m_\pi^2}\,\langle q'|J_\pi(0)|q\rangle.$$

One may show that

$$\langle q'|J_\pi(0)|q\rangle = \frac{1}{(2\pi)^3}\,\frac{1}{\sqrt{4q_0q_0'}}\,(-i)\,\varepsilon_{\mu\nu\sigma\lambda}\,e'_\mu\,e_\nu\,(-q)_\sigma\,q'_\lambda\,F\left[(q'-q)^2\right] =$$

$$= \frac{1}{(2\pi)^3}\,\frac{1}{\sqrt{4q_0q_0'}}\,(-i)\,\varepsilon_{\mu\nu\sigma\lambda}\,e'_\mu\,e_\nu\,q'_\sigma\,q_\lambda\,F. \tag{22}$$

As far as the matrix element $\langle q'|J_\pi(0)|q\rangle$ is taken in the pole at $(q'-q)^2 = -m_\pi^2$ then it is the same value of F which is encountered in the neutral pion decay enters (21).

Substituting (22) into (21) and reducing the spinors in the c.m.s., we get

$$\langle p'q'|S-1|pq\rangle = -\frac{gF}{(2\pi)^6}\,(2\pi)^4\,\delta(p'+q'-p-q)\,\frac{1}{(4q_0q_0')^{1/2}}\,\frac{M}{(p_0p_0')^{1/2}}$$

$$\frac{q}{2M}\,(\vec{\sigma},\vec{k}-\vec{k}')\,q\,q'\left[(\vec{e}\,\vec{s}\,') + (\vec{e}\,'\vec{s})\right] = \frac{igF}{(2\pi)^6}\,\frac{M}{(4q_0q_0'p_0p_0')^{1/2}}\,(2\pi)^4\,\frac{i}{2M} \tag{23}$$

$$\delta(p'+q'-p-q)\left[(\vec{\sigma}\cdot\vec{k})(\vec{e}\,\vec{s}\,') - (\vec{\sigma}\,\vec{k}')(\vec{e}\,'\vec{s}) - (\vec{\sigma}\,\vec{k}')(\vec{e}\,\vec{s}\,') + (\vec{\sigma}\,\vec{k})(\vec{e}\,'\vec{s})\right]$$

Comparing (23) with (9), we have for the contribution of the pole diagram

$$R_{1p} = R_{2p} = R_{3p} = R_{4p} = 0$$
$$R_{5p} = -R_{6p} = \frac{gF}{8\pi W} \frac{q^3}{(p-p')^2 + m_\pi^2} \tag{24}$$

Hence, we arrive at a conclusion that the contribution to the amplitude of the pole diagram from the exchange and decay of the pseudo-scalar neutral pion reduces to

$$R_{5p} - R_{6p} = \frac{gF}{8\pi} \frac{m_\pi}{W} \frac{q}{m_\pi} \frac{2q^2}{2q^2(1-\cos\theta) + m_\pi^2} =$$
$$= \frac{gF}{8\pi} \frac{q}{W} \frac{1}{1 + \frac{m_\pi^2}{2q^2} - \cos\theta} . \tag{25}$$

It should be emphasized that in virtue of (19)

$$R_{5p} - R_{6p} < 0$$

if we assume that $F(0)$ and $F(-m_\pi^2)$ are not very different.

The expression for the cross section (formula (16) in /5/) has the pole term in the combinations

$$\frac{1}{2} |R_5 - R_6|^2 (1-\cos\theta)^3 - \mathrm{Re}\,(R_3 - R_4)^* (R_5 - R_6)\,(1-\cos\theta)^2 . \tag{26}$$

The contribution of one pole diagram is of the form

$$I_0^p(\theta) = \frac{1}{2} |R_{5p} - R_{6p}|^2 (1-\cos\theta)^3 =$$
$$= 2 \frac{g^2}{4\pi} \frac{F^2}{64\pi} \left(\frac{q}{W}\right)^2 \frac{(1-\cos\theta)^3}{\left(1 + \frac{m_\pi^2}{2q^2} - \cos\theta\right)^2} =$$
$$= \frac{2}{m_\pi \tau} \left(\frac{q}{W}\right)^2 \frac{g^2}{4\pi} \left(\frac{1}{m_\pi}\right)^2 \frac{(1-\cos\theta)^3}{\left(1 + \frac{m_\pi^2}{2q^2} - \cos\theta\right)^2} ; \tag{27}$$

what coincides with Jacob and J. Mathews's results.

We can expect the decrease of the cross section at 90° because of adding the pole term only when the second term in (26) is negative.

As R_4 is great and negative due to a large anomalous magnetic moment of a proton, $Re\,(R_3 - R_4)$, is a positive quantity in the energy range under consideration. So, the second term in (26) is positive, if $R_{5p} - R_{6p} < 0$. Hence, under the assumption that Goldberger and Treiman's analysis is correct the pole diagram rather increase than decrease the theoretical value of the cross section.

If one makes use of the results of our analysis [7] $Re\,(R_5 - R_6)$, would turn out to be determined, apart from the limiting theorem by the photoproduction amplitudes E_2 and M_3 . Since the (isotropic) part of the pole amplitude contribution is automatically taken into account, it is necessary to add to the amplitude we have obtained earlier not the whole expression (25), but only the contribution of (25) to higher states, i.e. the difference

$$(R_5 - R_6)_p - \frac{1}{2}\int (R_5 - R_6)_p \sin\theta \, d\theta .$$

As a result of such a procedure which is necessary in order not to violate the unitarity of the S matrix, (at $\theta = 90^\circ$) the quantity y_0^{-1} $(y_0 = 1 + m_\pi^2/2q^2)$ is replaced by

$$y_0^{-1} - \frac{1}{2}\ln\left|\frac{y_0 + 1}{y_0 - 1}\right|$$

what at $q^2 = m_\pi^2$ $(y_0 = 3/2)$ leads to the substitution of 2/3 by - 0.14. Thus, the conribu - tion of the amplitude decreases as much as five times, and the sign of the contribution changes. So, a higher accuracy is necessary for the connection between the neutral pion decay amplitude and the amplitude of γ rays scattering on protons to display. It was shown recently that the lifetime of a neutral pion is [9] $(2 \pm 0.4) \cdot 10^{-16}$ sec, what decreases the magnitude of the pole diagram contribution as well.

The ambiguity in the analysis of the photoproduction cannot affect the conclusion on the sign of the interference term in (25) since this sign is determined by a low energi theorem for γN amplitude. The scattering amplitude at low frequencies which was first obtained by Low and Gell-Mann, and Goldberger [10] was considered anew in the Appendix where it was obtained as a contribution of one-nucleon terms (see [6]).

Note, in particular, that

$$T_5^{\,0} = \frac{e^2}{M}(1+\lambda)\,\frac{M^2 Q^2}{Q^4 - M^2\nu^2} \qquad (28)$$

Let us give one more, less strict, but simpler prof, that the sign of the pole diagram is determined correctly*.

The matrix element $\langle q' | J_\pi(0) | q \rangle$ may be put as

$$\langle q' | J_\pi(0) | q \rangle = i \varepsilon_{\mu\nu\sigma\lambda} e'_\mu e_\nu q_\sigma q'_\lambda \frac{1}{(2\pi)^3} F[(q-q')^2] =$$
$$= -\frac{2Q^2}{(2\pi)^3} F \frac{(e'\mathcal{P}')(eN) - (e'N)(e\mathcal{P}')}{(\mathcal{P}'^2 N^2)^{1/2}} \qquad (29)$$

so that

$$\langle p'q' | S-1 | pq \rangle = i(2\pi)^{-2} g\, \delta(p'+q'-p-q)\, i \bar{u}(p') \gamma_5 u(p)$$
$$\frac{2Q^2 F}{4Q^2+m_\pi^2} \frac{(e'\mathcal{P}')(eN) - (e'N)(e\mathcal{P}')}{(\mathcal{P}'^2 N^2)^{1/2}} \qquad (30)$$

hence

$$T_{5p} = \frac{gF}{2\pi} 2 \cdot \frac{Q^2}{4Q^2+m_\pi^2} . \qquad (31)$$

Now let us introduce the function

$$f(\nu, Q^2) = \frac{T_5(\nu, Q^2)}{Q^2} . \qquad (32)$$

If $f(\nu, Q^2)$ is considered as an analytical function of Q^2 for the fixed ν, then it follows from the Cauchy theorem and from (31)

$$f(\nu, Q^2) = \frac{gF}{\pi} \frac{1}{4Q^2+m_\pi^2} + J_Q \qquad (33)$$

where J_Q is the dispersion integral, whose lower limit is found to be $4m_\pi^2$. In the region $Q^2 \ll 4m_\pi^2$ the integral in (33) is small and $f(\nu, Q^2)$ may be approximated by the expressior

$$f(\nu, Q^2) \cong \frac{gF}{\pi} \frac{1}{4Q^2+m_\pi^2} . \qquad (34)$$

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* An analogous approaoh was used by Bernstein, Fubini, Gell-Mann, Thirring and one of the authors/11/ to obtain the Goldberger-Treiman's relation.

On the other hand, $f(\nu, Q^2)$ is also an analytical function of ν for the fixed Q^2. According to the Cauchy theorem and taking into account (28)

$$f(\nu, Q^2) = \frac{e^2(1+\lambda)}{M} \frac{M^2}{Q^4 - M^2\nu^2} + J_\nu \tag{35}$$

where J_ν is another dispersion integral. In the region $2\nu \lesssim m_\pi$ the pole term will prevail, and roughly

$$f(\nu, Q^2) \simeq \frac{e^2(1+\lambda)}{M} \frac{M^2}{Q^4 - M^2\nu^2}. \tag{36}$$

Evidently, (34) is not correct in the vicinity of $M^2\nu^2 \simeq Q^2$, while (36) does not take place at $4Q^2 = -m_\pi^2$. Yet, it is possible that in a certain region ν and Q^2 (34) and (36) are correct. Equating these expressions at $2\nu = m_\pi$ and $Q^2 = 0$, we get

$$F = -\frac{e^2(1+\lambda)}{g M} 4\pi , \tag{37}$$

what is very close to Goldberger and Treiman's formula obtained in quite another manner.

Indeed, from (17) at

$$\frac{g^2}{4\pi} I_1 \gg 1$$

we get

$$F = -4\pi \frac{e^2(1+\lambda)}{g} \frac{I_0 + \rho I_1}{I_1}$$

what coincides with (37) up to a numerical factor.

There are two different choices of the general phase for the γN scattering amplitude in the literature. According to one of them,, the Tompson limit is equal to $+e^2/M$, according to another one - it is $-e^2/M$. The error in the papers published is that the choice of the general phase factor of the one-pion amplitude does not correspond to the choice of the sign of the rest of the γ-n amplitude.

A straightforward comparison of the amplitude used by Jacob and Mathews, with (9) shows that the functions f_i introduced in /4/ are connected with R_i by

$$-f_1 = R_1 + R_2 \cos\theta$$

$$f_2 = R_2$$

$$f_3 = R_3 + R_4 \cos\theta + (R_5 + R_6)(1+\cos\theta) - (R_5 - R_6)(1-\cos\theta)$$

$$f_4 = R_4$$

$$f_5 = R_4 + R_5$$

$$f_6 = R_6 .$$

It is also clear from here, that the (relative) sign of the pole term used in /4/ is different from that proved in this paper.

Appendix

One-Nucleon Terms in Dispersion Relations

Having taken into account, that

$$T\left(e'j\left(\tfrac{z}{2}\right) e\cdot j\left(-\tfrac{z}{2}\right)\right) = \theta(z_0)\left[e'\cdot j\left(\tfrac{z}{2}\right) e\cdot j\left(-\tfrac{z}{2}\right)\right] + \left(e' j\left(\tfrac{z}{2}\right)\right)\left(e\cdot j\left(-\tfrac{z}{2}\right)\right) \tag{A.1}$$

we determine the retarded and advanced amplitudes

$$N^{ret/adv} = \pm 2\pi^2 i \left(\frac{p_0 p_0'}{M^2}\right)^{1/2} \int d^4 z\, e^{\pm iKz} \langle p' | \theta(\pm z_0)\left[e' j\left(\tfrac{z}{2}\right) e\cdot j\left(-\tfrac{z}{2}\right)\right] | p\rangle . \tag{A.2}$$

The matrix element of the current has the form

It is possible to express A° in terms of the basic invariants

$$A^{\circ} = \frac{(e'\mathcal{P}')(e\mathcal{P}')}{\mathcal{P}'^2} A_1^{\circ} + \frac{(e'N)(eN)}{N^2} A_2^{\circ} + \qquad (A.8)$$

$$+ \frac{(e'\mathcal{P}')(eN) - (e'N)(e\mathcal{P}')}{(\mathcal{P}'^2 N^2)^{1/2}} A_3^{\circ} + \frac{(e'\mathcal{P}')(eN) + (e'N)(e\mathcal{P}')}{(\mathcal{P}'^2 N^2)^{1/2}} A_4^{\circ}.$$

Comparing (A.7) and (A.8), we have

$$A_1^{\circ} \mathcal{P}'^2 = \frac{\varepsilon^2}{4} \delta(P_n^2 + M^2)\, \bar{u}(p') \left[(1+\lambda)\hat{\mathcal{P}}' + \frac{i\lambda}{M} \mathcal{P}'\cdot p' \right] \qquad (A.9)$$

$$\left[-i(\hat{\mathcal{P}} - \hat{\mathcal{K}}) + M \right] \left[(1+\lambda)\hat{\mathcal{P}}' + \frac{i\lambda}{M} \mathcal{P}'\cdot p \right] u(p).$$

It is easy to verify that

$$\mathcal{P}'\cdot p' = \mathcal{P}'(\mathcal{P} - Q) = \mathcal{P}'\cdot p = \mathcal{P}'^2 \quad ;$$

$$\bar{u}(p') \left[\hat{\mathcal{P}}' \left(-i(\hat{\mathcal{P}} - \hat{\mathcal{K}}) + M \right) \hat{\mathcal{P}}' \right] u(p) = \bar{u}(p') \Big\{ \left[i(\hat{\mathcal{P}} - \hat{\mathcal{K}}) + \right. \qquad (A.10)$$

$$\left. + M \right] \mathcal{P}'^2 - 2i\, \mathcal{P}'\cdot p\, \mathcal{P}' \Big\} u(p) = \bar{u}(p') \Big\{ -i\hat{\mathcal{K}} \mathcal{P}'^2 + 2\mathcal{P}'^2 M + 2i \frac{\mathcal{P}\cdot\mathcal{K}}{\mathcal{K}^2} \hat{\mathcal{K}} \mathcal{P}'^2 \Big\} u(p); \qquad (A.11)$$

$$\bar{u}(p') \hat{\mathcal{P}}' \left[-i(\hat{\mathcal{P}} - \hat{\mathcal{K}}) + M \right] u(p) = \bar{u}(p') \left[\left(\hat{\mathcal{P}} - \frac{\mathcal{P}\cdot\mathcal{K}}{\mathcal{K}^2} \hat{\mathcal{K}} \right) \left(-i(\hat{\mathcal{P}} - \hat{\mathcal{K}}) + \right. \right.$$

$$\left. + M \right) \Big] u(p) = \bar{u}(p') \left[M \left(iM - \frac{\mathcal{P}\cdot\mathcal{K}}{\mathcal{K}^2} \hat{\mathcal{K}} \right) - i(\mathcal{P}^2 + \mathcal{P}\cdot\mathcal{K}) + i \left(\hat{\mathcal{P}}\hat{\mathcal{K}} + \right. \right. \qquad (A.12)$$

$$\left. \left. + \frac{\mathcal{P}\cdot\mathcal{K}}{\mathcal{K}^2} \hat{\mathcal{K}}\hat{\mathcal{P}} \right) \right] u(p) \quad ;$$

$$\bar{u}(p')\left[\left(-i(\hat{\mathcal{P}}-\hat{\mathcal{K}})+M\right)\hat{\mathcal{P}}'\right]u(p)=\bar{u}(p')\left[M\left(iM-\frac{\mathcal{P}\cdot\mathcal{K}}{\mathcal{K}^2}\hat{\mathcal{K}}\right)-\right.$$

$$\left.-i(\mathcal{P}^2+\mathcal{P}\cdot\mathcal{K})+i\left(\hat{\mathcal{K}}\hat{\mathcal{P}}+\frac{\mathcal{P}\cdot\mathcal{K}}{\mathcal{K}^2}\hat{\mathcal{P}}\hat{\mathcal{K}}\right)\right]u(p)\quad; \tag{A.13}$$

$$\bar{u}(p')\left[-i(\hat{\mathcal{P}}-\hat{\mathcal{K}})+M\right]u(p)=\bar{u}(p')\left[i\hat{\mathcal{K}}+2M\right]u(p). \tag{A.14}$$

Using (A.10)-(A.14) and noting, that in the pole

$$(\mathcal{P}-\mathcal{K})^2=\mathcal{P}^2+\mathcal{K}^2-2\mathcal{P}\cdot\mathcal{K}=2\mathcal{K}^2-2\mathcal{P}\cdot\mathcal{K}-M^2=-M^2$$

or that

$$\mathcal{K}^2=\mathcal{P}\cdot\mathcal{K} \tag{A.15}$$

we get

$$A_1^o=\frac{\mathcal{E}^2}{4}\delta(2\mathcal{K}^2-2\mathcal{P}\mathcal{K})\,\bar{u}(p')\,(2M+i\hat{\mathcal{K}})\,u(p)=$$

$$=\frac{\mathcal{E}^2}{8M}\delta(\nu-Q^2/M)\,\bar{u}(p')\,(2M+i\hat{\mathcal{K}})\,u(p). \tag{A.16}$$

Anologously

$$A_2^o N^2=\frac{\mathcal{E}^2(1+\lambda)^2}{8M}\delta\left(\nu-\frac{Q^2}{M}\right)\bar{u}(p')\hat{N}\left[-i(\hat{\mathcal{P}}-\hat{\mathcal{K}})+M\right]\hat{N}u(p)=$$

$$=\frac{\mathcal{E}^2N^2(1+\lambda)^2}{8M}\delta(\nu-Q^2/M)\,\bar{u}(p')\left[-i\hat{\mathcal{K}}\right]u(p)\;;$$

$$A_2^o=-\frac{\mathcal{E}^2}{8M}\delta(\nu-Q^2/M)\,(1+\lambda)^2\,\bar{u}(p')\,i\hat{\mathcal{K}}\,u(p)\,; \tag{A.17}$$

$$(A_3^\circ + A_4^\circ)(\mathcal{P}'^2 N^2)^{1/2} = \frac{\varepsilon^2(1+\lambda)}{8M}\,\delta(\nu - \tfrac{Q^2}{M})\,\bar{u}(p')\Big[\hat{N}\,\big(-i\,(\hat{\mathcal{P}} -$$

$$-\hat{\mathcal{K}}) + M\big)\Big(\hat{\mathcal{P}}' + \frac{i\lambda}{M}\mathcal{P}'^2\Big)\Big]\,u(p) =$$

$$= \frac{\varepsilon^2(1+\lambda)}{8M}\,\delta(\nu - Q^2/M)\,\bar{u}(p')\Big[\frac{\mathcal{P}'^2}{M}\,\hat{N}\,(-iM + \hat{\mathcal{K}})\Big]\,u(p)\;;$$

(A.18)

$$(A_4^\circ - A_3^\circ)(\mathcal{P}'^2 N^2)^{1/2} = \frac{\varepsilon^2(1+\lambda)}{8M}\,\delta(\nu - Q^2/M)\,\bar{u}(p')\Big[\frac{\mathcal{P}'^2}{M}\,(\hat{\mathcal{K}} - iM)\,\hat{N}\Big]\,u(p)\;;$$

(A.19)

From (A.18) and (A.19)

$$A_4^\circ\;(\mathcal{P}'^2 N^2)^{1/2} = \frac{\varepsilon^2(1+\lambda)}{8M}\,\delta(\nu - Q^2/M)\,(-i\,\mathcal{P}'^2)\,\bar{u}(p')\,\hat{N}\,u(p) \qquad (A.20)$$

$$A_3^\circ\;(\mathcal{P}'^2 N^2)^{1/2} = \frac{\varepsilon^2(1+\lambda)}{8M}\,\delta(\nu - Q^2/M)\,\frac{\mathcal{P}'^2}{M}\,\bar{u}(p')\,\hat{N}\,\hat{\mathcal{K}}\,u(p).$$

One can show that

$$\bar{u}(p')\,\hat{N}\,\hat{\mathcal{K}}\,u(p) = (\mathcal{P}'^2 N^2)^{1/2}\; i\,\bar{u}(p')\,\gamma_5\,u(p)$$

$$i\,\bar{u}(p')\,\hat{N}\,u(p) = \mathcal{K}^2\,\bar{u}(p')\,\gamma_5\,\hat{\mathcal{K}}\,u(p).$$

If now we take into account that owing to (4) $(\mathcal{P}'^2 N^2)^{1/2} = \mathcal{P}'^2 Q^2$
then from (A.20) we obtain that

$$A_3^\circ = -\frac{\varepsilon^2(1+\lambda)}{8M}\,\delta(\nu - Q^2/M)\;i\,\bar{u}(p')\,\gamma_5\,u(p)$$

$$A_4^\circ = \frac{\varepsilon^2(1+\lambda)}{8M}\,\delta(\nu - Q^2/M)\;\bar{u}(p')\,\gamma_5\,\hat{\mathcal{K}}\,u(p). \qquad (A.21)$$

Finally, from (A.16), (A.17), and (A.21) we get

$$T_1^\circ = \frac{\varepsilon^2}{2\pi M}\,\frac{M^2 Q^2}{Q^4 - M^2\nu^2}\ ; \quad T_2^\circ = \frac{\varepsilon^2}{4\pi M}\,\frac{M^2\nu}{Q^4 - M^2\nu^2}\ ;$$

$$T_3^\circ = 0\ ; \quad T_4^\circ = -\frac{\varepsilon^2(1+\lambda)^2}{4\pi M}\,\frac{M^2\nu}{Q^4 - M^2\nu^2}\ ; \tag{A.22}$$

$$T_5^\circ = M T_6^\circ = \frac{\varepsilon^2(1+\lambda)}{4\pi M}\,\frac{M^2 Q^2}{Q^4 - M^2\nu^2}\ . \qquad \left(\frac{\varepsilon^2}{4\pi} = e^2 = \frac{1}{137}\right)$$

What coincides with the results obtained earlier with a correct sign. In making all the calculations of one-nucleon terms it was assumed that in the electromagnetic interactions the parity conservation holds. These results also remain valid for the CP - invariance only.

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Received by Publishing Department
on February 23, 1961