ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

S.M.Bilenky and R.M.Ryndin

D - 589

ON BREMSSTRAHLUNG OF LOW ENERGY QUANTA IN ELECTRON PROTON SCATTERING

ЖЭТФ, 1961, т40, в3, с 819.

Дубна 1960

S.M.Bilenky and R.M.Ryndin

D - 589

ON BREMSSTRAHLUNG OF LOW ENERGY QUANTA IN ELECTRON PROTON SCATTERING

Abstract

The bremsstrahlung of low energy γ quanta in e-p scattering is treated. It is shown that the first two terms of the amplitude expansion in powers of the photon energy are expressed in terms of the electromagnetic form-factors of a proton. The differential cross section for the process has been obtained in this approximation.

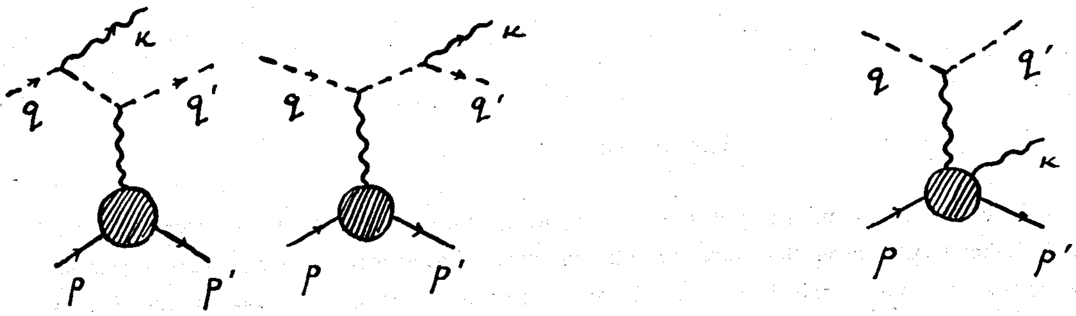
Introduction

A study of elastic scattering of high energy electrons by protons and nuclei is at present the main source of the information about the electromagnetic structure of nucleons. The experiments performed allowed to draw important conclusions about the behaviour of the electromagnetic form-factors of nucleons $F_1^{p,n}$ and $F_2^{p,n}$ as functions of the square of the four-dimensional momentum transfer. It is also interesting to study other processes what permits to obtain additional information about the form-factors F_1 and F_2 [1,2]. This paper treats the bremsstrahlung of low-energy γ quanta in $e-p$ scattering. It will be shown that the first two terms in the series the expansion of the differential cross section for this process in powers of the photon energy are expressed in terms of the electromagnetic form-factors of a proton and of their derivatives with respect to momentum transfer.

To obtain the amplitude of the process we applied a general method for considering the bremsstrahlung of low-energy quanta suggested by Low[3,4]. According to this method, that part of the amplitude of the process is first considered which has the pole at K equal to zero (K is the photon momentum). It is shown that the product of the exact renormalized vertex operator corresponding to the emission of a real γ quantum by an exact renormalized nucleon propagation function due to the Ward 's identity is expressed up to the terms of the order $\frac{1}{K}$ and the constant in terms of the charge, mass and the anomalous magnetic moment of the physical nucleon. Then, on the basis of the gauge invariance one can find the part of the amplitude which does not contain a pole. In conclusion the expression is given for the differential cross section of the process.

Amplitude of the Process

The bremsstrahlung in electron scattering by protons in the lowest approximation by e, but with account of strong interactions, is described by the following diagrams



1a 1b $\overline{\text{II}}$

Here q and q' are the 4-momenta of an electron before and after the collision, p and p' are the momenta of a proton, whereas κ is the momentum of a γ-quantum. Evidently, the nucleon vertex part of the diagrams 1a and 1b is determined by the form-factors describing the elastic scattering of electron by protons with a respective momentum transfer.

Let us write the element of the S-matrix as

$$S(p', q', \kappa; p, q) = -(2\pi)^4 i \left(\frac{M^2 m^2}{2 k_0 p_0 p_0' q_0 q_0'} \right)^{1/2} \varepsilon_\mu (T_\mu^{I} + T_\mu^{II}) \delta(p' + q' + \kappa - p - q)$$

where M and m are the proton and electron masses, ε_μ is the vector of photon polarization, $p_0 = -i p_4$, $q_0 = -i q_4$ etc., T_μ^{I} and T_μ^{II} are the contributions of the diagrams (1a, 1b) and $\overline{\text{II}}$, respectively.

For T_μ^{I}, we have

$$T_\mu^{I} = \bar{u}(q')\left[ie\gamma_\mu \frac{1}{i\gamma\cdot(q'+\kappa)+m}\, ie\gamma_\nu + ie\gamma_\nu \frac{1}{i\gamma\cdot(q-\kappa)+m}\, ie\gamma_\mu\right] u(q)\times$$

$$\times\ \bar{v}(p')\, ie\left[F_1((p'-p)^2)\gamma_\nu - \frac{\mu_p}{2M} F_2((p'-p)^2)\,\sigma_{\nu\rho}(p'-p)_\rho\right] v(p)\,\frac{1}{(p'-p)^2} \qquad (2)$$

Here μ_p, F_1 and F_2 is an anomalous magnetic moment (in nuclear magnetons), and the electromagnetic form-factors of the proton, $\sigma_{\mu\rho} = \frac{1}{2i}(\gamma_\mu\gamma_\rho - \gamma_\rho\gamma_\mu)$. The spinors u and v are normalized by $\bar{u}u = 1$; $\bar{v}v = 1$.

Let us turn now to the consideration of the diagrams of form II. Divide these diagrams into two classes A and B[5/]. All the diagrams in which the vertex with the emission of a real quantum is connected with the remaining part of the diagrams by the nucleon line are attributed to the class A. All the remaining graphs of form II are included into the class B.

Write T_μ^{II} as follows:

$$T_\mu^{II} = -\bar{u}(q')\, ie\gamma_\nu\, u(q)\left[T_{\nu\mu}^{A} + T_{\nu\mu}^{B}\right]\frac{1}{(q'-q)^2} \qquad (3)$$

where $T_{\nu\mu}^{A}$ and $T_{\nu\mu}^{B}$ describe the contribution of the diagrams of the classes A and B, respectively. For $T_{\nu\mu}^{A}$, we have the following expressions

$$T_{\nu\mu}^{A} = \bar{v}(p')\left[ie\,\Gamma_\mu(p', p'+\kappa)\, S(p'+\kappa)\, ie\,\Gamma_\nu(p'+\kappa, p) + \right.$$
$$\left. + ie\,\Gamma_\nu(p', p-\kappa)\, S(p-\kappa)\, ie\,\Gamma_\mu(p-\kappa, p)\right] v(p). \qquad (4)$$

Here S and Γ are the exact renormalized propagation function and the electromagnetic vertex operator of a proton.

We take interest in the emission of low-energy γ-quanta. Therefore, we expand the operators entering (4) in powers κ and restrict ourselves to the first two terms of expansion. Consider at first $S(p-\kappa)\,\Gamma_\mu(p-\kappa, p)\, v(p)$. The general expression for the propagation function may be put as

$$S(t) = \left[i\gamma\cdot t\, G(t^2) + M F(t^2)\right]^{-1} \qquad (5)$$

The values of the functions $G(t^2)$ and $F(t^2)$ and their derivatives $G'(t^2)$ and $F'(t^2)$ at $t^2 = -M^2$ are known to be related by [5].

$$F = G \tag{6}$$

$$F + 2M^2(F' - G') = 1. \tag{7}$$

By expanding $S(p-\kappa)$ in a power series of κ, and by using (6) and (7) and by keeping the first two terms of expansion, we get

$$S(p-\kappa) = \frac{1}{-2p\cdot\kappa F}\Big\{(-i\gamma\cdot p + M)F + i\gamma\cdot\kappa F - 2p\cdot\kappa(-i\gamma\cdot p\, G' + MF') + \\ + 2p\cdot\kappa\, M^2 F(-i\gamma\cdot p + M)\Big[\frac{2G'}{M^2} + F'' - G'' + \frac{F'^2 - G'^2}{F}\Big]\Big\} \tag{8}$$

The operator $\Gamma_\mu(t', t)$ pocesses the transformation properties of the 4-vector and under the charge conjugation is transformed as a current vector

$$C^{-1}\Gamma_\mu(t', t)C = -\Gamma_\mu^T(-t, -t') \tag{9}$$

Hence, the following expansion in power series is obtained [5,4]

$$\Gamma_\mu(p-\kappa, p) = \Gamma_\mu(p, p) - \frac{1}{2}\kappa_\rho \frac{\partial\Gamma_\mu(p,p)}{\partial p_\rho} + (p\cdot\kappa)\frac{G_1(p^2)}{M^3}\sigma_{\mu\rho}p_\rho \\ + \frac{G_2(p^2)}{2M}\sigma_{\mu\rho}\kappa_\rho + \frac{G_3(p^2)}{M^3}\sigma_{\lambda\rho}\kappa_\lambda p_\rho p_\mu + \frac{G_4(p^2)}{2M^2}\varepsilon_{\mu\lambda\rho\delta}p_\lambda\kappa_\rho\gamma_\delta\gamma_5 \tag{10}$$

The first two terms of this expansion are expressed in terms of the functions F and G and of their derivatives with the aid of the Ward's identity

$$\Gamma_\mu(p, p) = \frac{1}{i}\frac{\partial S^{-1}(p)}{\partial p_\mu} \tag{11}$$

The terms in (10), containing G_1 and G_3, fail to give any contribution to $S(p-\kappa)\Gamma_\mu(p-\kappa,p)\cdot v(p)$ as $(-i\gamma\cdot p+M)\sigma_{\mu\lambda}p_\lambda v(p)=0$.

By using also the relation

$$(-i\gamma\cdot p+M)\,\varepsilon_{\mu\lambda\rho\delta}\,p_\lambda\kappa_\rho\gamma_\delta\gamma_5 v(p)=(-i\gamma\cdot p+M)(-M)\,\sigma_{\mu\rho}\kappa_\rho\, v(p) \tag{12}$$

and throwing away, due to Lorentz condition, the terms proportional to κ_μ, we get the following expression with an accuracy up to the terms of the order $\frac{1}{\kappa}$ and the constant

$$S(p-\kappa)\,\Gamma_\mu(p-\kappa,p)\,v(p)=$$

$$=\frac{1}{-2p\cdot\kappa}\left[-2ip_\mu+(-i\gamma\cdot p+M)\sigma_{\mu\rho}\kappa_\rho\frac{F-1+G_2-G_4}{2M}+i\gamma\cdot\kappa\,\gamma_\mu\right]v(p) \tag{13}$$

If one considers the proton scattering by an external constant magnetic field, it is possible to show with the aid of (10)[5], that the quantity $F-1+G_2-G_4$ is equal to an anomalous magnetic moment of the proton μ_p. Evidently, expression (13) may be written as

$$ie\,S(p-\kappa)\,\Gamma_\mu(p-\kappa,p)\,v(p)=\frac{1}{i\gamma\cdot(p-\kappa)+M}\left(ie\gamma_\mu+\frac{ie}{2M}\mu_p\,\sigma_{\mu\rho}\kappa_\rho\right)v(p) \tag{14}$$

Thus, $ie\,S(p-\kappa)\,\Gamma_\mu(p-\kappa,p)\,v(p)$ is expressed, if one restricts to the terms of the order $\frac{1}{\kappa}$ and the constant, through the charge, magnetic moment and the mass of a physical proton. Note that the exact expression (14) we obtained coincides by its form with the corresponding expression obtained in the lowest approximation of the perturbation theory for the point proton with the Pauli anomalous magnetic moment. It can be shown in an analogous way that in the approximation we are considering

$$ie\,\bar{v}(p')\,\Gamma_\mu(p',p'+\kappa)\,S(p'+\kappa) = \bar{v}(p')\left(ie\gamma_\mu + i\mu_p\frac{e}{2M}\sigma_{\mu\rho}\kappa_\rho\right)\frac{1}{i\gamma\cdot(p'+\kappa)+M} \tag{15}$$

Then it is convenient to introduce the invariants

$$\begin{aligned} M_1^2 &= -(p-\kappa)^2 = M^2 + 2p\cdot\kappa \\ M_2^2 &= -(p'+\kappa)^2 = M^2 - 2p'\cdot\kappa \end{aligned} \tag{16}$$

With an accuracy up to the terms of the order $\frac{1}{\kappa}$ and the constant we have the relations

$$\begin{aligned} \frac{-i\gamma\cdot(p-\kappa)+M}{-2p\cdot\kappa} &= \frac{-i\gamma\cdot(p-\kappa)+M_1}{-2p\cdot\kappa} + \frac{1}{2M} \\ \frac{-i\gamma\cdot(p'+\kappa)+M}{2p'\cdot\kappa} &= \frac{-i\gamma\cdot(p'+\kappa)+M_2}{2p'\cdot\kappa} + \frac{1}{2M} \end{aligned} \tag{17}$$

Noting that

$$\begin{aligned} \gamma\cdot(p-\kappa)\,[-i\gamma\cdot(p-\kappa) + M_1] &= iM_1[-i\gamma\cdot(p-\kappa) + M_1]\,, \\ [-i\gamma\cdot(p'+\kappa) + M_2]\,\gamma\cdot(p'+\kappa) &= iM_2[-i\gamma\cdot(p'+\kappa) + M_2]\,. \end{aligned} \tag{18}$$

we get the following expression for $T^A_{\nu\mu}$

$$\begin{aligned} T^A_{\nu\mu} = \bar{v}(p')\Big[&ie\,\Gamma^0_\nu(p',p-\kappa)\frac{1}{i\gamma\cdot(p-\kappa)+M}\left(ie\gamma_\mu + \frac{ie}{2M}\mu_p\sigma_{\mu\rho}\kappa_\rho\right) \\ &+ \left(ie\gamma_\mu + \frac{ie}{2M}\mu_p\sigma_{\mu\rho}\kappa_\rho\right)\frac{1}{i\gamma\cdot(p'+\kappa)+M}\,ie\,\Gamma^0_\nu(p'+\kappa,p)\Big]v(p) \\ &+ \bar{v}(p')\left\{\left[ie\,\Gamma_\nu(p',p-\kappa) - ie\,\Gamma^0_\nu(p',p-\kappa)\right]\frac{ie\gamma_\mu}{2M}\right\}v(p) \\ &+ \bar{v}(p')\left\{\frac{ie\gamma_\mu}{2M}\left[ie\,\Gamma_\nu(p'+\kappa,p) - ie\,\Gamma^0_\nu(p'+\kappa,p)\right]\right\}v(p) \end{aligned} \tag{19}$$

where

$$\Gamma_\nu^{\circ}(t',t) = a\,\gamma_\nu + b\,\sigma_{\nu\rho}(t'-t)_\rho + c\,\sigma_{\nu\rho}(t'+t)_\rho \tag{20}$$

This operator is obtained from the general expression for the vertex part $\Gamma_\nu(t',t)$ by substituting the operator γt in the right-hand side by $i\sqrt{-t^2}$, and the operator $\gamma t'$ in the left-hand side by $i\sqrt{-t'^2}$. In formula (20) a, b and c are the functions of $-t^2$, $-t'^2$ and $(t-t')^2$, a and b do not change in substituting t by t', and c changes its sign. Note, that at $t^2 = t'^2 = -M^2$ the function a and b are equal to $F_1((t-t')^2)$ and $F_2((t-t')^2)$ respectively, and the function c vanishes.

Let us now proceed to the consideration of the contribution to the amplitude from the diagrams of the class B. The contribution $T^B_{\nu\mu}$ of these diagrams as $\kappa \to 0$ is tending to the constant, the limit is independent of how κ is tending to zero. The last two terms in formula (19) also possess the abovementioned properties. We denote the sum of these terms and $T^B_{\nu\mu}$ by $M^{(2)}_{\nu\mu}$. By $M^{(1)}_{\nu\mu}$ we shall mean the sum of the first two terms

$$M^{(1)}_{\nu\mu} = \bar{v}(p')\Big[ie\,\Gamma_\nu^{\circ}(p',p-\kappa)\frac{1}{i\gamma\cdot(p-\kappa)+M}\Big(ie\gamma_\mu + \frac{ie}{2M}\mu_p\,\sigma_{\mu\rho}\kappa_\rho\Big)$$

$$+\Big(ie\gamma_\mu + \frac{ie}{2M}\mu_p\,\sigma_{\mu\rho}\kappa_\rho\Big)\frac{1}{i\gamma\cdot(p'+\kappa)+M}\,\Gamma_\nu^{\circ}(p'+\kappa,p\Big]v(p) \tag{21}$$

Owing to the gauge invariance

$$\kappa_\mu\,(T^{I}_\mu + T^{II}_\mu) = 0 \tag{22}$$

It can be easily seen from formula (2), that $\kappa_\mu T^{I}_\mu = 0$. Therefore, from (21) and (22) it follows that

$$\kappa_\mu M^{(2)}_{\nu\mu} = -\kappa_\mu M^{(1)}_{\nu\mu} = e\,\bar{v}(p')\big[ie\,\Gamma_\nu^{\circ}(p',p-\kappa) - ie\,\Gamma_\nu^{\circ}(p'+\kappa,p\big]v(p), \tag{23}$$

In virtue of the energy-momentum conservation law $p' - p + \kappa = q - q'$ the operators Γ_ν^o entering (21) and (23) may be written as

$$\Gamma_\nu^o(p', p-\kappa) = a(M^2, M^2 + 2p\cdot\kappa, \varkappa^2)\gamma_\nu + b(M^2, M^2 + 2p\cdot\kappa, \varkappa^2)\sigma_{\nu\rho}\varkappa_\rho$$

$$+ c(M^2, M^2 + 2p\cdot\kappa, \varkappa^2)\sigma_{\nu\rho}(p' + p - \kappa)_\rho \quad ;$$

$$\Gamma_\nu^o(p'+\kappa, p) = a(M^2 - 2p'\kappa, M^2, \varkappa^2)\gamma_\nu + b(M^2 - 2p'\kappa, M^2, \varkappa^2)\sigma_{\nu\rho}\varkappa_\rho$$

$$+ c(M^2 - 2p'\kappa, M^2, \varkappa^2)\sigma_{\nu\rho}(p' + p + \kappa)_\rho \tag{24}$$

where $\varkappa = q - q'$.

The absence of the pole in $M_{\nu\mu}^{(2)}$ as $\kappa \to 0$ allows to use (23) for a unique determination of $M_{\nu\mu}^{(2)}$ up to the terms of the order of a constant. Indeed, by expanding $M_{\nu\mu}^{(2)}$ in powers of κ and restricting to the first term of expansion, we get

$$M_{\nu\mu}^{(2)} = -e\bar{v}(p')\left[ie\frac{\partial\Gamma_\nu^o(p', p-\kappa)}{\partial\kappa_\mu} + ie\frac{\partial\Gamma_\nu^o(p'+\kappa, p)}{\partial\kappa_\mu}\right]_{\kappa=0} v(p) \tag{25}$$

Keeping in the expansion $M_{\nu\mu}^{(1)}$ in powers of κ the corresponding terms we find by means of relation (24) that the derivatives with respect to the masses in the sum $M_{\nu\mu}^{(1)} + M_{\nu\mu}^{(2)}$ cancel. Finally we get the following expression for T_μ^{II}

$$T_\mu^{II} = -\frac{1}{\varkappa^2}\bar{u}(q')ie\gamma_\nu u(q)\cdot\bar{v}(p')\Big\{\left[ieF_1(\varkappa^2)\gamma_\nu - \frac{ie}{2M}\mu_p F_2(\varkappa^2)\sigma_{\nu\rho}\varkappa_\rho\right]\times$$

$$\times\frac{1}{i\gamma\cdot(p-\kappa)+M}\left(ie\gamma_\mu + i\frac{e}{2M}\mu_p\sigma_{\mu\rho}\kappa_\rho\right) + \tag{26}$$

$$+\left(ie\gamma_\mu + \frac{ie}{2M}\mu_p\sigma_{\mu\rho}\kappa_\rho\right)\frac{1}{i\gamma\cdot(p'+\kappa)+M}\Big[ieF_1(\varkappa^2)\gamma_\nu -$$

$$-\frac{ie}{2M}\mu_p\sigma_{\mu\rho}\varkappa_\rho F_2(\varkappa^2)\Big]\Big\}v(p).$$

Upon obtaining this formula we used the above-mentioned properties of the functions a, b and c. Thus, the amplitude of the bremsstrahlung of low-energy quanta in e - p scattering with an accuracy up to the terms of the order of $\frac{1}{K}$ and the constant is completely expressed in terms of the electromagnetic form-factors of a proton.

Note, that formula (26) coincides with the expression obtained by the dispersion relation method in the one-nucleon approximation[6].

Conclusion

In conclusion we give the expression for the differential cross section of the bremsstrahlung of low-energy γ- quanta obtained with the aid of the amplitude we have found/formulae (1),(2), and (26)/ As independent variables we choose the energy of an incident electron q_0, the energy of a γ quantum ω and three angles: the angle θ between the directions of the momenta $\vec{\kappa}$ and $\vec{q}$, the angle θ' between $\vec{\kappa}$ and $\vec{q}\,'$ and the angle φ between the directions of the normals to the planes $(\vec{\kappa}, \vec{q})$ and $(\vec{\kappa}, \vec{q}\,')$ (lab.system). All the rest variables must be expressed in terms of the independent ones. Then it is necessary to expand the cross section in powers of ω. Since the amplitude we obtained is correct only with an accuracy up to the terms of order $\frac{1}{\omega}$ and the constant we must keep in this expansion only the first two terms.

We have

$$d\sigma = \frac{d\sigma_0}{\omega} + d\sigma_1 \tag{27}$$

For $d\sigma_0$ we get the following expression

$$d\sigma_0 = d\sigma\cdot\omega\Big/_{\omega=0} = \frac{\alpha}{(2\pi)^2}\,\omega^2\left[\left(\frac{q'}{q'\cdot\kappa} - \frac{q}{q\cdot\kappa}\right) - \left(\frac{p'}{p'\cdot\kappa} - \frac{p}{p\cdot\kappa}\right)\right]^2 d\Omega_\kappa\, d\omega\, d\sigma_{sc} \tag{28}$$

Here $d\sigma_{sc}$ is the cross section for elastic scattering of electrons with the energy q_0 by protons, while $\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$. The factor standing in front of $d\sigma_{sc}$ in formula (27) is the probability of the photon radiation when the electron is scattering. For non-relativistic particles it turns into a usual expression for the probability of the dipole emission:

$$\frac{\alpha}{(2\pi)^2}\left[\frac{1}{m}(\vec{q}\,'-\vec{q})\times\vec{n}-\frac{1}{M}(\vec{p}\,'-\vec{p})\times\vec{n}\right]^2\frac{d\omega}{\omega}\,d\Omega_\kappa$$

where $\vec{n}$ is a unit vector in the direction of the photon emission.

In the case of the ultra-relativistic electrons we are interested in we have well-known expression for $d\sigma_{sc}$ [7].

$$d\sigma_{sc}=\left[F_1^2+\left(\frac{\mu_p F_2}{2M}\right)^2\varkappa_1^2+\frac{\varkappa_1^2}{2M^2}(F_1+\mu_p F_2)^2 \operatorname{tg}^2\frac{\theta_1}{2}\right]d\sigma_0 \tag{29}$$

Here

$$d\sigma_0=\frac{\alpha^2\cos^2\frac{\theta_1}{2}}{4q_0^2\left(1+\frac{2q_0}{M}\sin^2\frac{\theta_1}{2}\right)\sin^4\frac{\theta_1}{2}}$$

θ_1 is the angle between $\vec{q}$ and $\vec{q}\,'$, whereas

$$\varkappa_1^2=\varkappa^2\Big/_{\omega=0.}=\frac{4q_0^2\sin^2\frac{\theta_1}{2}}{1+\frac{2q_0}{M}\sin^2\frac{\theta_1}{2}}$$

Expression (28) for $d\sigma_0$ is quite evident; it is determined by the pole term of the amplitude. The method we used allows, in fact, to find the following term of the expansion – $d\sigma_1$. As a result of rather cumbersome calculations we get:

$$\begin{aligned}
d\sigma_1=\frac{\alpha^3}{(2\pi)^2}\Big\{&A_q^2\,\varkappa\cdot\kappa\left[f_1\left(2M^2+\tfrac{1}{2}\varkappa^2\right)-2f^2\varkappa^2\right]-\tfrac{1}{2}A_p^2\,\varkappa\cdot\kappa\, f_1\varkappa^2\\
&-2(A_p\cdot A_q)\,\varkappa\cdot\kappa\,(f_1M^2-f^2\varkappa^2)+f_1(p\cdot q+p\cdot q')(A_q-A_p)\cdot\Big[\Big(\frac{q'}{q'\kappa}+\frac{q}{q\kappa}\Big)\times\\
&\times(p'\kappa+p\kappa)-\Big(\frac{p'}{p'\cdot\kappa}+\frac{p}{p\cdot\kappa}\Big)(q\cdot\kappa+q'\cdot\kappa)-4(p-q')\Big]\\
&-2(A_q-A_p)^2 f_1\left[(p\cdot q)(\kappa q')+(pq')(\kappa q)+\tfrac{1}{2}(p\cdot\kappa)\varkappa^2\right]\\
&-2\varkappa\cdot\kappa((A_q-A_p)\cdot A_q)\Big[4\Big(2F_1F_1'+2\Big(\frac{\mu_p}{2M}\Big)^2F_2F_2'\varkappa^2+\Big(\frac{\mu_p}{2M}F_2\Big)^2\Big)(pq)(p\cdot q')\times\\
&\times\cos^2\frac{\theta_1}{2}-ff'\varkappa^4\Big]+
\end{aligned} \tag{30}$$

$$+(A_q-A_p)^2\Big[4\Big(2F_1F_1'\varkappa^2+2\big(\tfrac{\mu_p}{2M}\big)^2F_2F_2'\varkappa^4+\big(\tfrac{\mu_p}{2M}F_2\big)^2\varkappa^2+f_1\Big)(pq)\times$$

$$\times(pq')\cos^2\frac{\theta_1}{2}+f\varkappa^4(f'\varkappa^2+f)\Big]\frac{dq_0'}{d\omega}\frac{1}{q_0'}\omega\Big\}\frac{q_0'\omega}{\varkappa^4q_0M\left(1+\frac{2q_0}{M}\sin^2\frac{\theta_1}{2}\right)}\Bigg|_{\omega=0}\times$$

$$\times d\Omega_K\,d\omega\,d\Omega_{q'}+$$

$$+\frac{\alpha}{(2\pi)^2}\Big[\frac{4(\varkappa\cdot K)}{\varkappa^2}((A_q-A_p)\cdot A_q)\omega+(A_q-A_p)^2\frac{1}{q_0}\left(1+\frac{2q_0}{M}\sin^2\frac{\theta}{2}\right)\omega^2$$

$$+\frac{2}{(p'\cdot K)}\omega\Big]_{\omega=0}d\omega\,d\Omega_K\,d\sigma_{sc}+2\frac{\alpha}{(2\pi)^2}\Big[\Big(1+\frac{m^2\omega}{q_0'(q'\cdot K)}\cos\theta'\Big)\Big(\frac{m^2}{(q'\cdot K)^2}-$$

$$-\frac{(p'q')}{(p'\cdot K)^2}-\frac{(p'\cdot q)(q'\cdot K)}{(p'\cdot K)^2(q\cdot K)}\Big)+\frac{m^2\omega\cos\theta'}{(q'\cdot K)^2q_0}\Big(\frac{(qq')}{(q\cdot K)}+\frac{(p'q')}{(p'\cdot K)}-\frac{(pq')}{(p\cdot K)}\Big)-$$

$$-\frac{M^2(q'\cdot K)}{(p'\cdot K)^3}+\frac{(pq')}{(p\cdot K)(p'\cdot K)}-\frac{(p\cdot p')(q'\cdot K)}{(p\cdot K)(p'\cdot K)^2}+\frac{m^2}{(q'\cdot K)(p'\cdot K)}-\frac{(q\cdot q')}{(p'\cdot K)(q\cdot K)}\Big]\frac{dq_0'}{d\omega}\frac{1}{q_0'}\omega^2\Big|_{\omega=0}d\omega\,d\Omega_K\,d\sigma_{sc}$$

Here

$$A_q=\left(\frac{q'}{q'\cdot K}-\frac{q}{q\cdot K}\right);$$

$$A_p=\left(\frac{p'}{p'\cdot K}-\frac{p}{p\cdot K}\right);$$

$$f=F_1+\mu_pF_2;$$

$$f_1=F_1^2+\left(\frac{\mu_pF_2}{2M}\right)^2\varkappa^2;$$

$$\frac{dq_0'}{d\omega}\frac{1}{q_0'}\Big|_{\omega=0}=\frac{2\sin^2\frac{\theta'}{2}}{M\left(1+\frac{2q_0}{M}\sin^2\frac{\theta_1}{2}\right)}-\frac{1}{q_0}\left(1+\frac{2q_0}{M}\sin^2\theta/2\right);$$

$$F_1'=\frac{dF_1}{d\varkappa^2},\ \ldots$$

All the magnitudes in formula (30) are taken at $\omega = 0$. Thus, in the approximation we are considering the differential cross section for bremsstrahlung of quanta in e-p scattering is entirely deter - mined by the electromagnetic form-factors of a proton F_1 and F_2 and by their derivative with respect to the 4-momentum transfer. Thereby, the experimantal investigation of low-energy bremsstrahlung would allow to obtain additional information on the behaviour of these important magnitudes.

The authors are greateful to S.S.Gernstein, P.S.Isaev, A.A.Logunov, and J.A.Smorodinski for fruitful discussion of the problems treated in this paper.

Received by Publishing Department
on August 15, 1960.

References

1. W.K.H.Panofsky and E.A.Allton. Phys.Rev. 110, 1155 (1958).
2. А.А.Логунов, ДАН 117, 792 /1957/.
   A.A.Logunov and L.D.Solovyov. Nucl.Phys. 10,60 (1959).
   S.Fubini, Y.Nambu anf V.Watagin. Phys.Rev. 111, 329 (1958).
3. F.E.Low, Phys.Rev. 110, 974 (1958).
4. E.Kazes, Nuovo Cimento, 13, 1326 (1959).
5. M.Gell-Mann and M.L.Goldberger. Phys.Rev. 96, 1433 (1954).
6. P.S.Isaev and I.S.Zlatev. Nucl.Phys. 16, 608 (1960).
7. M.Rosenbluth. Phys.Rev. 79, 615 (1950).