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ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

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ON GAUGE TRANSFORMATIONS OF GREEN FUNCTIONS

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A b s t r a c t

The gauge transformation laws of many-particle Green functions are obtained. General Ward's identities follow from these laws.

1. I n t r o d u c t i o n

After Landau and Chalatnikov's work[1] devoted to gauge transformation laws of the Green functions $\langle T\psi(x)\bar{\psi}(y)\rangle$ and $\langle T\psi(x)\bar{\psi}(y)A_{\mu}(u)\rangle$ appeared many papers related to this question[1-5].

This laws follow only from the transformation properties of the Heisenberg's operators in changing gauge without any dependence on a particular kind of coupling.

In this paper the general theorem is proved, which established the gauge transformation laws of the Green functions with any number of whichever charged and neutral fields without assumptions about interaction.

Recently in electrodynamics Okubo derived the gauge transformation law of many-particle Green functions, which are somewhat differently defined. His proof however is valid in electrodynamics only and related to the expansion of the Green functions in the perturbation theory series.

With the help of the Green functions gauge transformation laws it is possible to get the most general and natural derivation of identities of Ward's type. This was done in [4,5] for the relation between the vertex part and one-particle Green function.

Below, in the last section, the general identities between many-particle Green functions are obtained and several particular examples of Ward-type identities are quoted.

2. Definitions of Green Functions

Let us write down the definitions of many-particle Green functions for which the gauge transformation laws will be obtained. The Green functions without photon ends are defined as a vacuum expectation of the T-product

$$G(x^1 \dots x^n, y^1 \dots y^n, z^1 \dots) = \langle T \Psi \rangle \equiv \langle T \psi_1(x^1) \dots \psi_n(x^n) \varphi_1(y^1) \dots \varphi_n(y^n) \chi_1(z^1) \dots \rangle \qquad (1)$$

where $\psi(x)$ and $\varphi(y)$ are operators of any lepton, meson and baryon charged fields which have the gauge transformation law

$$\psi'(x) = \exp\left[ie\Lambda(x)\right]\psi(x) \qquad \varphi'(y) = \varphi(y)\exp\left[-ie\Lambda(y)\right] \qquad (2)$$

and $\chi(z)$ are operators of any neutral fields except the electromagnetic one. The Green functions with electromagnetic field operators $A_\mu(u)$ are defined as

$$G_\mu(x^1 \dots, y^1 \dots, z^1 \dots, u) = \langle T \Psi A_\mu(u) \rangle \qquad (3)$$

$$G_{\mu_1\mu_2}(x^1 \dots, y^1 \dots, z^1 \dots, u^1 u^2) = \langle T \Psi A_{\mu_1}(u^1) A_{\mu_2}(u^2) \rangle - \langle T \Psi \rangle \langle T A_{\mu_1}(u^1) A_{\mu_2}(u^2) \rangle \qquad (4)$$

$$\begin{aligned} G_{\mu_1\mu_2\dots\mu_m}(x^1 \dots, y^1 \dots, z^1 \dots, u^1 u^2 \dots u^m) &= \langle T \Psi A_{\mu_1}(u^1) A_{\mu_2}(u^2) \dots A_{\mu_m}(u^m) \rangle - \\ &- \sum_{k>l} G_{\mu_1 \dots \mu_{l-1}\mu_{l+1} \dots \mu_{k-1}\mu_{k+1} \dots \mu_m} \langle T A_{\mu_l}(u^l) A_{\mu_k}(u^k) \rangle - \\ &- \sum_{i>j>k>l} G_{\underbrace{\mu_1 \dots \mu_m}_{m-4}} \langle T A_{\mu_i}(u^i) A_{\mu_j}(u^j) A_{\mu_k}(u^k) A_{\mu_l}(u^l) \rangle - \dots\dots\dots \end{aligned} \qquad (5)$$

Formally these expressions may be obtained by means of Schwinger's method[6] of functional differentiation with respect to the source current

$$G_{\mu_1\mu_2\ldots\mu_m}(x^1\ldots,y^1\ldots,z^1\ldots,u^1\ldots u^m)=\left.\frac{\delta^m\langle T\Psi\rangle}{\delta J_{\mu_1}(u^1)\delta J_{\mu_2}(u^2)\ldots\delta J_{\mu_m}(u^m)}\right|_{J=0} \tag{6}$$

where functional derivative is defined as

$$\frac{\delta\langle T\Psi\rangle}{\delta J_\mu(u)}=\langle T\Psi A_\mu(u)\rangle-\langle T\Psi\rangle\langle A_\mu(u)\rangle \tag{7}$$

and at $J = 0$ the vacuum expectations for odd number of $A_\mu(u)$'s vanish.

The Green functions defined by formulae (3) - (5) are transformed in changing of gauge simpler than vacuum expectations and the natural generalization of Ward-type indentities is obtained for them. These definitions correspond in the perturbation theory to the throwing away of the graphs containing unconnected parts all external ends of which are photon ones.

3. Gauge Transformations of Green Functions

In this section the Green function (5) in an arbitrary covariant gauge will be expressed in terms of Green functions in true Landau gauge, in which by definition the photon propagator has no longitudinal part and the equal-time commutator of electromagnetic field operators vanishes[5].

The transition from Heisenberg operators in Landau gauge ψ^τ, φ^τ, χ^τ and A^τ_μ to the operators in the arbitrary gauge ψ, φ, χ and A_μ is accomplished according to

$$\begin{aligned} &\psi(x)=\exp[ie\Lambda(x)]\psi^\tau(x) && \varphi(y)=\varphi^\tau(y)\exp[-ie\Lambda(y)] \\ &\chi(z)=\chi^\tau(z) && A_\mu(u)=A^\tau_\mu(u)+\frac{\partial\Lambda(u)}{\partial u_\mu} \end{aligned} \tag{8}$$

where $\Lambda(x)$ is suitably chosen hermitian operator, which can be represented as [1]

$$\Lambda(x)=\int d^4k\,\lambda(k^2)\left(a_k e^{ikx}+a^+_k e^{-ikx}\right) \tag{9}$$

(a_k and a_k^+ are annihilation and creation operators). One can believe that operator Λ acts in the Hilbert space different from one in which ψ^τ, $\bar\psi^\tau$, χ^τ and A_μ^τ operate[1,5] and, consequently, Λ commutes with them. It is worthewhile to note also that in the transitions to true gauge, in which by the definition the equal-time electromagnetic fields commutator vanishes, the choice of Λ is limited by the condition [5]

$$\langle [\Lambda(\vec{x},0),\dot{\Lambda}(0)]\rangle = 0 \, . \tag{10}$$

Then the photon propagator in an arbitrary true covariant gauge will be expressed as

$$D^c_{\mu\nu}(u^1-u^2) = \langle T A_\mu(u^1)A_\nu(u^2)\rangle = \langle T A^\tau_\mu(u^1)A^\tau_\nu(u^2)\rangle + \frac{\partial^2}{\partial u^1_\mu \partial u^2_\nu}\langle T\Lambda(u^1)\Lambda(u^2)\rangle =$$

$$= \int d^4q \, e^{iq(u^1-u^2)} \left[(q^2\delta_{\mu\nu} - q_\mu q_\nu) d_t(q^2) + q_\mu q_\nu d_\ell(q^2) \right] . \tag{11}$$

Now, we proceed to the derivation of the transformation law of Green function (1). According to what has been said above

$$G(x^1\ldots, y^1\ldots, z^1\ldots) = G^\tau(x^1\ldots, y^1\ldots, z^1\ldots)\langle T\exp(ie\Phi)\rangle \tag{12}$$

where

$$\Phi = \Lambda(x^1) + \ldots + \Lambda(x^n) - \Lambda(y^1) - \ldots - \Lambda(y^n) \, . \tag{13}$$

Owing to the choice of the operator $\Lambda(x)$ in the form (9) the vacuum expectation $\langle T\exp(ie\Phi)\rangle$ may be evaluated by means of the Wick's theorem. This theorem can be just applied to Φ, as Φ is linearly dependent on Λ. Thus,

$$\langle T\exp(ie\Phi)\rangle = \sum_{k=0}^{\infty} \frac{(ie)^{2k}}{(2k)!}\langle T\Phi^{2k}\rangle = \sum_{k=0}^{\infty}\frac{(ie)^{2k}}{(2k)!}(2k-1)!!\langle T\Phi^2\rangle^k = \exp\left(-\frac{e^2}{2}\langle T\Phi^2\rangle\right) \tag{14}$$

and the transformation law for G is obtained

$$G = \exp(e^2\rho)G^\tau \tag{15}$$

where

$$\begin{aligned}\rho &\equiv \rho(x^1\ldots, y^1\ldots) = -\tfrac{1}{2}\langle T\Phi^2\rangle = \\ &= \frac{1}{2}\sum_{i,j=1}^{n}\left[\langle T\Lambda(x^i)\Lambda(y^j)\rangle - \langle T\Lambda(x^i)\Lambda(x^j)\rangle - \langle T\Lambda(y^i)\Lambda(y^j)\rangle\right]\end{aligned} \tag{16}$$

Now we will show the following general theorem is valid.

Theorem. The Green function $G_{\mu_1\cdots\mu_m}$ in an arbitrary gauge is expressed in terms of Green functions in Landau gauge according to the law:

$$\begin{aligned}G_{\mu_1\mu_2\cdots\mu_m} = \exp(e^2\rho)\Big\{&G^\tau_{\mu_1\cdots\mu_m} + ie\sum_{\tau=1}^{m}\rho_{\mu_\tau}G^\tau_{\mu_1\cdots\mu_{\tau-1}\mu_{\tau+1}\cdots\mu_m} + \\ &+ (ie)^2\sum_{\tau_2>\tau_1=1}^{m}\rho_{\mu_{\tau_1}}\rho_{\mu_{\tau_2}}G^\tau_{\mu_1\cdots\mu_{\tau_1-1}\mu_{\tau_1+1}\cdots\mu_{\tau_2-1}\mu_{\tau_2+1}\cdots\mu_m} + \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ &+ (ie)^k\sum_{\tau_k>\tau_{k-1}\cdots>\tau_1=1}^{m}\rho_{\mu_{\tau_1}}\rho_{\mu_{\tau_2}}\cdots\rho_{\mu_{\tau_k}}G^\tau_{\mu_1\cdots\mu_{\tau_1-1}\mu_{\tau_1+1}\cdots\mu_{\tau_k-1}\mu_{\tau_k+1}\cdots\mu_m} + \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ &+ (ie)^m\rho_{\mu_1}\rho_{\mu_2}\cdots\rho_{\mu_m}G^\tau\Big\}\end{aligned} \tag{17}$$

where

$$\begin{aligned}\rho_{\mu_\tau} &\equiv \rho_{\mu_\tau}(x^1\ldots, y^1\ldots, u^\tau) = \langle T\frac{\partial\Lambda(u^\tau)}{\partial u^\tau_{\mu_\tau}}\Phi\rangle = \\ &= \sum_{j=1}^{n}\langle T\frac{\partial\Lambda(u^\tau)}{\partial u^\tau_{\mu_\tau}}[\Lambda(x^j) - \Lambda(y^j)]\rangle\end{aligned} \tag{18}$$

For the proof we note, that the definition of the many-particle Green functions (6) can be rewritten in the form

$$G_{\mu_1\mu_2\cdots\mu_m}=\frac{\delta^m\langle T\Psi\rangle}{\delta J_{\mu_1}(u^1)\delta J_{\mu_2}(u^2)\ldots\delta J_{\mu_m}(u^m)}\bigg|_{J=0}=$$

$$=\prod_{\tau=1}^{m}\left(\frac{\delta}{\delta J^\tau_{\mu_\tau}}+\frac{1}{i\alpha}\frac{\partial}{\partial u^\tau_{\mu_\tau}}\right)\langle T\Psi^\tau\rangle\cdot\frac{\langle T\exp\left[ie\Phi+i\alpha\sum_{\ell=1}^{m}\Lambda(u^\ell)\right]\rangle}{\langle T\exp\left[i\alpha\sum_{\ell=1}^{m}\Lambda(u^\ell)\right]\rangle}\Bigg|_{\substack{J^\tau=0\\ \alpha=0}} \qquad (19)$$

where the functional derivative $\frac{\delta}{\delta J^\tau_\mu}$ is defined by the rule (7) with the replacement of A_μ for A^τ_μ . $\frac{\delta}{\delta J^\tau_\mu}$ operates on $\langle T\Psi\rangle$ only. On the whole ntil J^τ_μ and parameter α are not equal to zero, the operator $\frac{\delta}{\delta J^\tau_\mu}+\frac{1}{i\alpha}\frac{\partial}{\partial u_\mu}$ generates $A_\mu(u)$ in the form of the sum $A^\tau_\mu+\frac{\partial\Lambda}{\partial u_\mu}$ according to the rule (7) in which the vacuum expectation $\langle T\ldots\rangle$ should be replaced by

$$\frac{\langle T\ldots\exp\left[i\alpha\sum\Lambda(u^\ell)\right]\rangle}{\langle T\exp\left[i\alpha\sum\Lambda(u^\ell)\right]\rangle}\,.$$

After the calculation analogous to (14) we obtain

$$\frac{\langle T\exp\left[ie\Phi+i\alpha\sum\Lambda(u^\ell)\right]\rangle}{\langle T\exp\left[i\alpha\sum\Lambda(u^\ell)\right]\rangle}=\exp\left[e^2\rho-\alpha e\sum_{\ell=1}^{m}\langle T\Phi\Lambda(u^\ell)\rangle\right]. \qquad (20)$$

Differentiation of this expression leads, evidently, to

$$\left(\frac{1}{i\alpha}\right)^k\frac{\partial^k}{\partial u^1_{\mu_1}\cdots\partial u^k_{\mu_k}}\exp\left[e^2\rho-\alpha e\sum_{\ell=1}^{m}\langle T\Phi\Lambda(u^\ell)\rangle\right]\bigg|_{\alpha=0}=$$

$$=(ie)^k\rho_{\mu_1}\rho_{\mu_2}\cdots\rho_{\mu_k}\exp(e^2\rho)\,. \qquad (21)$$

As

$$G^\tau_{\mu_1\mu_2\cdots\mu_k}=\frac{\delta^m\langle T\Psi^\tau\rangle}{\delta J^\tau_{\mu_1}(u^1)\delta J^\tau_{\mu_2}(u^2)\ldots\delta J^\tau_{\mu_k}(u^k)}\bigg|_{J^\tau=0} \qquad (22)$$

and

$$\prod_{\tau=1}^{m}\left(\frac{\delta}{\delta J^{\tau}_{\mu_\tau}}+\frac{1}{i\alpha}\frac{\partial}{\partial u^{\tau}_{\mu_\tau}}\right)=\frac{\delta^m}{\delta J^{\tau}_{\mu_1}\cdots\delta J^{\tau}_{\mu_m}}+\frac{1}{i\alpha}\sum_{\tau=1}^{m}\frac{\delta^{m-1}}{\delta J^{\tau}_{\mu_1}\cdots\delta J^{\tau}_{\mu_{\tau-1}}\delta J^{\tau}_{\mu_{\tau+1}}\cdots\delta J^{\tau}_{\mu_m}}\cdot\frac{\partial}{\partial u^{\tau}_{\mu_\tau}}+$$

$$+\left(\frac{1}{i\alpha}\right)^2\sum_{\tau_2>\tau_1=1}^{m}\frac{\delta^{m-2}}{\delta J^{\tau}_{\mu_1}\cdots\delta J^{\tau}_{\mu_{\tau_1-1}}\delta J^{\tau}_{\mu_{\tau_1+1}}\cdots\delta J^{\tau}_{\mu_{\tau_2-1}}\delta J^{\tau}_{\mu_{\tau_2+1}}\cdots\delta J^{\tau}_{\mu_m}}\cdot\frac{\partial^2}{\partial u^{\tau_1}_{\mu_{\tau_1}}\partial u^{\tau_2}_{\mu_{\tau_2}}}+ \qquad (23)$$

$$+\ldots+\left(\frac{1}{i\alpha}\right)^m\frac{\partial^m}{\partial u^{1}_{\mu_1}\cdots\partial u^{m}_{\mu_m}}$$

then the gauge law (17) follows from Eq.(19). The theorem is proved.

Let us make two comments.

Firstly, we may note, that the theorem can be proved without the prescription (6), (7), but using the initial definition (5) and applying the mathematical induction.

Secondly, it must be stressed, that the proof of the theorem is based essentially on the definition (9), which permits applying Wick's theorem to operators Λ. If one omits this definition, then the law obtained is not correct. For example, if we take*

$$\Lambda(x)=\Lambda_1(x)\Lambda_2(x) \qquad (24)$$

where $\Lambda_1(x)$ and $\Lambda_2(x)$ are hermitian operators of form (9), operating in different Hilbert spaces. For simplicity we assume that

$$\langle T\Lambda_1(x)\Lambda_1(y)\rangle_1=\langle T\Lambda_2(x)\Lambda_2(y)\rangle_2=F(x-y). \qquad (25)$$

Then the one-particle Green function transformation law is

$$\langle T\psi(x)\bar\psi(y)\rangle=\langle T\,\psi^{\tau}(x)\bar\psi^{\tau}(y)\rangle\left\{1+e^2\left[F^2(0)-F^2(x-y)\right]\right\}^{-1} \qquad (26)$$

....

* This example is only illustrative as $[A_\mu, A_\nu]$ is no longer c -number after such a transformation.

In this connection we note that the derivation of the law (15) for $\langle T\psi(x)\bar{\psi}(y)\rangle$ from group considerations in Evans, Feldman and Matthews's paper [5] is wrong, as they found the infinitesimal operator only for transition from the particular (Landau) gauge but not from an arbitrary one. Therefore, the transition from Eq.(3.15) to Eq.(3.16) in[5] is wrong, in spite of the fact,that gauge transformations forms Lee group indeed.So,in the example quoted just Eq.(3.15) is valid, but (3.16) is not. Eq.(3.16) takes place for operators $\Lambda(x)$ of the form (9) only, but for its justification it is necessary to know , in fact, the final result.

4. General Ward-type Identities

In the true Landau gauge

$$\frac{\partial}{\partial u^{\tau}_{\mu_{\tau}}} G^{\tau}_{\mu_1\cdots\mu_{\tau}\cdots\mu_k}=0 \qquad (27)$$

as a consequence of the Lorentz condition[5]

$$\frac{\partial A^{\tau}_{\mu}(u)}{\partial u_{\mu}}=0 \qquad (28)$$

and

$$[\psi^{\tau}(x),A^{\tau}_{\mu}(u)]=[\psi^{\tau}(y),A^{\tau}_{\mu}(u)]=[\chi^{\tau}(z),A^{\tau}_{\mu}(u)]=[A^{\tau}_{\nu}(u'),A^{\tau}_{\mu}(u)]=0 \qquad (29)$$

at $t_x=t_y=t_z=t_{u'}=t_u$.

Calculating divergence of both parts Eq.(17) and taking into account Eq.(27), we obtain general identities for Green functions in an arbi trary gauge -

$$\frac{\partial}{\partial u^{\tau}_{\mu_{\tau}}} G_{\mu_1\cdots\mu_{\tau}\cdots\mu_m} = ie\frac{\partial g_{\mu_{\tau}}}{\partial u^{\tau}_{\mu_{\tau}}} G_{\mu_1\cdots\mu_{\tau-1}\mu_{\tau+1}\cdots\mu_m} \qquad (30)$$

They have the form

$$\frac{\partial}{\partial u^{\tau}_{\mu_{\tau}}} G^{F}_{\mu_1\cdots\mu_{\tau}\cdots\mu_m}=-e\sum_{\ell=1}^{n}\left[D_c(u^{\tau}-x^{\ell})-D_c(u^{\tau}-y^{\ell})\right]G^{F}_{\mu_1\cdots\mu_{\tau-1}\mu_{\tau+1}\cdots\mu_m} \qquad (31)$$

$$D_c(x)=\frac{1}{(2\pi)^4}\int e^{ikx}\ \frac{dk}{k^2-i\varepsilon} \tag{31}$$

in the particular case of Feynman gauge [5] as the transition from Landau to Feynman gauge is realized by means of $\Lambda(x)$ for which

$$\langle T\Lambda(x)\Lambda(y)\rangle=\frac{-i}{(2\pi)^4}\int e^{ik(x-y)}\ \frac{dk}{(k^2)^2} \tag{32}$$

Operating on Eq.(31) by D'Alembert operator $\Box_{u^2}$ we get

$$\Box_{u^2}\frac{\partial}{\partial u^2_{\mu_2}}G^F_{\mu_1\cdots\mu_2\cdots\mu_m}=e\sum_{l=1}^{n}\left[\delta(u^2-x^l)-\delta(u^2-y^l)\right]G^F_{\mu_1\cdots\mu_{2-1}\mu_{2+1}\cdots\mu_m} \tag{33}$$

Gauge independent general Ward-type identities for processes with any number of charged and neutral particles and photons follow from identities (30) at proper definitions of higher vertex parts. Such a derivation of Ward identities is based on the gauge transformation law for field operators only, no assumptions about a particular form of renormalizable interactions are required.

As an example we consider the case of one charged particle $/\Psi=\psi(x)\bar\psi(y)/$ and m photons. Defining the vertex part $\Gamma_{\mu_1\mu_2\cdots\mu_m}$ as

$$G_{\mu_1\mu_2\cdots\mu_m}(x,y,u^1\ldots u^m)=(2\pi)^{-4(m+1)}e^m\int dp^1dp^2dq^1\ldots dq^m\exp\left(ip^1x-ip^2y+i\sum q^lu^l\right)\cdot$$

$$\cdot\delta\left(p^1-p^2+\sum_{l=1}^{m}q^l\right)\prod_{j=1}^{m}D^c_{\mu_j\nu_j}(q^j)S^c(p^1)\Gamma_{\nu_1\ldots\nu_m}(p^1p^2,q^1\ldots q^m)S^c(p^2) \tag{34}$$

we obtain from Eq. (30) after dividing by $q_2^2\,d_e(q_2^2)$ and by all $D^c_{\mu_j\nu_j}(q^j)$

$$S^c(p^1)q^\tau_{\nu_\tau}\Gamma_{\nu_1\dots\nu_\tau\dots\nu_m}(p^1,p^2,q^1\dots q^m)S^c(p^2)=$$

$$=S^c(p^1)\Gamma_{\nu_1\dots\nu_{\tau-1}\nu_{\tau+1}\dots\nu_m}(p^1,p^2-q^\tau,q^1\dots q^{\tau-1}q^{\tau+1}\dots q^m)S^c(p^2-q^\tau)-$$

$$-S^c(p^1+q)\Gamma_{\nu_1\dots\nu_{\tau-1}\nu_{\tau+1}\dots\nu_m}(p^1+q^\tau,p^2,q^1\dots q^{\tau-1}q^{\tau+1}\dots q^m)S^c(p^2) \qquad (35)$$

where $p^1-p^2+\sum_{\ell=1}^{m}q^\ell=0.$ For $m=1$ this identity can be written as

$$S^c(p^1)q_\nu\Gamma_\nu(p^1,p^1+q,q)S^c(p^1+q)=S^c(p^1)-S^c(p^1+q) \qquad (35a)$$

Let us consider one more particular example with two charge particles, namely the relation between the processes

$$\pi^+ + p \to \pi^+ + p$$
$$\pi^+ + p \to \pi^+ + p + \gamma$$

Introducing standard definitions

$$G(x^1x^2,y^1y^2)=(2\pi)^{-12}\int dp^1dp^2dp^3dp^4\exp\left(ip^1x^1+ip^2x^2-ip^3y^1-ip^4y^2\right)$$
$$\cdot\delta(p^1+p^2-p^3-p^4)\;G(p^1,p^2,p^3,p^4)$$

$$G_\mu(x^1x^2,y^1y^2,u)=(2\pi)^{-16}e\int dp^1dp^2dp^3dp^4\exp\left(ip^1x^1+ip^2x^2-ip^3y^1-ip^4y^2+iqu\right)\cdot \qquad (36)$$
$$\cdot\delta(p^1+p^2-p^3-p^4+q)D^c_{\mu\nu}(q)S^c(p^1)\Delta^c(p^2)\Gamma_\nu(p^1,p^2,p^3,p^4,q)S^c(p^3)\Delta^c(p^4)$$

we obtain the identity

$$S^c(p^1)\Delta^c(p^2)q_\mu\Gamma_\mu(p^1,p^2,p^3,p^4,q)S^c(p^3)\Delta^c(p^4)=$$

$$=G(p^1,p^2,p^3,p^4-q)+G(p^1,p^2,p^3-q,p^4)-G(p^1,p^2+q,p^3,p^4)-G(p^1+q,p^2,p^3,p^4) \qquad (37)$$

where $$p^1 + p^2 + q = p^3 + p^4 \quad .$$

We restrict ourselves to these examples.

Identities (35) which are the direct generalization of Ward identity, were found by E.Fradkin (1955)[2], who derived them as a consequence of Schwinger's equations system for Green functions in electrodynamics, i.e. with exploring particular form of the renormalized interaction. Recently the identities of the form (35) were derived by Kazes [7] in the perturbation theory.

Feynman gauge identities (33) have a direct relation to the identities for perturbation theory coefficient functions, obtained by Bogolubov and Shirkov (1955)[8] in quantum electrodynamics.

Nonperturbative proof of generalized Ward identity (Equation (35a) , which has been formulated by Green [9], was given by Takahashi[10] with the aid of the Feynman gauge identity (33), derived him for $m = 1$. Okubo[4] gave gauge idependent derivation of this identity in quantum electrodynamics by means of Cainaniello method, closely related to the perturbation theory. Finally Evans, Feldman and Matthews[5] derived the same identity as a consequence of the gauge transformation Green functions $\langle T\, \psi(x)\, \overline{\psi}(y) \rangle$ and $\langle T\, \psi(x) \overline{\psi}(y) A_\mu(u) \rangle$ laws[1]. In their derivation no references are necessary on the particular form of interaction.

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