ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

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ANALYSIS OF EXPERIMENTAL DATA ON THE TOTAL CROSS SECTIONS FOR PION-PROTON INTERACTION

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1. EXPERIMENTAL DATA

Pion scattering by protons is at present one of the most extensively studied processes involving strong interactions. The experimental data available in this field are so rich that they allow to make a complete joint analysis of all the information and to find some of the characteristics of this process with a considerably higher accuracy than by means of partial analyses. This paper is concerned, first of all, with the analysis of all the data on the total cross sections of the interaction of pions with protons, since, on the one hand, the total cross sections include the major part of the information about the process under study and, on the other, they are investigated experimentally most fully.

A complete list of all whatever reliable experimental results on the total cross sections which have been available to the authors of this paper by March 1960 is given in Tables 1 and 2. Yet, some remarks should be made concerning the layout of the material and on the nature of some of the quantities included.

In Column 1 are indicated the numbers, according to the list of references, to the papers in which the data analysed here are published, mentioned, or corrected. The references to experimental papers /1-74/ are given strictly chronologically. Therefore, to a certain extent, one can judge by the number of the reference, when the paper was written. The limiting values of the cross sections at zero kinetic energy of the pions have been calculated according to the results of the analysis /75/, the author of which made use of the papers /18, 20, 24 26, 28, 37, 43, 46, 50/ of our list and of some other independent data. We deliberately excluded, as well, some papers pertaining to the total cross sections only indirectly and, besides, giving insignificant information on them. Excluded are also all the data on scattering of negative pions from paper /61/ since it is well-known that the energy of the incident particles has been determined there utterly incorrectly. The authors of the present paper hope that they missed nothing and did not repeat the results of the same measurements. We apologize beforehand to the authors of the experimental papers for possible inaccuracy in treating the data that may be caused by an incomplete or, in our opinion, by an obscure description of experiments.

Column 2 presents the estimates of the mean kinetic energy of a pion beam and /below/ the estimate of the standard deviation of this mean value. Most of the authors of the experimental papers fail to give the data on the error in the estimation of this mean energy, therefore, the data listed were obtained partially by an additional analysis of the conditions of some experiments. Not all the figures for the standard deviation of the mean energy are quite reliable, but the major part of them produces no effect upon the results of the analysis (see discussion of Column 4). The values of half-width of the energy distribution of the particle beam usually mentioned by the authors are not presented in the Table since the corrections to the original data which are due to the curvature of the investigated cross section in the band of the beam width are almost everywhere negligibly small. In some cases necessary corrections to the energy averaging were included into the figures given in Column 6.

In Column 3 are listed the estimates of the total cross sections for pion interaction with protons which were reported by the authors of the experimental papers or obtained by combining the published data. Below are given the estimates of the standard deviations of these cross sections.

Column 4 shows the standard deviations of the cross sections caused by the errors in the determination of the mean energy of the beams (confluent errors). These standard errors were determined according to formulae of chapter 3 (confluent analysis) of paper /76/; it became possible to neglect everywhere the terms due to the curvature. Then $S_c(\sigma) = S_E \left|\frac{d\sigma}{dE}\right|$ where the derivative has been found on the basis of the results of a preliminary analysis of all the data. To take into account the confluent error, its square must be added to the square of the error given by the author. Therefore, the confluent errors are indicated only in the cases when they yield not too small a contribution to the final error. For negative mesons these errors are almost everywhere small, they affect the final results but slightly, and at the present stage of the analysis it does not seem reasonable to take them into account.

In Column 5 are indicated the corrections on the Coulomb pion-proton scattering and on the interference of this scattering with the nuclear one for the results of transition measurements. These corrections were calculated by using the formulae of paper /77/ and the preliminary data on the angular distributions of the pion - proton scattering. The corrections are not indicated for the cases when they had been already introduced by the authors of the experimental papers, when they turned out to be small or when the measurements were so inaccurate that they are not worth correcting.

Column 6 presents other corrections introduced into the initial cross sections, namely, the corrections on the small-angle nuclear scattering of particles not inducing any weakening of the beam due to the finite dimensions of the counters or which cannot be reliably identified in nuclear emulsion plates. In the same Column are also given the corrections arising from a specification of the data on the angular distributions which were used in calculating the corrections just mentioned, as well as the corrections on the averaging over particle energies.

In the cases when the total cross sections were obtained as a result of the measurement of the angular distributions of elastic pion scattering and charge-exchange scattering, a complete analysis of the angular distributions has been made anew with the introduction at each point of corrections on Coulomb scattering and interference, on the errors in the determination of the angles and on the averaging over the scattering angles. There was no need in repeating the analysis only when all the necessary corrections were introduced by the authors of an experimental paper. In some cases the analysis could not be repeated since relevant papers did not contain the necessary information. The tables of the coefficients of the angular distributions and the analysis of their energy dependence will be given elsewhere. Also, we are going to take into account the results of all relative measurements of angular distributions normalized to some values of the total cross sections. Such measurements, of course, could not be used in the present paper.

In Column 7 are enumerated the final estimates of the cross sections and (below) the estimates of the standard deviations of these cross sections. A part of the data is excluded from a further analysis since either the corresponding points fall out of the groups determined by the latest most accurate measurements, or the data have a relatively great error and a very small weight in all the ensemble of the data, or it became known that in obtaining these experimental results the assumptions used turned out later to be incorrect, or, finally, as it is often the case, there is a combination of these unfavourable circumstances. It was considered reasonable to exclude these data from the analysis, rather than to correct them by means of a displacement or of an increase of their errors, since the latter procedure is quite ambiguous.

It is necessary to explain in more detail the reasons for which some points were excluded from the analysis. At low energies three points are excluded from the data for σ_t^+ : the point from paper /43/ has already been accounted for in the determination of the cross section at E = 0 ; the point from paper /16/ cannot be reliably corrected for Coulomb scattering and for small-angle scattering; the point from paper /57/ falls out. At low energies 6 points are excluded from the data for σ_t^- . Out of these the point from paper /20/ is taken into account in the determination of the cross section at $E = 0$, two points fall out, and three are very weak. The point for σ_t^- at 333 MeV from the paper /56/ is excluded from the analysis since it falls out of a group of points found later on by the same authors and, besides, it is determined from angular distributions, which do not agree well with dispersion relations (see /78/). All the points from paper /47/ are excluded from the data for σ_t^- at high energies, as the major part of them is it in sharp contradiction with the latest and more accurate measurements in this region; the data for σ_t^+ from the same paper do not disagree with other experiments and are included in the analysis. The points of the report /68/ were excluded from the final variant of the analysis as they seem to be preliminary results of paper /71/. The reasons for excluding other points neglected are indicated above and evident from the figures.

As the electronic computer used at an early stage of the analysis had an insufficient extent of memory, some points at the same or at very close energies were combined . Such combinations of points are denoted by parentheses.

In Column 8 are presented the values of the momentum in the center-of-mass-system (in the units of μc), because just this quantity enters the interpolating expressions as an argument.

In Column 9 are given the values of $A \equiv \frac{\sigma_t}{4\pi}$; this ratio enters into further formulae, and is convenient for operating with scattering angular distributions.

Finally, in Column 10 are given the weights with which the individual points enter the analysis. The weights were calculated by the formula $w = \left(\frac{4\pi}{S_\sigma}\right)^2$. For all the points excluded from the analysis a zero weight is indicated. In the course of the analysis the limiting cross sections at $E = 0$ were assumed to be exact.

In Fig. 1 are shown all the points included into the list for σ_t^+ at the energies from zero up to 376 **MeV**, in Fig. 2, from **335 MeV** up to **3.86 BeV**. In Fig. 3 are plotted all the points for σ_t^- from zero up to **373 MeV**, in Fig. 4, from **363 MeV** up to **6.66 BeV**. The experimental points included into the analysis are indicated by full circles, the excluded ones, by empty ones. For each point the standard deviations S_σ and (for π^+) S_E , and the reference number of the experimental paper are shown. The standard deviations smaller than the radius of a circle are not plotted.

Table 1

Experimental data on the total scattering cross sections of positive pions on protons

Reference	E $\pm S_E$ MeV	σ $\pm S_\sigma$ Original mb	S_c mb	Coulomb correction mb	Other corrections mb	σ $\pm S_\sigma$ Final mb	η	A_0 mb/ster	W $(\text{mb/ster})^{-2}$
1	2	3	4	5	6	7	8	9	10
75	0	2,76				2,76	0	0,220	–
		0,53				0,53			
43	2I,5	3,0				3,0			0
	2	I,0				I,0			
I6	33	6,4				6,4			0
		2,I				2,I			
I5	37	II,8			0,75	I2,6	0,655	I,0	I00
	0,5	I,0			0,75	I,2			
57	37	6,5		angular		6,5			0
		0,5				0,5			
42	38	8,8		angular		7,9	0,664	0,63	83
		0,9				I,4			
II	40	I0,9	0,9	angular		I3,3	0,683	I,06	29
	3	3				2,3			
I6	44	9,8	I,5		2,6	I2,4	0,7I8	0,99	2I
	4	I,5			I,7	2,7			
I8	45	I2	0,9	angular		I0	0,727	0,8	34
	3	3				2,2			
7	53	20		angular		I9,2	0,794	I,53	I2
	I0	4				3,6			
4	56	20				20			
	2,7	I0				I0			
I6	56	I7,6	2,5	0,34	I,3	I9,2			
	4	2,2			I,3	3,6	0,830	I,3I	54
20	58	I5,8	I,9	angular		I5,5			
	3	I,5				2			
2	58	27,8			I	28,8			0
		2,5				2,5			
I6	70	I9,0	4,5	0,I4	2,26	2I,4	0,924	I,7I	5,2
	5	2,6			I,8	5,5			

I	2	3	4	5	6	7	8	9	I0
8	75	41			4	45			0
	I,7	I5				I5			
9	78	3I	3,I	angular		3I	0,98I	2,47	9,9
	3	3				4			
5	79	48		0,2	0,I	48,3	0,988	3,84	I,6
	7,5	I0				I0			
4	82	50				50	I,008	3,98	0,9
	I,3	I3				I3			
54	83	34	5,6	angular		34,5	I,0I5	2,75	3,3
	5	4				6,9			
53	I00	62,4		angular		59	I,I26	4,69	2,4
	5	8				8			
5	I09	80	2,8	0,4I	7,6	88			
	I,5	I0				II,4	I,I87	6,36	4
6	II0	74,5	5,7	angular		76,4			
	3	5				7,7			
30	II3	79	5,7	angular		80	I,207	6,37	3,8
	3	5				6,5			
5	II5	95		I,7	2,I	99	I,2I9	7,88	0,7
	I	I5				I5			
4	II8	9I				9I	I,237	7,2	3,7
	0,23	6				6			
52	I20	96,6	II	angular		97,5	I,249	7,75	I,I
	5	8,7				I2			
5	I27	I25		0,77	I3	I38	I,290	II,0	0,5
	I,4	I5				I7,5			
29	I28	I22			-I,7	I20,3	I,295	9,6I	2,7
	0	8				9,3			
5	I33	I35			23	I57			0
	0,75	I5				I9			
29	I35	I26			-0,9	I25,I			
		4				4			
6	I35	I2I		angular		I23	I,336	9,95	I0
	3	I9				I9			
4	I36	I52				I52	I,342	I2,I	0,6
	0,43	I6				I6			

I	2	3	4	5	6	7	8	9	I0
38	I40 0,6	I33 8		2		I35 8	I,364	I0,75	2,4
29	I42	I50 8				I50 8	I,380	I0,54	5
58	I43 2	I40,5 5	4,4			I40,5 7,4			
38	I44 0,7	I5I 4	I,5	2		I53 4,3	I,386	I2,I8	8,7
25	I45 3	I65 23		angular		I65 23			0
36	I46 3	I50 7	5,8			I50 9,I	I,397	II,94	I,9
39	I50 2	I64,5 4,6	3,8	I,I		I65,6 5,9	I,4I9	I3,22	8
39	I50 2	I66,6 5,0	3,8	angular		I66,6 6,3			
3I	I5I I,3	I5I I9	2,4	angular		I6I I9	I,424	I2,8	0,4
29	I52 0	I75 6			–I	I74 6	I,43	I3,85	4,3
29	I56 0	I70 5			–I,2	I68,8 5	I,45I	I3,44	6,2
36	I57 3	I62 7	5			I62 8,6	I,457	I2,9	2,3
58	I62 I	I70,5 3,5	I,4			I70,5 3,8	I,484	I3,57	II
38	I64 0,8	I69 5	I,2	0,93		I70 5,I	I,495	I3,55	6,I
33	I65 0	I88,2 5,4				I88,2 5,4	I,500	I5,08	6,6
33	I65	I99 II		angular		I96 II,5			
29	I66 0	I94 I2		0,6		I94,6 I2			
29	I66	I88 II		0,6		I88,6 II			

I	2	3	4	5	6	7	8	9	I0
29	I66 0	I87 7			−2	I85 7	I,505	I4,58	I2,5
23	I66 3,3	I82 6	2,I			I82 6,4			
36	I66 3,3	I76 7	2,I			I76 7,3			
58	I70 2	I98 3,5	I.2			I98 3,7			
39	I70 2	I94,9 6,0	I,2			I94,9 6,I	I,526	I5,67	2I
39	I70 2	20I,6 6,0	I,2	angular		I95 6,I			
29	I7I 0	2I0 I2			9	2I9 I2			
29	I7I 0	I96 6				I96 6	I,53I	I6,I6	I3,8
29	I7I 0	205 6				205 6			
36	I7I 3,4	204 6	2,I			204 6,4			
36	I73 3,4	205 6	I,I			205 6,I	I,542	I6,32	4,2
58	I73,5 2	I93,5 3,5				I93,5 3,5	I,544	I5,4I	I2
39	I74 1	I93 6		0,3		I93,3 6	I,547	I5,39	4,3
45	I76 2	I99,4 4,9		angular		I97,7 7,3	I,557	I5,74	3
58	I77 2	I98 5				I98 5.	I,562	I5,76	6
36	I8I 3,6	I87 7				I87 7	I,583	I4,89	3,I
29	I82 0	I66 20			8	I74 20			0
58	I83,5 2	I92 3,5				I92 3,5			
38	I84 0,9	I96 6		−0,28		I95,7 6	I,597	I5,35	I7

I	2	3	4	5	6	7	8	9	I0
29	I85 0	I88 8			-I	I87 8,5	I,603	I4,89	2,2
I7	I88 I	I59 34		angular		I58 34			0
34	I89 0	I94,I 5,2				I94,I 5,2			
34	I89 0	I98,3 8,5		angular		I93,7 8,3	I,623	I5,I8	II
36	I89 3,3	I82 7				I82 7			
38	I94 I	200 6		-0,86		I96 6	I,648	I5,85	4,2
58	I95 2	I74 4				I74 4	I,653	I3,85	I0
29	I96	202 I4				202 I4	I,658	I6,I	0,8
45	200 2	I77,9 3,7	2,5	angular		I83 6	I,678	I4,55	4,3
58	205 2	I78 4	2,8			I78 4,9	I,702	I4,I7	6,6.
38	209 I	I79 6	I,4	-I,64		I77,4 6,2	I,722	I4,I2	4,2
36	2I0 6	I48 20	9			I48 22			0
36	2I4 4,3	I4I 7	6,3			I4I 9,4	I,746	II,23	I,8
40	2I6,5 2	I5I,0 I8,2		angular		I65 I9	I,758	13.1	0,4
38	2I9 I,I	I56 7	I,6	-I,9		I54,I 7,2	I,770	I2,27	3,I
49	220 2	I40,9 4,2	3	angular		I40 5,2	I,775	II,29	7,5
36	222 4,4	I48 7	6,7			I48 9,7			
I3	225	I50 I8		angular		I42 I8	I,798	II,3	0,5

I	2	3	4	5	6	7	8	9	I0
38	229 I	I32 7	I,7	-2		I30 7,2	I,8I7	I0,34	3
45	240 0,8	I25,6 2,5	I,3	-2,2		I23,4 2,8	I,868	9,89	30
45	240 0,8	I27,2 3,6			angular	I25,4 3,8			
36	262 5,2	III 7	5,I			III 8,7	I,968	8,84	2,I
36	263 5,3	Io7 7	5,3			I07 8,8	I,972	8,52	2,0
45	270 0,9	85,2 3,0	0,75	-2,2		83 3,I	2,003	6,46	32
45	270 0,9	8I,3 3,I	0,75		angular	79,5 3,I			
36	280 9	88 11				88 11	2,047	7,0	1,3
36	298 6	75 5	3,8			75 6,2	2,124	5,97	4
45	307 1	65,7 2,2		-1,9		63,8 2,2	2,162	5,2	5,1
45	307 1	69 2,6			angular	67,7 2,5			
36	335 6,7	53 5	3,1			53 5,9	2,278	4,22	4,6
12	340 10	48 9				48 9	2,298	3,82	2
73	376 4	40,78 1,62				40,78 1,62	2,440	3,245	60
36	450 14	27,5 6	2			27,5 6,3	2,718	2,03	16
47	450 19	24,8 2,4	2,7			24,8 3,6			

1	2	3	4	5	6	7	8	9	10
73	469 5	23,67 1,08				23,67 1,08	2,786	1,884	135
64	500 5	17,0 3,4				17,0 3,4	2,894	1,35	14
47	550 23	16,1 2,5	0,6			16,1 2,5	2,064	1,28	24
73	567 6	17,37 0,82				17,37 0,82	3,120	1,382	235
73	626 6	15,16 0,72				15,16 0,72	3,309	1,206	300
61	628 6	13,7 1,3				13,7 1,3	3,326	1,09	100
73	664 7	14,77 0,67				14,77 0,67	3,426	1,175	350
47	670 27	14,5 2,0				14,5 2,0	3,445	1,15	39
61	685 7	14,4 1,3				14,4 1,3	3,497	1,15	100
36	700 21	17 3				17 3	3,536	1,35	17
61	734 7	17,1 1,3				17,1 1,3	3,643	1,36	100
73	770 8	19,44 0,80				19,44 0,80	3,741	1,547	250
61	788 8	18,3 1,3				18,3 1,3	3,791	1,46	100
47	790 32	19,5 2,0				19,5 2,0	3,798	1,55	39
61	798 8	16,5 1,3				16,5 1,3	3,822	1,31	100
74	816 8	23,39 0,61				23,39 0,61	3,871	1,861	425
73	818 8	21,36 0,81				21,36 0,81	3,876	1,700	240
73	838 8	22,42 0,83				22,42 0,83	3,932	1,784	230
73	863 9	21,85 0,86				21,85 0,86	4,000	1,739	210
74	866 9	23,34 0,68				23,34 0,68	4,008	1,857	340

1	2	3	4	5	6	7	8	9	10
74	915 9	23,13 0,74				23,13 0,74	4,140	1,841	290
61	947 10	21,3 1,3				21,3 1,3	4,219	1,70	100
74	965 10	23,80 0,78				23,80 0,78	4,270	1,894	260
47	1,00BeV 0,02	23,5 1,4	0,6			23,5 1,55	4,360	1,87	65
74	1,014 0,01	25,80 0,84				25,80 0,84	4,395	2,053	225
74	1,064 0,011	26,78 0,94				26,78 0,94	4,519	2,131	180
47	1,07 0,021	27,3 3,7	0,6			27,3 3,7	4,534	2,17	11
61	1,093 0,02	27,4 1,3				27,4 1,3	4,586	2,18	100
72	1,10 0,033	30,0 1,2				30,0 1,2	4,608	2,387	110
74	1,114 0,011	27,51 1,07				27,51 1,07	4,642	2,189	140
72	1,14 0,034	32,9 1,2				32,9 1,2	4,704	2,618	110
47	1,15 0,023	31,3 1,7	0,7			31,3 1,85	4,728	2,49	46
74	1,164 0,012	30,73 1,19				30,73 1,19	4,761	2,445	110
72	1,19 0,036	34,6 1,2				34,6 1,2	4,822	2,753	110
74	1,213 0,012	35,32 1,37				35,32 1,37	4,875	2,811	84
72	1,23 0,037	37,2 1,2				37,2 1,2	4,915	2,960	110
47	1,25 0,025	38,8 2,5				38,8 2,5	4,961	3,09	25
74	1,263 0,013	36,51 1,56				36,51 1,56	4,990	2,905	65

1	2	3	4	5	6	7	8	9	10
74	1,288 0,013	37,40 1,45				37,40 1,45	5,046	2,976	75
71	1,30 0,039	39,4 0,6				39,4 0,6	5,073	3,136	440
74	1,313 0,013	38,09 1,47				38,09 1,47	5,102	3,031	73
72	1,33 0,04	38,4 1,2				38,4 1,2	5,140	3,056	110
71	1,33 0,04	39,1 0,8				39,1 0,8	5,140	3,111	245
74	1,338 0,013	37,66 1,54				37,66 1,54	5,157	2,997	66
68	1,36 0,03	40,6 0,45				40,6 0,45	5,217	3,23	780
74	1,363 0,014	36,13 1,56				36,13 1,56	5,212	2,875	65
47	1,38 0,028	41,4 3,0				41,4 3,0	5,249	3,30	17
74	1,412 0,014	36,49 1,60				36,49 1,60	5,319	2,904	62
72	1,43 0,043	38,2 1,2				38,2 1,2	5,357	3,040	110
74	1,462 0,014	34,10 1,59				34,10 1,59	5,425	2,714	62
71	1,47 0,044	35,8 0,9				35,8 0,9	5,442	2,849	195
47	1,50 0,03	35,3 2,5				35,3 2,5	5,505	2,81	25
72	1,50 0,045	35,9 1,2				35,9 1,2	5,505	2,857	110
68	1,51 0,03	36,3 0,5				36,3 0,5	5,537	2,89	630
71	1,60 0,048	30,1 0,5				30,1 0,5	5,710	2,395	630
68	1,665 0,03	31,2 0,7				31,2 0,7	5,840	2,48	320
47	1,67 0,033	32,6 1,8				32,6 1,8	5,849	2,60	49

1	2	3	4	5	6	7	8	9	10
71	1,76 0,053	28,4 0,6				28,4 0,6	6,024	2,260	440
47	1,77 0,035	31,7 2,4				31,7 2,4	6,043	2,52	28
68	1,81 0,04	29,4 0,5				29,4 0,5	6,128	2,34	630
71	1,91 0,057	27,8 0,6				27,8 0,6	6,307	2.212	440
68	1,96 0,04	28,3 0,4				28,3 0,4	6,404	2,25	990
71	2,33 0,07	29,0 0,6				29,0 0,6	7,043	2,308	440
68	2,46 0,05	28,2 0,35				28,2 0,35	7,258	2,247	1290
69	2,76 0,06	28 4				28 4	7,74	2,22	10
71	2,83 0,085	29,2 0,5				29,2 0,5	7,837	2,324	630
68	3,11 0,06	29,1 0,48				29,1 0,48	8,260	2,316	690
71	3,44 0,10	29,2 0,4				29,2 0,4	8,706	2,324	980
71	3,86 0,12	29,3 0,4				29,3 0,4	9,251	2,332	980

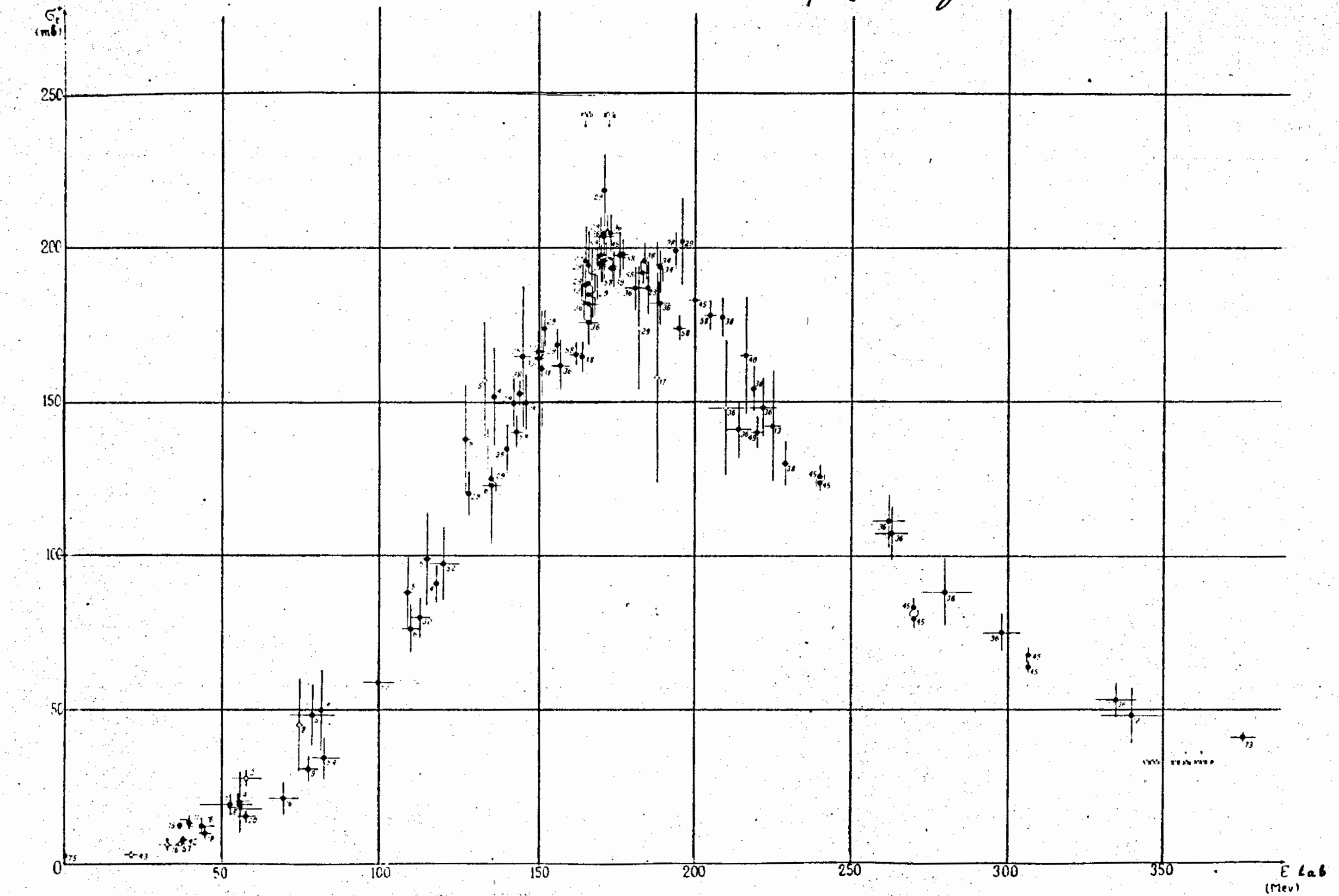

Fig. 1. Total cross sections for positive pion scattering on protons (in Figs. 1,2,3,4 the figures at the points denote the number of the reference; the vertical arrows indicate the thresholds for the production reactions).

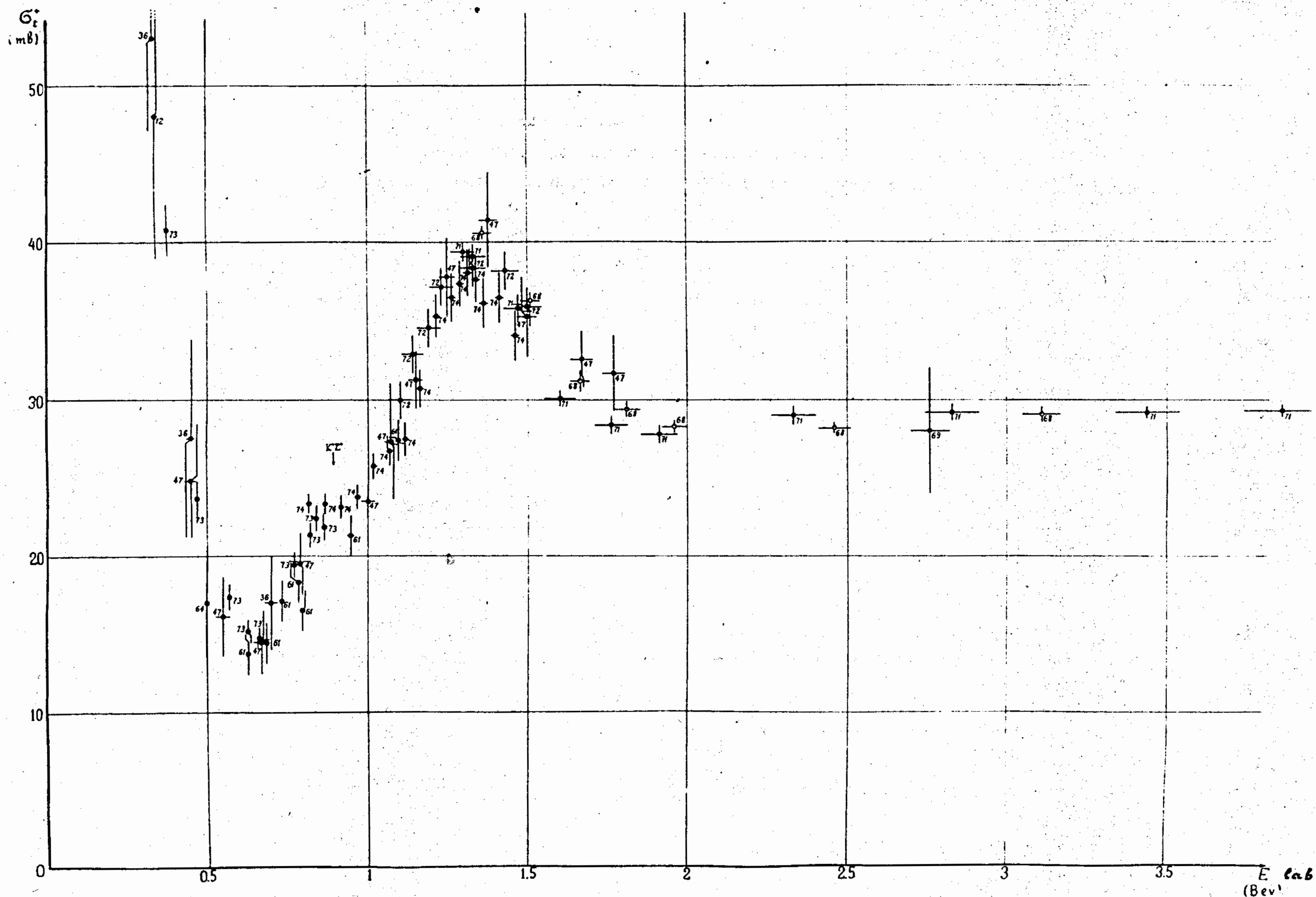

Fig. 2. Total cross sections for positive pion scattering on protons.

I	2	3	4	5	6	7	8	9	I0
5	I27	45		-0,73		44	I,290	3,52	0
		I0				I0			
65	I30	42,6				42,6			
		I,0				I,0	I,307	3,40	230
65	I30	42,7				42,7			
		1,5				I,5			
22	I33	46,9		-I,3		45,6	I,324	3,63	28
		2,4				2,4			
3	I35	52			-I	5I	I,336	4,05	4,4
		6				6			
6	I35	56,8				56,8	I,336	4,52	0
		3,2				3,2			
35	I40	44,3		-I,7		42,6	I,364	3,39	22
		2,7				2,7			
9	I44	48,I		angular		46,4	I,386	3,69	8
		4,5				4,5			
39	I50	53,2		angular		54,2	I,4I9	4,32	23
		2,6				2,6			
29	I52	60,7		-I,I		59,6			
		3,0				3,0			
65	I52	60,0				60,0	I,430	4,755	I10
		2,3				2,3			
65	I52	59,6				59,6			
		I,6				I,6			
22	I57	62,9		-I		6I,9	I,457	4,92	27
		2,4				2,4			
66	I58,2	56,4				56,4	I,463	4,488	40
		2,0				2,0			
33	I65	69,8		angular		67,7			
		3,8				3,8	I,499	5,37	80
33	I65	67,5				67,5			
		I,5				I,5			
I0	I69	63		angular		6I	I,52I	4,85	I0
		4				4			
39	I70	62,7		angular		62,4	I,526	4,96	80
		I,4				I,4			

I	2	3	4	5	6	7	8	9	I0
66	I7I,7	67,2 1,1				67,2 I,I	I,535	5,348	I30
3	I76	66 6			−2	64 6	I,557	5,I0	4,4
66	I78,4	67,2 I,I				67,2 I,I	I,570	5,348	I30
22	I79	65,9 2,5		−0,43		65,5 2,5	I,573	5,2I	25
35	I84	65,7 2,4		−0,46	0,36	65,6 2,4	I,598	5,22	27
66	I85,2	67,7 I,o				67,7 I,0	I,604	5,387	I60
27	I87	64,0 2,0		angular		64,0 2,0	I,6I3	5,07	I00
27	I87	63,5 I,6				63,5 I,6			
66	I89,9	67,8 0,8				67,8 0,8	I,628	5,395	250
22	I94	64,6 2,5		−0,I5		64,45 2,5	I,648	5,I3	25
I0	I94	74 5		angular		72 5	I,648	5,72	6
22	I95	63,I 2,5		−I,3		6I,8 2,5	I,653	4,92	25
66	I96,2	64,0 I,I				64,0 1,1	1,659	5,093	I30
35	I97	7I,2 2,5		−0,I3	0,43	7I,5 2,5	I,663	5,69	0
66	20I,0	63,8 I,0				63,8 I,0	I,683	5,077	I60
66	205,8	59,3 I,0				59,3 I,0	I,706	4,7I9	I60
23	209	57,2 2,9				57,2 2,9	I,722	4,55	I9
I0	2I0	64 5		angular		62 5	I,727	4,93	6
66	2I0,6	58,7 I,I				58,7 I,I	I,730	4,67I	I30

I	2	3	4	5	6	7	8	9	I0
22	2I5	55,5		0,I9		55,7	I,75I	4,435	33
		2,2				2,2			
66	2I5,4	55,6				55,6	I,753	4,425	I60
		I,0				I,0			
35	2I6	57,3		0,23	0,37	57,9	I,756	4,6I	25
		2,5				2,5			
3	2I7	60			–2	58	I,76I	4,6	4,4
		6				6			
23	2I7	54,5		angular		57,7	I,76I	4,5I	4,4
		6				6			
23	22o	52,I				52,I	}		
		2,3				2,3	}		
49	220	53,7				53,7	} I,775	4,24	I20
		I,5				I,5	}		
49	220	53,4				53,4	}		
		2,8				2,8	}		
66	220,2	52,2				52,2	I,776	4,I54	I60
		I,0				I,0			
66	225,0	50,2				50,2	I,798	3,995	200
		0,9				0,9			
35	226	53,3		0,38	0,32	54	I,803	4,30	39
		2,0				2			
66	228,3	48,2				48,2	I,8I4	3,836	200
		0,9				0,9			
66	23I,6	49,0				49,0	I,829	3,899	200
		0,9				0,9			
66	234,9	44,5				44,9	I,845	3,541	200
		0,9				0,9			
22	236	46,I		0,4		46,5	I,850	3,70	27
		2,4				2,4			
66	238,2	44,9				44,9	I,860	3,573	200
		0,9				0,9			
59	240	48,3		angular		48,5	I,868	3,86	I4
		3,3				3,3			
22	240	43,5		0,4		43,9	I,868	3,495	3I
		2,3				2,3			

I	2	3	4	5	6	7	8	9	I0
66	24I,5	42,7				42,7	I,875	3,398	200
		0,9				0,9			
66	244,8	43,I				43,I	I,890	3,430	200
		0,9				0,9			
66	248,I	4I,0				4I,0	I,905	3,263	200
		0,9				0,9			
66	25I,4	39,3				39,3	I,920	3,I27	200
		0,9				0,9			
66	254,7	39,8				39,8	I,935	3,I67	250
		0,8				0,8			
35	256	37,5		0,58	0,42	38,5	I,94I	3,064	44
		I,9				I,9			
29	258	38,2		0,5		38,7			
		3,4				3,4	1,950	3,087	260
66	258,0	38,8				38,8			
		0,8				0,8			
66	26I,4	36,8				36,8	I,965	2,928	250
		0,8				0,8			
36	265	44				44	I,98I	3,5	0
		6				6			
66	266,5	35,6				35,6	I,988	2,833	250
		0,8				0,8			
59	270	36,5		angular		36,I	2,00	2,79	27
		2,4				2,4			
66	27I,6	33,4				33,4	2,010	2,658	250
		0,8				0,8			
66	276,7	3I,I				3I,I	2,032	2,475	250
		0,8				0,8			
66	281,8	32,4				32,4	2,055	2,578	250
		0,8				0,8			
66	286,9	3I,6				3I,6	2,077	2,5I5	250
		0,8				0,8			
35	290	33,9		0,68	0,22	34,8	2,090	2,770	I09
		I,I		0,6		I,2			
66	292,0	30,5				30,5	2,099	2,427	250
		0,8				0,8			
66	297,2	29,3				29,3	2,I2I	2,332	250
		0,8				0,8			

I	2	3	4	5	6	7	8	9	I0
66	302,5	28,9				28,9	2,I44	2,300	250
		0,8				0,8			
66	307,7	28,I				28,I	2,I65	2,236	250
		0,8				0,8			
66	3I3,0	28,7				28,7	2,I87	2,284	322
		0,7				0,7			
66	3I8,2	27,0				27,0	2,209	2,I49	440
		0,6				0,6			
66	323,5	26,2				26,2	2,23I	2,085	440
		0,6				0,6			
66	328,2	26,4				26,4	2,250	2,I0I	440
		0,6				0,6			
4I	330	24		angular		24			0
		5				5			
56	333	28,8				28,8			
		I,8				I,8			0
66	334,2	26 ,0				26,0	2,275	2,069	440
		0,6				0,6			
35	335	25,7		0,67	0,23	26,6	2,278	2,II7	II0
		I,0				I,0			
36	340	23				23			0
		II				II			
66	345,0	24,0				24,9	2,3I8	I,98	160
		I,0				I,0			
66	36I,0	25,2				25,2	2,382	2,0I	I60
		I,0				I,0			
35	363	26,5			0,2	26,7	2,390	2,I3	59
		I,6				I,6			
73	373	28,9				28,9	2,429	2,30	8I
		I,4				I,4			
35	393	25,9			0,3	26,2	2,506	2,08	22
		2,7				2,7			
73	426	29,5				29,5	2,630	2,35	8I
		I,4				I,4			
I2	450	25				25	2,718	1,99	18
		3				3			
47	450	28,8				28,8			0
		2,7				2,7			

I	2	3	4	5	6	7	8	9	I0
73	468	30,0				30,0	2,782	2,39	II0
	5	I,2				I,2			
I4	470	27				27	2,792	2,25	6
		5				5			
47	500	3I,3				3I,3	2,894	2,49	0
		4,8				4,8			
36	5I0	20				20			0
		7				7			
73	5I8	34,9				34,9	2,956	2,78	93
	5	I,3				I,3			
47	550	37,4				37,4	3,064	2,98	0
		3,o				3,0			
73	567	44,6				44,6	3,120	3,55	44
	6	I,9				I,9			
73	59I	45,8				45,8	3,I98	3,64	55
	6	I,7				I,7			
36	600	23				23			0
		II				II			
73	604	45,5				45,5	3,240	3,62	49
	6	I,8				I,8			
47	6Io	37,0				37,0	3,259	2,94	0
		2,I				2,I			
73	6I6	45,I				45,I	3,278	3,59	55
	6	I,7				I,7			
73	643	44,4				44,4	3,362	3,53	33
	6	2,2				2,2			
73	665	39,2				39,2	3,43o	3,I2	8I
	7	I,4				I,4			
47	670	39,5				39,5	3,445	3,I4	0
		3,0				3,0			
36	700	42				42			0
		I0				I0			
73	7I9	35,I				35,I	3 ,592	2,79	70
	7	I,5				I,5			
73	749	37,6				37,6	3,680	2,99	62
	7	I,6				I,6			
73	769	37,4				37,4	3,738	2,98	40
	7	2,o				2,0			

I	2	3	4	5	6	7	8	9	I0
47	790	46,I				46,I			0
		3,4				3,4			
74	797	40,00				40,00	3,8I8	3,I83	I05
	8	I,22				I,22			
74	8I6	46,20				46,20	3,87I	3,676	70
	8	I,50				I,50			
73	8I9	47,9				47,9	3,879	3,8I	44
	8	I,9				I,9			
74	836	48,00				48,00	3,926	3,820	70
	8	I,50				I,50			
73	840	54,6				54,6	3,937	4,34	36
	8	2,I				2,I			
I4	840	47				47	3,937	3,74	6
		5				5			
74	856	53,22				53,22	3,98I	4,235	53
	9	I,72				I,72			
47	860	47,7				47,7	3,992	2,80	0
		2,7				2,7			
74	866	54,0I				54,0I	4,008	4,298	49
	9	I,84				I,84			
73	868	58,6				58,6	4,0I4	4,66	27
	9	2,4				2,4			
74	886	56,64				56,64	4,062	4,507	46
	9	I,84				I,84			
73	890	57,8				57,8	4,073	4,60	33
	9	2,2				2,2			
47	900	44,4				44,4	4,I00		0
		2,3				2,3			
74	9I5	55,07				55,o7	4,I40	4,382	49
	9	I,79				I,79			
73	9I8	54,5				54,5	4,148	4,34	27
	9	2,4				2,4			
73	943	50,4				50,4	4,2I4	4,0I	23
	9	2,6				2,6			
74	945	48,7				48,7	4,2I9	3,875	59
	9	I,63				I,63			
48	950	38				38			0
	50	3				3			

I	2	3	4	5	6	7	8	9	I0
74	965	45,58				45,58	4,270	3,627	60
	10	I,62				I,62			
47	970	45,I				45,I	4,283	3,58	0
		2,7				2,7			
73	972	44,7				44,7	4,288	3,56	33
	I0	2,2				2,2			
74	985	4I,37				4I,37	4,322	3,292	67
	10	I,53				I,53			
2I	I,00	48				48	4,360	3,82	0
		4				4			
47	I,00	46,0				46,0	4,360	3,66	0
		3,0				3,0			
73	I,0I4	39,6				39,6	4,395	3,I5	40
	0,010	2,0				2,0			
74	I,064	36,62				36,62	4,5I9	2,9I4	93
	0,010	I,30				I,30			
73	I,076	35,9				35,9	4,549	2,36	40
	0,011	2,0				2,0			
47	I,08	36,3				36,3	4,559	2,89	0
		2,6				2,6			
63	I,08	47,2				47,2		3,76	0
	0,03	3,I				3,I			
73	I,I50	35,5				35,5	4,728	2,82	40
	0,012	2,0				2,0			
74	I,I64	34,27				34,27	4,76I	2,727	90
	0,012	I,32				I,32			
47	I,25	29,2				29,2	4,93I	2,32	0
		3,7				3,7			
74	I,263	35,28				35,28	4,990	2,807	86
	0,013	I,35				I,35			
47	I,35	30,I				30,I	5,I84	2,39	0
		2,8				2,8			
74	I,363	32,99				32,99	5,2I2	2,625	79
	0,014	I,4I				I,4I			

I	2	3	4	5	6	7	8	9	I0
32	I,37	34,6 2,7				34,6 2,7	5,228	2,75	23
47	I,38	30,8 2,8				30,8 2,8	5,249	2,45	0
74	I,462 0,015	3I,40 I,50				3I,40 I,50	5,425	2,499	70
47	I,47	3I,4 I,8				3I,4 I,8	5,442	2,50	0
I9	I,50	34 3				34 3	5,505	2,7I	18
47	I,50	30,0 2,0				30,0 2,0			0
47	I,67	3I,4 3,9				3I,4 3,9	5,849	2,50	0
47	I,90	3I,3 I,6				3I,3 I,6	6,289	2,49	0
5I	4,I6	28,7 2,6				2,87 2,6	9,65	2,28	23
44	4,5	2o,4 3,5				20,4 3,5			0
55	5	22,5 2,4				22,5 2,4			0
70	6,66	30 5				30 5	I2,4	2,38	6,4

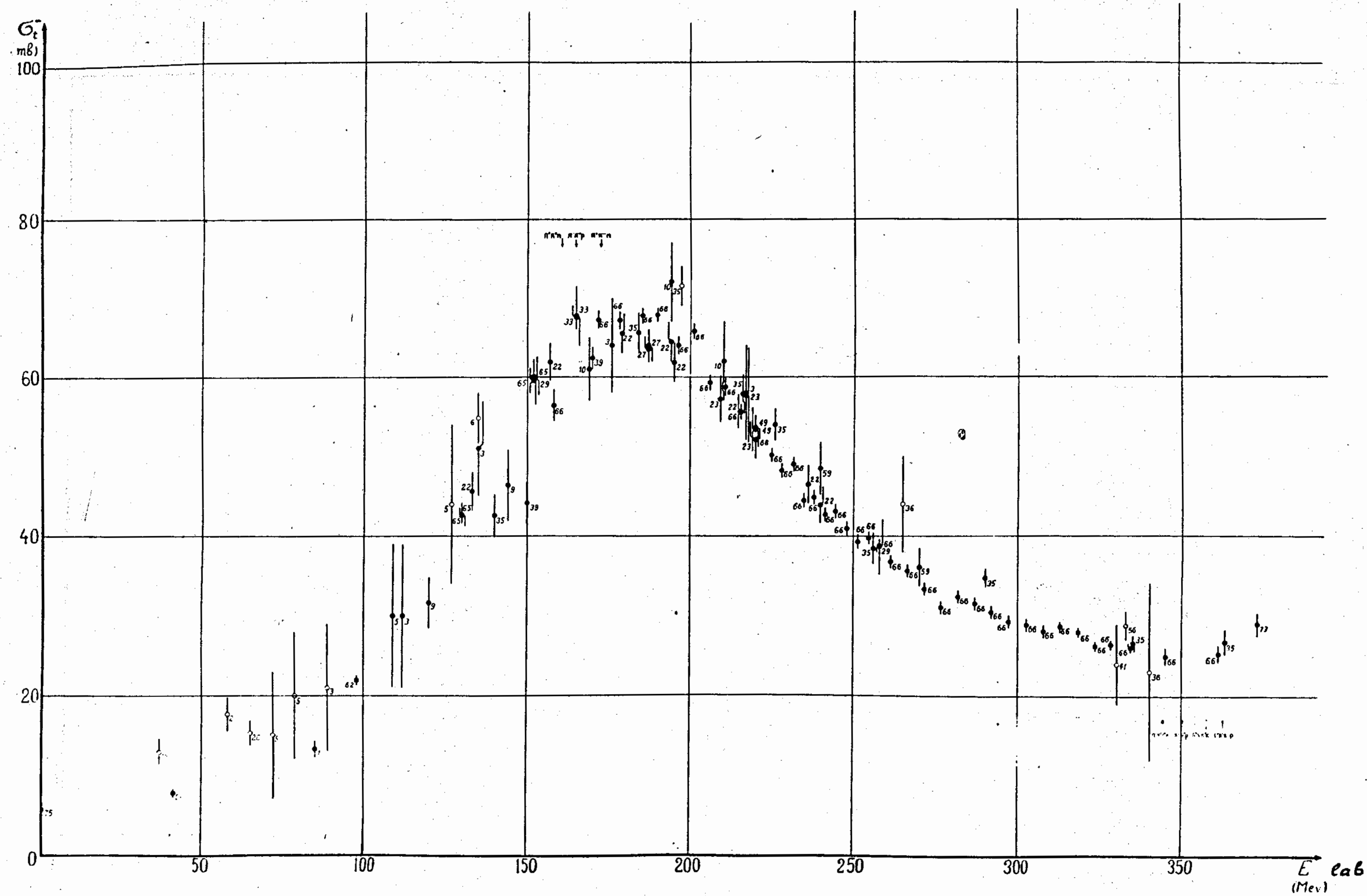

Fig. 3. Total cross sections for negative pion scattering on protons.

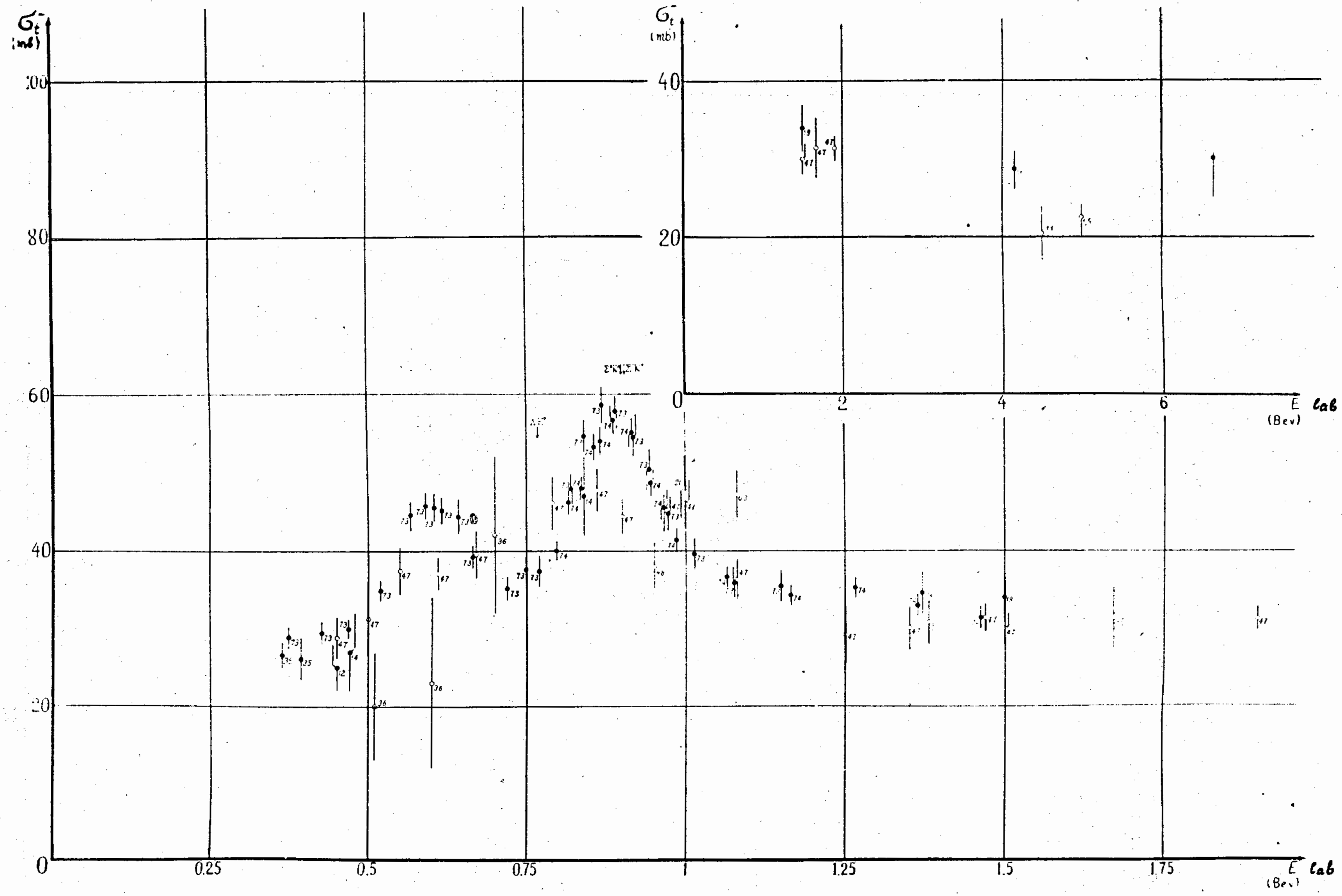

Fig. 4. Total cross sections for negative pion scattering on protons.

2. CHOICE OF THE INTERPOLATION FORMULA

A choice of a definite interpolation formula is not an auxiliary or a purely formal procedure. The interpolation formula is a physical hypothesis which is suggested on the basis of the facts already known and verified by a comparison with experimental data. In our case we are in need of a formula-hypothesis describing the behaviour of the scattering cross sections σ^+ and σ^- in the energy range from 0 up to ∞. For many reasons both of technical and principal nature, this formula must contain not many, at the most, 10-15 unknown parameters, by varying of which an agreement should be obtained with more than two hundred experimental points.

First of all, let us make a usual assumption about the behaviour of the cross sections σ^+ and σ^- at low energies ($E < 50$ MeV):

$$\sigma(E) \approx \sigma(0) + \alpha E^2 \qquad (2.1)$$

Further we assume that both cross sections σ^+ and σ^- are tending to the same constant at infinitely high energies, and namely, that

$$\sigma^{\pm}(E) \approx \sigma(\infty) \qquad (2.2)$$

at $E > 3$ BeV.

Besides, we accept the hypothesis asserting that the total interaction cross section may be presented as a sum of the two terms

$$\sigma = \sigma_{el} + \overline{\sigma}. \qquad (2.3)$$

The first component σ_{el} possesses all the properties of elastic scattering in the absence of inelastic one. The second term $\overline{\sigma}$ is increasing monotonously with energy everywhere, with the only possible exception of small regions near the thresholds /79, 80/.

At the energies lower than the threshold for the production of one additional pion $\overline{\sigma} = 0$, whereas at higher energies this component plays the main role for it includes both the inelastic and the diffraction scattering.

Further we assume the hypothesis about the resonant character of elastic scattering. This means that each term in the expansion in partial waves

$$\sigma_{el} = \sum_{\ell=0}^{\infty} (2j+1)\, S_{\ell,\, j=\ell+\frac{1}{2}} + \sum_{\ell=1}^{\infty} (2j+1)\, S'_{\ell,\, j=\ell-\frac{1}{2}} \qquad (2.4)$$

is limited by the inequality

$$0 \leq S(E) \leq 2\pi \lambda\!\!\!^{-2} \qquad (2.5)$$

and is described by the resonance formula $B(E)$ of the Breit-Wigner type. The only exception is made for the S scattering which was treated as a nonresonant one.

A list of resonance formulae derived on the basis of nonrelativistic models and the discussion of their structure may be found in the paper /81/. Relativistic resonance formulae with one and many levels which were obtained on the basis of concepts of the analytical properties of the S matrix are given in the paper by Ning Hu /82/. The same paper treats in detail the hypothesis about the resonant character of the scattering as a consequence of the analicity and unitarity of the S matrix.

For the angular momentum $\ell \geqslant 1$ we gave up the resonance formulae of paper /82/ and preferred the formulae of paper /81/, in which the momentum and the energy are replaced by relativistic ones because of two considerations.

First, the resonance formulae of paper /81/ behave at low energies as

$$B_{\ell}(E) \sim E^{2\ell} \quad at \quad E \to 0 \qquad (2.6)$$

what is an inevitable result of the short-acting character of nuclear forces. At the same time the simplest formulae of paper /82/ give the finite limiting value of the cross section at $E \to 0$ for $\ell > 0$, what contradicts the isotropy of the scattering at low energies.

Second, a parameter giving the asymmetry of the resonance curves which is necessary to be introduced to reach an agreement with the experiment in the range 0 - 300 MeV enters the formulae of paper /81/.

Finally, the cross section is written as

$$\sigma(E) = L(E) + \sum_{j,\ell} B_{j,\ell}(E) + A(E) \qquad (2.7)$$

where **L(E)** is a decreasing function describing, for the most part, the *S* scattering; **ΣB(E)** is the sum of the resonances. The number of these resonances in the energy range **0 - 1.5 BeV,** the corresponding values of ℓ and j, the widths and the resonance energies should be determined from experimental data. The function **A(E)** approximates the cross section $\bar{\sigma}$.

It is natural that the form of (2.7) is assigned to the scattering cross sections to states with a definite isotopic spin ; $\sigma^{3/2}$ and $\sigma^{1/2}$, whereas σ^{+} and σ^{-} are associated with $\sigma^{3/2}$ and $\sigma^{1/2}$ in a usual way:

$$\sigma^{+} = \sigma^{3/2}, \qquad \sigma^{-} = \frac{1}{3}\sigma^{3/2} + \frac{2}{3}\sigma^{1/2}. \qquad (2.8)$$

In further formulae we shall make use of the natural system of units in which

$$c = m_{\pi^{\pm}} = 1. \qquad (2.9)$$

It is more convenient to use instead of $E_{lab.}$ the momentum in the c.m.s. η which is connected with the kinetic energy of the πp system, entering B , by the equality

$$E_K = \sqrt{1+\eta^2} + \sqrt{M^2+\eta^2} - 1 - M, \qquad (2.10)$$

where $M = 6.719$ is the proton mass. The magnitude of $2\pi\lambda\!\!\!^{-2}$ in millibarns is

$$2\pi\lambda\!\!\!^{-2} = \frac{12.5.4}{\eta^2}\ mb \qquad (2.11)$$

The parameters being varied are denoted by a , those not varied, by c .

The function L the role of which was to approximate the cross sections near zero was chosen in the form

$$L = \frac{\sigma(0)}{1+|\alpha|\eta^2} \quad . \qquad (2.12)$$

For $\sigma(0)$ the values of [75] were used

$$\sigma^{3/2}(0) = 2.76 \pm 0.53\ mb \quad ; \quad \sigma^{1/2}(0) = 7.05 \pm 1.01\ mb . \qquad (2.13)$$

The resonance functions (coinciding up to notations with the function used in the paper[83]) had the form

$$B = (2j+1)\cdot 2\pi\lambda^2 \frac{\left(\frac{\Gamma}{2}\right)^2}{(E_K - a_1)^2 + \left(\frac{\Gamma}{2}\right)^2} \quad , \qquad (2.14)$$

where

$$\left(\frac{\Gamma}{2}\right)^2 = a_2 \left(\frac{\eta}{c}\right)^6 \left(\frac{1+a_3 c^2}{1+a_3 \eta^2}\right)^2 . \qquad (2.15)$$

The constant **C** is introduced into (2.15) in order to ensure that the parameter a_3 giving the asymmetry would not affect the width of the resonance curve as well.

Strictly speaking, the resonance formulae must have a different dependence of Γ on η^2 for different values of ℓ .

One and the same formula corresponding to $\ell = 1$ was used since, on the one hand, the contribution from far resonances (**1300 MeV** for $\sigma^{3/2}$ and **900 MeV** for $\sigma^{1/2}$) is very small at low energies, where appears the difference between these formulae, and, on the other hand, the values of ℓ for these resonances are not still determined finally.

The choice of the function $A(E)$ was somewhat more complicated since there is rather poor information about this function. A number of variants of this function were tried. It became clear that the curve $\sigma(E)$ obtained and the goodness of the approximation are weakly dependent upon the variant used. Finally the simplest formula was adopted:

$$A = \begin{cases} \sigma(\infty)\frac{2}{\pi}\,\mathrm{arc\,tg}\{(\eta-\eta_0)^2|\alpha|\} , & \eta > \eta_0 = 1.497 \\ 0 , & \eta < \eta_0 \end{cases} \qquad (2.16)$$

As $\sigma^{3/2}(\infty) = \sigma^{1/2}(\infty)$ the value was used

$$\sigma(\infty) = 29.2 \text{ mb} \qquad (2.17)$$

which was found by averaging of the last three points for σ^+ (paper /71/) and the two points for σ^- (papers /51/,/70/). At $E < 3$ **BeV** the curve $\sigma(E)$ turned out to be insensitive to a 10% change of $\sigma(\infty)$. An attempt has been made to include $\sigma(\infty)$ into the number of the alternating parameters, but it was found that it cannot be reliably determined by the available data (there is a tendency of an infinite increase of this limiting value). For a reliable determination of $\sigma(\infty)$ good experimental data are necessary up to the energies of **10-12 BeV**, where the number of inelastic channels of the reaction becomes great.

3. *IDENTIFICATION OF RESONANT STATES*

The work on the approximation of the cross sections by means of the formula (2.7) was started at the end of 1958, when the second maximum in the cross section σ^+ was hardly felt experimentally, and the cross section σ^- in the region **0.5-2 BeV** was represented for the most part, by the measurements of Cool, Piccioni and Clark /47/. These measurements, however, were found to be incorrect and distorted entirely the situation in this region.

We tried many versions of the curves with different number of resonances and with different ℓ and j. Each time a complete statistical analysis described in §4 was made. In particular, all the resonances were checked for asymmetry, which was found to be significant only for the main resonance $P_{3/2}$, T=3/2, and equal to zero for all the other ones.

At first it was necessary to get the curve $\sigma^+(E)$. It became clear from the first variants of the calculation that, for the second resonance in the cross section of σ^+ situated in the region of **1300 MeV,** the experiment does not contradict either the angular momentum j = 5/2, or the angular momentum j=3/2 as the curves $\sigma^+(E)$ approximating the cross section are not very different in these two cases.

It should be noted that if the variant $j = 3/2$ is correct, the second resonance may belong only to the D-wave, $\ell = 2$, since otherwise, when $\ell = 1$, due to the restriction (2.5), the second and the third resonances would begin to interfere, and the cross section would become much less than the experimental one.

To give up the variant $j = 5/2$ it became possible only after the results of the measurements made at Saclé /73/,/74/ and at Berkeley /71/ had been published. These results determined strictly the width of the second resonance. If this width is attributed to the Breit curve with $j = 5/2$, then we would get a considerably more sharp and high maximum than is consistent with the experimental data. At the same time the resonance $D_{3/2}$ is in good agreement with experiment.

After the first successful approximation of the cross section σ^+, the "experimental" points for $\sigma^{1/2}$ were calculated

$$\sigma^{1/2}_{\text{"exp"}} = \frac{3}{2}\sigma^-_{exp} - \frac{1}{2}\sigma^+_{appr} . \quad (3.1)$$

It was found impossible to draw either a curve of type (2.7) with one resonance, or without resonances at all through the points among which the points from paper /47/ played an important part.

Hundreds of times more probable, although contradicting the experimental data qualitatively, was found the curve with two resonances for the angular momenta $j = 3/2$ and $j = 5/2$, respectively. It is interesting to note that both the resonance energies and the widths of these resonances were determined rather correctly and in accordance with the contemporary values from the incorrect data of Cool, Piccioni and Clark /47/. This indicates that the cross sections found by these authors are, as a matter of fact, cross sections averaged over very large energy intervals.

Just a week later, Phys.Rev.Letters 2, n.3 arrived at the Joint Institute for Nuclear Research were the measurements /61/ were reported. The points from this paper lay on a two-humped curve which looked like that we just had obtained. The more unexpected turned out the result that these points were in sharp disagreement with formula (2.7). This contradiction could not be overcome even if we assume a strong interference of σ_{el} and σ_{nonel}.

One can easily become sure of this if one sees the figure presented to the Kiev Conference (see /84/). In this figure the dashed line is obtained by approximating the data /61/ by the resonance formulae with $j = 3/2$ and $j = 5/2$.

At the Kiev Conference of 1959 were presented the precise data of Pontecorvo /66/, as well as preliminary results of the groups working at Saclé and at Berkeley. Besides, it became clear that in some experiments on the cross sections σ^- in the range 0-50 MeV an incorrect value of Panofsky ratio was made use of. Therefore, only the latest measurements should be taken into account. An increased accuracy of the values of the cross section σ^- made an individual approximation of the cross sections σ^+ and σ^- disadvantageous, since the procedure (3.1) began to lead to an appreciable loss of information. Therefore, a program has been made up for an electronic computer. This program consisted in a simultaneous determination of all the parameters of the curves σ^+ and σ^- which then were looked for only jointly.

The preliminary data of the group working at Saclé which Professor A.Berthelot sent us allowed to investigate the region of 0.5 - 1.5 **BeV** more seriously. Later on Professor Falk-Variant kindly sent us twice precise and supplemented data prior to publication, a fact which accelerated our work and made it considerably easier. We take the opportunity of thanking him personally and all the experimentators of this group.

The French data supported the hypothesis made before that the resonance in the cross section $\sigma^{1/2}$ in the range of **900 MeV** corresponds to the angular momentum $j = 5/2$. Alongside with American measurements of σ^+ at higher energies /71/, the French data /73/,/74/ allowed to establish finally the situation in the cross section σ^+ in the region of **0.7 - 4 BeV.**

Much more complicated than it was assumed earlier on the basis of more rough measurements, was found the region of **500-800 MeV** in which we failed to approximate well either the cross section σ^+, or $\sigma^{1/2}$.

In this region in the cross section σ^+ there is a deep minimum. At the same time the experimental points do not lie on a smooth curve, but have a distinct break, possibly, an incorrect one, and, perhaps, of a threshold origin* (see Fig. 9). In the latter case formula (2.7) should, in principle, be made more complicated by adding threshold terms

$$\sigma(E) = L + \sum_{\ell,j} B_{\ell,j} + A + \sum_t \Pi_t(E) \qquad (3.2)$$

where the threshold terms Π_t are different from zero only in small vicintities of the threshold energies E_t.

Fortunately, it is not necessary to vary simultaneously all the parameters of formula (3.2) as the parameters of the threshold terms which have a local nature must not be correlated with the parameters of the functions L, B, A (see also § 4). Therefore the function $\sigma(E)$ can be looked for in two stages : at first, to find L,B,A by putting $\sum \Pi = 0$, and then to find $\Pi_t(E)$ by assuming L,B,A to be fixed functions, what the authors hope to do further.

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* We mean the thresholds for the reactions $\pi^+ p \rightarrow K^+\Sigma^+$, $\pi^- p \rightarrow \Lambda^0 K^0$, $\pi^- p \rightarrow \Sigma^0 K^0$, $\pi^- p \rightarrow \Sigma^- K^+$ lying at the energies 891.2; 767.5; 899.1 and 903.6 MeV, respectively.

The first maximum in the cross section $\sigma^{1/2}$ lies in the same energy range. It may be due only to the resonance in the state $P_{3/2}$ for the angular momenta j = 3/2 and higher are in sharp contradiction with experiment. However, the curve approximating σ^- lies at **600 MeV** by **4.5 mb** below the experimental points. Since the nature of this divergence is still obscure, for obtaining the curve which would agree with experiment and valid for the calculation of the dispersion integrals, a systematic divergence in this region was eliminated artificially by a formal substitution of the factor 2.8 for the factor $2j+1 = 2$.

In identifying the resonance states only the data on the total cross sections were made use of, and no additional information was involved, such as the information contained in the angular distributions and in the data on pion photoproduction. Additional information may help to overcome the ambiguity, if any, in the determination of $\ell = j \pm 1/2$.

In spite of the fact that a good agreement between the interpolating curve and the experimental data is not yet obtained everywhere, it is worthwhile to note a complete success of identifying the resonant part of the cross sections $\sigma^+(E)$ and $\sigma^-(E)$ and an expediency of further research in this direction. In particular, one may hope to get information on inelastic processes in π-p scattering if the form of the function $A(E)$ is more specified by applying resonance formulae for the description of the generation of pions and of other particles. Especially interesting results may be found in the regions where a full description of the energy dependence of the cross sections σ^+ and σ^- is not yet available.

4. *METHOD OF ANALYSIS*

The best values of the parameters α and their errors were found by the method of least squares adapted to the solution of problems with many non-linear parameters.

The quadratic form minimized was as follows

$$M = \sum_{i=1}^{n^+} [\sigma_i^+ - y^+(\eta_i, \alpha)]^2 w_i + \sum_{i=n^++1}^{n^++n^-} [\sigma_i^- - y^-(\eta_i, \alpha) \cdot N_i]^2 w_i + (N_0 - N)^2 w_{N_0} \quad (4.1)$$

where σ_i^+, σ_i^- are the experimental values of the cross sections (see Table 1,2). $y^+ = y^{3/2}$ $y^- = \frac{1}{3} y^{3/2} + \frac{2}{3} y^{1/2}$ are approximating curves, and w_i are the weights equal to the inverse variances of the measurements (see Table 1,2). The normalization factors N_i, N, N_0 are introduced into formula (4.1) to take into account the fact that the data of paper /66/ (39 points) have a general normalizing factor (a systematic error) $N_0 = 1 \pm 0.016$, which should be taken into consideration separately from the relatively small independent errors of these points. Therefore, in (4.1) $N_i = N$ for the points of paper /66/, $N_i = 1$ for all the rest points, $W_{N_0} = \frac{1}{(0.016)^2}$ is the weight of the measurement N_0. In other experiments the series of measurements are not so large, the relative accuracy is less, and, therefore, their normalization was not introduced. In calculating the last version $n^+ + n^- = 138 + 126 = 264$ experimental values of the total cross sections were used. The expressions for $y^{3/2}$ and $y^{1/2}$ had the form

$$y^{3/2} = L^+(a) + B^+_{j=3/2}(a_1, a_2, a_3) + B^+_{j=5/2}(a_1, a_2) + A^+(\alpha) , \qquad (4.2)$$

$$y^{1/2} = L^-(a) + B^-_{j=1/2}(a_1, a_2) + B^-_{j=5/2}(a_1, a_2) + A^-(\alpha) . \qquad (4.3)$$

Thus, along with the normalization factor $m = 14$ simultaneously variable parameters entered the formula for M.

The calculations were made by means of an electronic computer. To minimize the form M, the linearization method /85/ was applied. The idea of this method consists in the following. The exact equations of the minimum of the form $M = \sum_i M_i$*

$$\frac{\partial M}{\partial \alpha_k} = \sum_i \frac{\partial M_i}{\partial y_i} \frac{\partial y_i}{\partial \alpha_k} = 0 \qquad (4.4)$$

are replaced by a linear system of equations

$$\lambda \sum_i \frac{\partial M_i}{\partial y_i} \frac{\partial y_i}{\partial \alpha_k} + \sum_i \left| \frac{\partial^2 M_i}{\partial y_i^2} \right| \frac{\partial y_i}{\partial \alpha_k} \sum_{\ell=1}^{m} \Delta \alpha_\ell \frac{\partial y_i}{\partial \alpha_\ell} = 0 \qquad (4.5)$$

by solving of which one can find the step $\Delta \alpha_\ell$.

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* The summation over involves further all the terms in expression (4.1), including the last one $(N_0 - N)^2 W_{N_0}$.

In the equation (4.5) the number λ is the normalization of the step. It was chosen by the electronic computer so that the step in either direction would not exceed some prescribed values. For sufficiently small λ the iteration process $a_k^{(n+1)} = a_k^{(n)} + \Delta a_k$ always converges to the minimum of M/85/ (in practice, only 5-10 iterations are needed).

When solving the equations (4.5), one calculates the matrix Z_{kl}^{-1} inverse to the matrix Z_{kl}, the latter one is equal to

$$Z_{kl} = \sum_i w_i \frac{\partial y_i}{\partial a_k} \frac{\partial y_i}{\partial a_l} . \tag{4.6}$$

The matrix Z^{-1} is an estimate of the error matrix of the parameters a. It contributes to the calculation of the estimates of the error corridors of the curves y

$$S_y^2(\eta) = \sum_{k=1}^{m} \sum_{l=1}^{m} \frac{\partial y(\eta)}{\partial a_k} \frac{\partial y(\eta)}{\partial a_l} Z_{kl}^{-1} \tag{4.7}$$

and of the estimate of the error corridor of the dispersion integrals Y

$$S_Y^2(\omega) = \sum_{k=1}^{m} \sum_{l=1}^{m} \frac{\partial Y(\omega)}{\partial a_k} \frac{\partial Y(\omega)}{\partial a_l} Z_{kl}^{-1} . \tag{4.8}$$

The estimates of the coefficients of correlation r_{kl} of the parameters a

$$r_{kl} = \frac{Z_{kl}^{-1}}{\sqrt{Z_{kk}^{-1} Z_{ll}^{-1}}} \tag{4.9}$$

can be found trivially from the non-diagonal terms of the error matrices Z^{-1}. These coefficients reflect the pair coupling of these parameters. Besides, the correlation factors R/85/ which reflect the connection between the given parameter a_k with all the other ones were calculated. By definition, the correlation factor is equal to the ratio of the variance D_k of the parameter a_k to its variance $\bar{D}_k$ in case if all the remaining parameters would be fixed (be known exactly)

$$R_k = \frac{D_k}{\bar{D}_k} = Z_{kk}^{-1} Z_{kk} . \tag{4.10}$$

Evidently,

$$R_K \geqslant 1 . \qquad (4.11)$$

Besides, for large correlations r_{kl} we have

$$R_k \to \infty \qquad \text{for } |r_{kl}| \to 1 . \qquad (4.12)$$

Large correlations may arise only in the case if the regions in which the derivatives of the approximating function by these parameters are different from zero, are strongly overlapped. On the other hand, if for some pair of parameters the intersection of these regions is small compared with the largest of them, these parameters will be weakly coupled. Therefore, the parameters of the threshold additions, Π_K, due to their local character, must be so weakly connected with the remaining parameters that one can neglect this coupling and look for them separately.

Large correlations of the parameters are extremely undesiarable from all points of view. In some cases one succeeds in reducing them artificially without changing essentially the interpolation formula, but changing only the way of the introduction the parameters. So, for instance, in the formula for $\left(\frac{\Gamma}{2}\right)^2$ (2.15) the constant **c** was introduced which was chosen so that the parameter a_3 would not affect appreciably the width of the resonance curve. This procedure diminished hundreds times the respective correlation factors R_{a_2} and R_{a_3}.

The quality of the approximation was checked both by a direct comparison of the curves $y(\eta)$ with the experimental values σ_i and by a calculation of the magnitude

$$\nu^2 = \frac{M_{min}}{n-m} = \frac{M_{min}}{n^+ + n^- + 1 - 14} = \frac{M_{min}}{251} \qquad (4.13)$$

(unity takes into account the normalization experiment of paper /66/), which must be close to unity, if the spread of points with respect to the curves has the same nature as the errors of the points themselves. Due to the presence of the region 690-900 MEV where the proper discription of the data on σ^+, σ^- was not yet found, the value of v^2 was found impossible to make smaller than 2.4.

5. RESULTS

The cross sections $\sigma^+(E)$ and $\sigma^-(E)$ which are in best agreement with experiment are shown in Figs. 5-8 . The solid curve refers to the last variant, its width corresponding to the error corridor $S(E)$. The error corridor has the same meaning as the standard errors of measurements usually indicated : for each fixed energy E the true cross section lies inside the corridor of errors with a probability of 68%. It is natural that the corridor of errors has this well-defined meaning only for those regions of energies where the curves $\sigma^+(E)$ and $\sigma^-(E)$ agree with experiment. The dashed lines show the curves obtained for other values of j for one of the resonances.

Since the curves $\sigma^+(E)$, $\sigma^-(E)$ for the most part may be used for the normalization of relative measurements, for which the accuracy of Figs. 5-8 is insufficient, the most probable values of the total cross sections are presented also in Table 3. In the first Column of this Table the laboratory kinetic energy of a pion E in **MeV** is written down, in the second one, the cross sections $\sigma(E)$ with the corridor of errors $S(E)$. In those regions where the agreement with experiment is not reached , as well as energies greater than 3 **BeV** to which the cross section was extrapolated the corridor of errors is not given , and the values of the cross sections are taken in brackets.

Table 4 presents the data on the parameters of the interpolating function $\sigma(E)$. The first Column of this Table shows the value for the projection of the isotopic spin T. The second Column indicates the form of the function, the indices $^+$and$^-$denote the belonging to the states T=3/2 and T=½ respectively . The values of the constants entering the formula are shown in Column 3; $E_{max.}$ and $E_{res.}$ indicated for the resonant curves are, respectively, the laboratory kinetic energies of pions for which the resonant curves reach the maximum value σ_{max} and the value $2\pi\lambda\!\!\!^{-2}(2j+1)$. Column 4 contains the values of the parameters and their standard deviations. The correlation factors R are listed in Column 5; throughout numeration introduced in the sixth Column corresponds to the numeration of the parameters in the error matrix.

In the last presented version A the value 0.9 having no physical meaning was attributed to the angular momentum j of the first resonance in $\sigma^{1/2}$. If the correct value j = ½ is left for this angular momentum (version B), then the agreement with experiment becomes worse, but the corresponding a_1 and a_2 gain a certain physical interest. The remaining parameters of the version B are close to those of the A and, thus, are not presented.

Table 5 contains the error matrix and the correlations. Along the diagonal and above it one can see the elements of the error matrix $Z^{-1}_{\kappa\ell}$. Below the diagonal, instead of the elements $Z^{-1}_{\ell\kappa} = Z^{-1}_{\kappa\ell}$ are written down the coefficients of correlation $z_{\kappa\ell}$. Correlations less than 10% are omitted as they are of no practical importance.

Table 3.

Interpolated values of the cross sections

E MeV	σ^+ mb	$\pm\delta_{\sigma^+}$ mb	E BeV	σ^+ mb	$\pm\delta_{\sigma^+}$ mb
0	2.76		I	23.24	0.I8
20	4.65		I.05	25.50	0.24
30	6.93		I.I0	28.58	0.33
40	I0.I6		I.I5	(32.49)	
50	I4.4I		I.I9	(35.93)	
60	I9.87	0.73	I.25	(40.07)	
70	26.79	0.84	I.275	40.90	0.28
85	40.58	0.97	I.30	4I.I3	0.26
I00	59.29	I.08	I.33	40.68	0.26
II0	75.5	I.I	I.338	40.47	0.26
I20	94.5	I.2	I.36	39.5I	0.27
I35	I27.5	I.2	I.38	38.9I	0.27
I50	I6I.02	I.04	I.42	37.I9	0.29
I57	I74.56	0.92	I.46	35.54	0.30
I65	I86.47	0.8I	I.50	34.I5	0.30
I70	I9I.3I	0.76	I.55	32.76	0.29
I74	I93.7I	0.74	I.60	3I.69	0.27
I79	I94.74	0.74	I.65	30.89	0.25
I8I	I94.56	0.74	I.70	30.29	0.23
I85	I93.29	0.75	I.76	29.76	0.2I
I89	I90.87	0.75	I.8I	29.4I	0.I9
I95	I85.35	0.74	I.85	29.23	0.I8
200	I79.32	0.72	I.9I	29.00	0.I7
209	I66.56	0.69	I.96	28.85	0.I6
220	I49.39	0.67	2.00	28.77	0.I5
240	II9.93	0.73	2.I5	28.57	0.I3
255	I0I.II	0.78	2.33	28.5I	0.I2
270	86.06	0.79	2.46	28.52	0.II
290	70.23	0.78	2.60	28.60	0.I0
305	6I.0I	0.75	2.76	28.62	0.I0
320	53.66	0.7I	3.00	(28.78)	(0.09)
335	47.57	0.67	3.44	(28.90)	(0.09)
340	45.84	0.66	3.86	(29.06)	(0.08)
360	39.84	0.6I	5.00	(29.42)	
380	35.I8	0.56	6.66	(29.62)	
400	3I.49	0.52	I0	(29.8I)	
450	25.26	0.43	50	(29.48)	
470	23.59	0.40			
500	2I.66	0.36			
550	I9.58	0.30			
590	I8.62	0.26			
630	(I8.09)				
670	(I7.88)				
700	(I7.88)				
740	(I8.02)				
770	(I8.30)				
800	(I8.62)				
840	(I9.I7)				
880	(I9.88)				
9I5	20.64	0.I6			
965	22.0I	0.I6			

E MeV	σ^- mb	$\pm S_{\sigma^-}$ mb
0	(5.62)	
I0	(5.5)	
20	(5.8)	
30	(6.4)	
40	(7.3)	
50	(8.5)	
70	I2.37	0.29
85	I6.76	0.32
I00	22.82	0.34
II0	28.I2	0.36
I20	34.32	0.38
I30	4I.36	0.40
I40	48.95	0.4I
I50	56.23	0.40
I57	60.69	0.39
I65	64.52	0.39
I70	66.I8	0.39
I75	67.I0	0.39
I79	67.35	0.39
I85	66.92	0.39
I90	65.92	0.39
I95	64.42	0.39
200	62.52	0.38
2I0	58.02	0.36
220	53.I7	0.33
230	48.48	0.3I
240	44.24	0.29
250	40.48	0.27
265	35.94	0.26
285	33.55	0.25
290	30.75	0.24
300	29.32	0.24
3I5	27.67	0.25
330	26.6I	0.26
340	26.09	0.27
350	27.70	0.27
360	25.42	0.27
375	25.20	0.27
390	25.I6	0.27
425	25.75	0.29
450	26.85	0.39
465	27.89	0.49
485	29.80	0.66
500	3I.6I	0.83
525	36.56	I.07
550	42.23	I.05
570	45.79	0.7I
590	46.87	0.26
600	(46.42)	
605	(46.00)	
6I0	(45.24)	
620	(44.3I)	
640	(4I.67)	
665	(38.85)	
680	(37.59)	

E MeV	σ^- mb	$\pm S_{\sigma^-}$ mb
700	(36.46)	
750	36.25	0.44
770	37.38	0.40
790	39.46	0.45
8I5	43.87	0.6I
835	49.09	0.69
850	53.49	0.65
865	57.I4	0.52
873	58.77	0.46
880	59.47	0.43
890	59.49	0.42
900	58.44	0.46
9I5	55.5I	0.55
940	49.63	0.67
965	44.82	0.65
985	4I.94	0.59
BeV		
I.00	40.35	0.54
I.04	37.64	0.42
I.08	36.38	0.33
I.I0	36.05	0.29
I.I5	36.I6	0.26
I.20	36.76	0.23
I.25	37.29	0.I9
I.29	37.3I	0.I8
I.32	37.I0	0.I8
I.35	36.57	0.I7
I.37	36.2I	0.I7
I.38	36.03	0.I7
I.43	35.I0	0.I7
I.46	34.59	0.I7
I.50	34.0I	0.I7
I.60	32.95	0.I6
I.70	32.32	0.I4
I.80	3I.92	0.I3
I.90	3I.67	0.I2
2.00	3I.5I	0.I2
2.50	3I.2I	0.I0
3.00	(3I.I7)	(0.08)
4.00	(3I.06)	(0.06)
5.00	(3I.I8)	
6.00	(3I.I6)	
6.66	(3I.I5)	
I0	(30.98)	
50	(29.84)	

Table 4

Version A

1 T	2 F	3 Additional Information		4 parameters	5 R	6 №
$\frac{3}{2}$	L	$\sigma^+(0)=2.76\pm0.53$ mb		$a=0.17\pm0.20$	29	1
	B	$j=3/2$	$E_{max}=179$ MeV	$a_1=1.1342\pm0.0024$	2	2
		$l=1$	$E_{res}=195.4$ MeV	$a_2=0.0706\pm0.0018$	6	4
		$c=1.249$	$\sigma_{max}=192.7$ mb	$a_3=0.947\pm0.070$	7	3
	B	$j=3/2$	$E_{max}=1.29$ BeV	$a_1=5.901\pm0.018$	1.1	5
		$l=2$	$E_{res}=1.31$ BeV	$a_2=0.229\pm0.019$	3	6
		$c=4.36$	$\sigma_{max}=19.5$ mb	$a_3=0$		
	A	$\sigma^+(\infty)=29.2$ mb	$\eta_0=1.497$	$a=0.1255\pm0.0090$	20	7
$\frac{1}{2}$	L	$\sigma^{1/2}(0)=7.05\pm1.0$ mb		$a=0.42\pm0.13$	8	8
	B	$j=0.9$	$E_{max}=587$ MeV	$a_1=3.082\pm0.021$	1.2	9
		$(l=1)$	$E_{res}=593$ MeV	$a_2=0.152\pm0.022$	3	10
		$c=3.23$	$\sigma_{max}=34.4$ mb	$a_3=0$		
	B	$j=5/2$	$E_{max}=886$ MeV	$a_1=4.3279\pm0.0084$	1.1	11
		$l=2(3)$	$E_{res}=889$ MeV	$a_2=0.0573\pm0.0057$	2.6	12
		$c=3.8$	$\sigma_{max}=45.5$ mb	$a_3=0$		
	A	$\sigma^{1/2}(\infty)=29.2$ mb	$\eta_0=1.497$	$a=1.320\pm0.093$	5.5	13
				$N=0.9988\pm0.0064$	4	14

Version B

$\frac{1}{2}$	B	$j=1/2$	$E_{max}=585$ MeV	$a_1=3.112\pm0.043$	1.9
		$l=1$	$E_{res}=600$ MeV	$a_2=0.325\pm0.071$	5
		$c=3.23$	$\sigma_{max}=24.4$ mb	$a_3=0$	

Table 5

(42^{-I}=0.42·$I0^{-I}$;...)

	I	2	3	4	5	6	7	8	9	I0	II	I2	I3	I4
I	42^{-I}	78^{-4}	-66^{-2}	20^{-3}	29^{-3}	-46^{-3}	$I6^{-2}$	-53^{-2}	$-II^{-3}$	46^{-3}	$-I4^{-4}$	-97^{-4}	-54^{-2}	28^{-3}
2	I6%	58^{-5}	29^{-4}	26^{-5}	83^{-6}	$I0^{-5}$	24^{-5}	$I5^{-4}$	-20^{-6}	75^{-5}	$I2^{-6}$	-50^{-6}	-67^{-4}	-88^{-6}
3	-46%	I8%	50^{-2}	$I8^{-4}$	$-I2^{-3}$	-24^{-3}	-98^{-4}	26^{-2}	92^{-5}	-94^{-4}	39^{-5}	$I3^{-4}$	34^{-2}	23^{-4}
4	52%	58%	I4%	33^{-5}	$I5^{-5}$	$-I3^{-5}$	74^{-5}	69^{-4}	27^{-7}	80^{-5}	$I6^{-6}$	-40^{-6}	-28^{-4}	34^{-5}
5					33^{-3}	-66^{-4}	60^{-5}	-32^{-4}	$-I8^{-5}$	56^{-5}	34^{-5}	26^{-5}	$-I0^{-3}$	$I2^{-5}$
6	-I2%		-I8%		-I9%	38^{-3}	-83^{-4}	II^{-3}	30^{-4}	33^{-4}	-96^{-5}	$-I9^{-5}$	-23^{-3}	-62^{-5}
7		II%	-I5%	45%		-48%	80^{-4}	$-2I^{-3}$	-74^{-5}	44^{-5}	$I2^{-7}$	-38^{-5}	-66^{-4}	$I2^{-4}$
8	-20%		28%	29%			-I8%	$I7^{-I}$	$I2^{-3}$	49^{-3}	34^{-4}	52^{-4}	-55^{-2}	55^{-3}
9									46^{-3}	$I2^{-3}$	$I8^{-4}$	-46^{-4}	59^{-4}	$I6^{-5}$
I0	I0%	I4%		20%				I7%	25%	49^{-3}	$I9^{-4}$	-87^{-4}	-63^{-3}	32^{-4}
II									I0%	I0%	70^{-4}	$4I^{-5}$	-57^{-5}	$I2^{-5}$
I2									-38%	-64%		32^{-4}	89^{-4}	$-I8^{-7}$
I3	-29%	-30%	52%	-I6%		-I3%		-46%		-30%		I7%	87^{-2}	50^{-4}
I4	2I%			29%			2I%	66%		23%				$4I^{-4}$

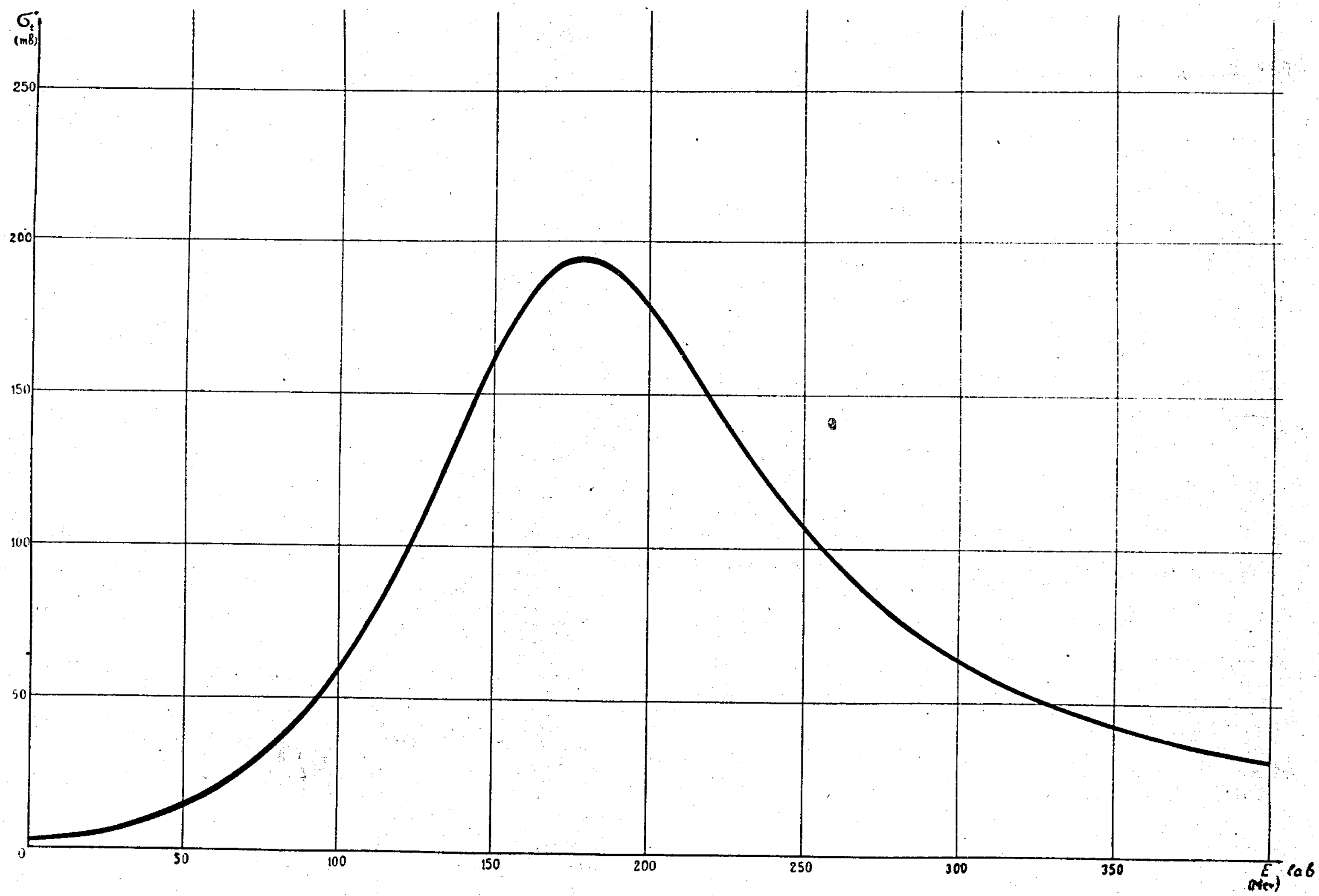

Fig. 5. Interpolated total cross section for positive pion scattering on protons. The thickness of the curve corresponds to the corridor of errors.

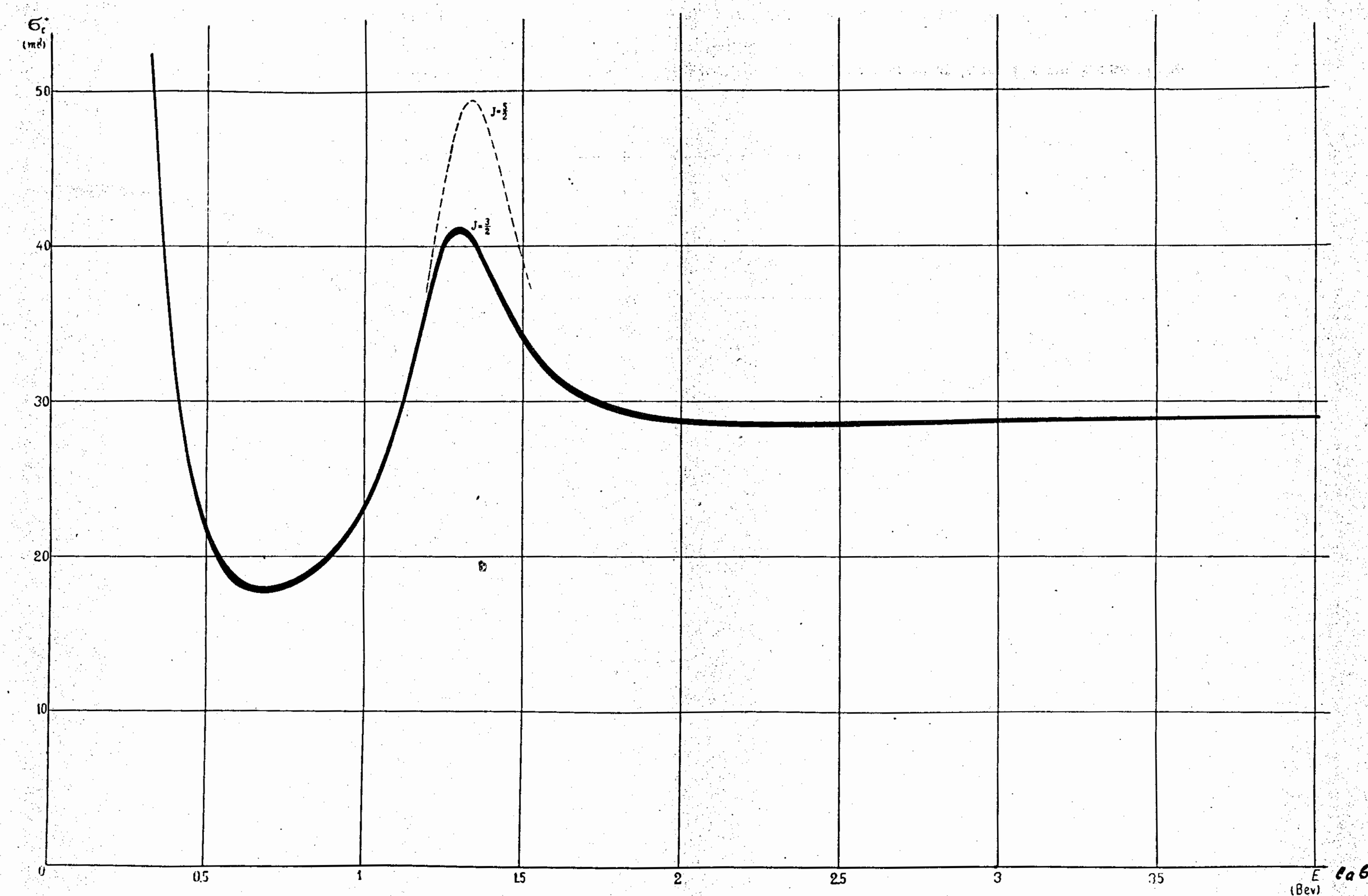

Fig. 6. Interpolated total cross section for positive pion scattering on protons. The thickness of the curve corresponds to the corridor of errors.

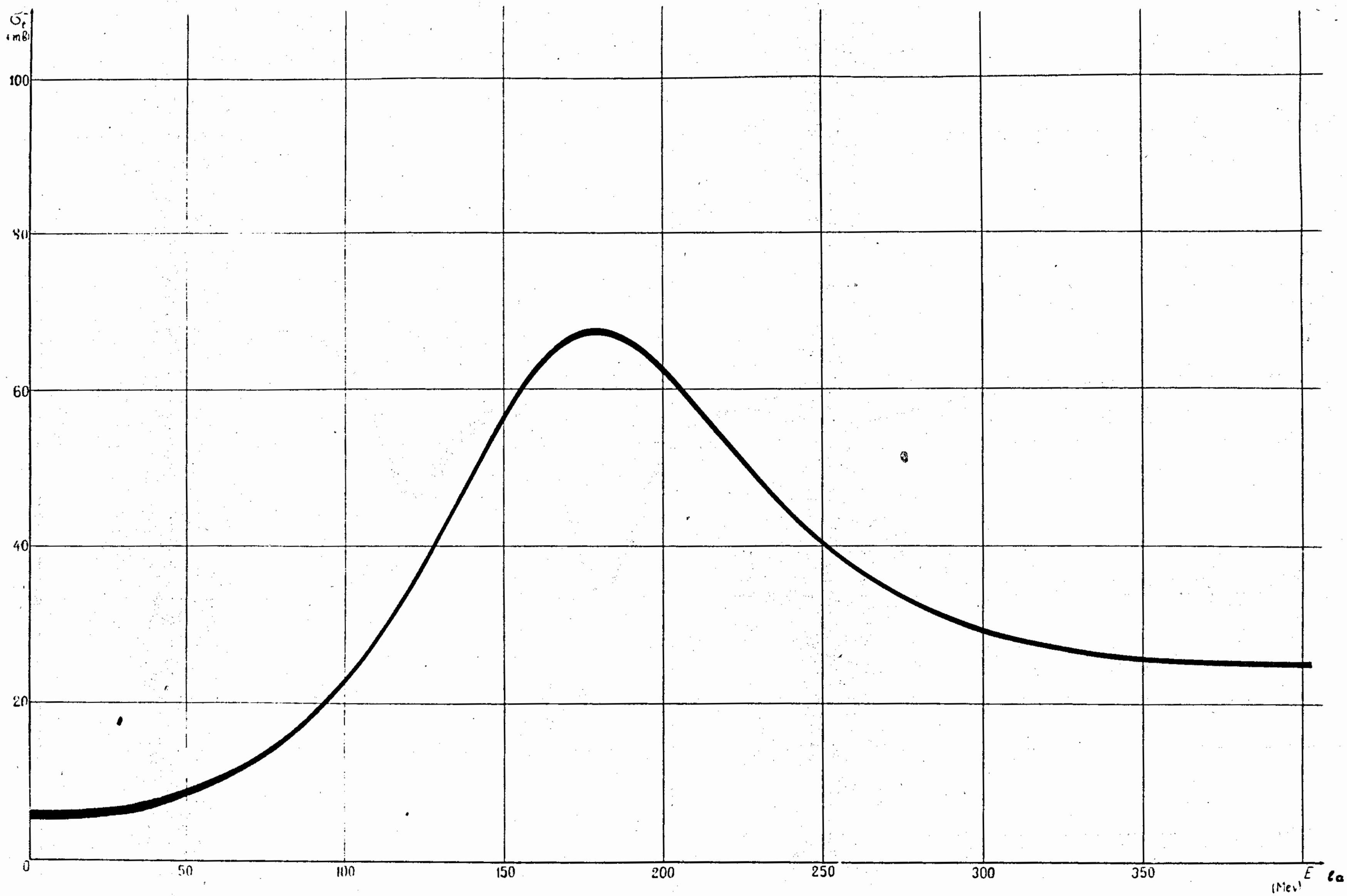

Fig. 7. Interpolated total cross section for negative pion scattering on protons. The thickness of the curve corresponds to the corridor of errors.

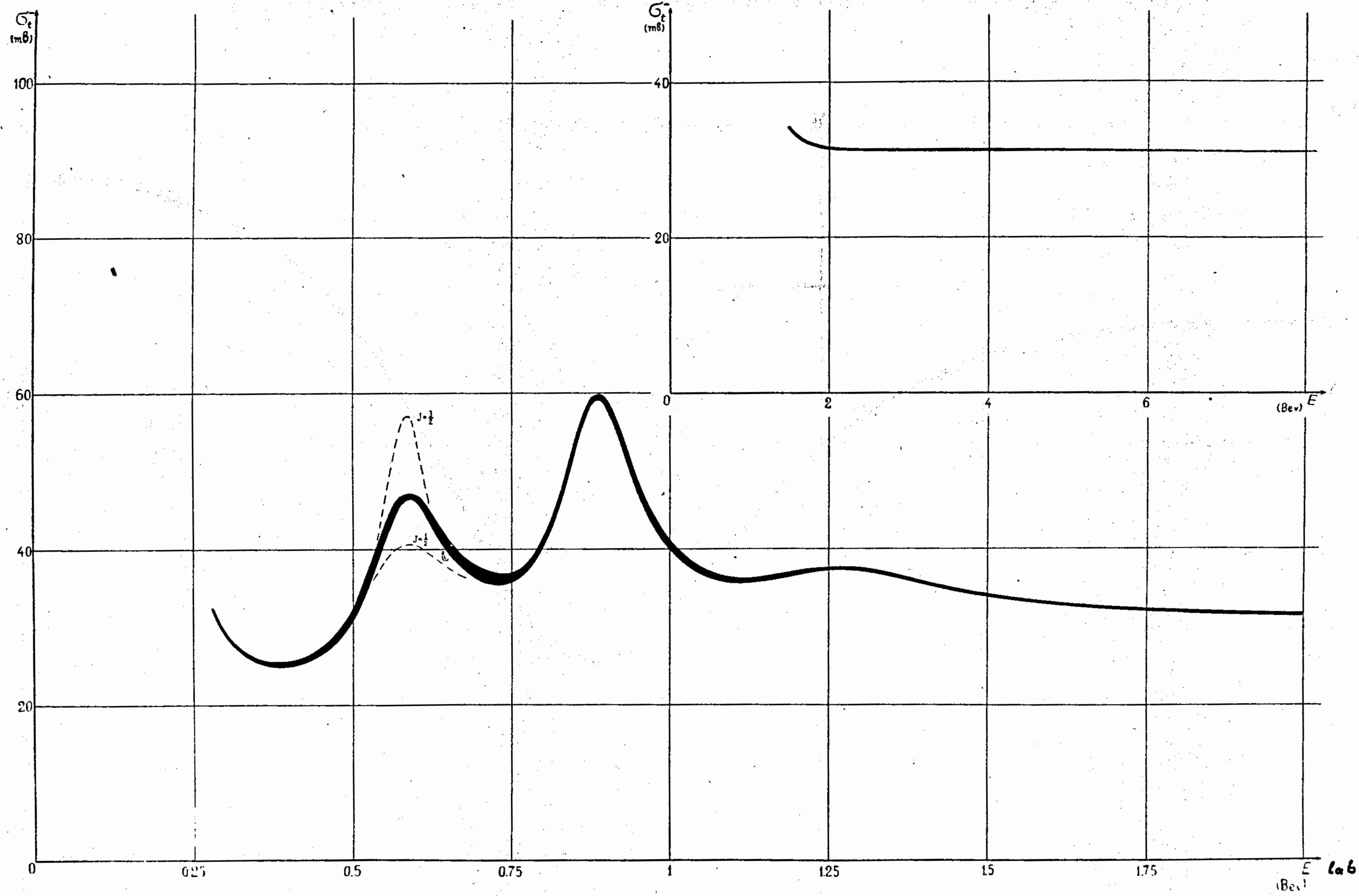

Fig. 8. Interpolated total cross section for negative pion scattering on protons. The thickness of the curve corresponds to the corridor of errors.

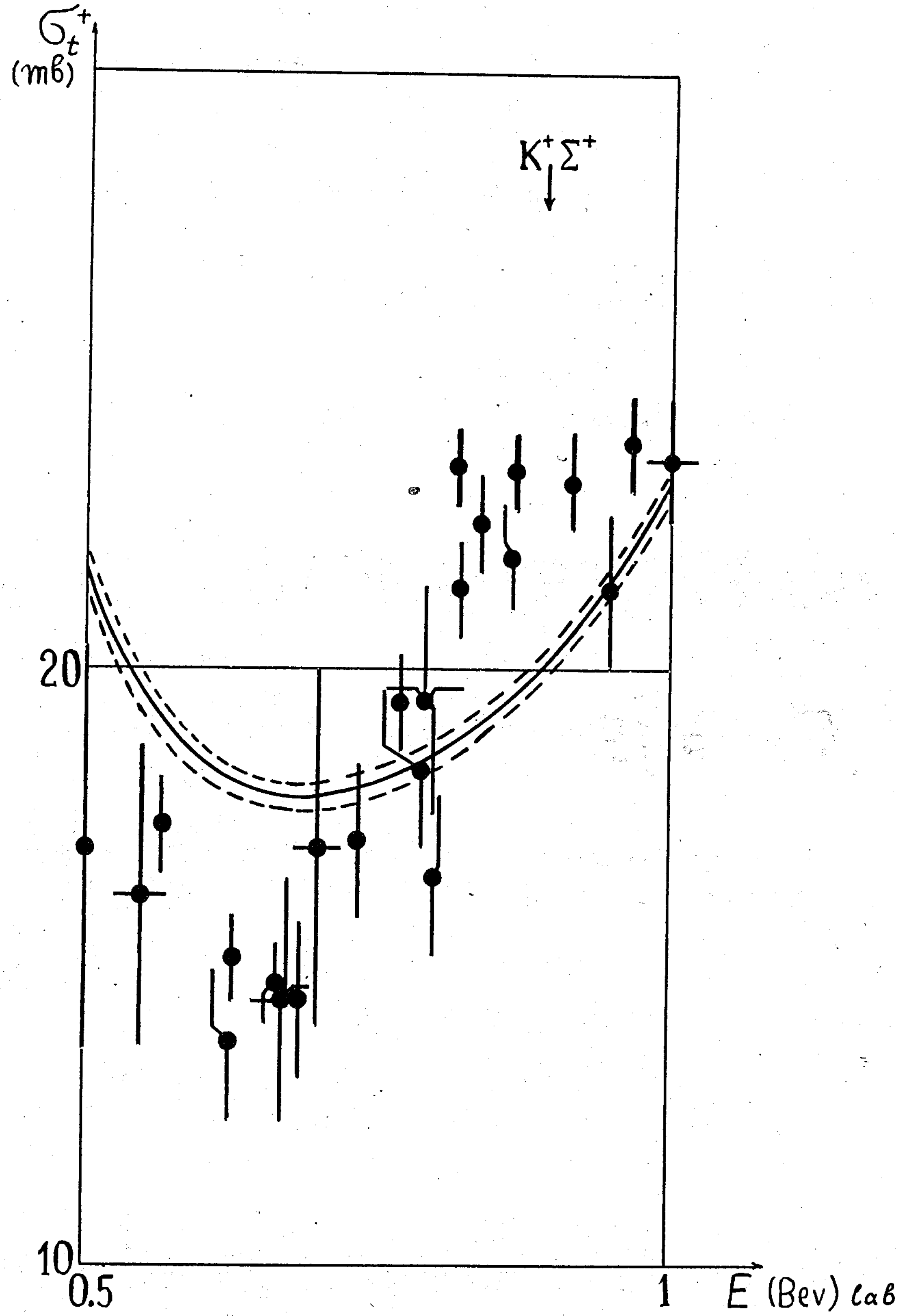

Fig. 9. The break in the total cross section for positive pion scattering on protons in the range of 600-900 MeV. The dashed lines show the corridor of errors of the interpolating curve.

6. DISPERSION INTEGRALS

The analytic expressions for $\sigma^{\pm}(E)$ obtained above may be used for checking the dispersion relations for the forward $\pi - p$ scattering .

Let us write them down as

$$D_{\pm}^{b}(\omega) = \pi_{\pm}(\omega) + F_1^{\pm}(\omega) + F_2^{\pm}(\omega) \tag{7.1}$$

where

$$\pi_{\pm}(\omega) = \frac{1}{2}\left(\frac{K}{K_b}\right)_{\omega=1} \cdot \frac{K_b}{K}\left\{(\omega+1)D_{\pm}^{b}(1) + (\omega-1)\left[-D_{\mp}^{b}(1)\right]\right\} + \frac{K K_b}{\omega \mp \frac{1}{2M}} f^2 , \tag{7.2}$$

$$F_1^{\pm}(\omega) = \frac{K K_b}{4\pi^2}\int_0^{\infty}\frac{\sigma^{\pm}(\omega')}{\omega'-\omega}\frac{d\omega'}{K'} \quad ; \quad F_2^{\pm}(\omega) = \frac{K K_b}{4\pi^2}\int_0^{\infty}\frac{\sigma^{\pm}(\omega')}{\omega'+\omega}\frac{d\omega'}{K'} . \tag{7.3}$$

Here K stands for the momentum of a pion in the laboratory system, $\omega = \sqrt{K^2+1}$, M is the mass of the nucleon in the units of m_{π} , the index "b" denotes that the value is considered in the centre-of-mass system.

The function $\pi_{\pm}(\omega)$ depends upon the couplifig constant f^2 and the subtraction parameters $D_{\pm}^{b}(1)$ which are determined by the scattering lengths

$$D_{+}^{b}(1) = a_3 \quad , \quad D_{-}^{b}(1) = \frac{2}{3}a_1 + \frac{1}{3}a_3 . \tag{7.4}$$

Formula (7.8) contains the principal value of the integral $F_1(\omega)$ and the regular integral $F_2(\omega)$. The calculation of them was made numerically by the same method which can be elucidated on the computation of $F_1(\omega)$. The integrand for $F_1(\omega)$ contains the alternating function $\frac{1}{\omega'-\omega}$ with a pole at the point $\omega'=\omega$. The integrals over the intervals $(0, \omega)$ and (ω, ∞) entering $F_1(\omega)$ have large magnitude and opposite signs. Therefore the magnitude $F_1(\omega)$ is determined by the difference of large numbers. To avoid the loss of accuracy during the computation, $F_1(\omega)$ is presented as

$$\frac{4\pi^2}{KK_6} F_1^{\pm}(\omega) = \int_{2\omega}^{\infty} \frac{f(\omega')}{\omega'-\omega} d\omega' + \int_0^{\omega} \frac{f(\omega') - f(2\omega-\omega')}{\omega'-\omega} d\omega', \qquad (7.5)$$

where

$$f(\omega') = \frac{6^{\pm}(\omega')}{K'} .$$

The first of these integrals is regular, the second one has an uncertainty of the type $\frac{0}{0}$ at the point $\omega=\omega'$. In its computation the integration was performed up to the point $\omega-\varepsilon$, and the quantity $f(\omega+\varepsilon) - f(\omega-\varepsilon)$ dependent upon the derivative of the function $f(\omega)$ at the point ω was added. It can be seen here that the principal value integral is sensitive to the form of the curve/86/.

Since the error matrix is known for the parameters entering formula (7.1) the calculation of the root-mean-square error of the functions $f_1(\omega)$ and $f_2(\omega)$ is not difficult and may be carried out by formulae (4.8). The derivatives entering (4.8) were calculated by analogy to $F(\omega)$ themselves.

The values of the functions $F_1^{\pm}(\omega)$, $F_2^{\pm}(\omega)$, $D_{\pm}(\omega)$ are given in Tables 6,7 *. The functions $F_2^{\pm}$ are changing monotonously with energy and weakly depend upon the details of the behaviour of $\sigma^{\pm}(\omega)$. The function $F_1^{\pm}(\omega)$ in contrast to $F_2^{\pm}(\omega)$, undergoes sharp changes in the region of the maxima of the total cross sections what is most clearly seen on the example of $F_1^{+}(\omega)$ in the region of the resonance $P_{3/2}, T=\frac{3}{2}$. The sharp decrease of $F_1^{+}(\omega)$ completely determines the behaviour of $D_{+}^{b}(\omega)$ in the region $2 \leq \omega \leq 2.5$.

In earlier investigations similar results were obtained for $F_1^{-}(\omega)$. However, the steepness in the region of the resonance changed considerably. This is due to the fact that the cross section 6^{-} was

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* The latter four Columns of Table 7 exhibit the integrals of $6^{1/2}$ and their errors (see (2.8).

known insufficiently accurately, and therefore, the form of the maximum at $\sim$*200 MeV* was chosen in a different way. This arbitrariness strongly affected the final expression. The investigations performed by B.Pontecorvo's group filled up the gap in our knowledge about $\sigma^{1/2}$.

Within the framework of the accepted hypothesis about the analytical form of $\sigma^{\pm}(\omega)$ the best curve has been found, i.e. the arbitrariness in the choice of the form of $\sigma^{-}(\omega)$ has been removed.

The use of new experimental data, as well as an accurate calculation of $F^{\pm}_{1,2}(\omega)$ leads, as is seen from Figs. 10, 11, to an agreement between the calculated values of $D^{b}(\omega)$ and the experimental ones. The difference between the experimental points for $D_{\pm}(\omega)$ and the curve is of the same order as the corrections to the dispersion relations /87-91) (for mesoatoms etc.) which should be accurately introduced before discussing the problem whether or not causality is kept to. A better agreement can be hardly expected at the given stage of comparison. The assumption about the constancy and the magnitude of the cross sections at infinity affects slightly the behaviour of. $D(\omega)$ at low energies what was established by direct calculation.

The discussion of all the consequences which can be derived from the results set forth in this paper will be given in another paper.

The authors take the opportunity of thanking I.M.Silin for participating in developing the method and the program of the analysis and for the calculations, as well as Professor J.A.Smorodinski and the Corresponding Member of the USSR Academy of Sciences B.M.Pontecorvo for constant interest in this research.

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Table 6

ω	$D_+^b(\omega)$	$D_-^b(\omega)$	ω	$D_+^b(\omega)$	$D_-^b(\omega)$
1	-0.105	0.07500	2.25	0.1373	0.1064
1.05	-0.09155		2.30	-0.0092	0.06983
1.10	-0.05211	0.07656	2.35		
1.15	-0.02357	0.07943	2.40	-0.2411	+0.00097
1.20	0.006420	0.08358	2.45	-0.3411	-0.02779
1.25	0.03790	0.08893	2.50	0.4257	-0.05158
1.30	0.07093	0.09548	2.55	0.4975	-0.07032
1.35	0.1056	0.1031	2.60	0.5542	-0.08430
1.40	0.1418	0.1119	2.65	0.5984	-0.09383
1.45	0.1796	0.1216	2.70	0.6318	-0.09980
1.50	0.2189	0.1324	2.75	0.6550	-0.1025
1.55	0.2596	0.1440	2.80	0.6728	-0.01023
1.60	0.3012	0.1563	2.85	0.6834	-0.09957
1.65	0.3429	0.1691	2.90	0.6899	-0.09807
1.70	0.3840	0.1821	2.95	0.6923	-0.09373
1.75	0.4231	0.1948	3.00	0.6916	-0.08856
1.80	0.4581	0.2064	3.20	0.6710	-0.06231
1.85	0.4866	0.2162	3.40	0.6365	-0.03286
1.90	0.5051	0.2230	3.60	0.5993	-0.002780
1.95	0.5099	0.2256	3.80	0.5602	+0.02797
2.00	0.4972	0.2226	4.00	0.5235	+0.06036
2.05	0.4633	0.2130	4.20	0.4887	+0.09571
2.10	0.4061	0.1959			
2.15	0.3259	0.1716			
2.20	0.2259	0.1412			

Table 7

ω	F_2^+	ΔF_2^+	F_1^+	ΔF_1^+	F_2^-	ΔF_2^-	F_1^-	ΔF_1^-	$F_2^{1/2}$	$\Delta F_2^{1/2}$	$F_1^{1/2}$	$\Delta F_1^{1/2}$
1.05			0,00019	0,00020	0,002591	0,000014	0,004084	0,000094	0,001614	0,000026	−0,00012	0,00016
1.10	0,009128	0,000059	0,02721	0,00042	0,005213	0,000029	0,00900	0,00017	0,003256	0,000052	−0,00010	0,00028
1.15	0,01375	0,000089	0,04404	0,00063	0,007866	0,000043	0,01472	0,00024	0,004924	0,000078	+0,00006	0,00038
1.20	0,01841	0,00012	0,06293	0,00082	0,01055	0,000058	0,02123	0,00029	0,00662	0,00010	0,00038	0,00045
1.25	0,02310	0,00015	0,08385	0,00098	0,01326	0,000073	0,02852	0,00032	0,00833	0,00013	0,00086	0,00049
1.30	0,02783	0,00018	0,1068	0,0011	0,01599	0,000087	0,03660	0,00035	0,01007	0,00016	0,00149	0,00051
1.35	0,03258	0,00021	0,1318	0,0012	0,01875	0,00010	0,04545	0,00037	0,01183	0,00016	0,00228	0,00052
1.40	0,03735	0,00024	0,1587	0,0013	0,02153	0,00012	0,05507	0,00040	0,01362	0,00021	0,00323	0,00051
1.45	0,04214	0,00027	0,1876	0,0015	0,02432	0,00013	0,06543	0,00044	0,01542	0,00023	0,00435	0,00048
1.50	0,04695	0,00030	0,2182	0,0017	0,02715	0,00015	0,07652	0,00049	0,01724	0,00026	0,00563	0,00045
1.55	0,05178	0,00032	0,2505	0,0019	0,02998	0,00016	0,08823	0,00057	0,01908	0,00028	0,00709	0,00041
1.60	0,05662	0,00036	0,2839	0,0022	0,03283	0,00018	0,1004	0,00066	0,02094	0,00031	0,00873	0,00038
1.65	0,06148	0,00038	0,3177	0,0026	0,03571	0,00019	0,1129	0,00080	0,02282	0,00034	0,01055	0,00036
1.70	0,06635	0,00041	0,3511	0,0030	0,03859	0,00021	0,1254	0,00093	0,02471	0,00036	0,01255	0,00037
1.75	0,07123	0,00044	0,3826	0,0034	0,04149	0,00022	0,1374	0,00106	0,02662	0,00039	0,01476	0,00042
1.80	0,07612	0,00047	0,4103	0,0038	0,04441	0,00023	0,1482	0,0012	0,02855	0,00042	0,01717	0,00045
1.85	0,08103	0,00050	0,4315	0,0043	0,04733	0,00025	0,1570	0,0013	0,03049	0,00044	0,01980	0,00058
1.90	0,08593	0,00053	0,4429	0,0047	0,05027	0,00026	0,1627	0,0015	0,03244	0,00047	0,02266	0,00070
1.95	0,09085	0,00056	0,4408	0,0051	0,05322	0,00028	0,1641	0,0016	0,03441	0,00050	0,02577	0,00084
2.00	0,09577	0,00059	0,4212	0,0056	0,05618	0,00029	0,1598	0,0018	0,03639	0,00053	0,02915	0,00098
2.05	0,1007	0,00062	0,3805	0,0062	0,05915	0,00031	0,1487	0,0020	0,03838	0,00055	0,03279	0,00114
2.10	0,1056	0,00064	0,3167	0,0067	0,06213	0,00032	0,1301	0,0021	0,04038	0,00058	0,03639	0,00131
2.15	0,1106	0,00068	0,2300	0,0070	0,06512	0,00034	0,1041	0,0023	0,04240	0,00061	0,0412	0,0015
2.20	0,1155	0,00071	0,1234	0,0071	0,06811	0,00035	0,0719	0,0023	0,04442	0,00063	0,0462	0,0017
2.25	0,1204	0,00073	0,0286	0,0070	0,07112	0,00037	0,0354	0,0023	0,04646	0,00066	0,0517	0,0019
2.30	0,1253	0,00076	−0,1242	0,0067	0,07413	0,00038	0,0030	0,0022	0,04851	0,00068	0,0575	0,0022
2.35	0,1303	0,00079	−0,2501	0,0065					0,05057	0,00071	0,0638	0,0024
2.40	0,1352	0,00082	−0,3684	0,0063	0,08016	0,00041	−0,0759	0,0021	0,05264	0,00073	0,0704	0,0027
2.45	0,1402	0,00084	−0,4745	0,0063	0,08319	0,00043	−0,1068	0,0021	0,05472	0,00076	0,0771	0,0030
2.50	0,1451	0,00088	−0,5661	0,0064	0,08623	0,00044	−0,1327	0,0021	0,05680	0,00079	0,0840	0,0032
2.55	0,1500	0,00090	−0,6428	0,0065	0,08927	0,00046	−0,1536	0,0022	0,05890	0,00082	0,0910	0,0035
2.60	0.1549	0,00093	−0,7054	0,0065	0,09231	0,00047	−0,1698	0,0022	0,06100	0,00084	0,0981	0,0037
2.65	0,1598	0,00096	−0,7554	0,0063	0,09535	0,00049	−0,1816	0,0023	0,06311	0,00087	0,1052	0,0039
2.70	0,1647	0,00099	−0,7945	0,0062	0,09840	0,00050	−0,1899	0,0024	0,06522	0,00089	0,1124	0,0041
2.75	0,1697	0,00101	−0,8244	0,0059	0,1014	0,00051	−0,1951	0,0024	0,06735	0,00092	0,1196	0,0043

ω	F_2^+	ΔF_2^+	F_1^+	ΔF_1^+	F_2^-	ΔF_2^-	F_1^-	ΔF_1^-	$F_2^{1/2}$	$\Delta F_2^{1/2}$	$F_1^{1/2}$	$\Delta F_1^{1/2}$
2.80	0,1746	0,00104	−0,8469	0,0057	0,1045	0,00053	−0,1979	0,0025	0,06948	0,00095	0,1266	0,0045
2.85	0,1795	0,00107	−0,8634	0,0054	0,1076	0,00054	−0,1988	0,0026	0,07162	0,00097	0,1336	0,0046
2.90	0,1844	0,0011	−0,8752	0,0052	0,1106	0,00056	−0,1980	0,0027	0,07376	0,00100	0,1405	0,0047
2.95	0,1893	0,0011	−0,8831	0,0050	0,1137	0,00057	−0,1962	0,0027	0,07591	0,00102	0,1471	0,0048
3.00	0,1941	0,0011	−0,8880	0,0048	0,1168	0,00059	−0,1935	0,0028	0,07807	0,00105	0,1537	0,0049
3.20	0,2136	0,0013	−0,8889	0,0046	0,1290	0,00065	−0,1778	0,0030	0,0867	0,0011	0,1778	0,0050
3.40	0,2329	0,0014	−0,8755	0,0048	0,1413	0,00070	−0,1592	0,0032	0,0955	0,0012	0,1989	0,0052
3.50	0,2427	0,0014	−0,8669	0,0049	0,1475	0,00073	−0,1497	0,0033	0,0909	0,0013	0,2088	0,0055
3.60	0,2521	0,0015	−0,8579	0,0051	0,1536	0,00076	−0,1403	0,0036	0,1043	0,0014	0,2184	0,0059
3.70	0,2617	0,0015	−0,8488	0,0052	0,1597	0,00079	−0,1308	0,0039	0,1087	0,0014	0,2282	0,0065
3.80	0,2712	0,0016	−0,8399	0,0054	0,1659	0,00081	−0,1210	0,0042	0,1132	0,0014	0,2384	0,0070
3.90	0,2807	0,0016	−0,8313	0,0055	0,1720	0,00084	−0,1109	0,0046	0,1176	0,0015	0,2494	0,0077
4.00	0,2901	0,0017	−0,8230	0,0057	0,1781	0,00087	−0,1002	0,0050	0,1221	0,0015	0,2613	0,0083
4.10	0,2996	0,0017	−0,8151	0,0058	0,1842	0,00090	−0,0888	0,0055	0,1265	0,0016	0,2744	0,0090
4.20	0,3089	0,0018	−0,8076	0,0060	0,1903	0,00092	−0,0766	0,0058	0,1310	0,0016	0,2889	0,0095
4.30	0,3183	0,0018	−0,8005	0,0061	0,1964	0,00095	−0,0638	0,0060	0,1355	0,0017	0,3046	0,0098
4.40	0,3275	0,0019	−0,7938	0,0062	0,2025	0,00098	−0,0505	0,0060	0,1400	0,0017	0,3212	0,0098
4.50	0,3368	0,0019	−0,7875	0,0063	0,2086	0,0010	−0,0374	0,0056	0,1445	0,0018	0,3377	0,0093
4.60	0,3460	0,0020	−0,7815	0,0064	0,2150	0,0010	−0,0258	0,0046	0,1490	0,0018	0,3521	0,0080
4.70	0,3552	0,0020	−0,7759	0,0065	0,2208	0,0011	−0,0181	0,0036	0,1535	0,0019	0,3607	0,0062
4.80	0,3643	0,0021	−0,7705	0,0066	0,2268	0,0011	−0,0179	0,0038	0,1580	0,0019	0,3584	0,0063
4.90	0,3734	0,0021	−0,7657	0,0067	0,2328	0,0011	−0,0294	0,0068	0,1626	0,0020	0,3389	0,010
5.00	0,3825	0,0021	−0,7609	0,0068	0,2389	0,0011	−0,055	0,011	0,1671	0,0020	0,298	0,016
5.10	0,3915	0,0022	−0,7563	0,0069	0,2449	0,0012	−0,095	0,014	0,1716	0,0021	0,236	0,022
5.20	0,4006	0,0022	−0,7519	0,0070	0,2509	0,0012	−0,141	0,015	0,1761	0,0021	0,165	0,024
5.30	0,4096	0,0023	−0,7477	0,0071	0,2569	0,0012	−0,185	0,015	0,1806	0,0022	0,096	0,022
5.40												
5.50	0,4274	0,0024	−0,7401	0,0072	0,2689	0,0012	−0,245	0,010	0,1897	0,0022	0,002	0,016
5.60					0,2749	0,0013	−0,2598	0,0077	0,1942	0,0023	−0,021	0,012
5.70	0,4451	0,0025	−0,7333	0,0073	0,2808	0,0013	−0,2660	0,0057	0,1987	0,0023	−0,0324	0,010
5.80	0,4539	0,0025	−0,7301	0,0073	0,2868	0,0013	−0,2661	0,0049	0,2032	0,0024	−0,0342	0,0084
5.90	0,4626	0,0026	−0,7271	0,0073	0,2927	0,0014	−0,2619	0,0051	0,2077	0,0024	−0,0292	0,0086
6.00	0,4713	0,0026	−0,7243	0,0074	0,2986	0,0014	−0,2543	0,0059	0,2123	0,0025	−0,0193	0,0097
6.10	0,4799	0,0026	−0,7215	0,0074	0,3045	0,0014	−0,2441	0,0068	0,2168	0,0025	+0,0054	0,011
6.20	0,4886	0,0027	−0,7188	0,0074	0,3104	0,0014	−0,2318	0,0077	0,2213	0,0026	+0,012	0,012
6.30	0,4971	0,0027	−0,7162	0,0074	0,3162	0,0015	−0,2176	0,0085	0,2256	0,0026	+0,032	0,013

ω	F_2^+	ΔF_2^+	F_1^+	ΔF_1^+	F_2^-	ΔF_2^-	F_1^-	ΔF_1^-	$F_2^{1/2}$	$\Delta F_2^{1/2}$	$F_1^{1/2}$	$\Delta F_1^{1/2}$
6.40	0,5057	0,0028	-0,7137	0,0074	0,3221	0,0015	-0,2018	0,0091	0,2303	0,0026	0,054	0,014
6.50	0,5142	0,0028	-0,7112	0,0074	0,3279	0,0015	-0,1846	0,0093	0,2348	0,0027	0,079	0,014
6.60	0,5227	0,0029	-0,7087	0,0074	0,3377	0,0015	-0,1670	0,0091	0,2393	0,0027	0,104	0,014
6.70	0,5311	0,0029	-0,7063	0,0074	0,3396	0,0016	-0,1502	0,0084	0,2438	0,0028	0,128	0,012
6.80	0,5395	0,0029	-0,7039	0,0074	0,3454	0,0016	-0,1370	0,0068	0,2483	0,0028	0,146	0,011
6.90	0,5479	0,0030	-0,7014	0,0074	0,3512	0,0016	-0,1323	0,0047	0,2528	0,0028	0,1523	0,0075
7.00	0,5563	0,0030	-0,6990	0,0074	0,3569	0,0016	-0,1431	0,0040	0,2572	0,0029	0,1348	0,0065
7.10	0,5646	0,0031	-0,6964	0,0074	0,3627	0,0017	-0,1781	0,0072	0,2617	0,0029	0,081	0,011
7.20	0,5729	0,0031	-0,6939	0,0074	0,3684	0,0017	-0,242	0,011	0,2662	0,0030	-0,016	0,016
7.40	0,5893	0,0032	-0,6885	0,0075	0,3799	0,0017	-0,424	0,016	0,2752	0,0031	-0,292	0,022
7.60												
7.80												
8.00	0,6380	0,0037	-0,6703	0,0076	0,4139	0,0018	-0,6312	0,0052	0,3019	0,0033	-0,6116	0,0083
8.20	0,6540	0,0035	-0,6637	0,0076	0,4251	0,0019	-0,6238	0,0046	0,3107	0,0034	-0,6038	0,0072
8.40	0,6698	0,0035	-0,6574	0,0076	0,4363	0,0020	-0,6123	0,0047	0,3196	0,0034	-0,5897	0,0074
8.60	0,6855	0,0036	-0,6520	0,0074	0,4474	0,0020	-0,6013	0,0048	0,3284	0,0035	-0,5758	0,0077
8.80	0,7012	0,0037	-0,6494	0,0072	0,4585	0,0020	-0,5924	0,0050	0,3372	0,0036	-0,5640	0,0079
9.00	0,7167	0,0039	-0,6509	0,0069	0,4695	0,0021	-0,5868	0,0050	0,3459	0,0037	-0,5548	0,0080
9.20	0,7321	0,0039	-0,6595	0,0070	0,4805	0,0021	-0,5851	0,0049	0,3547	0,0037	-0,5479	0,0080
9.40	0,7474	0,0039	-0,6783	0,0077	0,4914	0,0022	-0,5881	0,0050	0,3634	0,0038	-0,5431	0,0081
9.60	0,7626	0,0040	-0,7100	0,0089	0,5023	0,0022	-0,5967	0,0051	0,3721	0,0039	-0,5401	0,0081
9.80	0,7777	0,0041	-0,7554	0,0100	0,5131	0,0023	-0,6108	0,0054	0,3808	0,0040	-0,5385	0,0081
10.0	0,7927	0,0041	-0,812	0,011	0,5238	0,0023	-0,6295	0,0056	0,3894	0,0040	-0,5381	0,0081
10.2	0,8076	0,0042	-0,875	0,011	0,5346	0,0023	-0,6509	0,0057	0,3980	0,0041	-0,5387	0,0081
10.4	0,8224	0,0043	-0,938	0,011	0,5452	0,0024	-0,6727	0,0057	0,4066	0,0041	-0,5403	0,0081
10.6	0,8371	0,0043	-0,994	0,011	0,5558	0,0024	-0,6933	0,0057	0,4152	0,0043	-0,5426	0,0082
10.8	0,8517	0,0044	-1,043	0,010	0,5664	0,0025	-0,7114	0,0056	0,4238	0,0043	-0,5456	0,0082
11.0	0,8662	0,0045	-1,0825	0,0097	0,5769	0,0025	-0,7269	0,0055	0,4323	0,0044	-0,5496	0,0082

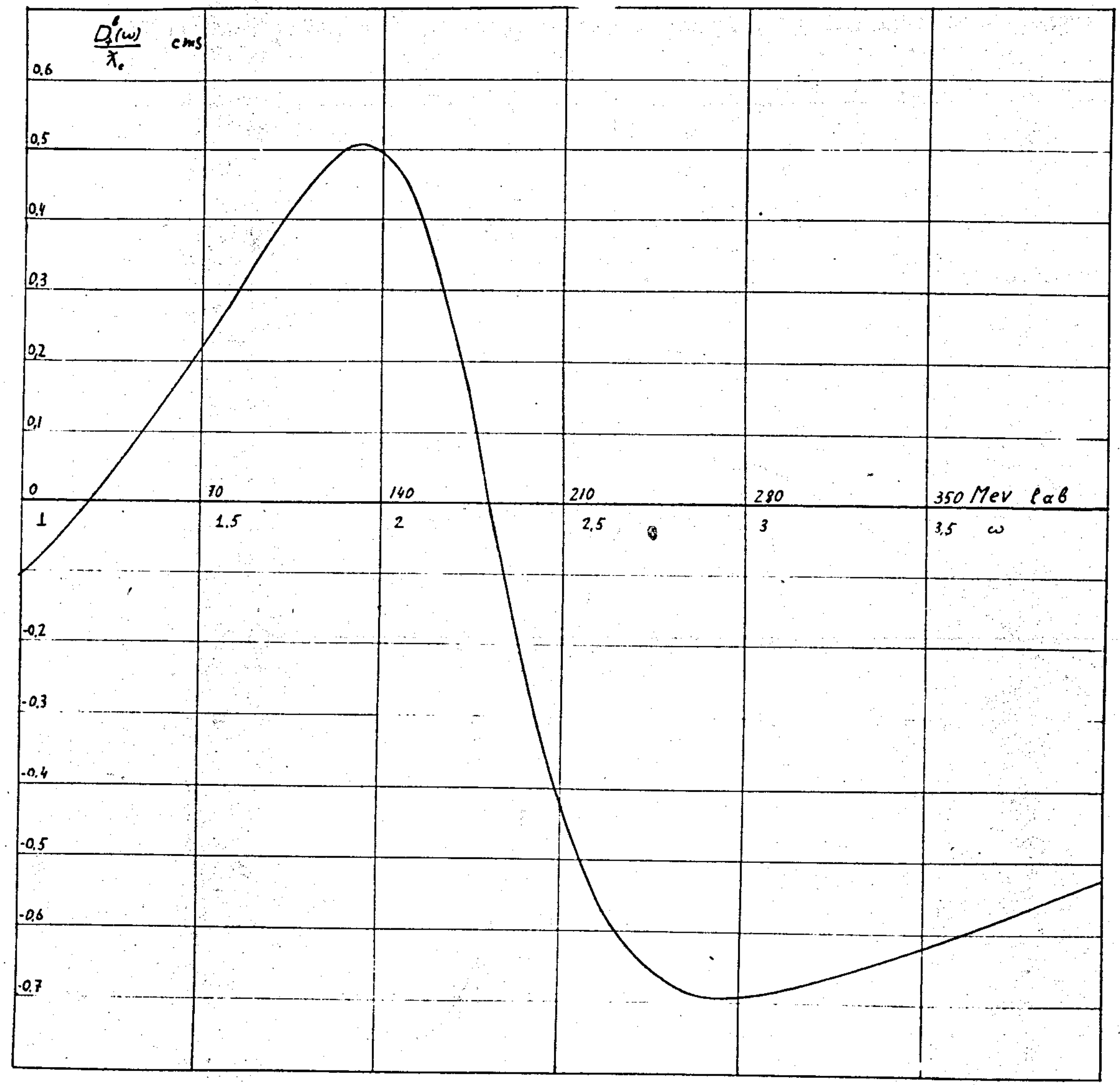

Fig. 10. The real part of the forward $\pi^+ p$ scattering amplitude .

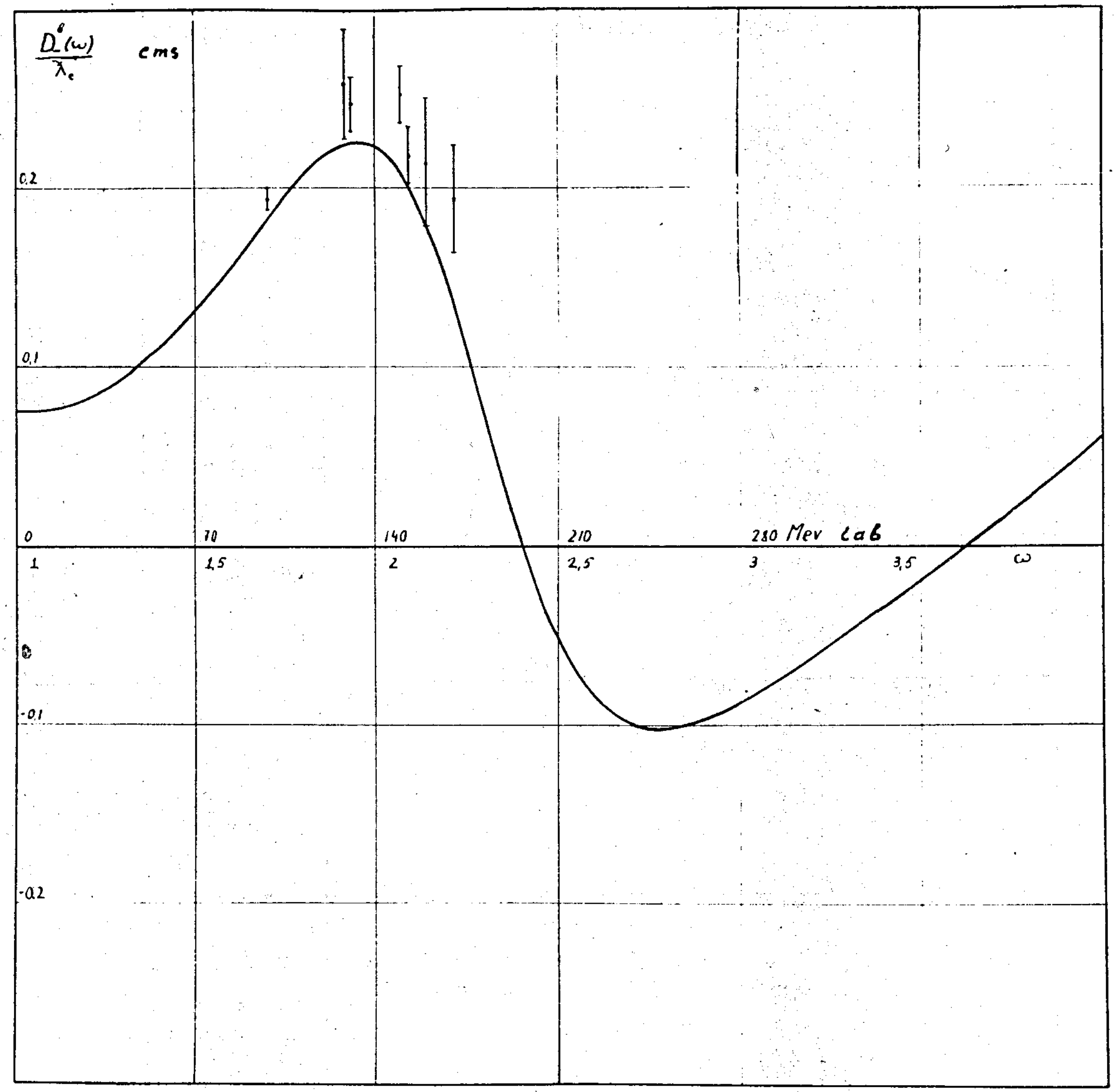

Fig. 11. The real part of the forward $\pi^- p$ scattering amplitude. A comparison with experiment.

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