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DETERMINATION OF THE PION-NUCLEON INTERACTION CONSTANT BY THE DIFFERENTIAL CROSS SECTION OF ELASTIC PP-SCATTERING

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S u m m a r y

In order to determine f^2 the data of $\sigma_{pp}(\vartheta)$ have been treated at energies 147, 330 and 380 MeV. The obtained values of f^2 at $E =$ 147 and 380 MeV do not contradict the value $f^2 = 0.08$. No success is achieved in attempts to obtain satisfactory agreement of $\sigma_{pp}(\vartheta)$ with the value of $f^2 = 0.08$ at an energy 330 MeV.

The analysis of experimental data on neutron scattering by protons /1/ has shown that the differential cross section $\sigma_{np}(\vartheta)$ in a wide range of energies from 90 to 630 MeV does not contradict the value of the renormalized constant of pion-nucleon interaction $f^2 = 0.08$ within experimental errors. Owing to the fact that such an investigation of data on proton scattering by protons can provide interesting results an analogous treatment of $\sigma_{pp}(\vartheta)$ has been performed at the energies 147 /2/, 330 /3/ and 380 /4/ MeV. It should be noted, however, that the use of $\sigma_{pp}(\vartheta)$ for the determination of f^2 by the method employed in /1/ is rather impeded due to a necessity to take the Coulomb scattering into account. The introduction of corrections for the coulomb scattering considerably reduces the accuracy of experimental data especially in the region of small angles and besides is possible only after performing phase shift analysis. At present suitable data on phase shifts in elastic (pp) - scattering are known only for energies 310 /5/ and 150 /6/ MeV. This circumstance, properly speaking, determined the choice of data to be treated.

The account of Coulomb effects was performed by the method suggested in /5/. For this purpose $\bar{R}$- matrix was written in the following form:

$$\bar{R} = \bar{S} - 1 = \bar{S} - \bar{S}_c + \bar{S}_c - 1 = \bar{\alpha} + \bar{R}_c$$

$\bar{\alpha}$ is the matrix the elements of which are expressed by the total bar phase shifts δ_e and the phase shifts of pure Coulomb scattering Φ_e ; $\bar{R}_c$ is the matrix of Coulomb scattering.

The matrix elements α which determine the scattering amplitude are written as

$$\alpha_e = e^{2i\delta_e} - e^{2i\Phi_e} \approx (1+2i\Phi_e)(e^{2i\delta_e^N} - 1)$$

for singlet scattering; for triplet scattering

$$\alpha_{ej} = e^{2i\delta_{ej}} - e^{2i\Phi_e} \approx (1+2i\Phi_e)(e^{2i\delta_{ej}^N} - 1) \quad \text{for } \ell = j$$

$$\alpha_{j\pm1,j} = \cos 2\varepsilon_j \exp(2i\delta_{j\pm1,j}) - \exp(2i\Phi_{j\pm1}) \approx$$

$$\approx (1+2i\Phi_{j\pm1})(\cos 2\varepsilon_j \, e^{2i\delta^N_{j\pm1,j}} - 1)$$

$$\alpha^j = i \sin 2\varepsilon_j \exp[i(\delta_{j+1,j} + \delta_{j-1,j})] \approx$$
$$\approx i \sin 2\varepsilon_j [1 + i(\Phi_{j+1} + \Phi_{j-1})] \exp[i(\delta^N_{j+1,j} + \delta^N_{j-1,j})].$$

ε_j is the mixing parameter ; $\delta^N_e = \delta_e - \Phi_e$; $\delta^N_{j\pm1,j} = \delta_{j\pm1,j} - \Phi_{j\pm1}$; the values of δ are taken from/5,6/.

The obtained corrections for a Coulomb scattering (Fig. 1) were substracted from differential cross sections. The errors made were determined by the errors of phase shifts under the assumption that the latter are independent*. The absence of error matrix in papers/5,6/ does not allow to take into account phase shift correlations. This leads to some increase of the error in the determination of the Coulomb effect contribution to the scattering cross section. It may be shown that the weighted mean squares of the corridors of errors $\overline{\Delta^2_c}$ and $\overline{\Delta^2}$ obtained when correlations are and are not taken into account, satisfy the following relation:

$$\overline{\Delta^2} / \overline{\Delta^2_c} = K$$

where $K = \frac{1}{m} \sum K_i$ is the mean correlation factor/1/, m is the number of varied parameters.

For 330 MeV the contribution from Coulomb scattering was determined both by the first and the second sets of phase shifts/5/. It turned out that the results coincided.

On the grounds of the known analytical properties of (pp) -scattering amplitude as in/1/ for $\sigma_{np}(\vartheta)$ the nuclear part of the (pp) -scattering cross section was represented in the form:

$$\sigma_{pp}(\vartheta) = a_1 b^2 \left[\frac{1}{(x_o - x)^2} + \frac{1}{(x_o + x)^2}\right] + a_2 \left[\frac{1}{x_o - x} + \frac{1}{x_o + x}\right] + \sum_{n=0}^{n_{max}} A_n x^{2n} \quad (1)$$

where $b = \frac{\mu^2}{2k^2}$, μ is the pion mass, k is the particle momentum in the centre-of-mass system, $x_o = 1 + b$, $x = \cos\vartheta$; a and A_n are unknown coefficients. The coefficients a , A_n were determined by the least squares method.The normalization of $\sigma_{pp}(\vartheta)$ was chosen so that $a_1 = f^4$.

Since the accuracy of experimental data in the region of small angles was comparatively low, the number of sum terms in expression (1) was difficult to determine. For evaluation of n max in/1/ the following considerations were used. It is easy to show that contribution to the polarization $P(\vartheta)\sigma_{pp}(\vartheta)$ from the amplitude terms having a singuliarity with $x = \pm x_o$ is equal to zero.

* A quarter of the total interval given in/6/ was taken as an error of phase shifts at 147 MeV (a relative error is equal to ~20 %).

Indeed :

$$P(\vartheta)\,\sigma_{pp}(\vartheta) = \frac{\sqrt{2}}{4}\,\mathrm{Re}\left[i\,(M_{10}-M_{01})(M_{11}-M_{1-1}+M_{00})\right] \qquad (2)$$

$$\sigma_{pp}(\vartheta) = \frac{1}{4}|M_{ss}|^2 + \frac{1}{2}|M_{11}|^2 + \frac{1}{2}|M_{1-1}|^2 + \frac{1}{2}|M_{10}|^2 + \frac{1}{2}|M_{01}|^2 + \frac{1}{4}|M_{00}|^2. \qquad (3)$$

Making use of the Mandelstam representation as well as of the data of the paper /7/, one can separate in the nuclear part of each partial amplitude M_{ik} the terms having a singularity with $X = \pm X_0$ and write them in the following form:

$$M_{11} = f^2 m\left[\frac{1}{2}\,\frac{1-x}{x_0-x} + \frac{1}{2}\,\frac{1+x}{x_0+x}\right] + \sum_{\text{odd } K} \alpha^{11}_K P_K(x)$$

$$M_{00} = f^2 m\left[\frac{x}{x_0-x} - \frac{x}{x_0+x}\right] + \sum_{\text{odd } k} \alpha^{00}_K P_K(x)$$

$$M_{1-1} = f^2 m\left[\frac{1}{2}\,\frac{1+x}{x_0-x} + \frac{1}{2}\,\frac{1-x}{x_0+x}\right] + e^{-2i\varphi} \sum_{\text{odd } K} \alpha^{1-1}_K P^{(2)}_K(x)$$

$$M_{ss} = f^2 m\left[-\frac{1}{x_0-x} + \frac{1}{x_0+x}\right] + \sum_{\text{even } k} \alpha^{ss}_K P_K(x)$$

$$M_{10} = f^2 m\left[-\frac{\sqrt{2}}{2}\,\frac{\sin\vartheta}{x_0-x} + \frac{\sqrt{2}}{2}\,\frac{\sin\vartheta}{x_0+x}\right] + e^{-i\varphi} \sum_{\text{odd } K} \alpha^{10}_K P^{(1)}_K(x)$$

$$M_{01} = f^2 m\left[-\frac{\sqrt{2}}{2}\,\frac{\sin\vartheta}{x_0-x} + \frac{\sqrt{2}}{2}\,\frac{\sin\vartheta}{x_0+x}\right] + e^{i\varphi} \sum_{\text{odd } K} \alpha^{01}_K P^{(1)}_K(x)$$

m is the mass of nucleon.

Substituting M_{ik} to (2) and (3) we shall obtain

$$P(\vartheta)\,\sigma_{pp}(\vartheta) = \sin\vartheta \sum_{n=1}^{n_{max}} C_n\, X^{2n-1} \qquad (4)$$

$$\sigma_{pp}(\vartheta) = f^4 b^2 \left[\frac{1}{(x_0-x)^2} + \frac{1}{(x_0+x)^2}\right] + a_2 \left[\frac{1}{x_0-x} + \frac{1}{x_0+x}\right] + \sum_{n=0}^{n_{max}} A_n x^{2n} \qquad (5)$$

Normalization is the same as in (1). Thus, the terms having singuliarities with $x = \pm x_0$ do not contribute into $P(\vartheta)\,\sigma_{pp}(\vartheta)$ and from the comparison of (4), (5) it is clear that the maximum degrees x in $\sigma_{pp}(\vartheta)$ and $P(\vartheta)\,\sigma_{pp}(\vartheta)$ differ by a unity*. Consequently, knowing the angular dependence of polarization one can state orbital moments beginning from which the pole term of one-pion graph gives the main contribution to the scattering cross section. The data on the angular dependence of $P(\vartheta)\,\sigma_{pp}(\vartheta)$ given in /2/ show that at 147 MeV n_{max} in expression (1) for $\sigma_{pp}(\vartheta)$, apparently, does not exceed two. With increasing energy up to 315 MeV the $^3P_2 - {}^3F_2$ interference /8/ influences more and more on the character of $P(\vartheta)\,\sigma_{pp}(\vartheta)$ and, consequently, n_{max} is equal to three in this case. It is worthy of notice, however, that the polarization $P(\vartheta)\,\sigma_{pp}(\vartheta)$ is determined by the cross products of the amplitudes M_{ik} so that small contributions of waves with large ℓ can become more apparent in the angular distribution $P(\vartheta)\,\sigma_{pp}(\vartheta)$ than in $\sigma_{pp}(\vartheta)$.

Results

The results of the treatment are presented in Table 1. The coefficients a_1 obtained at 380 and 147 MeV provide the values 0.066 ± 0.014 and 0.070 ± 0.015 for f^2 with $\chi^2 = \frac{\chi^2}{\bar{\chi}^2} = 0.6$ and 1.6; $n_{max} = 1$ and 0, respectively. These values are well consistent with the results obtained when treating $\sigma_{np}(\vartheta)$ /1/ and do not contradict $f^2 = 0.08$. The increase of n_{max} by a unity changes a_1 slightly in both cases. However, the errors of the coefficients a and A quickly rising with the increase of n_{max} do not allow to treat the data with greater n_{max}. For $E_p =$ 330 MeV the coefficient a_1 is obtained approximately an order greater, and $f^2 = 0.19 \pm 0.02$ ($n_{max} = 2$). In this case the change of the number of terms in the expression for $\sigma_{pp}(\vartheta)$ weakly influences on the value of the first coefficient also. The criterium of goodness of fit remains constant and unsatisfactory when n_{max} changes in expression (1) from two to four ($\chi^2 = \frac{\chi^2}{\bar{\chi}^2} \simeq 3$). The attempt to fit the experimental data with the fixed coefficient $a_1 = f^4 = 0.064$ increases χ^2 up to 3.9 and besides provides $A_{n_{max}} < 0$. All this indicates apparently that in the experimental data of $\sigma_{pp}(\vartheta)$ there are appreciable errors which have not been taken into account. Indeed, comparing Fig. 2 to Fig. 3 one sees that experimental points at $E_p =$ 330 MeV have a considerably larger spread around the calculated curve than at $E_p =$ 380 MeV. It is worthy of notice, however, that when discussing the obtained res. 'ts

* There is similar connection in (n-p)-scattering.

Table 1.

E_p MeV	$a_1 \cdot 10^2 = f^4 \cdot 10^2$	$a_2 \cdot 10^2$	$A_n \cdot 10^2$, $n=0$	$n=1$	$n=2$	$n=3$	$n=4$	$v^2 = \frac{\chi^2}{\bar{\chi}^2}$
330/3/	3,45±0,49 (4I8)	-0,48±0,I6 (202I)	I,8I±0,I6 (5I25)	-0,I8±0,I5 (737)	I,23±0,30 (I44I)			2,95
	3,72±0,89 (I349)	-0,30±0,I0 (229I7)	I,00±0,I7 (4638)	0,26±0,23 (I844)	0,5I±0,9 (2667)	0,6±0,7 (4934)		3,0
	2,46±2,02 (7.10^3)	-0,I3±0,26 (169.10^3)	0,74±0,47 (35.10^3)	0,I2±0,3I ($3,2.10^3$)	0,5I±0,4 (2267)	0,60±0,70 (4934)	-2,00±3,00 (64.10^3)	3,0
	0,64 фиксирован	0,027±0,0I2 (360)	0,45±0,22 (78)	-0,48±0,I0 (348)	I,42±0,3I (I526)	I,53±0,32 (I036)		3,9
380/4/	0,44±0,I6 (98)	-0,006±0,07 (535)	0,5I±0,0I3 (63)	0,72±0,002 (68)				0,6
	0,35±0,33 (420)	-0,0005±0,02 (4046)	0,5±0,04 (54I)	0,07±0,03 (7I,0)	-0,03±0,I0 (728)			0,62
I47/2/	0,50±0,20 (6I)	-0,0I±0,03 (202)	0,5I±0,05 (6I)					I,6
	0,76±0,54 (454)	-0,07±0,I0т (3489)	0,56±0,I6 (787)	0,09±0,I7 (224)				

Correlation factors are given in brackets /1/.

L.I. Lapidus pointed out that a leap of the coefficient a_1 at $E_p =$ 330 MeV might be caused also by contribution to the real part of the (pp) -scattering amplitude caused by a sharp increase of the cross section of pion production in (pp) collisions near the reaction threshold /9/

Table 2 presents the values of f^2 obtained when treating $\sigma_{pp}(\vartheta)$ and $\sigma_{np}(\vartheta)$ /1/ by the above-described method.

Table 2.

Kind of interaction	np	pp	pp	pp	np	np
Energy E MeV	90	147	330	380	380–400	630
f^2	0,060±0,006	0,070±0,015	0,18±0,02	0,066±0,014	0,065±0,007	0,044±0,012

It is seen from Table 2 that at all the energies ranging from 90 up to 630 MeV (except $E_p \approx 330$ MeV) the values obtained do not apparently contradict the value 0.08 though systematically being below this value.

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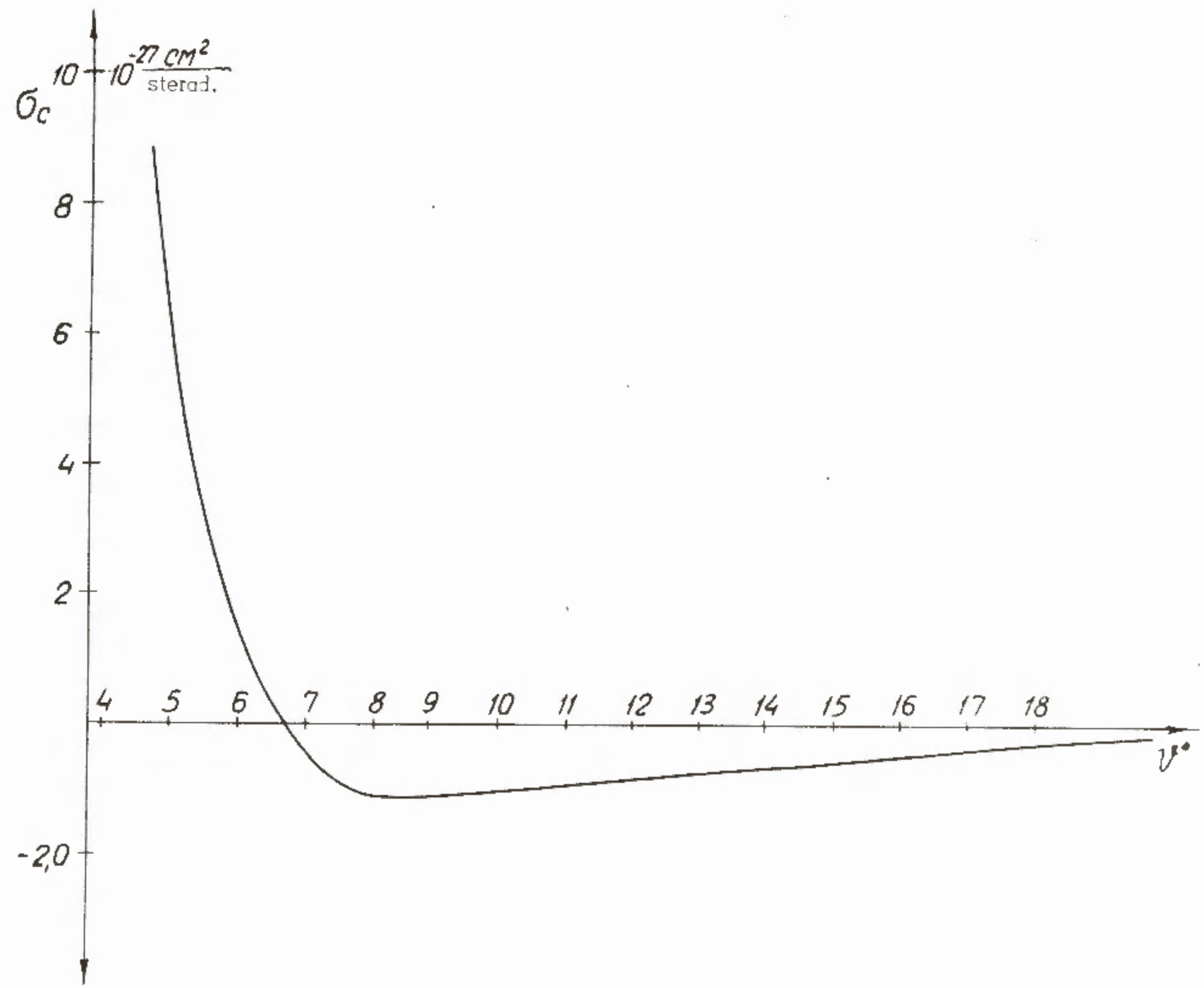

F i g. 1. Dependence of corrections for the Coulomb scattering on the scattering angle ($E_p =$ 380 MeV).

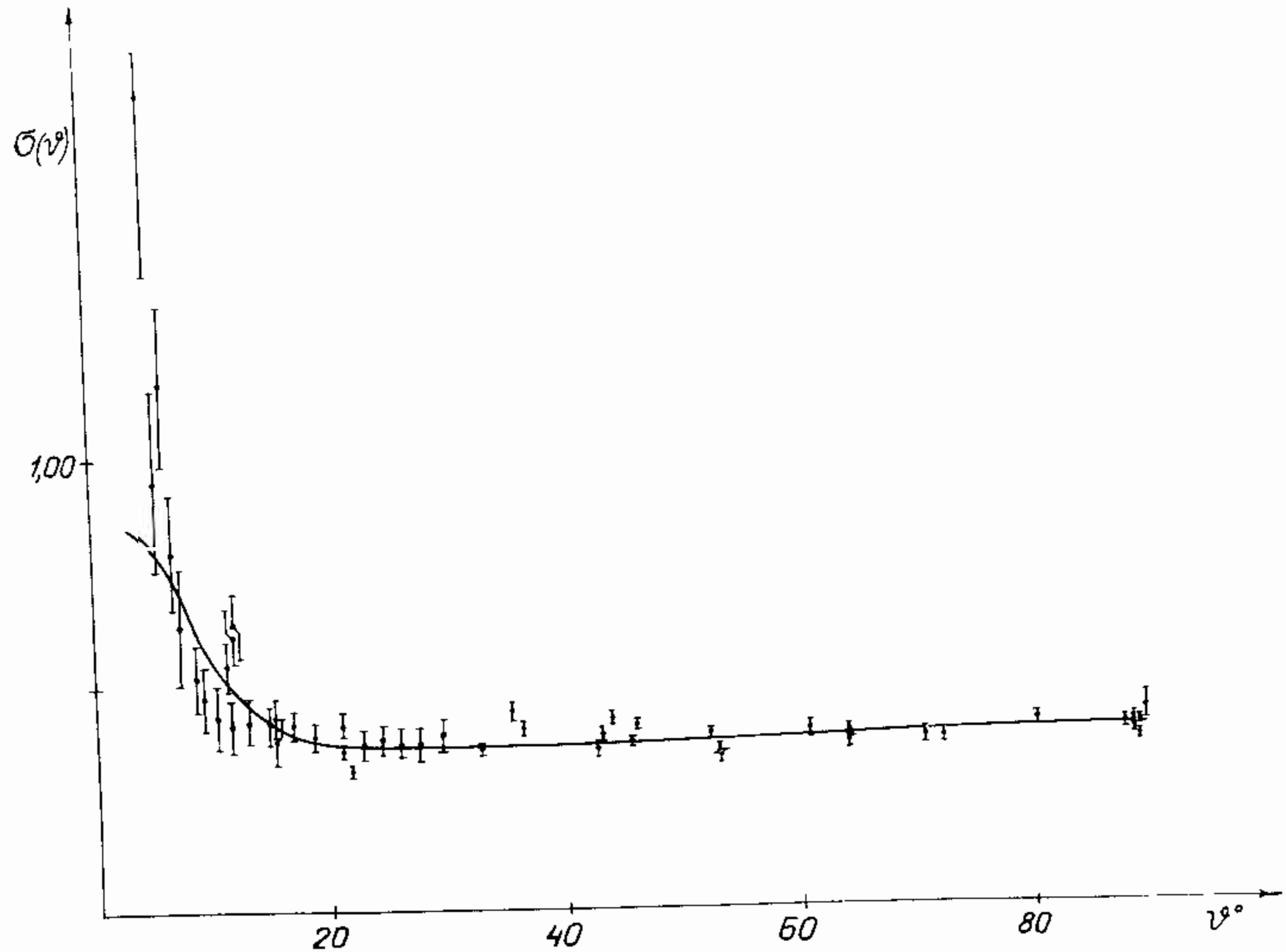

Fig. 2. Dependence of the nuclear part of $\sigma_{pp}(\vartheta)$ on the scattering angle at 330 MeV.

The solid curve is calculated by formula (1).

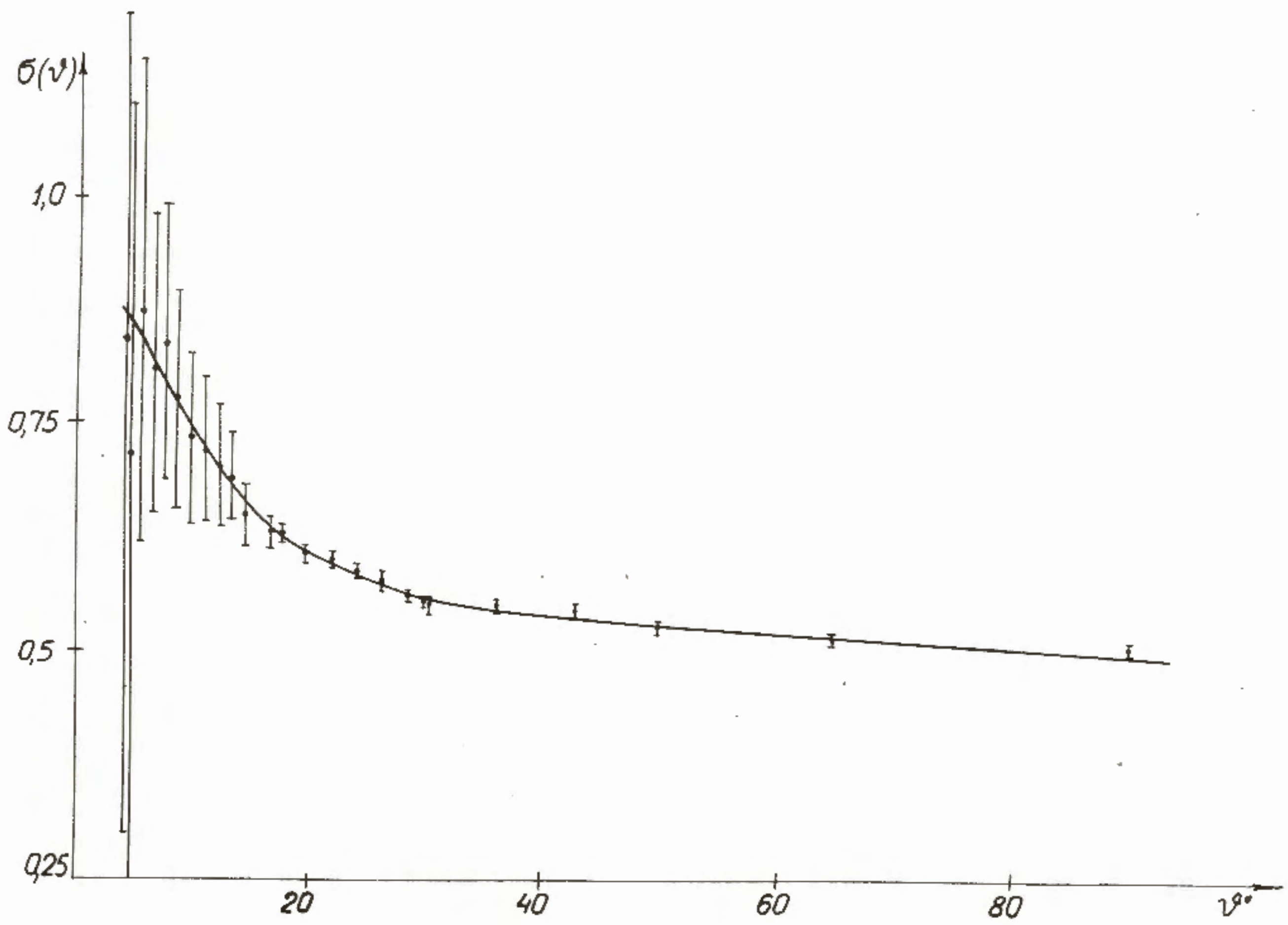

Fig. 3. Dependence of the nuclear part of $\sigma_{pp}(\vartheta)$ on the scattering angle at $E_p =$ 380 MeV.
The solid curve is calculated by formula (1).