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ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

ЛАБОРАТОРИЯ ЯДЕРНЫХ ПРОБЛЕМ

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ON THE COMPLETE SET OF EXPERIMENTS ON THE DETERMINATION OF THE AMPLITUDE RATIOS OF PIONS PRODUCTION BY NUCLEONS IN DIFFERENT ISOTOPIC SPIN STATES

ЖЭТФ, 1961, т40, в2, с 805.
Intern. Conf. on High Energy Physics. Rochester, 1960, p 194-195.

Дубна 1960 год

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ON THE COMPLETE SET OF EXPERIMENTS ON THE DETERMINATION OF THE AMPLITUDE RATIOS OF PIONS PRODUCTION BY NUCLEONS IN DIFFERENT ISOTOPIC SPIN STATES

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ABSTRACT

The complete set of experiments is discussed which is concerned with the determination of the amplitude ratios of pion production by nucleons in different isotopic spin states.

The number of experiments carried out up to now at 660 MeV nucleon energy is inadequate to determine all the ratios between the amplitudes.

One of the experiments under discussion can be rather a sensitive test on the correctness of the resonance theory of π-mesons production by nucleons.

INTRODUCTION

Attempts to create the theory of π-meson production by nucleons made by some authors ended usually in phenomenological treatments which provided quantitative explanation of available experimental data. Thus, e.g. Watson and Brueckner [1] performed the partial analysis of meson production by nucleons. They assumed that a small number of states with definite orbital moments should take part in the process. The matrix elements of processes were assumed to be constant values; the principle of charge invariance, conservation laws and also the influence of attractive interaction of nucleons in the final state were taken into account. Basing on such a phenomenological approach Rosenfeld [2] as well as Gell-Mann and Watson [3] performed the analysis of the data of experiments carried out at the energy near the threshold. They used Brueckner's [4] hypothesis that at a nucleon energy close to the threshold one of the secondary nucleons and a π-meson produced are mainly in the resonance state (3/2, 3/2).

S.Mandelstam[5] extended this phenomenological theory to the region of higher energies (from 400MeV up to 600 MeV). In his consideration matrix elements were supposed to be constant values; resonance πN-interaction (3/2, 3/2) was introduced; all the kinematic factors and the rules of addition of nuclear angular momenta and particle spins were taken into consideration. According to this theory which provides the best explanation of all the experimental data obtained with a proton beam in the above-mentioned energy region, there should not be π-meson production by nucleons even of small intensities at states with isotopic spin equal to zero ($T_{NN} = 0$). However, experimental data [6,7] show that the prediction of Mandelstam's resonance theory is in contradiction with the experiment.

Can we draw a conclusion from this on the incorrectness of Mandelstam's resonance theory? In order to solve the problem, it is necessary to take into consideration not only resonance interaction in the πN –system with isotopic spin $T = 3/2$ but also with isotopic spin $T = 1/2$. Undoubtedly, in the most general case both the resonances will be observed. But Mandelstam's resonance model is compatible only with the predominance of the resonance $T = 3/2$. Thus, the resonance theory can be tested directly if one determines ratios between the probabilities of these two resonance interactions. However, the number of experiments performed up to now is inadequate to find the required ratios. All these circumstances made us perform anew a more detailed discussion on the complete set of experiments concerned with the determination of ratios between the amplitudes of π –meson production by nucleons in different isotopic spin states.

I. GENERAL FORMALISM

As is known, the phenomenological description of π –meson production by nucleons in the isotopic spin comes to the introduction of three independent amplitudes. The probability of any process of π– meson production by two nucleons is clearly expressed by three independent amplitudes. If the amplitude of meson production in the collision of two nucleons N_1 and N_2 is expressed by $M(N_1 N_2 \to N_1' N_2' \pi)$, where N_1' and N_2' are secondary nucleons, this amplitude will change with the interchange of nucleons N_1, N_2 or N_1', N_2'. If the charge symmetry is taken into account the number of different processes of π –meson production by nucleons amounts to seven [8]

In one of the presentations when the subsystem of two nucleons is introduced in the initial state, the amplitudes of different processes are expressed as follows

$$\left.\begin{aligned}
&M(pp \to np\pi^+) = \frac{1}{\sqrt{2}} F_{10} + \frac{1}{2} F_{11}; \quad && M(pn \to nn\pi^+) = \frac{1}{\sqrt{6}} F_{01} - \frac{1}{2} F_{11}; \\
&M(pp \to pn\pi^+) = -\frac{1}{\sqrt{2}} F_{10} + \frac{1}{2} F_{11}; && M(np \to nn\pi^+) = \frac{1}{\sqrt{6}} F_{01} + \frac{1}{2} F_{11}; \\
&M(pp \to pp\pi^0) = -\frac{1}{\sqrt{2}} F_{11}; && M(np \to np\pi^0) = -\frac{1}{2\sqrt{3}} F_{01} + \frac{1}{2} F_{10}; \\
& && M(np \to pn\pi^0) = -\frac{1}{2\sqrt{3}} F_{01} - \frac{1}{2} F_{10};
\end{aligned}\right\} \quad (1)$$

Here F_{1j} denotes three amplitudes of reactions in isotopic space. The first subscript shows isotopic spin of the two nucleon system in the initial state, the second one – in the final state.

In another presentation when the subsystem of a π –meson and a nucleon is introduced in the final state, the amplitudes of different processes according to [9] are

$$\left.\begin{aligned}
&M(pp\to np\pi^+)=\frac{\sqrt{3}}{2}A_{13}; &&M(pn\to nn\pi^+)=-\frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{6}}A_{13}+\frac{1}{\sqrt{3}}A_{11}-\frac{1}{\sqrt{3}}A_{01}\right];\\
&M(pp\to pn\pi^+)=-\frac{1}{2\sqrt{3}}A_{13}+\sqrt{\frac{2}{3}}A_{11}; &&M(np\to nn\pi^+)=\frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{6}}A_{13}+\frac{1}{\sqrt{3}}A_{11}+\frac{1}{\sqrt{3}}A_{01}\right];\\
&M(pp\to pp\pi^0)=-\frac{1}{\sqrt{6}}A_{13}-\sqrt{\frac{1}{3}}A_{11}; &&M(np\to np\pi^0)=\frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{3}}A_{13}-\frac{1}{\sqrt{6}}A_{11}-\frac{1}{\sqrt{6}}A_{01}\right];\\
& &&M(np\to pn\pi^0)=-\frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{3}}A_{13}-\frac{1}{\sqrt{6}}A_{11}+\frac{1}{\sqrt{6}}A_{01}\right];
\end{aligned}\right\}\quad (2)$$

where A_{ij} - are amplitudes of reactions in isotopic space. The first subscript shows isotopic spin of the two–nucleon system in the initial state, and the second subscript $j=2T_{\pi N}$, where $T_{\pi N}$ is isotopic spin of the subsystem consisting of a π – meson and a nucleon.

Differential cross sections of the quoted processes are written in the following way

$$\left.\begin{aligned}
\text{I.}\ \ & d\sigma(pp\to np\pi^+)=\frac{1}{2}|F_{10}|^2+\frac{1}{4}|F_{11}|^2+\frac{1}{\sqrt{2}}|F_{11}|\cdot|F_{10}|\cos\phi_{10,11}=\\
&\qquad =\frac{3}{4}|A_{13}|^2;\\
\text{II.}\ \ & d\sigma(pp\to pn\pi^+)=\frac{1}{2}|F_{10}|^2+\frac{1}{4}|F_{11}|^2-\frac{1}{\sqrt{2}}|F_{11}|\cdot|F_{10}|\cos\phi_{10,11}=\\
&\qquad =\frac{1}{3}\left[\frac{1}{4}|A_{13}|^2+2|A_{11}|^2-\sqrt{2}\cdot|A_{13}|\cdot|A_{11}|\cos\varphi_{13}\right];\\
\text{III.}\ \ & d\sigma(pp\to pp\pi^0)=\frac{1}{2}|F_{11}|^2=\\
&\qquad =\frac{1}{3}\left[\frac{1}{2}|A_{13}|^2+|A_{11}|^2+\sqrt{2}|A_{13}|\cdot|A_{11}|\cos\varphi_{13}\right];\\
\text{IV.}\ \ & d\sigma(pn\to nn\pi^+)=\frac{1}{6}|F_{01}|^2+\frac{1}{4}|F_{11}|^2-\frac{1}{\sqrt{6}}|F_{01}|\cdot|F_{11}|\cos\phi_{01,11}=\\
&\qquad =\frac{1}{6}\left[\frac{1}{2}|A_{13}|^2+|A_{11}|^2+|A_{01}|^2+\sqrt{2}|A_{13}|\cdot|A_{11}|\cos\varphi_{13}-\right.\\
&\qquad \left.-2|A_{11}|\cdot|A_{01}|\cos\varphi_{01}-\sqrt{2}\cdot|A_{13}|\cdot|A_{01}|\cos\varphi_{03}\right];
\end{aligned}\right\}\quad (3)$$

$$\text{V}.\ d\sigma(np\to nn\pi^+) = \frac{1}{6}|F_{01}|^2 + \frac{1}{4}|F_{11}|^2 + \frac{1}{\sqrt{6}}|F_{01}|\cdot|F_{11}|\cos\phi_{01,11} =$$
$$= \frac{1}{6}\Big[\frac{1}{2}|A_{13}|^2 + |A_{11}|^2 + |A_{01}|^2 + \sqrt{2}|A_{13}|\cdot|A_{11}|\cos\varphi_{13} +$$
$$+ 2|A_{11}|\cdot|A_{01}|\cos\varphi_{01} + \sqrt{2}|A_{13}|\cdot|A_{01}|\cos\varphi_{03}\Big];$$

$$\text{VI}.\ d\sigma(np\to np\pi^0) = \frac{1}{12}|F_{01}|^2 + \frac{1}{4}|F_{10}|^2 - \frac{1}{2\sqrt{3}}|F_{01}|\cdot|F_{10}|\cos\phi_{01,10} =$$
$$= \frac{1}{6}\Big[|A_{13}|^2 + \frac{1}{2}|A_{11}|^2 + \frac{1}{2}|A_{01}|^2 - \sqrt{2}|A_{13}|\cdot|A_{11}|\cos\varphi_{13} -$$
$$- 2|A_{13}|\cdot|A_{01}|\cos\varphi_{03} + |A_{11}|\cdot|A_{01}|\cdot\cos\varphi_{01}\Big];$$

$$\text{VII}.\ d\sigma(np\to pn\pi^0) = \frac{1}{12}|F_{01}|^2 + \frac{1}{4}|F_{10}|^2 + \frac{1}{2\sqrt{3}}|F_{01}|\cdot|F_{10}|\cos\phi_{01,10} =$$
$$= \frac{1}{6}\Big[|A_{13}|^2 + \frac{1}{2}|A_{11}|^2 + \frac{1}{2}|A_{01}|^2 + \sqrt{2}|A_{13}|\cdot|A_{11}|\cos\varphi_{13} +$$
$$+ \sqrt{2}|A_{13}|\cdot|A_{01}|\cos\varphi_{03} - |A_{11}|\cdot|A_{01}|\cos\varphi_{01}\Big].$$

2. METHOD OF MEASURING INTERFERENCE EFFECTS

Processes (I) and (II), (IV) and (V) as well as (VI) and (VII) in expression (3) differ from each other due to interference between two amplitudes. The establishment of difference between the two processes gives rather essential additional data. Consider methods permitting to observe this difference experimentally.

A. If a proton and a neutron are interchanged in the initial state [processes (IV) and (V)] , the difference between these two processes can be determined owing to charge symmetry by comparing the processes $pn\to nn\pi^+$ and $pn\to pp\pi^-$ at the same π-meson angle.

B. If a neutron and a proton are interchanged in the final state [processes (I) and (II) or (VI) and (VII)] , it is necessary to register simultaneously two particles, e.g. a π -meson and one of the nucleons, in order to establish difference between the probabilities of these processes. Here, we must decide which of the nucleons should be called the first one. Let us stick to the determination given by Fermi [10]. According to this determination the first nucleon is that the impulse projection of which to the π-meson impulse direction in the centre of mass system of two colliding nucleons is the greatest by its algebraic value.

So far as it is necessary to determine only ratios between the amplitudes F (or A) and no determination of spin and angular dependence of these amplitudes is required, then the aim of our experiments should be the measurement of the corresponding total cross sections of the processes.

If only a π -meson is registered in the experiment, no difference between processes (I) and (II) as well as (VI) and (VII) should be established. We shall measure only summary probabilities of each pair of processes.

The corresponding total cross sections are

$$\left.\begin{aligned} \sigma(pp\to\pi^+) &= \int \frac{d\sigma}{d\Omega}(pp\to np\pi^+)\,d\Omega + \int \frac{d\sigma}{d\Omega}(pp\to pn\pi^+)\,d\Omega\,; \\ \sigma(np\to\pi^0) &= \int \frac{d\sigma}{d\Omega}(np\to np\pi^0)\,d\Omega + \int \frac{d\sigma}{d\Omega}(np\to pn\pi^0)\,d\Omega. \end{aligned}\right\} \quad (4)$$

In order to determine difference between processes (I) and (II) as well as between (V) and (VII) it is necessary to measure the total cross section of each of the two processes separately.

I) the process when a proton is the first nucleon, and
2) when a neutron is the first nucleon.

In a general case, each spin component of wave functions of the final state of the process $NN\to NN'\pi$ is a function of π -meson impulse, the direction of π -meson emission (θ_π,φ_π) as well as the direction of nucleon flight $(\theta_{12},\varphi_{12})$. The angles $(\theta_{12},\varphi_{12})$ are counted from the vector of π -meson impulse in the centre-of-mass system of two secondary nucleons. When discussing the problem on the total cross sections, it is suffice to consider the differential cross section averaged over the asimuthal angles φ_π and φ_{12}. If one confines oneself only to S- and P-waves of a π- meson and nucleons, these averaged cross sections will be of the form

$$\left.\begin{aligned} d\sigma^{pp}_{np\pi^+}(\theta_\pi,\theta_{12};p_\pi) &\sim a_0(p_\pi) + a_1(p_\pi)\cos\theta_{12}\cos\theta_\pi + a_2(p_\pi)\cos^2\theta_{12} + \\ &\quad + a_3(p_\pi)\cos^2\theta_\pi + a_4(p_\pi)\cos^2\theta_{12}\cos^2\theta_\pi\,; \\ d\sigma^{pp}_{pn\pi^+}(\theta_\pi,\theta_{12};p_\pi) &\sim a_0(p_\pi) - a_1(p_\pi)\cos\theta_{12}\cos\theta_\pi + a_2(p_\pi)\cos^2\theta_{12} + \\ &\quad + a_3(p_\pi)\cos^2\theta_\pi + a_4(p_\pi)\cos^2\theta_{12}\cos^2\theta_\pi\,, \end{aligned}\right\} \quad (5)$$

where $a_i(p_\pi)$ are functions of π -meson impulse.

It is seen from (5) that the measure of distinguishing between the probabilities of the processes $pp \to np\pi^+$ and $pp \to pn\pi^+$ is the coefficient a_1 . However, the total cross section of the processes $pp \to np\pi^+$ and $pp \to pn\pi^+$ does not depend upon this coefficient since the difference between the differential cross sections of processes under investigation changes its sign to the opposite one as a result of transformations $\theta_\pi \to (\pi - \theta_\pi)$ and $\theta_{12} \to (\pi - \theta_{12})$. Hence, it follows that the difference of differential cross sections of these processes should be measured only in the angle range $0 < \theta_\pi < \pi/2$, $0 < \theta_{12} < \pi/2$, the averaging over relative asimuthal angles joining the cross section as cos $(\varphi_\pi - \varphi_{12})$ and cos 2(φ_π - φ_{12}) being performed [II].

Thus, the subtracted total cross sections $\Delta\sigma$ to be determined below correspond to the difference between the probabilities of processes (I) and (II)

$$\Delta\sigma_{10,11} = 4\int_0^{\pi/2}\int_0^{\pi/2}\left[d\sigma^{pp}_{np\pi^+}(\theta_\pi,\theta_{12}) - d\sigma^{pp}_{pn\pi^+}(\theta_\pi,\theta_{12})\right] d\Omega(\theta_\pi)\, d\Omega(\theta_{12}), \qquad (6)$$

and in the same way - (VI) and (VII)

$$\Delta\sigma_{01,10} = 4\int_0^{\pi/2}\int_0^{\pi/2}\left[d\sigma^{np}_{np\pi^0}(\theta_\pi,\theta_{12}) - d\sigma^{np}_{pn\pi^0}(\theta_\pi,\theta_{12})\right] d\Omega(\theta_\pi)\, d\Omega(\theta_{12}). \qquad (7)$$

The difference between processes (IV) and (V) can be written simplier

$$\Delta\sigma_{01,11} = 2\int_0^{\pi/2}\left[d\sigma^{pn}_{nn\pi^+}(\theta_\pi) - d\sigma^{np}_{nn\pi^+}(\theta_\pi)\right] d\Omega(\theta_\pi) =$$

$$= 2\int_0^{\pi/2}\left[d\sigma^{np}_{pp\pi^-}(\theta_\pi) - d\sigma^{np}_{nn\pi^+}(\theta_\pi)\right] d\Omega(\theta_\pi). \qquad (8)$$

As in the previous two cases the subtracted cross section determined in (8) is a measure of the coefficient „b" in the meson angular distribution

$$d\sigma(np \to \pi^-) = a + b\cos\theta_\pi + c\cdot\cos^2\theta_\pi ,$$

the contribution of which to a usual total cross section is, naturally, equal to zero.

3. RATIOS BETWEEN THE AMPLITUDES WITH DIFFERENT ISOTOPIC SPIN STATES

In order to find three amplitudes F_{ij} or A_{ij} - three absolute values and three phase ratios between them (three dimensional case) - one may employ any six independent equations of seven possible:

$$\left.\begin{aligned}
&\sigma(pp\to\pi^+) = |F_{10}|^2 + \tfrac{1}{2}|F_{11}|^2; && \sigma(np\to\pi^0) = \tfrac{1}{6}|F_{01}|^2 + \tfrac{1}{2}|F_{10}|^2;\\
&\sigma(pp\to\pi^0) = \tfrac{1}{2}|F_{11}|^2; && \sigma(np\to\pi^+) = \tfrac{1}{3}|F_{01}|^2 + \tfrac{1}{2}|F_{11}|^2;\\
&\Delta\sigma_{10,11} = \sqrt{2}\,\Omega_{10,11}; && \Delta\sigma_{11,01} = \sqrt{\tfrac{2}{3}}\,\Omega_{11,01};\\
& && \Delta\sigma_{10,01} = \sqrt{\tfrac{4}{3}}\,\Omega_{10,01}
\end{aligned}\right\} \quad (9)$$

where $\Omega_{ij,kl} = |F_{ij}|\cdot|F_{kl}|\cos\phi_{ij,kl}$.

One of the equations connecting usual total cross sections of the processes is not independent since there is one ratio between the total cross sections:

$$\sigma(pp\to\pi^+) + \sigma(np\to\pi^+) + \sigma(np\to\pi^-) = 2\left[\sigma(pp\to\pi^0) + \sigma(np\to\pi^0)\right].$$

The amplitudes F_{ij} are obtained from the experimentally observed values with the help of the relations

$$\left.\begin{aligned}
&|F_{10}|^2 = \sigma(pp\to\pi^+) - \sigma(pp\to\pi^0); && \Omega_{10,11} = \tfrac{1}{\sqrt{2}}\Delta\sigma_{10,11};\\
&|F_{11}|^2 = 2\sigma(pp\to\pi^0); && \Omega_{11,01} = \sqrt{\tfrac{3}{2}}\Delta\sigma_{11,01};\\
&|F_{01}|^2 = 3\left[\sigma(np\to\pi^+) + \sigma(np\to\pi^-) - \sigma(pp\to\pi^0)\right]; && \Omega_{10,01} = \sqrt{3}\,\Delta\sigma_{10,01}
\end{aligned}\right\} \quad (10)$$

If F_{ij} are found, the values A_{ij} can be determined from the following relations

$$\left.\begin{array}{lll}\vec{A}_{13}=\sqrt{\frac{2}{3}}\vec{F}_{10}+\sqrt{\frac{1}{3}}\vec{F}_{11}; & A_{13}^2=\frac{2}{3}F_{10}^2+\frac{1}{3}F_{11}^2+\frac{2\sqrt{2}}{3}\Omega_{10,11}; & \omega_{13}=\frac{\sqrt{2}}{3}F_{11}-\frac{\sqrt{2}}{3}F_{10}^2+\frac{1}{3}\Omega_{10,11};\\ \vec{A}_{11}=-\sqrt{\frac{1}{3}}\vec{F}_{10}+\sqrt{\frac{2}{3}}\vec{F}_{11}; & A_{11}^2=\frac{1}{3}F_{10}^2+\frac{2}{3}F_{11}^2-\frac{2\sqrt{2}}{3}\Omega_{10,11}; & \omega_{03}=\sqrt{\frac{2}{3}}\Omega_{10,01}+\sqrt{\frac{1}{3}}\Omega_{11,01};\\ \vec{A}_{01}=\vec{F}_{01}; & A_{01}^2=F_{01}^2; & \omega_{01}=-\sqrt{\frac{1}{3}}\Omega_{10,01}+\sqrt{\frac{2}{3}}\Omega_{11,01}\end{array}\right\}$$

or in a distinct form by means of the observed values

$$\left.\begin{array}{l}|A_{13}|^2=\frac{2}{3}\sigma(pp\to\pi^+)+\frac{2}{3}\Delta\sigma_{10,11}; \quad \omega_{13}=\frac{\sqrt{2}}{3}\left[3\sigma(pp\to\pi^0)-\sigma(pp\to\pi^+)\right]+\frac{1}{3\sqrt{2}}\Delta\sigma_{10,11}\\ |A_{11}|^2=\sigma(pp\to\pi^0)+\frac{1}{3}\sigma(pp\to\pi^+)-\frac{2}{3}\Delta\sigma_{10,11}; \quad \omega_{01}=\Delta\sigma_{11,01}-\Delta\sigma_{10,01};\\ |A_{01}|^2=3\left[\sigma(np\to\pi^+)+\sigma(np\to\pi^-)-\sigma(pp\to\pi^0)\right]; \quad \omega_{03}=\sqrt{2}\,\Delta\sigma_{10,01}+\frac{1}{\sqrt{2}}\Delta\sigma_{11,01}\end{array}\right\} \quad (12)$$

where $\omega_{ij}=|A_{1i}|\cdot|A_{j1}|\cos\varphi_{ij}$.

4. EXPERIMENTS WHICH HAVE BEEN PERFORMED WITH A NEUTRON BEAM

The investigation of the reactions $np\to pp\pi^-$ and $np\to nn\pi^+$ at 600 MeV neutron energy[7] permits to conclude that $|F_{01}|\neq 0$. From the same data it follows that $\Delta\sigma_{01,11}$ is very close to zero in this energy range. This means that the amplitudes F_{01} and F_{11} are nearly orthogonal with respect to each other. It is required to measure the subtracted cross section $\Delta\sigma_{01,10}$ with a neutron beam. This will allow to find a phase ratio and to improve essentially the reliability of the value $|F_{01}|$.

5. EXPERIMENTS WHICH HAVE BEEN PERFORMED WITH A PROTON BEAM

The cross sections $\sigma(pp\to\pi^+)$[12] and $\sigma(pp\to\pi^0)$[13] have been measured with a 600 MeV proton beam. These data permit to determine $|F_{10}|$ and $|F_{11}|$. The values A_{13} and A_{11} are of great interest at 660 MeV proton energy, i.e., in the resonance region (3/2, 3/2) where A_{13} should predominate. However, the relation between A_{13} and A_{11} can be found if one measures $\Delta\sigma_{10,11}$.

According to Mandelstam's resonance model, $|A_{11}| = 0$, from where the well-known ratio

$$\frac{\sigma_+}{\sigma_0} = 5$$

arises as a consequence.

In fact we observe experimentally the values

$$\frac{\sigma_+}{\sigma_0} = 3{,}4$$

which is possible to explain theoretically in a somewhat artificial way. Practically, it means that $A_{11} \neq 0$.

For the sake of more detailed consideratior of the problem on permissable values of A_{13} and A_{11} it is worth while to introduce the following notations :

$$\alpha = \frac{\sigma_+}{\sigma_c}, \quad k = \frac{|A_{13}|}{|A_{11}|}.$$

Fig. 1 presents the region of permissable values α and k which is filled with a family of curves:

$$\alpha = \frac{4 + 5k^2 - \sqrt{8}\, k \cos\varphi_{13}}{2 + k^2 + \sqrt{8}\, k \cos\varphi_{13}} \tag{13}$$

with the parameter $\cos\varphi_{13}$. This region is limited from below by the curve

$$\alpha_{\varphi_{13}=0} = \frac{4 + 5k^2 - \sqrt{8}\, k}{2 + k^2 + \sqrt{8}\, k}, \tag{14}$$

which has the horizontal asymptotes $\alpha = 2(k \to 0)$ and $\alpha = 5(k \to \infty)$ and reaches the minimum $\alpha = 1$ with $k = \frac{1}{\sqrt{2}}$.

From above the region is limited by two branches of the curve

$$\alpha_{\varphi_{13}=\pi} = \frac{4 + 5k^2 + \sqrt{8}\, k}{2 + k^2 - \sqrt{8}\, k}, \tag{15}$$

which has two horizontal asymptotes $\alpha = 2(k \to 0)$ and $\alpha = 5(k \to \infty)$ and the common vertical asymptote with $k = \sqrt{2}$.

The curves $\alpha = \alpha(k; \cos\varphi_{13})$ have their extremum when

$$\alpha = \frac{5k^4 + k^2 - 4}{k^4 - k^2 - 2} \tag{16}$$

except the monotonic curve $\alpha_{1/2} = 5 - \frac{6}{k^2+2}$, corresponding to $\varphi_{13} = \pi/2$.

From Fig. 1 it is seen that if $\alpha = \frac{\sigma_+}{\sigma_0} = 3.4$, then the permissable values of k^2 are in the interval

$$\frac{1}{20} < k^2 < 64 \qquad (17)$$

As for other peculiarities of this region, it should be noted that if $\cos\phi_{10,11} = \pm 1$, then $\cos\varphi_{13} = \pm 1$, respectively. However $\phi_{10,11} = \pi/2$ corresponds to $\varphi_{13} = \pi/2$ only for $\alpha = 3$. In other cases the orthogonality $\vec{F}_{10}$, $\vec{F}_{11}$ does not mean the orthogonality $\vec{A}_{13}$, $\vec{A}_{11}$. The following relation

$$\cos\varphi_{13} = \frac{\sqrt{\alpha-1}\cos\phi_{10,11} + (3-\alpha)}{\sqrt{\alpha^2 + 3\alpha - 2(3-\alpha)\sqrt{\alpha-1}\cos\phi_{10,11} - 8(\alpha-1)\cos^2\phi_{10,11}}}, \qquad (18)$$

takes place. If $\alpha = \frac{\sigma_+}{\sigma_0} = 5$, the permissable values of k^2 are limited by the interval $\frac{1}{8} < k^2 < \infty$. Thus, the fact that $\frac{\sigma_+}{\sigma_0} = 5$ should not be considered as a proof of the resonance theory correctness.

Attention should be payed also to the fact that if k is constant but the relative phase A_{13} and A_{11} changes due to some reasons, then as a result of it the value α will change. From the point of view of the authors such a process can take place in -meson production on coupled nuclear nuclei, in particular, on a deutron.

Fig.1 shows also the curve corresponding to $\Delta\sigma_{10,11} = 0$, $\alpha = \frac{3k^2}{2-k^2}$, i.e., to the case of the symmetric emission of a proton and neutron relative to a $\pi^{\pm}$-meson impulse.

It is of interest to note that if α is rather large (~10 and more), that takes place in the energy range close to the threshold, then $|A_{13}| \sim 1{,}5 \cdot |A_{11}|$, and the difference of phases (to an accuracy of π) is very small. Nearly the same picture may be expected if one takes into consideration that near zero π-meson energy the phases A_{13} and A_{11} should be small and, hence, their difference should be small too.

6. SCHEME OF THE EXPERIMENT TO BE PERFORMED WITH WITH A660 MEV PROTON BEAM

The scheme of the experiment for measuring $\Delta\sigma_{10,11}$ to be performed with a 660 MeV proton beam is characterized by the following values: $\theta_{\pi}^{c.m.s.} = 55^{\circ}$ (at this angle it is easy to combine the differential and total cross sections under the condition that S- and P-mesons should predominate; if $E_{\pi}^{c.m.s.} =$ 80 MeV, then $\theta_{\pi}^{lab.} = 30^{\circ}$, $E_{\pi}^{lab.} = 170$ MeV. If one chooses $\theta_{12}=30^{\circ}$, the angles of flight of the first and second protons are rather convenient ($\theta_{p1}^{lab.} = 33^{\circ}$ and $\theta_{p2}^{lab} = 20^{\circ}$) while their energies are $E_{p1}^{lab.} = 180$ MeV, $E_{p2}^{lab.} = 160$ MeV.

If one changes θ_{12}, then either $\theta_{p2}^{lab.}$ will be greatly overlapped with respect to a π -meson angle or the angle $\theta_{p1}^{lab.}$ will be very small.

There is another variant:
$\theta_{\pi}^{c.m.s.}=125^{\circ}$, $\theta_{12}^{c.m.s.}= 39^{\circ}$, $E_{\pi}^{lab} = 57$ MeV, $\theta_{p1}^{lab} = 26^{\circ}$, $\theta_{p2}^{lab} = 18^{\circ}$, $\theta_{\pi}^{lab} = 83^{\circ}$, $E_{p1}^{lab} = 264$MeV, $E_{p2}^{lab} = 190$MeV. In this case proton angles are somewhat smaller but π^{+}-mesons are easier to register in view of the smaller background at 83°.

Fig. 2 shows a diagram presenting permissable values $|A_{13}|^2$, $|A_{11}|^2$ and $\Omega_{3,1}$ with respect to the permissable values of the subtracted cross section $\Delta\sigma_{10,11}$. Along the absciss and ordinate coordinates there are the above values in units of 10^{-27} cm ^{2}. It is seen from Fig. 2 that if $\Delta\sigma_{10,11}=0$, then $|A_{13}|^2 \approx |A_{11}|^2$ and these amplitudes are nearly orthogonal. The greatest permissable cross section $\Delta\sigma_{10,11}$ amounts to $9.9.10^{-27}$ cm^2 at 660 MeV proton energy.

CONCLUSION

I. In order to solve the problem on the correctness of the resonance theory of π -meson production by nucleons at 660 MeV nucleon energy, it is necessary and sufficient to measure the subtracted total cross section $\Delta\sigma_{10,11}$ concerned with asymmetry in proton and neutron emission relative to π^{+} -meson direction.

2. The region of permissable values $\frac{|A_{13}|}{|A_{11}|}$ with different values $\alpha = \frac{\sigma(pp\to\pi^{+})}{\sigma(pp\to\pi^{0})}$ is considered.
3. The experimental scheme for measuring $\Delta\sigma_{10,11}$ at 660MeVenergy is given

The authors wish to express their gratitude to L.I.Lapidus for interest to the work and discussions.

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Received by
Publishing Department
on August 13, 1960

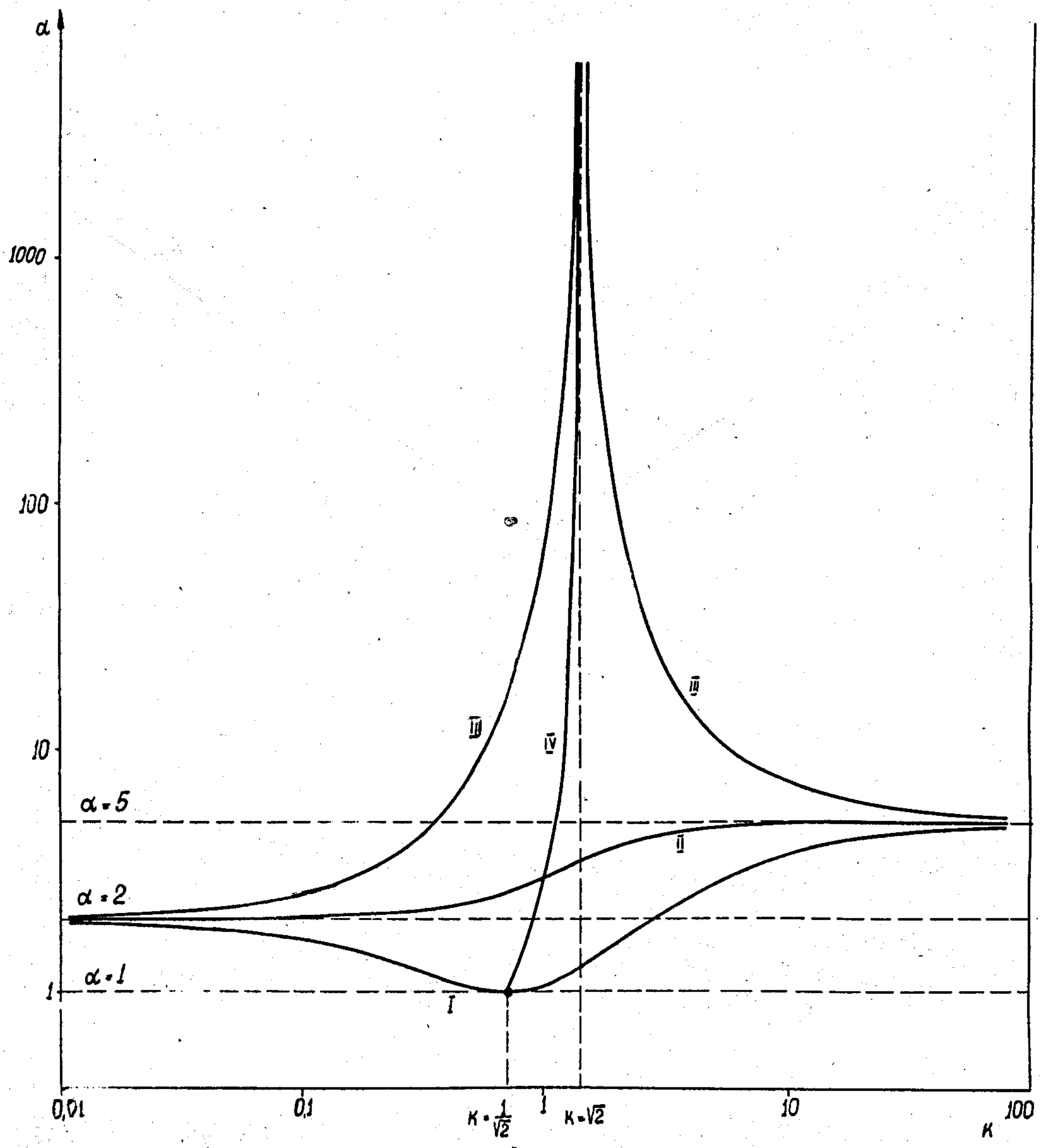

Fig. 1. The permissable region of the values $\alpha = \frac{\sigma(pp \to \pi^+)}{\sigma(pp \to \pi^0)}$ and $k = \frac{|A_{13}|}{|A_{11}|}$.

Curve I corresponds to $\varphi_{13} = 0$

curve II - to $\varphi_{13} = \pi/2$

curve III - to $\varphi_{13} = \pi$

curve IV - to $\Delta\sigma_{10;11} = 0$.

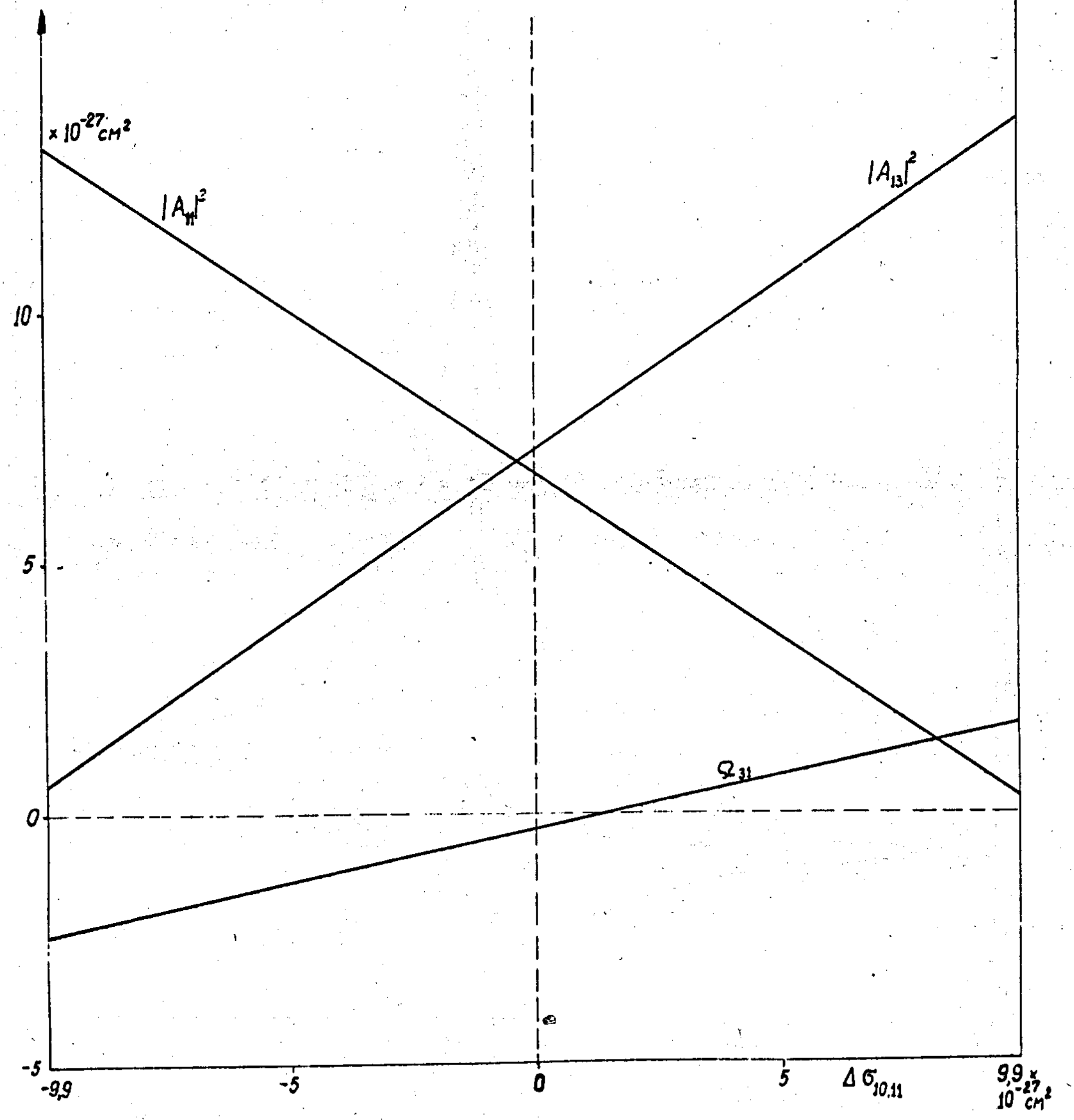

Fig. 2. The dependence of the permissable values $|A_{13}|^2$, $|A_{11}|^2$ and $\Omega_{3,1}$ on the value of the possible values of the subtracted total cross section $\Delta\sigma_{10,11}$ at 660 MeV proton energy ($\alpha = 3.4$).