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ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

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PHASE-SHIFT ANALYSIS OF P-P-SCATTERING AT 95,150 AND 310 MEV

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I. Introduction

The aim of this report is to give the results of the phase-shift analysis of p-p scattering at energies of 95, 150, 310 MeV. The analysis was being made by a new numerical method (the method of "ravines") suggested by I.M.Gelfand which allowed to make progress in clearing up the character of the acceptable regions of phase space (solutions) and the problem of unambiguty. There are three problems to be solved in the course of the analysis:

1. To clear up whether the solutions already reported [1-3] which were found by usual gradient method, constitute the whole set of possible solutions. It was also essential to analyse the limits of accuracy for possible values of the phase shifts usually given by the error matrix.

2. To clear up to role of the "modified analysis" [2-4] in which the pole (one-pion) contribution to the scattering amplitude is taken into account. At the same time it is necessary to clear up to what extent the modified analysis permits to make accurate the values of the phase shifts without increasing the accuracy of experimental data and to what extent it allows to decrease the number of necessary experiments [5].

3. To clear up whether it is possible from the available experimental data to obtain the peripheral phase-shifts for which the main contribution must be given by the one-pion approximation and to check thereby, the accuracy of one-pion approximation obtained by calculating the two-pion phase shifts [6]

2. Method of „Ravines"

The phase-shifts which give the best description of the experimental data were calculated by minimizing the square deviation

$$\chi^2(\delta) = \sum \left[\frac{y(\delta) - y_{exp}(\delta)}{\Delta_{exp}} \right]^2 \qquad (1)$$

Here $y(\delta)$ are the values of the functions being measured (cross sections etc) with varied values of the phase-shifts δ, $y_{exp}(\delta)$ are the corresponding measured values; the sum is extended over all the selected points of the measured curves. So, the problem reduces formally to the determination of the local minima of function (1) in the many-dimensional phase space of the parameters δ. It is clear that it is practically impossible to investigate uniformly the whole space when there is a great number of phase shifts. Even if there are 9 parameters and 10-15 values of each are being chosen , we are led to the necessity of investigating about 10^{10} points. Therefore, a method is generally accepted now of random selection of the points with a subsequent descent by the gradient. The final stations of such routes (local minima) are called solutions. The accuracy of the solution is estimated according to the error matrix (see, e.g. [1-3]).

Evidently, such a method fails to provide for a sufficient guarantee that there are found all the regions with low values of χ^2. Generally it is more suitable for finding separate holes which must not necessarily correspond to true solutions rather than for obtaining the information on the topography of the surface (1) what is need for clearing up the ambiquity of the solution. The method is not convenient from the practical point of view since the route follows small roughness of the relief having usually no physical meaning. The route cannot also avoid separate "hollows" in which it sticks. For this reason the solutions belonging actually to one region may appear as separate solutions with small limits set by the error matrices.

There are no above-mentioned shortcomings in the method of "ravines" *. A characteristic feature of this method is the "jumps" of a finite length along the "ravines" of low values of χ^2. If a net of ravines is not too tangled, then this method makes it possible to find rather quikly all the regions with low values of χ^2. This holds for "properly arranged" functions, and the functions of many variables encounted in practical problems are usually of such a kind.

In making the phase shift analysis it is necessary to define properly what solutions should be considered acceptable. In this paper we accepted that the solution is feasible if the corresponding χ^2 does not exceed the twofold exceptation value $\chi^2 \leqslant 2\,\bar{\chi}^2$. A comparison of the curves shows that they agree with the experimental ones fairly well. It should be emphasized that for the available experimental data** it would be reasonable even to weaken the criterion of the acceptability allowing, for instance, the solutions with the values $\chi^2 \leqslant 3\,\bar{\chi}^2$. Obviously, this would lead to a certain extension of the region of solutions.

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* A detailed description of this method will be published separately.

** Note that the data for 98 MeV published recently, are different from those we used , by some statistical errors at certain scattering angles. An analogous situation is known to hold for the depolarization at 150 MeV.

3. One-Pion "Tail"

In constructing the square deviation (1) all the phase shifts corresponding to large orbital mo - menta were taken into account in the one-pion approximation, as suggested in [4]. The lower value of the momentum beginning with which the phase-shifts were "fixed" in the one-pion approximation were chosen according to the estimates of two-pion contribution obtained by Galanin et al[6]. The one-pion "tail" was calculated for the meson-nucleon constant $g^2=14.5$.

In the investigations of Berkeley group g^2 was also varied and its extremal value corresponding to the minimum of χ^2 was recognized as the experimental value of this constant. Such a procedure cannot be regarded consistent. Indeed, a substitution of the real "tail" by the one-pion is correct only for sufficiently large ℓ. Therefore, at every stage of calculations one should look for small corrections to the calculated phase-shifts by varying χ^2 with respect to them. Since the calculated phase-shifts are not actually varied then the necessary corrections are taken into account to a certain extent automatically by altering the parameter g^2. As a result the value for g^2 must be different from the real one and tend to it as the number of the varied phase-shifts increases (provided the accuracy of experimental data allows). The accuracy of available data is not high and a specification of the analysis by varying g^2 is beyond the limits of experimental error.

As independently varied parameters were used the proper phase shifts with an account of the Coulomb interaction (BB-shifts in Stapp and others notations [1]). However, to facilitate the introduction of the one-pion " tail " and the comparison with the one-pion approximation the real parts of the scattering matrix

$$\frac{1}{2} S^{J}_{\ell,\ell'}$$

were made use of :

$$\xi_2 = \mathrm{Re}\,\frac{S^2_{1,3}}{2i}\,; \quad \eta_3^2 = \mathrm{Re}\left(\frac{S^2_{3,3} - e^{i\varphi_3}}{2i}\right); \quad \eta_3^4 = \mathrm{Re}\left(\frac{S^4_{3,3} - e^{i\varphi_3}}{2i}\right).$$

(φ_3 is the Coulomb phase shift) coinciding up to the second order corrections with nuclear phase shifts (NB in Stapp's notations).

4. An Analysis for 95 MeV

By the beginning of our investigations for this energy there was only information on the cross-section σ and polarization P which were analysed with five varied parameters*

$$\delta_0(^1S_0),\quad \delta_2(^1D_2),\quad \delta_1^0(^3P_0),\quad \delta_1^1(^3P_1),\quad \delta_1^2(^3P_2)$$

Starting from F phase-shifts and the parameter ξ_2 the one-pion approximation was put into operation. The analysis developed a rather wide and complicated region of solutions. Additional data on the depolarization D /7/ recently published did not result in an essential decrease of the region of solutions.

The obtained region may be very roughly characterized by the boundaries of the change of phase shifts what corresponds to the 5-dimensional parallelipiped described around the region of solutions. In this way, however, one fails to demonstrate some correlations (not very essential) between the boundaries of different phase shifts. In Table 1 are shown the boundaries thus obtained.

To clear up the possibility of determining the peripheral phase shifts an analysis was repeated by varying 9 parameters (additionally ξ_2 and F phase shifts). This extends the dimensions of the region to some extent and the values $\chi^2 \simeq 10$ (for $\bar{\chi}^2 \simeq 24$) were obtained in the minima. This makes one think that from the available data it is impossible to obtain reliably the mixing parameter ξ_2 and F phase-shifts and allows only to drawn a conclusion that pion values for these phase shifts do not contradict the experiment.

In considering the topography of the obtained region one can see that it has a tendency of dividing into two ones, since in the places of the supposed division there is a low "ridge" with $\chi^2 \approx 3\bar{\chi}^2$. The inclusion of the new data will lead, very likely to even more evident division into two regions which we shall call regions N1 and N2. By analogy to the analysis for 150 MeV (see below) the measurement of the rotation of the polarization R must be effective in this sence. As an esample, in Table 1 are given the solutions from the regions N1 and N2.

5. Analysis for 150 MeV

A 9 parametric analysis of the data for σ, P, D, R (the depolarization was taken from the measurement of the Harward Laboratory). In contrast to an analysis at 95 MeV there were found two regions sharply separated, the boundaries of which are indicated in Table 2. The inclusion of the data on the rotation of the polarization R played the main part in obtaining comparatively narrow regions. A minor role

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* The calculations on the cross section and polarization for 95 MeV have been carried with close assistance of V.A.Borovikov.

of depolarization is also indicated by the fact that in the substitution of the Harward data by appreciably different data of Harwell we have not obtained a large shift of regions.

The solution found by Stabler and Lomon [3] lies in region N1. It is to be emphasized, however, that the limits they pointed out are several times to smaller Narrow limits for ξ_2 and F phase shifts proved to be also of interest. To clear up the role of F phase shifts the calculations were also performed in which the mixing parameter ξ_2 and still higher phase shifts were assumed to be zero. In another variant these phase shifts by analogy to 95 MeV - analysis were replaced by the theoretical ones (one-pion $\xi_2 = 0.075$, $\eta^2_3 = 0.03$, $\delta^3_3=-0.02$ and one two-pion value [6] $\eta^4_3= 0.015$). In both variants it did not become possible to obtain solutions with $\chi^2<150$. This confirms the correctness of the narrow limits for these phase shifts and points out the importance not only of the phase shifts themselves but also the deviations from the one-pion values.

6. Analysis for 310 MeV

A 9 parametric analysis of the data for σ, P, D, R and A has been made. All eight solutions (more exactly their corresponding regions) were found by the method of ravines. Other regions with low χ^2 were not found. However, the limits [1,2] obtained earlier turned out to be several to small. Solutions 1 and 3 lie in one region (region N1) by analogy to solutions 2 and 4 (region N2) and to solutions 5 and 6. Solution 7 is clearly separated from these three regions, whereas solution 6 is closely adjoint to region N1 and separated from it by the „ridge" with $\chi^2=130$. In Table 3 are indicated the boundaries of regions N1 and N_2 N2 by twofold and threefold expectation values $\overline{\chi^2}=26$, as well some solutions lying outside the limits of the above- mentioned errors.

7. Conclusion

1. When experimental data are insufficient the phase shift analysis yields as a solution rather large and complicated regions which cannot be described by indicating the local minima and error matrices. The limits for possible values of the phase shifts obtained earlier by the generally accepted procedure are underestimated.

2. A "modified" analysis does not reduce the multiplicity of the solution and does not allow a decrease of the number of necessary experiments. But it leads to a certain decrease of the permissible regions weakening thereby the requirements to the accuracy of experimental points. Smoothing the goodness-of-fit curves, it allows one also to restrict oneself to the measurements of a small number of scattering angles.

3. Regions N1 at different energies are the continuation of one another. The same may be siad about regions N2. Solutions 5-8 and 7 for 310 MeV, when continued to lower energies vanish and fail to give new regions, what may indicate that these regions does not contain real solutions et al. Solution N 1

at 95 MeV yield the values for ^3P phase shifts, close to the one-pion and the positive ^1S phase shift what corresponds to the sign of the ^1S phase shift in the region of effective range approximation. Solution N2 does not possess these properties.

4. The available data allow to distinguish "weakly peripheral" phase shifts corresponding to the impact parameters $r_0 \sim \frac{1}{\mu}$ ($\mathfrak{D}$ and F phase shifts at 150 MeV). These phase shifts are in qualitative agreement with the theoretical ones, however at the present time it is impossible to make quantitative comparison.

5. To obtain an unambiguous and sufficiently accurate analysis it is necessary to perform additional experiments and a further increase the accuracy of the known data.

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Table I

Boundaries of the region of solutions for 95 MeV and some solutions

Phase-shifts	Common boundaries of region N1 and N2	Two solutions from region $\chi^2=20$	$\chi^2=24$	Solution from region N2 $\chi^2=34$
$^1S_0$	-0,45 ↔ 0,45	0,17	0,35	-0,25
$^1D_2$	-0,07 ↔ 0,16	-0,06	0,105	0,13
$^3P_0$	-0,35 ↔ 0,6	0,45	0,25	-0,23
$^3P_1$	-0,23 ↔ 0.16	-0,046	-0,20	0,13
$^3P_2$	0,15 ↔ 0,32	0,27	0,28	0,28

Table 2

Boundaries of Regions of Solutions for 150 MeV and some solutions

Phase shifts	Boundaries of region N1 with $\chi^2 \leq 2\overline{\chi^2}$	Boundaries of region N1 with $\chi^2 \leq 3\overline{\chi^2}$	Solution from region N1 with $\chi^2 = 37$	Boundaries of region N2 with $\chi^2 \leq 3\overline{\chi^2}$	Solution from region N2 with $\chi^2 = 58$
$^1S_0$	0,2 ↔ 0,4	0,2 ↔ 0,45	0,296	-0,6 0,I	-0,I55
$^1D_2$	0,09 ↔ 0,I6	0,08 ↔ 0,I6	0,I34	0,05 0,I7	0,I39
$^3P_0$	0,02 ↔ 0,2	0 ↔ 0,25	0,I32	-0,5 -0,35	-0,4I5
$^3P_1$	-0,35 ↔ -0,27	-0,35 ↔ -0,27	-0,295	0,04 0,I3	0,090
$^3P_2$	0,25 ↔ 0,3	0,24 ↔ 0,3	0,280	0,3 0,36	0,32I
ξ_2	-0,06 ↔ -0,02	-0,06 ↔ -0,02	-0,034	-0,I -0,06	-0,079
η_3^2	-0,04 ↔ 0,02		-0,024		0,007
$^3F_3$	-0,02 ↔ 0,07				
η_3^1	-0,0I ↔ 0,03		0,0I6		-0,034
			-0,00I		-0,007

Table 3

Boundaries of regions of solutions for 310 MeV and some solutions

Phase shifts	Boundaries of region N1 with $\chi^2 \leq 2\overline{\chi^2}$	Boundaries of region N1 with $\chi^2 \leq 3\overline{\chi^2}$	Five solutions from region 2 $\chi^2=30$	43	45	49	50
$^1S_0$	-0,4 ↔ 0	-0,45 ↔ 0,05	-0,204	-0,267	-0,387	-0,I4	-0,I58
$^1D_2$	0,I8 ↔ 0,27	0,I6 ↔ 0,28	0,240	0,228	0,248	0,25	0,200
$^3P_0$	-0,45 ↔ 0	-0,5 ↔ 0,I	-0,2I7	-0,324	-0,245	-0,0I3	-0,223
$^3P_1$	-0,55 ↔ -0,35	-0,55 ↔ -0,35	-0,427	-0,43I	-0,365	-0,384	-0,5I7
$^3P_2$	0,26 ↔ 0,4	0,25 ↔ 0,45	0,309	0,27I	0,304	0,374	0,273
ξ_2	-0,07 ↔ 0,02	-0,07 ↔ 0,02	-0,0I8	-0,009	0,00I	-0,0I4	-0,027
η_3^2	-0,05 ↔ 0,07		0,029	0,030	0,030	-0,040	0,0I7
$^3F_3$	-0,II ↔ -0,00		-0,045	-0,077	-0,076	-0,027	-0,036
η_3^4	0,0I ↔ 0,08		0,067	0,064	0,077	0,0I6	0,055

Phase shifts	Гран Boundaries of region N2 with $\chi^2 \leq 2\overline{\chi^2}$	Boundaries of region N2 with $\chi^2 \leq 3\overline{\chi^2}$ 2	Five solutions from region 2 $\chi^2=36$	43	45	48	49
$^1S_0$	-0,65 ↔ -0,2	-0,7 ↔ -0,I	-0,5I5	-0,5I5	-0,4I7	-0,4I3	-0,250
$^1D_2$	0,05 ↔ 0,I5	0,03 ↔ 0,I6	0,I02	0,I42	0,I02	0,096	0,034
$^3P_0$	-0,85 ↔ -0,25	-I ↔ -0,I5	-0,46I	-0,48I	-0,697	-0,33	-0,805
$^3P_1$	-0,23 ↔ -0,05	-0,25 ↔ -0,04	-0,I22	-0,II9	-0,I87	-0,I93	-0,I75
$^3P_2$	0,35 ↔ 0;45	0,34 ↔ 0,46	0,430	0,406	0,364	0,45	0,37I
ξ_2	-0,I5 ↔ -0,I	-0,I5 ↔ -0,I	-0,I08	-0,II7	-0,I33	-0,I32	-0,I34
η_3^2	-0,03 ↔ 0,07		0,004	-0,008	0,009	0,008	0,036
$^3F_3$	0 ↔ 0,04		0,023	0,0I5	0, 0I5	0,028	0,0II
η_3^4	0,03 ↔ 0,I		0,063	0,007	0,069	0,076	0,068

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