ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

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$\pi\pi$ INTERACTIONS IN πN-COLLISIONS AT HIGH ENERGIES OF PIONS

Intern. Conference on high energy Physics. Rochester, 1960. p 357.
Nucl. Phys., 1961, v 22, N 3, p 410.

Дубна 1960

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D-576

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In this paper the role of $\pi\pi$ interaction in πN collisions is studied when the momentum of the primary pion is about 7 Bev. It has been shown that in elastic πN scattering the contribution of $\pi\pi$ interaction is not great, and that it is fully camouflaged by the diffraction scattering from the nucleon core. In case of inelastic collisions the contribution of $\pi\pi$ interaction is already quite noticeable and, therefore, it is possible to determine the cross sections for $\pi\pi$ interaction from the experimental data on multiple pion production. The distribution of the transverse momentum of the recoil nucleon has been calculated in the processes $\pi + N \to N + 2\pi$ and of Λ^0 for the process $\pi + N \to \Lambda^0 + K + \pi$. It has been also shown that this distribution is in agreement with the experimental data.

1. Elastic Collisions

At high energies elastic pion scattering on nucleons reduces essentially to diffraction scattering. Therefore, the possibility of detecting $\pi\pi$ interaction from the measurements of elastic pion scattering on nucleons depends upon the relative role of the peripheral elastic $\pi\pi$ interactions and diffraction scattering induced by central opaque parts of the nucleon (core)/1/. In this section we are going to consider the relative role of these two contributions to elastic scattering.

We represent the wave function of the pion-nucleon system $\phi(x,y)$ (x - are the nucleon coordinates, y - are the pion coordinates) as a sum

$$\phi(x,y) = p(x,y) + q(x,y). \qquad (1)$$

At the same time $q(x,y)$ satisfies the equation

$$q(x,y) = \phi^0(x,y) + K_c\, q(x,y) + K_\pi\, q(x,y). \qquad (2)$$

Here $\phi^0(x,y)$ is the primary wave, $p(x,y)$ is the contribution of the renormalized diagram which may be divided into two parts connected only by one nucleon line (see Fig. 1a), K_π and K_c are the renormalized integral operators. The operator K_π represents the $\pi\pi$ interaction (diagram Fig. 1b), the operator K_c gives the contribution of all the rest diagrams (see, e.g. Fig. 1c)*.

If we divide $q(x,y)$ into two parts

$$q(x,y) = q_c(x,y) + q_\pi(x,y) \qquad (3)$$

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* The theory of Eq. (2) is developed by N. Zimmermann/2/. We give it here in somewhat changed notations.

where $q_c(x,y)$ is due to the core scattering and satisfies the equation

$$q_c(x,y) = \phi^\circ(x,y) + K_c\, q_c(x,y). \tag{4}$$

Then the wave $q_\pi(x,y)$ is due to $\pi\pi$ interaction and satisfies the equation

$$q_\pi(x,y) = K_\pi q_c(x,y) + K_c q_\pi(x,y) + K_\pi q_\pi(x,y). \tag{5}$$

In this Eq. the second term represents the diffraction on the core of the wave scattered by the pion, while the third term the reaction of the wave scattered by the pion. Therefore, the main term will be the first one, and, approximately, one may put

$$q_\pi(x,y) \cong K_\pi\, q_c(x,y). \tag{6}$$

At high energies of pion the influence of the nucleon core can be approximately described by the optical model. Therefore,

$$p(x,y) + q_c(x,y) = \phi^\circ(x,y) + \phi_d(x,y). \tag{7}$$

where $\phi_d(x,y)$ is the wave of the diffraction scattering. Thus, our task, since we are interested only in the influence of $\pi\pi$ interaction, reduces to the integration of the expression

$$q_\pi(x,y) \cong K_\pi\left(\phi^\circ(x,y) + \phi_d(x,y) - p(x,y)\right). \tag{8}$$

The similarity of integral (8) and the Feynmann's one allows to determine approximately $q_\pi(x,y)$ in terms of the corresponding Feynmann integral, if instead of an incident plane wave we use the wave distorted by the core $\phi^\circ(x,y) + \phi_d(x,y) - p(x,y)$.

Thus, we get from (1), (3), (7)

$$\phi(x,y) = \phi^\circ(x,y) + \phi_d(x,y) + q_{\pi_1}(x,y) + q_{\pi_2}(x,y) + q_{\pi_3}(x,y). \tag{9}$$

where $\phi^\circ(x,y) = \phi_1^\circ(x)\,\phi_2^\circ(y)$ is a plane incident wave, while

$$\phi_d(x,y) + q_{\pi_1}(x,y) + q_{\pi_2}(x,y) + q_{\pi_3}(x,y)$$

is a scattered wave. At the same time $q_{\pi_1}(x,y)$, $q_{\pi_2}(x,y)$, $q_{\pi_3}(x,y)$

are due to the terms $\Phi^{0}(x,y)$, $\Phi_d(x,y)$, $P(x,y)$ respectively (see (8)). The diffraction wave $\Phi_d(x,y)$ in the pion and nucleon center-of-mass-system, in accordance with/3/, m be put as

$$\Phi_d(x,y) = \frac{e^{-iE_m x_0 - ik_0 y_0}}{4\pi} \int_S \left(\vec{\gamma}\frac{\partial}{\partial \vec{r}} - \gamma_4 E_m - m\right)\gamma_3 \frac{e^{ip|\vec{r}-\vec{\rho}|}}{|\vec{r}-\vec{\rho}|} \left\{1-\Omega(\rho)\right\} u_{\vec{p}_i} d\vec{\rho}$$

$$= \frac{e^{-iE_m x_0 - ik_0 y_0}}{(2\pi)^3} \int_S \int (ip'_\mu \gamma_\mu - m) \frac{e^{i\vec{p}'\cdot\vec{r} - i\vec{p}'\cdot\vec{\rho}}}{p'^2+m^2-E^2-i\varepsilon} \gamma_3 u_{\vec{p}_i} d^3\vec{p}' \times \left\{1-\Omega(\rho)\right\} d\vec{\rho} \qquad (10)$$

m - is the nucleon mass, μ - is the pion mass, $\vec{r} \equiv \vec{y} - \vec{x}$,

$\vec{p}' = (p\sin\theta\cos\phi,\ p\sin\theta\sin\phi,\ p\cos\theta)$ is the scattered momentum in the c.m.s., $\vec{p}_i = (0, 0, p)$ is the initial momentum. $E_m = \sqrt{p^2+m^2}$, $k_0 = \sqrt{p^2+\mu^2}$, $d\vec{\rho} = \rho\, d\psi\, d\rho$ - the integration is being made over all the plane S, perpendicular to $\vec{p}_i$ and passing through the center of the core

$$\Omega(\rho) = \begin{cases} 1 & \text{at} \quad \rho > R_0 \\ e^{-K\sqrt{R_0^2-\rho^2}} & \text{at} \quad \rho \le R_0 \end{cases}$$

$R_0 =$ the radius of the core

At $r \gg R_0$

$$\Phi_d(x,y) \cong \frac{1}{4\pi}\left(i\vec{\gamma}\cdot\vec{p}' - \gamma_4 E_m - m\right)\frac{e^{ipr}}{r}\int_S e^{-ip\rho\cos\psi\sin\theta}\left\{1-\Omega(\rho)\right\}\gamma_3 u_{\vec{p}_i} d\vec{\rho} \qquad (10a)$$

θ is the angle between the directions $\vec{p}'$ and $\vec{p}_i$, it coincides with the direction $\vec{r}$.

The calculation of the first integral in (8) leads to the result

$$q_{\pi_1}(x,y) = \frac{e^{ipr}}{r}\sum_{\sigma=\pm\frac{1}{2}} u_{\vec{p}'\sigma} \frac{3}{8\pi^3}\frac{p}{k_0}\frac{1}{v} g_0^2 \lambda_0 \left(\bar{u}_{\vec{p}'\sigma}\cdot u_{\vec{p}_i}\right) I_1(\theta) \qquad (11)$$

where σ is the spin index, v is the relative velocity in the c.m.s., θ is defined as before, whereas $m\, I_1(\chi)$ is equal to

$$m\,I_1(\chi)=\int_0^1\!\!\int_0^1 \frac{m^2 x y^2\,dx\,dy}{\mu^2-xy\mu^2+x^2y^2m^2+4(1-x)(y-y^2)\sin^2\frac{\chi}{2}\,p^2} \qquad (12)$$

Similarly

$$q_{\pi_2}(x,y)=\frac{e^{ipr}}{r}\sum_{\sigma=\pm\frac{1}{2}} u_{\vec{p}'\sigma}\,\frac{3}{8\pi^3}\,\frac{p}{k_0}\,\frac{1}{v}\,g_0^2\,\lambda_0\,(\bar{u}_{\vec{p}'\sigma}\,I_2(\theta)\,u_{\vec{p}_i}) \qquad (13)$$

where

$$m\,I_2(\theta)=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}d\psi\int_{-1}^{+1}d(-\cos\theta')\int_0^{2\pi}d\phi'\int_0^{R_0}[\rho' d\rho'\, m\,I_1(\chi)\,p\,e^{ip\rho'\sin\theta'\cos\psi}\,\times$$

$$\times\;\frac{(-p\cos\theta'-p\gamma_2\gamma_2\sin\theta'\sin\phi'-p\gamma_1\gamma_3\sin\theta'\cos\phi'-p)}{2\,(2\pi)^2}\left\{1-\Omega(\rho')\right\}] \qquad (14)$$

Here χ is the angle between the directions $\vec{r}$ and (θ',ϕ'). The time depending phases are identical for $\Phi_d(x,y)$, $q_{\pi_1}(x,y)$, $q_{\pi_2}(x,y)$ and we omit them.

Neglecting $q_{\pi_2}(x,y)$ and $q_{\pi_3}(x,y)$, we obtain the expressions for the scattered wave (spinor)

$$\frac{e^{ipr}}{r}\left[A_\theta\,(i\,\vec{\gamma}\cdot\vec{p}-\gamma_4 E_m-m)\,\gamma_3\,u_{\vec{p}_i}+\sum_{\sigma=\pm\frac{1}{2}} B\,I_1(\theta)\,(\bar{u}_{\vec{p}'\sigma}\cdot u_{\vec{p}_i})\,u_{\vec{p}'\sigma}\right]$$

$$A_\theta=\frac{1}{4\pi}\int_S e^{ip\rho'\cos\psi\sin\theta}\,d\vec{\rho}'\,\left\{1-\Omega(\rho')\right\} \qquad (15)$$

$$B=\frac{p}{k_0}\,\frac{1}{v}\,g_0^2\,\lambda_0\,\frac{3}{8\pi^3}$$

From here, for the differential scattering cross section, we find

$$\left(\frac{d\sigma}{d\Omega}\right)_{elas}=\left(\frac{d\sigma}{d\Omega}\right)_{diffra}+\left(\frac{d\sigma}{d\Omega}\right)_{\pi_1} \qquad (16)$$

It is worth while noting that the interference terms are wholly absent. After averaging over the initial polarizations and summing over the finite ones, the cross section $\left(\frac{d\sigma}{d\Omega}\right)_{\pi_1}$ is found to be

$$\left(\frac{d\sigma}{d\Omega}\right)_{\pi_1} = \frac{1}{2E_m^2} B^2 (I(\theta))^2 \left[(1-\cos\theta)p^2 + 2m^2\right] \qquad (17)$$

In the following table are listed the values ($\left(\frac{d\sigma}{d\Omega}\right)_{\pi_1}$) in the c.m.s. for the momentum of the incident meson $P_{lab} = 6.8$ BeV/c :

θ°	0°	12°	24°	36°
$\left(\frac{d\sigma}{d\Omega}\right)_{\pi_1}$, $\frac{mb}{ster}$	2,96	1,74	1,02	0,62

It was assumed that the renormalized interaction constant $g_0^2/4\pi = 15$, whereas the pion interaction constant $|\lambda_0|$ is equal to $4\pi \times 0.3$ *.

In Fig. 2 the calculated cross section is compared with the experimental one.

As is seen, $\left(\frac{d\sigma}{d\Omega}\right)_{\pi_1}$ increases for forward scattering, but not at all so sharply as the experimental curve. Further, at $P_{lab} = 6.8$ BeV/c, $\left(\frac{d\sigma}{d\Omega}\right)_{\pi_1}$ is much smaller than the experimental value. In general, at $p \to \infty$, $\left(\frac{d\sigma}{d\Omega}\right)_{\pi_1}$ decreases as $\frac{1}{p^2}$ at $\theta = 0$, and as $\frac{1}{p^4}$ at $\theta \neq 0$.

Hence, for high energies ($\gtrsim$ 5–6 BeV lab.system) the contribution of $\pi\pi$ interaction to elastic πN collision is not great and within the experimental error /4/.

The contrary is the case for smaller pion momenta $P_{lab} \leq 1 \sim 2 \frac{BeV}{c}$. For instance, by $P_{lab} = 1.1$ BeV/c, $\left(\frac{d\sigma}{d\Omega}\right)_{\pi_1, \theta=0} \sim 10$ mb. At the same energy the experimental value $\left(\frac{d\sigma}{d\Omega}\right)_{exp, \theta=0}$ is less than 10 mb/steradian. It should be concluded from here that the role of $\pi\pi$ interaction in elastic πN scattering at $P_{lab} < 1$–2 BeV/c is very appreciable.

Consider now the importance of some simplifications we made. First of all, let us note that if in the core besides the absorption there is a refraction, then the quantity $\{1 - \Omega(\rho)\}$ would have an imaginary part. Under this condition the interference term in (16) would not have vanished. The estimates have shown that if the imaginary part $\{1 - \Omega(\rho)\}$ amounts to 5% of the real one, then $\left(\frac{d\sigma}{d\Omega}\right)_{interf} < 0.5$ mb/ster. (at $\theta = 0$), that is also very small if compared with the scattering from the core.

Further, we neglected the waves $q_{\pi_2}(x,y)$ and $q_{\pi_3}(x,y)$. If we retain the term $q_{\pi_2}(x,y)$, then the scattered wave has the form

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* This value is given by Mandelstam and Chew for low energies. If the momentum of the incident pion in the lab.system is equal to 6.8 BeV./c, then in the rest system of the incident and virtual pions the momentum is only 680 MeV/c. Therefore, it seems that the given value of $|\lambda_0|$ can be used without a large error (see 2,4).

$$\frac{e^{ipr}}{r}\Big[A_\theta\,(i\vec{\gamma}\cdot\vec{p}-\gamma_4 E_m - m)\,\gamma_5\, u_{\vec{p}_i} + \sum_{\sigma=\pm\frac{1}{2}} B\,(\bar{u}_{\vec{p}'\sigma}\,|\,I_1(\theta)+I_2(\theta)\,|\,u_{\vec{p}_i})\,u_{\vec{p}'\sigma}\Big]. \qquad (18)$$

To estimate the influence of $I_2(\theta)$ we put $\Omega(\rho)=0$, $\rho \le R_0$ and $\Omega(\rho)=1$, $\rho > R_0$ then

$$I_2(\theta) = \int_0^\pi d(-\cos\theta')\int_0^{2\pi} d\varphi'\, I_1(\chi)\,\frac{R_0 J_1(pR_0\sin\theta')}{\sin\theta'}\times$$

$$\times\,\frac{(-p\cos\theta' - p\gamma_2\gamma_3\sin\theta'\sin\varphi' - p\gamma_1\gamma_3\sin\theta'\cos\varphi' - p)}{8\pi} \qquad (19)$$

(χ - is the angle between the directions (θ,φ) and (θ',φ'))

Since

a) $I_1(\chi)$ has the maximum at $\chi=0$, and decreases rapidly when $\chi\neq 0$,

b) $|J_1(pR_0\sin\theta')|/\sin\theta'$ has the maximum at $\theta'=0$ and decreases rapidly when $\theta'\neq 0$

then it is easy to see that $|I_2(\theta)|_{max} = |I_2(0)|$ (if $\theta=0$, then $\chi=\theta'$) and rapidly decreases when θ deviated from $\theta=0$.

The value $|I_2(0)|$ in (19) amounts to $\sim 0.4\times I_1(0)$ (for $R_0 = 0.7\frac{\hbar}{\mu c}$ and p_{lab} = 6.8 BeV/c) and the sign of $I_2(0)$ opposite to $I_1(0)$, i.e.

$$m\,I_2(0) = \int_0^\pi d\theta'\,\frac{-p(\cos\theta'+1)}{4}\,m\,I_1(\theta')\,R_0 J_1(pR_0\sin\theta') \cong -0.5$$

$$m\,I_1(0) + m\,I_2(0) = 0.7 \qquad \left(\text{with } \lambda_0 = 4\pi\times 0.3,\ R_0 = 0.7\frac{\hbar}{\mu c}\right).$$

In this case the contribution of $\pi\pi$ interaction to elastic πN scattering is

$$\left(\frac{d\sigma}{d\Omega}\right)_{\pi_1+\pi_2,\,\theta=0} = \frac{1}{2E_m^2}\,B^2\,(I_1(0)+I_2(0))^2\,[(1-\cos\theta)\,p^2+2m^2] \cong 1\,\frac{mb}{ster}$$

i.e. with account of $q_{\pi_2}(x,y)$ the above-mentioned conclusions do not change.

For the isotopic state $T=\frac{3}{2}$, $p(x,y)=0$; whereas for $T=\frac{1}{2}$, $p(x,y)\neq 0$.

The module $p(x,y)$ is somewhat smaller than $\Phi_\alpha(x,y)$. Hence, the above conclusions for elastic collision also remain unchanged with account of the influence both of $q_{\pi_2}(x,y)$ and $q_{\pi_3}(x,y)$.

Thus, it is confirmed that the contribution of $\pi\pi$ interaction to elastic πN - scattering at an energy of $>$ 5–6 BeV is not great. Therefore, from the data on elastic πN scattering at high energy of pions it is difficult to determine $\pi\pi$ interaction.

2. Inelastic Collision of $\pi + N \to N + 2\pi$ Type

Now we show that the contribution of $\pi\pi$ interaction to inelastic πN scattering is not small and may be used to obtain data on $\pi\pi$ interaction.

In order to take into account the contributions from different factors at arbitrary momenta of final pions q_1 and q_2 we adopt the representation similar to Mandelstam's (see Fig. 3).

$$
\begin{aligned}
s &= -(p+q)^2 \\
t &= -(p-p')^2 \\
\bar{s} &= -(q-p')^2 \\
\mathfrak{M}^2 &= -(q_1+q_2)^2 = -Q^2
\end{aligned}
\tag{20}
$$

i.e. we consider two finite mesons as a single particle with the mass $\mathfrak{M}^{(*)}$. For each value of $\mathfrak{M}^2$ different q_1, q_2 $(q_1+q_2 = Q)$ may be possible. Below are treated the matrix elements as functions of the variables s, t, $\bar{s}$, at any fixed values of the other parameters.
Here

$$
s + \bar{s} + t = 2m^2 + \mu^2 + \mathfrak{M}^2 = M^2 \tag{21}
$$

when $\mathfrak{M}^2$ is fixed, then only two of s, $\bar{s}$, t are free. We know from dispersion relations for $A(s,\bar{s},t)$ (one of the invariant parts of the amplitude of the process shown in Fig. 3) with $s =$ const, $t =$ const and $\bar{s} =$ const that $A(s,\bar{s},t)$ has three poles : $t = \mu^2$, $s = m^2$, $\bar{s} = m^2$ and the cuts $t \geq 4\mu^2$, $s \geq (m+\mu)^2$ and $\bar{s} \geq (m+\mu)^2$.

Thus, following Mandelstam, we suppose that $A(s,\bar{s},t)$ is representable as follows/5/

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*Here we have neglected that diagram, in which the nucleon firstly emits a pion, then absorbs the incident pion, and finally emits another pion. But the contribution of this diagram is of the same order of contribution of pole $\bar{s} = m^2$, therefore it is small (see below).

$$A(s,\bar{s},t)=\frac{\alpha}{\mu^2-t}+\frac{\beta}{m^2-s}+\frac{\gamma}{m^2-\bar{s}}+$$

$$+\frac{1}{\pi^2}\int_{9\mu^2}^{\infty}\int_{(m+\mu)^2}^{\infty}\frac{A_{23}(\bar{s}',t')\,dt'd\bar{s}'}{(t'-t)(\bar{s}'-\bar{s})}+\frac{1}{\pi^2}\int_{9\mu^2}^{\infty}\int_{(m+\mu)^2}^{\infty}\frac{A_{31}(t',s')\,dt'ds'}{(t'-t)(s'-s)} \qquad (22)$$

$$+\frac{1}{\pi^2}\int_{(m+\mu)^2}^{\infty}\int_{(m+\mu)^2}^{\infty}\frac{A_{12}(s',\bar{s}')\,ds'd\bar{s}'}{(s'-s)(\bar{s}'-\bar{s})}$$

where the spectral functions $A_{23}(\bar{s}',t')$, $A_{31}(t',s')$, $A_{12}(s',\bar{s}')$ do not vanish only in the regions $(t'>9\mu^2,\ \bar{s}'>(m+\mu)^2)$, $(t'>9\mu^2,\ s'>(m+\mu)^2)$, $(s'>(m+\mu)^2,\ \bar{s}'>(m+\mu)^2)$ respectively.

From (22) (when $\mathcal{M}^2$ is fixed)

$$A_{\mathcal{M}^2}(s,\bar{s},t)=\frac{\alpha}{\mu^2-t}+\frac{\gamma}{m^2-\bar{s}}+\frac{1}{\pi}\int_{9\mu^2}^{\infty}dt'\,\frac{\mathcal{A}_3(t',s)}{t'-t}+\frac{1}{\pi}\int_{(m+\mu)^2}^{\infty}d\bar{s}'\,\frac{\mathcal{A}_2(\bar{s}',s)}{\bar{s}'-\bar{s}} \qquad (23)$$

where

$$\mathcal{A}_3(t',s)=\frac{1}{\pi}\int_{(m+\mu)^2}^{\infty}ds'\,\frac{A_{31}(t',s')}{s'-s}-\frac{1}{\pi}\int_{-\infty}^{M^2-(m+\mu)^2-t'}ds'\,\frac{A_{23}(\bar{s}',t')}{s'-s}\,,$$

$$\mathcal{A}_2(\bar{s}',s)=\frac{1}{\pi}\int_{(m+\mu)^2}^{\infty}ds'\,\frac{A_{12}(s',\bar{s}')}{s'-s}-\frac{1}{\pi}\int_{-\infty}^{M^2-9\mu^2-\bar{s}'}ds'\,\frac{A_{23}(\bar{s}',t')}{s'-s}\,.$$

Eq. (23) is just the dispersion relation for $A(s,\bar{s},t)$ with fixed s.
In the c.m.s. the kinematic consideration yields/6/

$$p^2=\frac{[s-(m+\mu)^2][s-(m-\mu)^2]}{4s} \qquad p=|\vec{p}|=|\vec{q}| \qquad (24)$$

$$p'^2=\frac{[s-(m+\mathcal{M})^2][s-(m-\mathcal{M})^2]}{4s} \qquad p'=|\vec{p}'|=|\vec{Q}|$$

$$2t = M^2 - s + 4pp'z - \frac{(m^2 - \mathfrak{M}^2)(m^2 - \mu^2)}{s}$$

$$2\bar{s} = \dot{M}^2 - s - 4pp'z + \frac{(m^2 - \mathfrak{M}^2)(m^2 - \mu^2)}{s} \qquad (25)$$

$z = \cos\theta$, θ is the angle between $\vec{p}$ and $\vec{p}'$. It is seen from here that possible values of $\mathfrak{M}^2$ lie within the limits $4\mu^2 \leq \mathfrak{M}^2 \leq (\sqrt{s} - m)^2 = (E - m)^2$ (E = the total energy), z lies in $-1 \leq z \leq 1$.

We see from (24) that $p' = 0$ when $\mathfrak{M}^2 = (E - m)^2$. Besides, at definite values of s and $\dot{\mathfrak{M}}^2$ the physically possible t lie within the limits

$$t_1 = \frac{1}{2}\left(M^2 - s + 4pp' - \frac{(m^2 - \mathfrak{M}^2)(m^2 - \mu^2)}{s} \right) \qquad (z = 1)$$

$$t_2 = \frac{1}{2}\left(M^2 - s - 4pp' - \frac{(m^2 - \mathfrak{M}^2)(m^2 - \mu^2)}{s} \right) \qquad (z = -1)$$

In Fig. 4 two extreme cases with $\mathfrak{M}^2 = (E - m)^2$ and $\mathfrak{M}^2 = 4\mu^2$ are shown, when $s = (26.6)^2\mu^2 = 707.6\ \mu^2$ (the pion momentum in the lab. system is $\simeq$ 6.8 BeV/c).

It is seen if $\mathfrak{M}^2$ increases from $4\mu^2$ up to $(E - m)^2$

a) the lines $\bar{s} = m^2 = 46.3\ \mu^2$ and $\bar{s} = (m + \mu)^2 = 61$ are gradually displacing to the right,

b) the physically possible interval (t_1 , t_2) on the line $s = 707.6\ \mu^2$ is decreasing gradually from ($t_1 \cong -0.26\ \mu^2$, $t_2 \cong -612.4\ \mu^2$) up to ($t_1 \cong -100.1\ \mu^2$, $t_2 \cong -100.1\ \mu^2$).

c) the distances of the interval (t_1 , t_2) on the line s=const from the poles and cuts are gradually increasing.

From here one can draw the following qualitative conclusions:

a) the main contribution is in the region of the value of $\dot{\mathfrak{M}}^2$ not very far from $4\mu^2$, and the value of t - not very far from $t = \mu^2$. Indeed, the closeness from the pole $t = \mu^2$ makes it very probable that γ' has the backward direction ($z = 1$);

b) the next important contribution is made by the cut $t \geq 9\mu^2$, which gives the contribution essentially to the scattering in the backward direction ($z = 1$);

c) the contributions of the pole $\bar{s} = m^2$ and of the cut $\bar{s} \geqslant (m+\mu)^2$ are very small, i.e., the probability that p' is directed in the forward direction ($z = -1$) is very small.

We can also see from Fig. 4 that the smallest values of the denominators of the terms of the pole $t = \mu^2$, of the cut $t \geqslant 4\mu^2$, of the pole $\bar{s} = m^2$ etc to Eq. (23) are equal to $|t-\mu^2| \sim 1$, $|t-4\mu^2| \sim 10$, $|\bar{s} - m^2| \sim 50$ etc, respectively.

Therefore, for the first approximation it is sufficient to take into account the contribution of the pole $t = \mu^2$ only, i.e.,

$$\langle f|S|i\rangle = \frac{(2\pi)^4}{2^{3/2}} \delta^4(p+q-p'-q_1-q_2) \frac{g_0(\bar{u}_f \gamma_5 u_i)}{(p'-p)^2+\mu^2} \frac{(\pi_1\pi_2|\pi\pi')}{\sqrt{\omega_1\omega_2\omega}}$$

After summing over the finite nucleon polarizations and averaging over the initial ones, we get for the cross section $\sigma_{\pi+N\to N'+\pi_1+\pi_2}$:

$$\sigma_{\pi+N\to N'+\pi_1+\pi_2} = \frac{g_0^2}{(2\pi)^5} \frac{1}{16 v_{\pi N}} \int \delta^4(p+q-p'-q_1-q_2) \left|(\pi_1\pi_2|\pi\pi')\right|^2 \times \tag{26}$$

$$\times \frac{1}{p_0 p_0'} \frac{p_0p_0' - pp'\cos\theta - m^2}{[2p_0p_0' - 2pp'\cos\theta - 2m^2 + \mu^2]^2} \frac{d^3p'\, d^3q_1\, d^3q_2}{\omega_1\omega_2\omega}$$

$$\left.\begin{array}{ll} p, p_0 & \text{is the momentum and the energy of the initial nucleon} \\ p', p_0' & \text{is the momentum and the energy of the recoil nucleon} \\ v_{\pi N} & \text{is the relative velocity between the initial pion and the nucleon} \\ q_1, \omega_1;\ q_2, \omega_2 & \text{are the momenta and the energy of final pions } \pi_1, \pi_2 \\ \omega & \text{is the energy of an incident pion,} \end{array}\right\} \text{In c.m.s.}$$

m, μ are the nucleon and pion masses,

θ is the angle between $\vec{p}$ and $\vec{p}\,'$

$(\pi_1\pi_2|\pi\pi')$ resembles the invariant factor in the matrix element

$$\langle \pi_1\pi_2|S|\pi\pi'\rangle = \frac{(2\pi)^4}{4} \frac{(\pi_1\pi_2|\pi\pi')}{\sqrt{\omega\omega'\omega_1\omega_2}} \delta^4(q+q'-q_1-q_2) \tag{27}$$

We shall consider the amplitude $(\pi_1\pi_2|\pi\pi')$ to be constant and later on discuss this assumption. Under this assumption we have:

$$\sigma_{\pi+N\to N'+\pi_1+\pi_2} \simeq \frac{g_0^2}{(2\pi)^4}\frac{1}{16\, v_{\pi N}}\frac{1}{p_0\omega}\int\frac{1}{p_0'}\frac{(p_0p_0'-pp'\cos\theta-m^2)}{[2p_0p_0'-2pp'\cos\theta-2m^2+\mu^2]^2}f(p')\,d^3p' \tag{28}$$

$$f(p') = \frac{|(\pi_1\pi_2|\pi\pi')|^2}{2\pi}\int\delta^4(p+q-p'-q_1-q_2)\frac{d^3q_1\,d^3q_2}{\omega_1\,\omega_2} =$$

$$= |(\pi_1\pi_2|\pi\pi')|^2\sqrt{1-\frac{4\mu^2}{(E-p_0')^2-p'^2}} \qquad (E=\omega+p_0) \tag{29}$$

where $\dfrac{f(p')}{f(0)} = \sqrt{1-\dfrac{4\mu^2}{(E-p_0')^2-p'^2}} \approx 1 \qquad (0<p'<p'_{max})$

and suddenly falls down to zero at $p'\approx p'_{max}$.

$$p'_{max} = \frac{1}{2E}\sqrt{(E+m+2\mu)(E+m-2\mu)(E-m+2\mu)(E-m-2\mu)}$$

From (28) it is possible to show that

1) $$\frac{\partial\sigma_{\pi+N\to N'+\pi_1+\pi_2}}{\partial\Omega_{p'}} = \frac{g_0^2\,|(\pi_1\pi_2|\pi\pi')|^2}{(2\pi)^4\;16\,v_{\pi N}}\frac{1}{p_0\omega}\int\frac{p'^2}{p_0'}\frac{(p_0p_0'-pp'\cos\theta-m^2)}{[2p_0p_0'-2pp'\cos\theta-2m^2+\mu^2]^2}\sqrt{1-\frac{4\mu^2}{(E-p_0')^2-p'^2}}\,dp' \tag{30}$$

has a sharp maximum at $\theta=0$ i.e., the recoil nucleons are directed sharply in the backward direction. Further,

2) the distribution of the total p' in the πN c.m.s.

$$\frac{\partial\sigma_{\pi+N\to N'+\pi_1+\pi_2}}{\partial p'} = \frac{g_0^2\,|(\pi_1\pi_2|\pi\pi')|^2}{(2\pi)^3\;16\,v_{\pi N}}\frac{1}{p_0\omega}\frac{p'^2}{p_0'}\times$$

$$\times \sqrt{1-\frac{4\mu^2}{(E-p_0')^2-p'^2}}\left[\frac{1}{4pp'}\log\left(1+\frac{4pp'}{(2p_0p_0'-2pp'-2m^2+\mu^2)}\right)-\frac{\mu^2}{(2p_0p_0'-2m^2+\mu^2)^2-4p^2p'^2}\right] \quad (31)$$

has the maximum near $p' \simeq p'_{max}$ (see Fig. 5). (If we take into acclunt the contribution of the cut $t > 9\mu^2$, then the maximum of the distribution in Fig. 5 will be somewhat displaced to the left).

3) The distribution of the transverse momentum of the recoil nucleon $p'_\perp$ is expressed by the formula in the πN c.m.s.:

$$\frac{\partial \sigma_{\pi+N\to N'+\pi_1+\pi_2}}{\partial p'_\perp} = \frac{g_0^2\,|(\pi_1\pi_2|\pi\pi')|^2}{(2\pi)^3\ 16\ v_{\pi N}}\ \frac{1}{p_0\omega}\ \times$$

$$\times \int_{-\sqrt{p'^2_{max}-p'^2_\perp}}^{\sqrt{p'^2_{max}-p'^2_\perp}} \frac{p'_\perp}{p'_0}\,\frac{(p_0p_0'-pp'_z-m^2)}{(2p_0p_0'-2pp'_z-2m^2+\mu^2)^2}\sqrt{1-\frac{4\mu^2}{(E-p_0')^2-p'^2}}\;dp'_z \quad (32)$$

$$\left(p'^2 = p'^2_\perp + p'^2_z\right)$$

The transverse momentum of the recoil nucleon $p'_\perp$ correspondin~ to the distribution maximum of $p'_\perp$ almost does not change with the pion energy : $p'_{\perp\,opt} \approx 330-350\ \frac{MeV}{c}$ (see the distribution curves in Fig. 6).

4) The total cross section for the reaction $\pi + N \to N' + \pi_1 + \pi_2$, according to (28), is equal to (in the πN c.m.s.):

$$\sigma_{\pi+N\to N'+\pi_1+\pi_2} = \frac{g_0^2\,|(\pi_1\pi_2|\pi\pi')|^2}{(2\pi)^3\ 16\ v_{\pi N}}\ \frac{1}{p_0\omega}\int_0^{p'_{max}}\frac{p'^2}{p_0'}\sqrt{1-\frac{4\mu^2}{(E-p_0')^2-p'^2}}\ \times$$

$$\times\left[\frac{1}{4pp'}\log\left(1+\frac{4pp'}{(2p_0p_0'-2pp'-2m^2+\mu^2)}\right)-\frac{\mu^2}{(2p_0p_0'-2m^2+\mu^2)^2-4p^2p'^2}\right]dp' \quad (33)$$

$$= \frac{g_0^2\,|(\pi_1\pi_2|\pi\pi')|^2}{(2\pi)^3\ 16\ v_{\pi N}}\ \frac{1}{p_0\omega}\times 0{,}5$$ (momentum of incid π $p_{lab} \approx 6.8\ \frac{\text{BeV}}{c}$)

In this integral the main contribution is from the region p' near p'_{max}. Therefore, it will be reasonable in formula (33) to take the value of $/(\pi_1\pi_2 \;/\; \pi\pi')/^2$ corresponding to $p' = p'_{max}$. This corresponds to $\pi\pi$ scattering when $|q| = 12.4\,\mu c$, $|q'| \cong 0$ in the lab. system, or $|q| = |q'| =$ 680 MeV/c in the center-of-mass system of π and π'.

Further,

$$\sigma_{\pi+\pi' \to \pi_1+\pi_2} = \frac{1}{2\varepsilon_{\pi\pi}^2}\,\frac{1}{16}\,\frac{1}{(2\pi)^2}\int \frac{|(\pi_1\pi_2|\pi\pi')|^2}{\omega^4}\, q^2\, d\Omega$$

(in the c.m.s.)

$$\cong \frac{1}{64\pi\,\omega^2}\,|(\pi_1\pi_2\,|\,\pi\pi')|^2 \qquad (34)$$

(considering $|(\pi_1\pi_2|\pi\pi')|^2$ to be independent of the angle). Therefore, the ratio of the cross sections $\sigma_{\pi+\pi'\to\pi_1+\pi_2} \;/\; \sigma_{\pi+N\to N'+\pi_1+\pi_2}$, when the momentum of the primary pion is $\cong$ 6.8 BeV, is found to be equal to 3.

On the other hand, the cross section for $p_\perp \leq 1\,\mu c$ ($\cong$ 140 MeV/c) turns out to be

$$\sigma^{(p_\perp \leq \mu c)}_{\pi+N\to N'+\pi_1+\pi_2} \approx \frac{1}{20}\,\sigma_{\pi+N\to N'+\pi_1+\pi_2}$$

when the momentum of the primary pion is $\cong$ 6.8 BeV/c, there fore

$$\sigma_{\pi+\pi'\to\pi_1+\pi_2} \;/\; \sigma^{(p_\perp \leq \mu c)}_{\pi+N\to N'+\pi_1+\pi_2} \cong 60$$

According to preliminary data the cross sections for $\pi^- p$ inelastic scattering for all final channels is $\sigma_{\pi^- p\,(total)} \approx 25\,mb$ * and $\sigma^{(p_\perp \leq \mu c)}_{\pi^- p\,(total)} \approx 2{,}6\,mb$ **. If

$\sigma_{\pi^-+p\to n+\pi^++\pi^-}$ and $\sigma_{\pi^-+p\to p+\pi^0+\pi^-} \cong 20\%\;\sigma_{\pi^- p\,(total)}$ i.e. if

$\sigma_{\pi^-+p\to n+\pi^++\pi^-}$ and $\sigma_{\pi^-+p\to p+\pi^0+\pi^-} \cong 5\,mb$;

$\sigma^{(p_\perp \leq \mu c)}_{\pi^-+p\to n+\pi^++\pi^-}$ and $\sigma^{(p_\perp \leq \mu c)}_{\pi^-+p\to p+\pi^0+\pi^-} \cong 0.5\,mb$,

then the cross section for elastic $\pi\pi$ scattering $\sigma_{\pi^-+\pi^+\to\pi^-+\pi^+}$

$\sigma_{\pi^-+\pi^0\to\pi^-+\pi^0}$ is 15-30 mb.

(Taking $\lambda_0 = 4\pi \times 0.3$ and using the method of the perturbation theory, we also get $\sigma_{\pi+N\to N'+\pi_1+\pi_2}$ in the same order as $\sigma_{\pi^- p\,(total)}$. This means that

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* According to the measurements in the propane cloud chamber in the meson beam with the momentum 6.8 BeV/c The private communication of Wang Kan-chang, Lab. of High Energies, JINR

** According to the measurements in emulsion, in the meson beam with the momentum 6.8 BeV/c. The private communication of R.M. Lebedev, Lab.of High Energies, JINR.

$\lambda_0 = 4\pi \times 0{,}3$ is correct in an order for the given case in section 1).

Note that if the factor $|(\pi_1\pi_2 | \pi\pi')|^2$ is not considered to be constant, then instead of (28), we have

$$\sigma_{\pi+N\to N'+\pi_1+\pi_2} \approx \frac{g_0^2}{(2\pi)^4 16 v_{\pi N}} \frac{1}{p_0\omega} \int \frac{1}{p_0'} \frac{p_0p_0' - pp'\cos\theta - m^2}{(2p_0p_0' - 2pp'\cos\theta - 2m^2 + \mu^2)^2} F'(p',\theta') d^3p' \qquad (35)$$

where

$$F'(p',\theta) = \frac{1}{2\pi} \int |(\pi_1\pi_2 | \pi\pi')|^2 \delta^4(p+q-p'-q_1-q_2) \frac{d^3q_1 d^3q_2}{\omega_1 \omega_2} \qquad (36)$$

In this case the conclusions 1), 2), 3) remain correct. Indeed, $f(p')$ in (29) is the first approximation for $F'(p',\theta)$ in (36), the conclusions 1), 2) and 3) are mainly determined by the factor $\frac{1}{p_0'} \frac{p_0p_0' - pp'\cos\theta - m^2}{(2p_0p_0' - 2pp'\cos\theta - 2m^2 + \mu^2)^2}$ and are not very sensitive to $f(p')$ and to $F'(p',\theta)$.

It follows from this consideration that if we also take into account the processes of multiple pion production in pion-pion interaction corresponding to diagram in Fig. 7, then the conclusions 1) and 3) on the distribution of the recoil momentum remain valid, since they are not sensitive to the vertex of $\pi\pi$ interaction. (vertex λ_0). Besides, in this case maximum of the distribution of total recoil momentum in Fig. 5 shifts further to the left according to conservation laws.

We have also made the calculation of the transverse recoil momentum of the Λ^0 -particle for the process shown in Fig. 8. This diagram gives the momentum of Λ^0 sharply in the backward direction. At the same time it turned out that the transverse momentum is found to be too large for the pseudoscalar vertex ($\Lambda^0 N K$) and too low for the scalar one. For the mixture of the pseudoscalar and scalar type ($1 \pm \gamma_5$) when the momentum of the primary pion is 6.8 BeV/c, $p_{\Lambda_{\perp\,opt}}$ is found to be $\approx$ 413 MeV/c, whereas the experimental value $p_{\Lambda_{\perp\,opt}} \approx 410 \pm 42\ \frac{MeV}{c}$ *

$$\frac{\partial\sigma}{\partial p'_{\Lambda\perp}} \propto \int_{-\sqrt{p'^2_{max} - p'^2_{\Lambda\perp}}}^{\sqrt{p'^2_{max} - p'^2_{\Lambda\perp}}} \frac{1}{\sqrt{p'^2_{\Lambda\perp} + p'^2_{\Lambda z} + m_\Lambda^2}} \sqrt{1 - \frac{2(m_K^2 + \mu^2)}{\left(E - \sqrt{p'^2_{\Lambda\perp} + p'^2_{\Lambda z} + m_\Lambda^2}\right)^2 - (p'^2_{\Lambda\perp} + p'^2_{\Lambda z})}} \times$$

$$\times \frac{p_0\sqrt{p'^2_{\Lambda\perp} + p'^2_{\Lambda z} + m_\Lambda^2} - p\,p'_{\Lambda z}}{\left(2p_0\sqrt{p'^2_{\Lambda\perp} + p'^2_{\Lambda z} + m_\Lambda^2} - 2p\,p'^2_{\Lambda z} - m^2 - m_\Lambda^2 + m_K^2\right)^2} p'_{\Lambda\perp} dp'_{\Lambda z} \qquad (37)$$

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* Ding Da-tsao. Private communication.

where m, m_Λ, m_K, μ are the masses of the nucleon, Λ^0- particle, K-meson and π meson)

$$p'_{max} = \frac{1}{2E}\sqrt{\left[(E+m_\Lambda)^2-(2\mu^2+2m_K^2)\right]\left[(E-m_\Lambda)^2-(2\mu^2+2m_K^2)\right]} \qquad (38)$$

p_0 and E are determined as earlier.

In Fig. 9 is given the distribution $p'_{\Lambda\perp}$ in the generation of Λ_0 and K particles. Note that the interactions of the type $1\pm\gamma_5$ will lead to the polarization of Λ^0 particles.

Summary

1. The contribution of $\pi\pi$ interaction to elastic pion scattering on a nucleon at the pion energy $>$ 5–6 BeV is not great, it is essential at an energy of $\lesssim$ 1 – 2 BeV.

2. In the same region of pion energies ($>$ 5 – 6 BeV) the contribution of $\pi\pi$ - interaction to inelastic pion scattering is quite appreciable. From the preliminary data on $\pi + N \to N' + \pi_1 + \pi_2$ process one can estimate the cross section for $\pi\pi$ - interaction to be $\approx$ 15 – 30 mb (at momentum $\cong 680 \frac{MeV}{c}$ in c.m.s. of $\pi\pi$)

3. The transverse momentum of the recoil nucleon in πN - collision is weakly dependent upon the pion energy. In the interval 7-1000 BeV/c, $p_{\perp_{opt}} \cong$ 330-350 MeV/c.

4. For the generation of Λ^0 particles the transverse momentum of the Λ^0 - particle is in agreement with experiment under the assumption that the vertex ($\Lambda_0 N K$) - interaction has the structure ($1 \pm \gamma_5$). In this case one should expect the polarization of the generated Λ^0 particles.

The authors express their gratitude to Kim Ze-phen for the numerical calculations. One of the authors (Wang Yung) thanks A.V. Efremov, Tzu Huan-yuan, Chou Kuang-chao, and Ho Tso-hsiu for helpful discussions and valuable advice.

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Recieved by Publishing Department
on July 15, 1960.

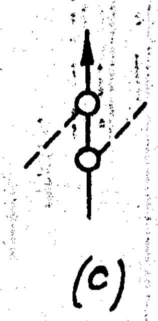
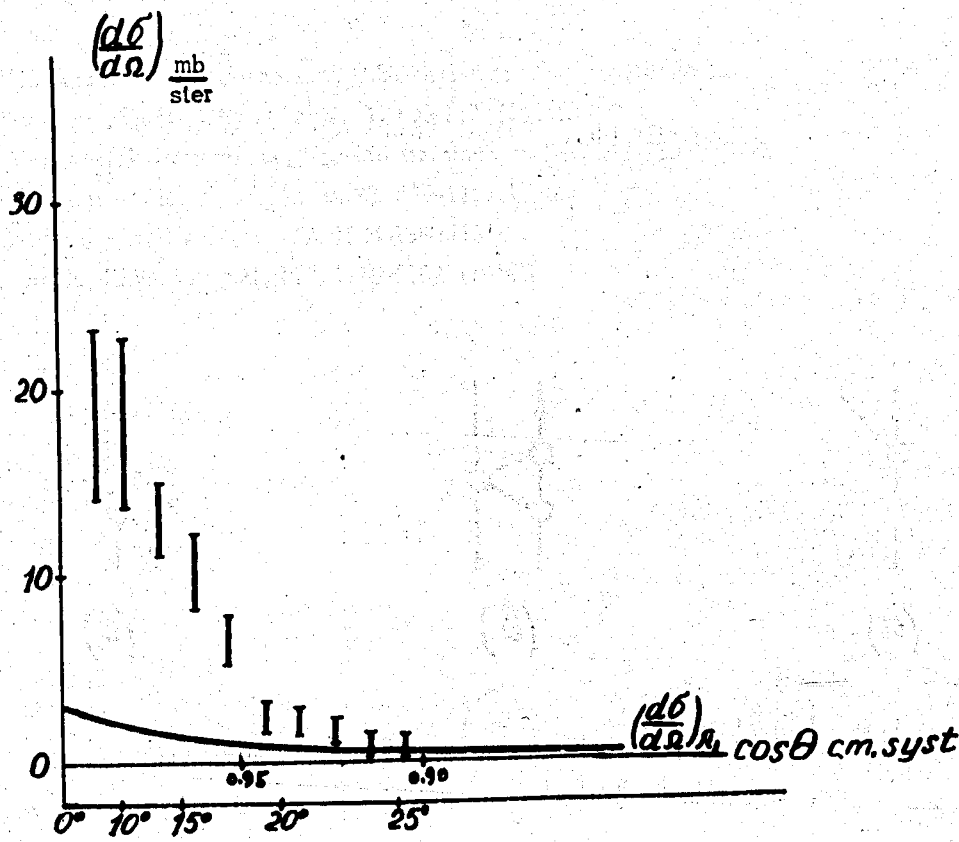


Fig. 1.



I experimental data of $\pi^- p$ collision

with $P_{lab} = 6.8 \frac{Bev}{c}$ (see /4/).

— curve of $\left(\frac{d\sigma}{d\Omega}\right)_{\pi_1}$ with $\lambda_0 = 4\pi \times 0.3$

Fig. 2.

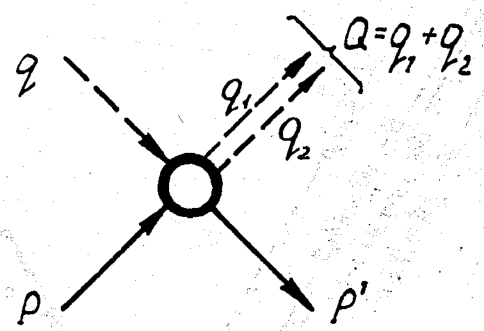


Fig. 3.

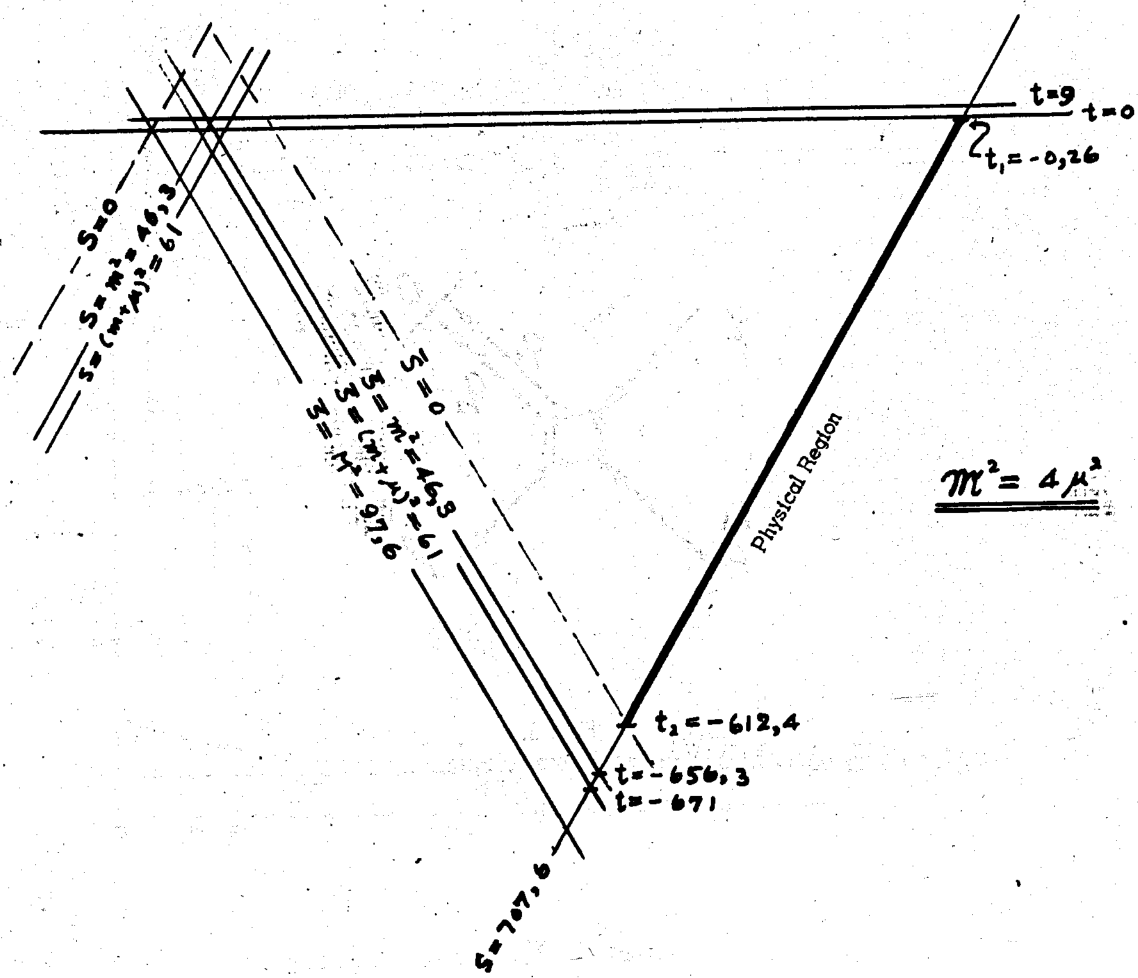

t=9
t=0
$t_1 = -0,26$
S=0
S = m² = 46,3
S = (m+μ)² = 61
Physical Region
$M^2 = 4\mu^2$
$t_2 = -612,4$
t = -656,3
t = -671
S = 707,6

Fig. 4a

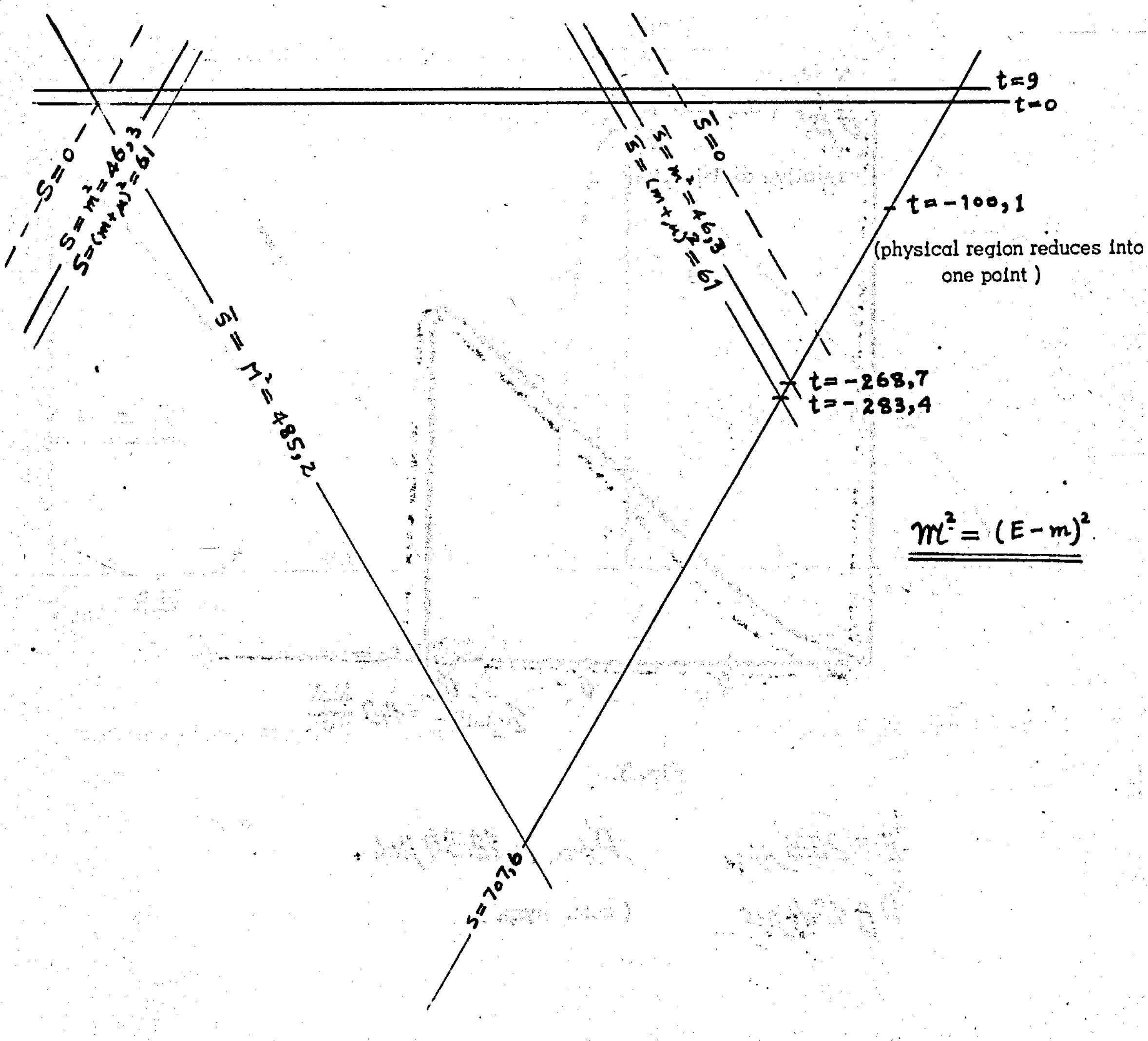

t=9
t=0
S=0
S=m²=46,3
S=(m+μ)²=61
S=M²=485,2
S=0
S=m²=46,3
S=(m+μ)²=61
t=-100,1
(physical region reduces into one point)
t=-268,7
t=-283,4
S=707,6
$\mathcal{M}^2 = (E-m)^2$

Fig. 4b

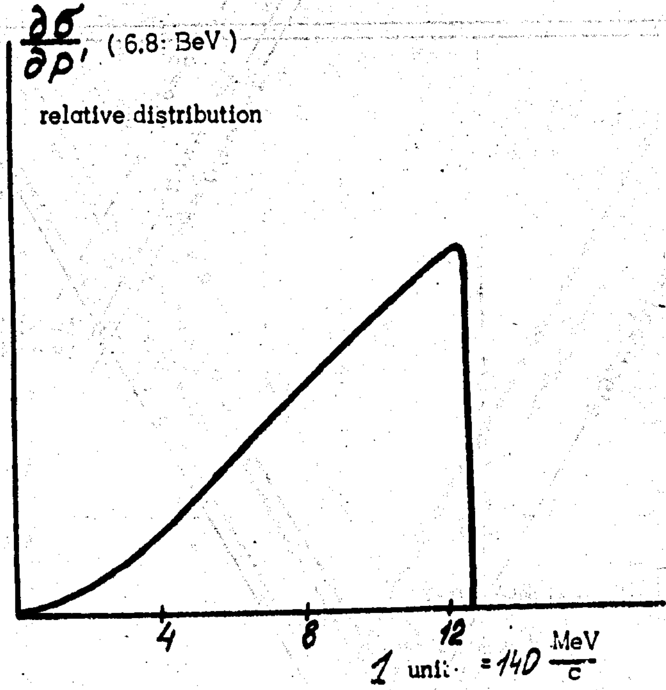
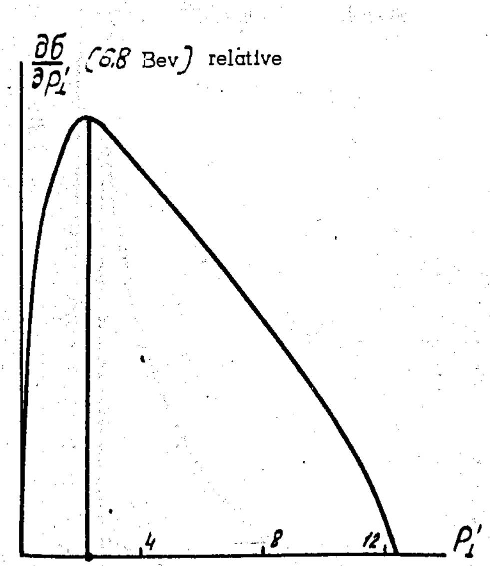


Fig. 5.

$E = 26.6\,\mu$ $\qquad$ $p'_{max} = 12.34\,\mu c$

$p = 12.4\,\mu c$ $\qquad$ (c.m. syst.)

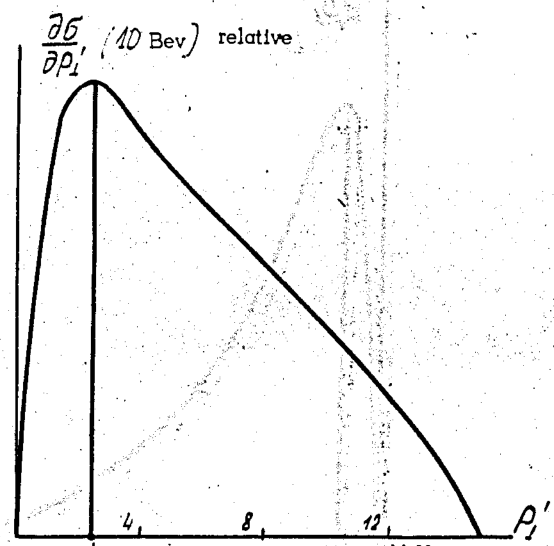


$p'_{1\,opt} = 2.39$ units

$= 335 \frac{MeV}{c}$ (1 unit = 140 $\frac{MeV}{c}$)

momentum of pion $\approx 6.8 \frac{Bev}{c}$ (1 .syst)

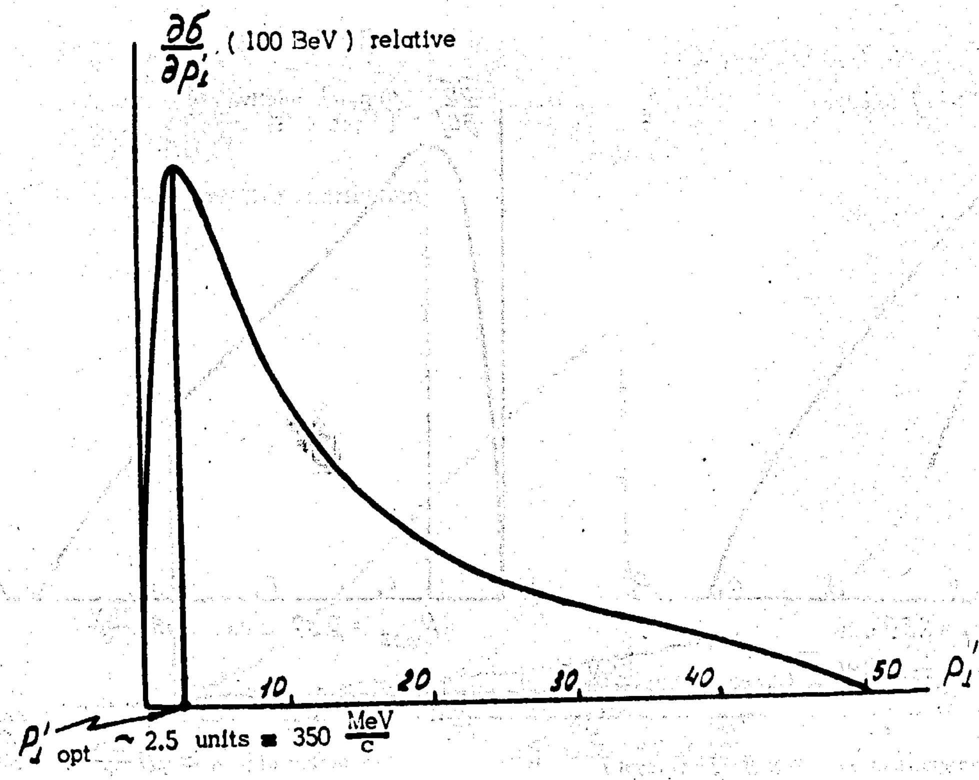
Fig. 6a



$p'_{1\,opt} = 2.37$ units $= 332 \frac{MeV}{c}$

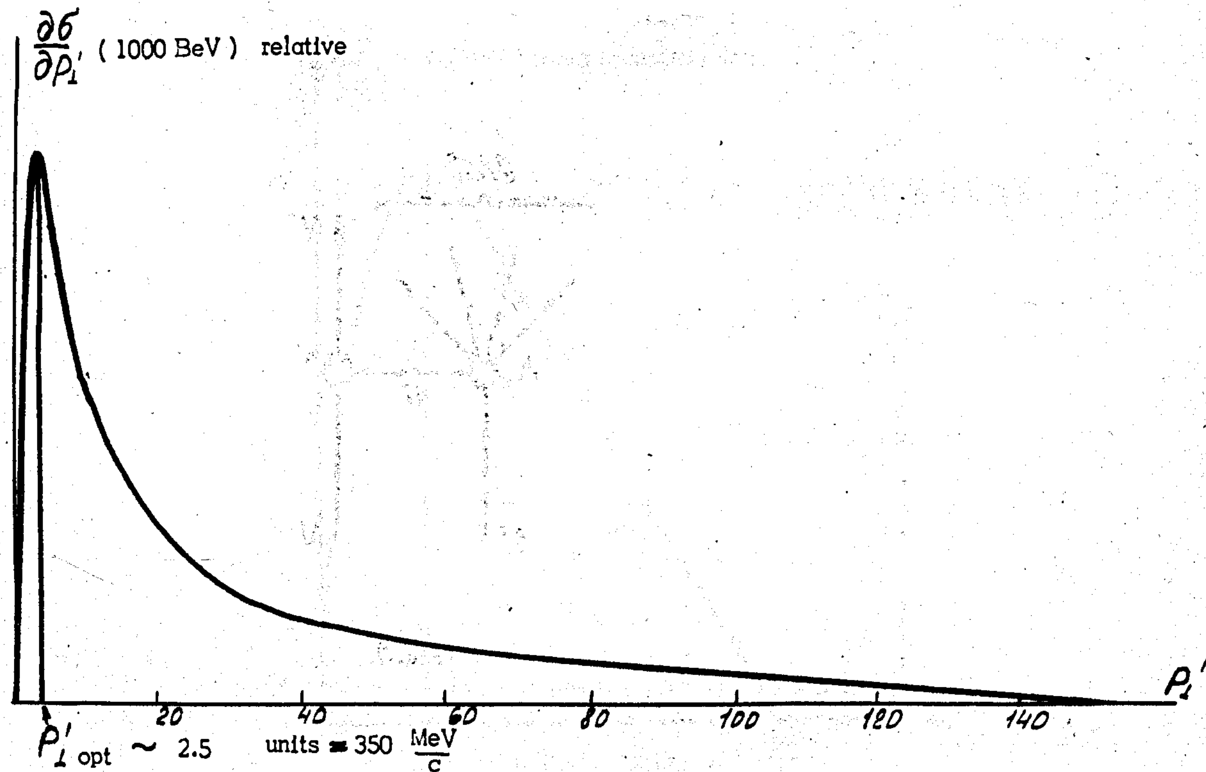
momentum of pion $\approx 10 \frac{Bev}{c}$ (1. syst.)

Fig. 6b.



momentum of pion ≈ 100 $\frac{BeV}{c}$ (l.syst)

Fig. 6c.



momentum of pion ≈ 1000 $\frac{BeV}{c}$ (l.syst)

Fig. 6d.

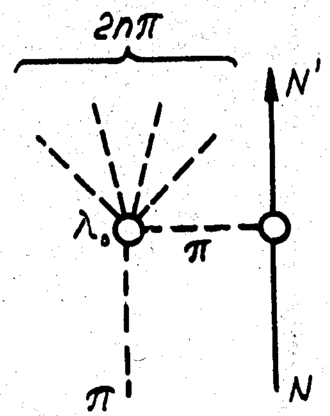


Fig. 7.

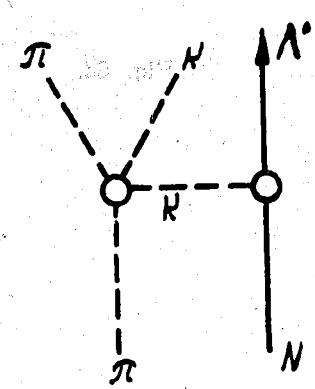


Fig. 8.

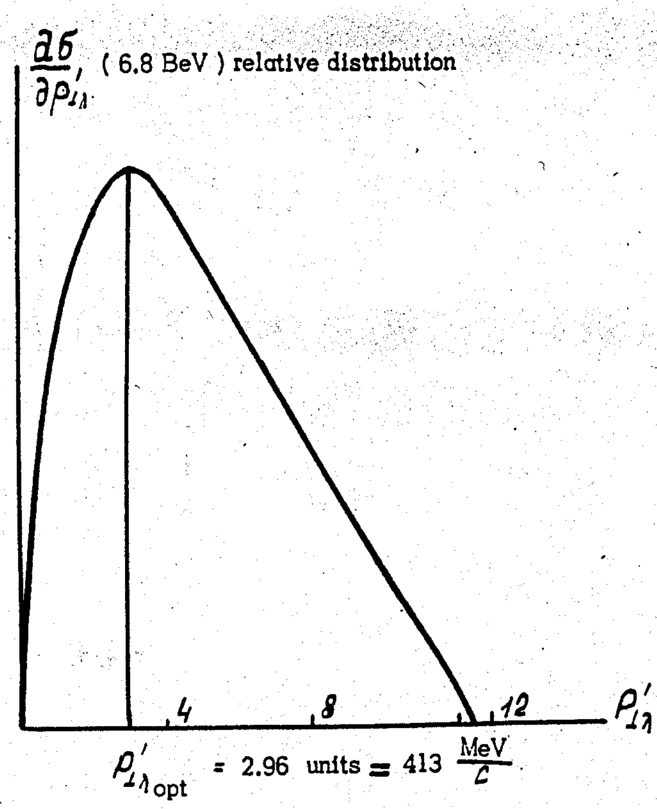

$\frac{\partial\sigma}{\partial p'_{\perp\Lambda}}$, (6.8 BeV) relative distribution
4
8
12
$p'_{\perp\Lambda}$
$p'_{\perp\Lambda\,opt}$ = 2.96 units = 413 $\frac{MeV}{c}$

Fig. 9