ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

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It has been recently shown by Peirls, Carruthers, and Bethe/1/,/2/ that the resonances in $\pi - p$ scattering at 600 and 900 MeV can be understood if one takes into account the resonance at $E \cong 200$ MeV and $\pi - \pi$ resonance. In this work we used a similar method to investigate K–N scattering, and a number of resonances in K–N scattering have been found. Consider one of the channels of inelastic K–N scattering involving three particles in the final state (the two of them, e.g. the particle with the momenta p_1 and p_2 strongly interact). In the general case, the cross section for such a reaction may be given by the following expression:

$$\sigma(E) = \frac{1}{(2\pi)^5 v} \cdot \int |\langle f| T|i\rangle|^2 \delta^4(P_i - p_1 - p_2 - p_3)\, d^3p_1 d^3p_2 d^3p_3$$

If the interaction between particles with momenta p_1 and p_2 has no resonant character, then

$$\sigma_0(E) = \frac{1}{(2\pi)^5 v} \int |\langle f_0| T| i\rangle|^2 \delta^4(P_i - p_1 - p_2 - p_3)\, d^3p_1 d^3p_2 d^3p_3$$

$$= \frac{1}{(2\pi)^5 v} \overline{|\langle f_0|T|i\rangle|^2_E} \int \delta^4(P_i - p_1 - p_2 - p_3)\, d^3p_1 d^3p_2 d^3p_3$$

where $\overline{|\langle f_0|T|i\rangle|^2_E}$ is the mean value of $|\langle f_0| T|i\rangle|^2$ at the given energy E.

On the other hand, if the interaction between these particles is resonant, then

$$|\langle f| T | i\rangle|^2 = a(\vec{K}, \vec{l})\, \overline{|\langle f_0| T| i\rangle|^2_E},$$

where

$$\vec{K} = (\vec{p}_1 + \vec{p}_2) \qquad \vec{l} = \frac{1}{2}(\vec{p}_1 - \vec{p}_2).$$

It follows that

$$\frac{\sigma(E)}{\sigma_0(E)} = \frac{\int a(\vec{K}, \vec{l})\, \delta^4(P_i - p_1 - p_2 - p_3)\, d^3p_1 d^3p_2 d^3p_3}{\int \delta^4(P_i - p_1 - p_2 - p_3)\, d^3p_1 d^3p_2 d^3p_3} \qquad (1)$$

Suppose that the cross section $\sigma_0(E)$ is changing slowly with the energy E, this is equivalent to the assumption that the cross section $\sigma(E)$ is varying rapidly only in the region of resonance. Then, we can obtain from (1) that in this case $\sigma(E)$ will reach maximum value at the energy $E = E_{max}$, where the ratio $\frac{\sigma(E)}{\sigma_0(E)}$ reaches maximum.

In order to find the value of E_{max} let us take $a(\vec{K}, \vec{l})$ in the following simpler form: (i) $a(\vec{K}, \vec{l}) = A(M)$, M = isobar mass $= \sqrt{K_0^2 - \vec{K}^2}$. (ii) $A(M) \approx 1$, when M is outside from the region of resonance

$(M_o-\Delta,\ M+\Delta)$, $A(M) = A \gg 1$, when M is in the region of resonance $(M_o-\Delta,\ M_o+\Delta)$ (iii) the region of resonance is narrow enough, so that $\Delta \ll M_o$, therefore, in this region we can put $:\sqrt{K^2 - \vec{K}^2} \doteq M_o$ - isobar mass. At the same time $|\vec{K}| = |\vec{P}_3|$, and it is determined by the total energy of the system E in the c.m.s. Thus, the problem reduces to finding the maximum of the expression (in the c.m.s.) :

$$\frac{A \int_{L(E)} d^3 l \int d^3 k\, \delta(E - \sqrt{\vec{K}^2 + M_o^2} - p_{3c})\ \delta^3(\vec{K} + \vec{p}_3)\, d^3 p_3}{\int \delta^4(P_i - p_1 - p_2 - p_3)\, d^3 p_1\, d^3 p_2\, d^3 p_3} \qquad (2)$$

$$= \frac{\frac{\pi A}{E^2}\sqrt{(E^2 - m_3^2 - M_o^2)^2 - 4m_3^2 M_o^2}\ \{ (E^2 - m_3^2 - M_o^2) - \frac{1}{2E^2}\left[(E^2 - m_3^2 - M_o^2)^2 - 4m_3^2 M_o^2 \right] \} \int_{L(E)} d^3 l}{\int \delta^4(P_i - p_1 - p_2 - p_3)\, d^3 p_1\, d^3 p_2\, d^3 p_3}$$

Here $L(E)$ is the region in the vector space of $\vec{l}$ which corresponds to the value of M lying between $(M_o - \Delta,\ M_o + \Delta)$. Note further, that $\int_{L(E)} d^3 l$ is the volume confined between the two surfaces, where the external one corresponds to the value $M = M_o + \Delta$ and the internal to $M = M_o - \Delta$. The degree of elongation of these surfaces depends upon the magnitude of $|\vec{K}|$, in other words, upon the velocity of the isobar $(p_1 + p_2)$ (for example, in the c.m.s. of the isobar, $|\vec{K}| = 0$), and we have spherical surfaces). From formula (2) we can obtain the following relations between the resonance energy E and the mass of the 'isobar' M :

Reaction<1>	Resonant kinetic energy of K –meson in the lab. system	Isobar mass<2> (μ – mass of pion)
1) $K^- + p \to \Lambda^o + \overline{\pi + \pi}$	250 - 300 MeV	$M_o(\pi\pi) \sim 3\,\mu$ [3]
2) $K^- + p \to \overline{\Lambda^o + \pi} + \pi$	250 - 300 MeV	$M_o(\Lambda^o \pi) \sim 10\,\mu$ ($Q \sim 115$ MeV) [7]

(1 and 2 are very close to each other, , therefore we can expect a more complicated interference)

3) $K + p \to \overline{n + \pi} + K$ $\tilde{K} + p \to \overline{n + \pi} + \tilde{K}$	~ 600 MeV	$M_o(n\pi) \sim 9\,\mu$ (With resonant state $j = T = \frac{3}{2}$)
4) $K + p \to n + \overline{\pi + K}$ $\tilde{K} + p \to n + \overline{\pi + K}$	~1000 - 1200 MeV	$M_o(\pi K) \sim 5.5$<3> $- 7.2$<4> μ
5) $K^- + p \to \overline{\Lambda + K^o} + \tilde{K}^o$	~1700 MeV	$M_o(\Lambda K^o) \sim 12.6\,\mu$ <5>

<1> The particles between which the resonant interaction is assumed are indicated by brackets.

<2> Q is the kinetic energy of the decay products of the 'isobar' in its rest system.

An experimental check of these results is more interesting. At present we are not able to determine the absolute values of the cross sections. However, if the resonance does exist, and is large enough, then, due to the unitarity of the amplitude, the cross section for elastic scattering will have also the resonance at the same energy.

From the consideration of isotopic invariance we can easily obtain the numerical ratio for some reactions given above. For instance, the ratio of the cross sections for the reactions:

$$K^+ + p \to p + \pi^+ + K^0, \qquad K^+ + p \to p + \pi^0 + K^+, \qquad K^+ + p \to n + \pi^+ + K^+$$

(cf. 3 in the Table) is approximately equal to 9:2:1, whereas the ratio of the cross sections of the reactions: $K^- + p \to p + \pi^0 + K^-$, $K^- + p \to n + \pi^+ + K^-$, $K^- + p \to n + \pi^0 + \tilde{K}^0$, $K^- + p \to p + \pi^- + \tilde{K}^0$ approximately to 2:1:2:1. This cannot be done in the cases where the isotopic spin of the 'isobar' is unknown. In these cases further experimental investigations are necessary<6>.

Our method is also valid in the case where the narrow maximum is observed in the momentum spectra of the particle with momentum p_3.

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<3> From the measurements in the propane chamber /4/.

<4> From the measurements in the propane chamber /5/.

<5> The estimate was obtained from the condition that the cross section for $\pi^- + p \to \Lambda + K^0$ has the maximum at the energy of a π^--meson $E \approx 1,1$ Bev /6/ in the lab. system.

<6> Since the resonant interaction is observed in the experiment between π^- and K^0 mesons, then we can suppose that the isotopic spin of the 'isobar' (πK) $T = 3/2$ /5/.