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ЛАБОРАТОРИЯ ЯДЕРНЫХ ПРОБЛЕМ

ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

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ON THE PROBLEM OF THE DETERMINATION OF PERMISSABLE PHASE SHIFT REGIONS IN THE PHASE SHIFT ANALYSIS BY THE 'RAVINE' METHOD

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In performing the phase shift analysis of experimental data the sum is minimized:

$$\chi^2(\delta_1 \delta_2 \dots \delta_m) = \sum_{i=1}^{i=n} \left[\frac{Y_i(\delta_1, \delta_2, \dots \delta_m) - Y_{i\,\mathrm{exp.}}}{\sigma_i} \right]^2 \qquad (1)$$

where $Y_i(\delta_1, \delta_2, \dots \delta_m)$ are theoretical values of the quantaties which depend on the unknown parameters (phase shifts) $\delta_1 . \delta_2, \dots \delta_m$; Y_i exp. is experimentally found value of the same quantaties; σ_i is the root-mean-square error of the measurements of the given quantaties.

When the number of the parameters is large, the minimum χ^2 is obtained in multi-dimensional space with the help of an electronic computer by gradient discending from the initial point chosen at random. The values of the phase shifts $\delta_{01}, \delta_{02}, \dots \delta_{0m}$, corresponding to the minimum χ^2 determine the theoretical curves which best of all satisfy the set of experimental values of the quantaties ($Y_1, Y_2, \dots Y_n$). On finding the values $\delta_{01}, \delta_{02}, \dots \delta_{0m}$ which are normal asymptotically and present the best evaluations of true phase shift values, the problem of the dispersion of these evaluations is solved. Standard deviations for phase shifts are usually obtained by investigating the functional $\chi^2(\delta_1, \delta_2, \dots \delta_m)$ in the direct neighbourhood of the minimum point. At this point the quotient derivatives $\frac{d\chi^2}{d\delta_1} = \frac{d\chi^2}{d\delta_2} = \dots = \frac{d\chi^2}{d\,\delta_m} = 0$ and the rise of the functional with the change of the parameters corresponding to the minimum point is characterized by the second order quotient derivatives.

In references /1,2/ in order to find the region of permissable values of phase shifts a new method has been applied which is based on the determination of the bottom relief of the functional $\chi^2(\delta_1, \delta_2, \dots \delta_m)$. With this aim specially I.M. Gelfand has developed a numerical method, the so-called 'ravine' method, when an electronic computer makes steps of finite length along the 'ravines' of small values of χ^2 without stopping at the minimum points of the functional. On finding the 'ravine' bottom relief of the functional the permissable region of phase shift values was determined proceeding from the condition that the increasing values of χ^2 should not exceed the level $\chi^2 = 2\bar{\chi}^2$. Obtained in this way, the regions of permissable solutions proved to be considerably larger than the regions /3–5/ determined in the case of gradient discending on the basis of the investigation of the functional structure in the neighbourhood of the minimum points χ^2.

Chosen in references /1,2/ the condition of the determination of the permissable phase shift region as the region surrounding the minimum point of the functional $\chi^2(\delta_1, \delta_2, \dots \delta_m)$, in which the value

$\chi^2 \leqslant 2\ \overline{\chi}^2$, might seem to be quite reasonable. The value $\chi^2 = \sum_1^n (\frac{\Delta i}{\sigma_i})^2$ is ruled by the so-called χ^2 – distribution which is characterized by the average value $\overline{\chi}^2 = n - m$ and the dispersion $D = 2(n - m)$.

If the series of independent measurements of the quantities $Y_1, Y_2, \dots Y_n$ are repeated many times and then for each of the series one finds the minimum value of the squares sum (1), then the obtained χ^2_{min} are distributed according to the χ^2 distribution with the above-indicated $\overline{\chi}^2$ and D. Thus, on average < 16% of the obtained values of χ^2_{min} will exceed the level $\chi^2 = 2(n - m)$ chosen in references/1,2/. If the chosen region of the values χ^2 is effectively filled with the values χ^2_{min} corresponding to the repeated series of measurements at the same points, this region χ^2 might seem to be permissable also for the values χ^2, obtained when phase shifts deviate from the values corresponding to the obtained minimum.

However, the direct determination of the root-mean-square deviations of the phase shifts from the likelihood function of the given set of the experimental values of the quantaties $Y_1, Y_2, \dots Y_n$ shows the groundlessness of using the χ^2 – distribution for the determination of the permissable phase shift region. Indeed, the likelihood function, which is proportional to the probability of obtaining such a set, in the case for instance of the normal law of error distribution of measuring the values Y_i has the following form:

$$P(Y_1, Y_2, \dots Y_n ; \delta_1, \delta_2, \dots \delta_m) = \frac{1}{(2\pi)^{n/2} \prod_1^n \sigma_i} e^{\frac{-\chi^2 (y_1, y_2, \dots y_n ; \delta_1, \delta_2, \dots \delta_m)}{2}} \qquad (2)$$

At the point $(\chi\delta_{01}, \delta_{02}, \dots \delta_{0m})$ we have the maximum value of the likelihood function $P_{max} = \frac{1}{(2\pi)^{n/2} \prod_1^n \sigma_i} e^{\frac{-\chi^2_{min}}{2}}$. According to the determination the root-mean-square deviations of phase shifts are such deviations from $\delta_{01}, \delta_{02}, \dots \delta_{0m}$, with which the probability of the set and consequently the value of the likelihood function decreases $e^{1/2}$ times.

From expression (2) it is seen that such a decrease of the likelihood function takes place when $\chi^2 = \chi^2_{min} + 1$. This level which exceeds the minimum value by a unity determines the region of the values χ^2 corresponding to the root-mean-square deviations of phase shifts. The error matrix employed in a conventional method determines this region on the basis of the investigation of the functional $\chi^2(\delta_1, \delta_2, \dots \delta_m)$ at the minimum point (the determination of the second order quotient derivatives from the functional at the point $\delta_{01}, \delta_{02}, \dots \delta_{0m}$).

If one returns to the consideration of the results of the phase shift analysis for the repeated independent series of measurements at the same points, then apart from the fact that the obtained values χ^2_{min} will suffer a considerable spread according to the χ^2 - distribution, it should be added that the obtained phase shift values for the minima will assemble in a comparatively narrow region, corresponding to the level $\chi^2 = \chi^2_{min} + 1$. With increasing the number of the measured values in the set (n) the spread of the values χ^2_{min} will increase, while the region of coordinates in the phase shift multi-dimensional space corresponding to these minima will naturally get narrower. At the same time the regions of phase shift values corresponding to $\chi^2 = c\,(n - m)$ will not decrease with increasing n.

Thus, as a result of the unfortunate misunderstanding the large regions of phase shift solutions obtained in reference/1/ have been taken to be the regions of permissable phase shift values*. At the same time the determination of the regions with small values of χ^2 - by the 'ravine' method is rather advisable as a fast preliminary investigation of the functional χ^2 which essentially simplifies further search for the minima by the method of gradient discending. The results obtained in references/1,2/ for 95 MeV may prove to be useful later on just as such a preliminary investigation.

A question may arise on the complete solution of the problem by the 'ravine' method including the determination of the minimum points of the functional and the root-mean-square phase shift deviations corresponding to $\chi^2 = \chi^2_{min} + 1$. However, in this case in the regions of small values of χ^2 the steps made by the computer should be decreased and the calculating accuracy of the value χ^2 should be improved. The advisability of the complete solution of the problem by the 'ravine' method is not obvious since the rate of such an analysis with the given requirements will sharply decrease.

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* The author of the present note had a chance to report paper/1/ at the Rochester conference, to answer numerous questions and to explain the method of investigating the 'ravine' bottom relief of the functional with the help of an electronic computer. In this connection it should be mentioned that no one of the participants of the conference discussed the problem of the correctness of the determination of the permissable phase shift region by the values $\chi^2 < 2.(n - m)$. But the incorrectness of such a determination drew the author's attention only quite recently.

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