ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

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INTEGRAL EQUATIONS FOR KN-SCATTERING

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Abstract

Starting with the Mandelstam representation integral equations for the lowest partial waves of $K N$ - and $\tilde{K} N$ - scattering are obtained on the basis of dispersion relations for forward and backward scattering. In the approximation used, the two systems of integral equations are not coupled. As an estimate for the $d_{3/2}$-wave an expression is given which depends only on s- and p- waves.

I

Recently, in a paper by MacDowell[1] the analytic properties of partial waves for K N- scattering were investigated on the basis of the Mandelstam-representation. In the present paper integral equations are set up for s-, p- and d- waves of these processes.

In contrast to the program outlined in paper[1] and to the Chew-Mandelstam approach for $\pi\pi$ - scattering, we obtain equations for partial waves using fixed angle dispersion relations. Thus we can avoid serious difficulties ocurring in reference[2], which were investigated in[3,4].

Especially, in the present paper we get partial wave equations from dispersion relations for forward and backward scattering.

The kinematical cut arising from the square root dependence of the invariant variables s and $\bar{s}$ on k^2 can in principle be removed by symmetrization with respect to the square root (see reference[5]). However, as a consequence of the particular kinematics of the K N -processes, we here can take into account the nearest singularities on the negative cut without symmetrization by introducing a cut -off at $-m_K^2 \approx -13\mu^2$.

According to the general idea of Mandelstam we consider the following reactions

$$(I) \qquad K + N \rightarrow K' + N' ,$$

$$(II) \qquad \tilde{K}' + N \rightarrow \tilde{K} + N' ,$$

$$(III) \qquad \tilde{K}' + K \rightarrow \tilde{N} + N' .$$

The unitarity condition in two particle approximation for reaction II leads further to the process

$$\tilde{K} + N \to Y + \pi$$

(Y: Λ or Σ - hyperon) and for the third reaction (taking only lowest mass states) to the amplitudes

$$\tilde{K} + K \to \pi + \pi ,$$

$$\pi + \pi \to N + \tilde{N} ,$$

which were investigated in[6,7]. All these reactions have to be considered as known amplitudes; in $\tilde{K}N \to Y\pi$ it would be possible, for example, to take into account the pole contributions or to put in experimental dates analysed in[8].

II

The matrix elements for the processes I, II, III are written as

$$S_{fi} = \delta_{fi} + \frac{i}{(2\pi)^2}\,\delta\left(p_1 + q_1 - p_2 - q_2\right)\frac{M}{\sqrt{4\,p_1^0 p_2^0 q_1^0 q_2^0}}\;\bar{u}\,T\,u ,$$

where the Green function has the structure

$$T = A + \tfrac{1}{2}(q_1 + q_2)\gamma B , \qquad (\gamma q = \gamma_0 q_0 - \vec{\gamma}\vec{q}).$$

For A and B we have

$$A = A^{(+)} + \vec{\tau}_K \vec{\tau}_N A^{(-)} .$$

The $A^{(\pm)}$, $B^{(\pm)}$ are in the following way connected with the isotopic spin amplitudes of the corresponding processes

$$\begin{pmatrix} A^0 \\ A^1 \end{pmatrix} = \begin{pmatrix} 1 & -3 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} A^{(+)} \\ A^{(-)} \end{pmatrix} , \qquad (1)$$

$$\begin{pmatrix} A^{0} \\ A^{1} \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} A^{(+)} \\ A^{(-)} \end{pmatrix}, \qquad (\mathrm{II})$$

$$\begin{pmatrix} A^{0} \\ A^{1} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} A^{(+)} \\ A^{(-)} \end{pmatrix}. \qquad (\mathrm{III})$$

The matrix elements for the reactions I, II can be written as

$$\bar{u} T u = \frac{4\pi W}{M} \chi^{+}_{N'} \left\{ f_1 + \frac{(\vec{\sigma}\vec{q}_1)(\vec{\sigma}\vec{q}_2)}{K^2} f_2 \right\} \chi_N ,$$

where W is the total energy, K^2 the square of momentum in the c.m.-system of the given process; the $f_{1,2}$ and A, B are connected by the relations

$$(\mathrm{I}) \quad \begin{aligned} f_1 &= \alpha A + \beta B , \\ f_2 &= -\gamma A + \delta B , \end{aligned} \qquad (\mathrm{II}) \quad \begin{aligned} \bar{f}_1 &= \alpha A - \beta B , \\ \bar{f}_2 &= -\gamma A - \delta B , \end{aligned} \qquad (2.1)$$

where

$$\alpha = \frac{p_0 + M}{8\pi W} , \qquad \gamma = \frac{p_0 - M}{8\pi W} , \qquad (2.2)$$

$$\beta = \frac{(p_0 + M)(W - M)}{8\pi W} , \qquad \delta = \frac{(p_0 - M)(W + M)}{8\pi W} ,$$

(p_0 ~ nucleon energy).

The connection of $f_{1,2}$ with the partial waves is given by

$$f_1(k^2,z) = \Sigma \left\{ f_{\ell+} P'_{\ell+1}(z) - f_{\ell-} P'_{\ell-1}(z) \right\} ,$$
$$f_2(k^2,z) = \Sigma \left\{ f_{\ell-} - f_{\ell+} \right\} P'_\ell(z) . \qquad (2.3)$$

Taking into account partial waves up to $f_{2-} = f_{d\frac{3}{2}}$ we get the following expressions, where $f_{1,2}(\pm) \equiv f_{1,2}(k^2, z = \pm 1)$:

$$f_0 = \frac{1}{2}\left\{ f_1(+) + f_1(-) \right\} + \frac{1}{6}\left\{ f_2(+) - f_2(-) \right\} ,$$
$$f_{1-} = \frac{1}{6}\left\{ f_1(+) - f_1(-) \right\} + \frac{1}{2}\left\{ f_2(+) + f_2(-) \right\} ,$$
$$f_{1+} = \frac{1}{6}\left\{ f_1(+) - f_1(-) \right\} ,$$
$$f_{2-} = \frac{1}{6}\left\{ f_2(+) - f_2(-) \right\} . \qquad (2.4)$$

The unitarity condition for reaction I gives

$$\mathrm{Im}\, f_{\ell\pm} = k \left| f_{\ell\pm} \right|^2 , \qquad (2.5)$$

for II we get

$$\mathrm{Im}\, \bar{f}_{\ell\pm} = \bar{k} \left| f_{\ell\pm} \right|^2 + \bar{k}_Y \left| F_{\ell\pm} \right|^2 , \qquad (2.6)$$

where $F_{\ell\pm}$ are the partial waves of $\bar{K} N \to Y\pi$, and $\bar{k}_Y$ is given by

$$\bar{k}^2 = \frac{1}{4\bar{s}} \left\{ \bar{s} - (\mathcal{M}_Y + \mu)^2 \right\} \left\{ \bar{s} - (\mathcal{M}_Y - \mu)^2 \right\} ,$$

whereas

$$k^2 = \frac{1}{4\bar{s}} \left\{ \bar{s} - (M + m)^2 \right\} \left\{ \bar{s} - (M - m)^2 \right\} ;$$

here $\mathcal{M}_Y, \mu$ denote the masses of hyperons and π-mesons respectively, M and m those of nucleons and K-mesons.

$$
\text{(II)} \qquad
\begin{aligned}
s &= M^2 + m^2 - 2\bar{k}^2\bar{z} - 2\sqrt{(\bar{k}^2+M^2)(\bar{k}^2+m^2)}\,,\\
\bar{s} &= M^2 + m^2 + 2\bar{k}^2 + 2\sqrt{(\bar{k}^2+M^2)(\bar{k}^2+m^2)}\,,\\
t &= -2\bar{k}^2(1-\bar{z})\,, \quad \{\bar{z} = \cos\bar{\varphi}\}\,;
\end{aligned}
\qquad (2.11)
$$

$$
\text{(III)} \qquad
\begin{aligned}
s &= M^2 - m^2 - 2q^2 + 2x\sqrt{q^2p^2}\\
\bar{s} &= M^2 - m^2 - 2q^2 - 2x\sqrt{q^2p^2}\\
t &= 4(m^2+q^2) = 4(M^2+p^2)\,, \quad \{x = \cos\vartheta\}\,.
\end{aligned}
\qquad (2.12)
$$

III

The Mandelstam double representation for the functions $B^{(1)}(s, \bar{s}, t)$ has the form

$$
\begin{aligned}
B^{(1)}(s,\bar{s},t) = {} & P_\Lambda + \begin{pmatrix} 3 \\ -1 \end{pmatrix} P_\Sigma + \frac{1}{\pi^2}\int_{(M+m)^2} ds' \int_{(\mathfrak{M}_\Lambda+\mu)^2} d\bar{s}' \, \frac{b_{12}^{(1)}(s',\bar{s}')}{(s'-s)(\bar{s}'-\bar{s})} \; + \\
& + \frac{1}{\pi^2}\int_{(\mathfrak{M}_\Lambda+\mu)^2} d\bar{s}' \int_{4\mu^2} dt' \, \frac{b_{23}(\bar{s}',t')}{(\bar{s}'-\bar{s})(t'-t)} + \frac{1}{\pi^2}\int_{4\mu^2} dt' \int_{(M+m)^2} ds' \, \frac{b_{34}(t',s')}{(t'-t)(s'-s)}\,,
\end{aligned}
\qquad (3.1)
$$

where the pole terms are

$$
4P_Y = \frac{g_Y^2}{\mathfrak{M}_Y^2 - \bar{s}}\,.
$$

The renormalized coupling-constants g_Λ^2 and g_Σ^2 are defined as residue of the pole terms of $B(s, \bar{s}, t)$ for the isotopic transitions $0 \rightarrow 0$ and $1 \rightarrow 1$ of the second process.

For $A^{(1)}$ hold analogous representations where the corresponding pole terms are multiplied

For the partial waves of reaction III and for $\pi\pi \longrightarrow N\tilde{N}$ we have

$$J_{++} = \frac{1}{p p_0} \sum \left(l+\tfrac{1}{2}\right)(pq)^{l} P_l(x) , \qquad (2.7)$$

$$J_{+-} = q \sum \frac{l+\frac{1}{2}}{\sqrt{l(l+1)}} (pq)^{l-1} f_-^{l} P_l^{(1)}(x) ,$$

where J_{++} and J_{+-} are the helicity states of these reactions (see[7]). For the process III J_{++} and J_{+-} are connected with A, B by

$$J_{++} = J_{--} = \frac{1}{8\pi p_0}\left\{ -pA + qMxB \right\} ,$$

$$J_{+-} = -J_{-+} = \frac{q}{8\pi}\sqrt{1-x^2}\, B . \qquad (2.8)$$

For the partial waves of reaction III the unitarity condition gives

$$\mathrm{Im}\, f_{\pm}^{l} = \frac{q_1^{2l+1}}{q^{0}}\, \Pi^{*l}\, T_{\pm}^{l} \qquad (2.9)$$

Π^{l} and $T_{\pm}^{l}$ are the partial waves of $K\tilde{K} \to \pi\pi$ and $\pi\pi \to N\tilde{N}$ respectively, and

$$q_1^2 = q^2 + m^2 - \mu^2 .$$

Here, k^2 , $\bar{k}^2$ and q^2 denote the squares of momentum for the respective processes in their c.m. - systems. The invarlatnt variables introduced by Mandelstam[9] have in the c.m. -systems of the reactions I, II and III the following form

$$s = M^2 + m^2 + 2k^2 + 2\sqrt{(k^2+M^2)(k^2+m^2)} ,$$

(I) $$\bar{s} = M^2 + m^2 - 2k^2 z - 2\sqrt{(k^2+M^2)(k^2+m^2)} , \qquad (2.10)$$

$$t = -2k^2(1-z) , \quad \{ z = \cos\varphi \} ;$$

by ($M \pm \mathfrak{M}_Y$); here the signs ($\pm$) refer to scalar and pseudoscalar K-mesons, respectively.

The bounderies of the region in which the spectral functions are not equal to zero can be calculate on the basis of the results of reference[10/]. It is found that the nearest boundary is determined by a diagram of the type shown in Fig. 1. In the plane of the invariant variables we obtain the curves of Fig.2, where A has the coordinates

$$s \approx -20\mu^2 \ , \quad \bar{s} \approx 113\mu^2 \ , \quad t \approx 25\mu^2 .$$

The curve Γ is symmetric under the exchange of s and $\bar{s}$ Γ' , Γ'' correspond to other diagrams.

The double representation gives for the functions $A^{(\pm)}(k^2, z)$, $B^{(\pm)}(k^2, z)$ the cuts of Fig.3 (reaction I). The line P represents the two poles at $k^2 = k_Y^2$, where for $z = +1$ $k_\Lambda^2 = -11{,}0\mu^2$, $k_\Sigma^2 = -12{,}2\mu^2$.

Fig. 4 gives the cuts of reaction II. Here, P denotes poles at $\bar{k}_\Lambda^2 = -8.9\mu^2$ and $k_\Sigma^2 = -7.2\mu^2$. From the value $-\lambda = -5.4\mu^2$ starts the cut of $\bar{K} N \to Y\pi$; this cut is the consequence of the inequality $(\mathfrak{M} + \mu)^2 < (M + m)^2$.

In equation (3.1) the integration over t begins with $4\mu^2$. On the other hand the kinematical cut from the square root begins with the mass of the K-meson; that means the interval from $-\mu^2$ up to $-m^2 = -13\mu^2$ on the negative axis of k^2 or $\bar{k}^2$ remains free from this kinematical cut.

In the following, therefore we shall restrict the integration over negative k^2 or $\bar{k}^2$ to the interval $[-m^2, -\mu^2]$.

Note that all kinematical coefficients $\alpha(k^2)$, $\beta(k^2)$ etc. are real in this interval; they also give no new singularities.

According to the analytic properties shown in fig.3 and 4 we can apply the Cauchy – theorem to the functions $A^{(\pm)}(k^2, z = \pm 1)$, $B^{(\pm)}(k^2, z = \pm 1)$ and $A^{(\pm)}(\bar{k}^2, \bar{z} = \pm 1)$, $B^{(\pm)}(\bar{k}^2, \bar{z} = \pm 1)$.

As an example we write the dispersion relations for $B^{(\pm)}(k^2, +1)$

$$B^{(\pm)}(k^2, +1) = \sum_Y B_Y^{(\pm)}(k^2) + \frac{1}{\pi}\int_0^\infty \frac{dk'^2}{k'^2 - k^2} \, \mathrm{Im}\, B^{(\pm)}(k'^2, +1) + $$

$$+ \frac{1}{\pi}\int_{-m^2}^{-\mu^2} \frac{dk'^2}{k'^2 - k^2} \, \mathrm{Im}\, B^{(\pm)}(k'^2, +1) . \qquad (3.2)$$

Here $$\sum_Y B_Y^{(1)}(k^2) = \frac{g_\Lambda^2}{4h(k_\Lambda^2)(k_\Lambda^2-k_\Lambda^2)} + \begin{pmatrix}3\\-1\end{pmatrix}\frac{g_\Sigma^2}{4h(k_\Sigma^2)(k_\Sigma^2-k^2)}$$

and

$$h(k^2) = \left|\frac{d\bar{s}(k^2)}{dk^2}\right|.$$

We remark, that in $B^{(1)}$ $(k^2, -1)$ there are no pole terms and for the second process the integration from the right hand cut begins with $-\lambda$.

Now, using the abbreviations $\alpha' = \alpha(k'^2)$, $\alpha_Y = \alpha(k_Y^2)$ etc. and

$$\sum_Y A_Y^{(1)} = \sum_Y (M \pm \mathfrak{M}_Y)\; B_Y^{(1)}$$

(in the following we suppress $\sum_Y$) we obtain from (3.2), (2.4) and (2.1) the equations for the first partial waves of process I:

$$f_0^{(1)}(k^2) = \frac{1}{6}(3\alpha_Y-\gamma_Y)A_Y^{(1)} + \frac{1}{6}(3\beta_Y+\delta_Y)B_Y^{(1)} + \frac{1}{\pi}\int_0^\infty \frac{dk'^2}{k'^2-k^2}\,\mathrm{Im}\, f_0^{(1)}(k'^2) +$$
$$+\frac{1}{\pi}\int_{-m^2}^{-\mu^2}\frac{dk'^2}{k'^2-k^2}\left\{\frac{3\alpha'-\gamma'}{6}\,\mathrm{Im}\, A^{(1)}(k'^2,+1) + \frac{3\beta'+\delta'}{6}\,\mathrm{Im}\, B^{(1)}(k'^2,+1) +\right.$$
$$\left.+\frac{3\alpha'+\gamma'}{6}\,\mathrm{Im}\, A^{(1)}(k'^2,-1) + \frac{3\beta'-\delta'}{6}\,\mathrm{Im}\, B^{(1)}(k'^2,-1)\right\}, \qquad (3.3)$$

$$f_{1-}^{(1)}(k^2) = \frac{1}{6}(\alpha_Y-3\gamma_Y)A_Y^{(1)} + \frac{1}{6}(\beta_Y+3\delta_Y)B_Y^{(1)} + \frac{1}{\pi}\int_0^\infty\frac{dk'^2}{k'^2-k^2}\,\mathrm{Im}\, f_{1-}^{(1)}(k'^2) +$$
$$+\frac{1}{\pi}\int_{-m^2}^{-\mu^2}\frac{dk'^2}{k'^2-k^2}\left\{\frac{\alpha'-3\gamma'}{6}\,\mathrm{Im}\, A^{(1)}(k'^2,+1) + \frac{\beta'+3\delta'}{6}\,\mathrm{Im}\, B^{(1)}(k'^2,+1)\right. \qquad (3.4)$$
$$\left.-\frac{3\gamma'+\alpha'}{6}\,\mathrm{Im}\, A^{(1)}(k'^2,-1) + \frac{3\delta'-\beta'}{6}\,\mathrm{Im}\, B^{(1)}(k'^2,-1)\right\},$$

$$f_{1+}^{(1)}(k^2) = \frac{1}{6}\alpha_Y A_Y^{(1)} + \frac{1}{6}\beta_Y B_Y^{(1)} + \frac{1}{\pi}\int_0^\infty\frac{dk'^2}{k'^2-k^2}\,\mathrm{Im}\, f_{1+}^{(1)}(k'^2) +$$
$$+\frac{1}{\pi}\int_{-m^2}^{-\mu^2}\frac{dk'^2}{k'^2-k^2}\left\{\frac{\alpha'}{6}\,\mathrm{Im}\, A^{(1)}(k'^2,+1) + \frac{\beta'}{6}\,\mathrm{Im}\, B^{(1)}(k'^2,+1) -\right.$$
$$\left.-\frac{\alpha'}{6}\,\mathrm{Im}\, A^{(1)}(k'^2,-1) - \frac{\beta'}{6}\,\mathrm{Im}\, B^{(1)}(k'^2,-1)\right\}, \qquad (3.5)$$

$$f_{2-}^{(1)}(\kappa^2) = -\frac{1}{6}\gamma_Y A_Y^{(1)} + \frac{1}{6}\delta_Y B_Y^{(1)} + \frac{1}{\pi}\int_0^\infty \frac{d\kappa'^2}{\kappa'^2-\kappa^2}\, \mathrm{Im}\, f_{1-}^{(1)}(\kappa'^2) +$$

$$+ \frac{1}{\pi}\int_{-m^2}^{-\mu^2} \frac{d\kappa'^2}{\kappa'^2-\kappa^2}\left\{ -\frac{\gamma'}{6}\,\mathrm{Im}\,A^{(1)}(\kappa'^2,+1) + \frac{\delta'}{6}\,\mathrm{Im}\,B^{(1)}(\kappa'^2,+1) + \right.$$

$$\left. + \frac{\gamma'}{6}\,\mathrm{Im}\,A^{(1)}(\kappa'^2,-1) - \frac{\delta'}{6}\,\mathrm{Im}\,B^{(1)}(\kappa'^2,-1)\right\}. \qquad (3.6)$$

In these equations $\mathrm{Im} f_{\ell\pm}$ is defined by (2.5) . To express the integrals over negative κ^2 by the processes II and III we need the connection between the variables of the first process with those of the second and third:

$$\bar{z}(\kappa^2,z) = 1 - \frac{\kappa^2}{\bar{\kappa}^2}(1-z),$$

$$\bar{\kappa}^2(\kappa^2,z) = \kappa^2\, \frac{M^2+m^2+\kappa^2(1+z)+2z\sqrt{(\kappa^2+M^2)(\kappa^2+m^2)}}{M^2+m^2-2\kappa^2 z-2\sqrt{(\kappa^2+M^2)(\kappa^2+m^2)}}\,; \qquad (3.7)$$

$$x(\kappa^2,z) = -\frac{2\kappa^2(1+z)+4\sqrt{(\kappa^2+M^2)(\kappa^2+m^2)}}{\sqrt{4M^2+2\kappa^2(1-z)}\,\sqrt{4m^2+2\kappa^2(1-z)}}\,,$$

$$q^2(\kappa^2,z) = -m^2 - \frac{\kappa^2}{2}(1-z). \qquad (3.8)$$

Therefore

$$\mathrm{Im}\,B(\kappa^2, z=+1) = \mathrm{Im}\,B\{\bar{\kappa}^2(\kappa^2,+1), \bar{z}=+1\}, \qquad (3.9a)$$

where $\mathrm{Im}\, B(\bar{\kappa}^2, +1)$ can be expressed with the help of (2.1), (2.4) and (2.6) through the partial waves $\bar{f}_{\ell\pm}$.

Correspondingly

$$\mathrm{Im}\,B(\kappa^2, z=-1) = \mathrm{Im}\,B\{q^2(\kappa^2,-1), x=-1\} \qquad (3.9b)$$

can be expressed with (2.7), (2.8) and (2.9) through the partial waves $f_{\pm}^{\ell}$ of the third reaction.

For the second process we obtain analogous equations for $\bar{f}_{\ell\pm}$. The pole contributions in the equations for $\bar{f}_{s}^{(1)}(\bar{\kappa}^2)$ and $\bar{f}_{P\frac{1}{2}}^{(1)}(\bar{\kappa}^2)$ are

$$\alpha_Y A_Y^{(1)} - \beta_Y B_Y^{(1)} \qquad (3.10)$$

and

$$-\gamma_Y A_Y^{(1)} - \delta_Y B_Y^{(1)}, \qquad (3.11)$$

respectively; the equations for $\bar{f}^{(1)}_{p\frac{3}{2}}(\bar{k}^2)$ and $\bar{f}^{(1)}_{d\frac{3}{2}}(\bar{k}^2)$ have no pole contributions.

Because of the connection between the three reactions we have for the negative region

$$\mathrm{Im}\, B(\bar{k}^2, \bar{z} = +1) = \mathrm{Im}\, B\left\{k^2(\bar{k}^2, 1), z = +1\right\}, \tag{3.12a}$$

$$\mathrm{Im}\, B(\bar{k}^2, \bar{z} = -1) = \mathrm{Im}\, B\left\{q^2(\bar{k}^2, -1), x = -1\right\}, \tag{3.12b}$$

where $\mathrm{Im}B(k^2, +1)$ can be expressed with (2.1), (2.4), (2.5) by the partial waves $f_{l\pm}$, and $\mathrm{Im}B(q^2, -1)$ again by $f_{\pm}^{l}$.

IV. In view of the fact that the partial wave development of the imaginary part of the scattering amplitude converges better than the development of the real part, we can in (3.9) and (3.12) neglect the contributions of the d–waves*. Then (3.3), (3.4) and (3.5) do not depend in explicit form on d–waves; the same approximation in (3.6) leads to an expression for f_{1-} which depends only on s– and p–waves and which should be sufficient to find the order of the f_{2-} -waves.

We note that the integral equations obtained for K N– and $\bar{K}$ N – scattering are not coupled in our approximation. In a following paper we shall set up integral equations without the necessary of a cut-off. The question of subtractions will also be considered.

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* Such a procedure is discussed in detail in [4] for the case of $\pi\pi$–scattering.

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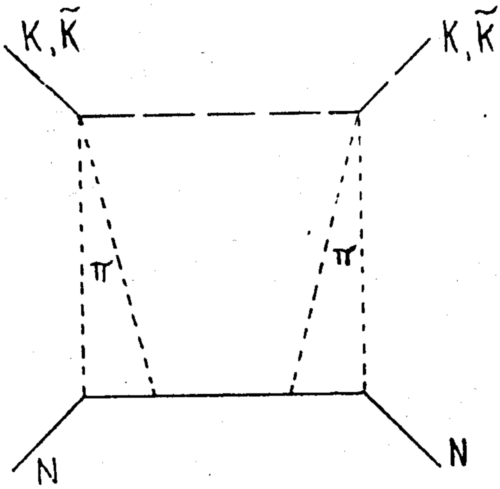

Fig.1

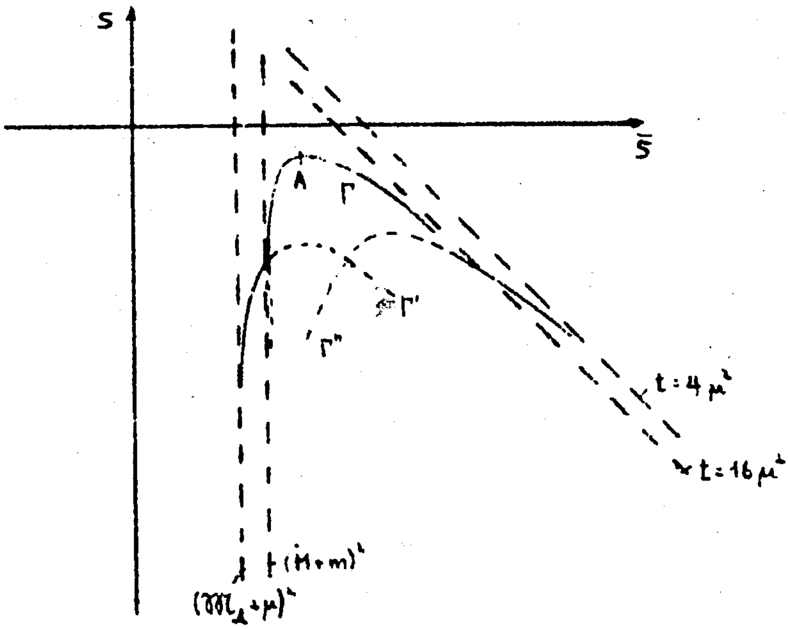

Fig.2

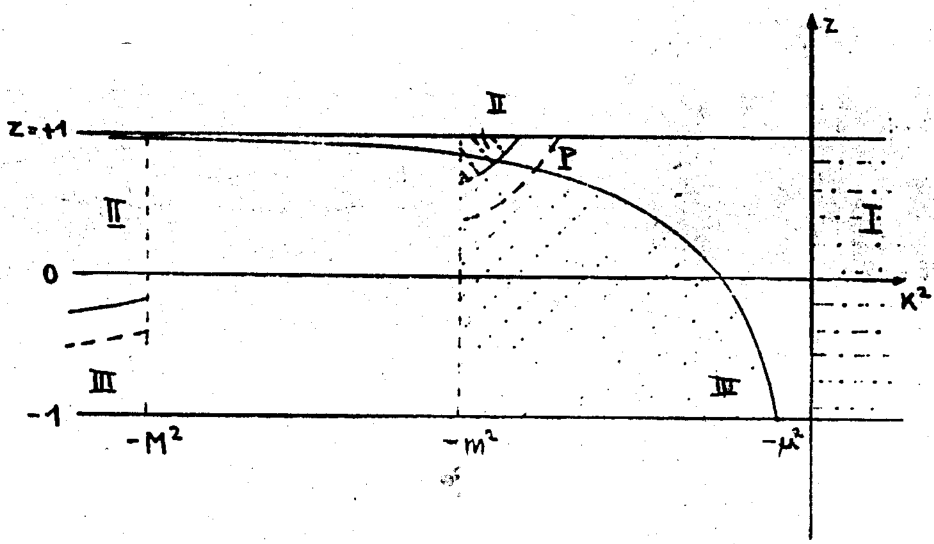

Fig.3

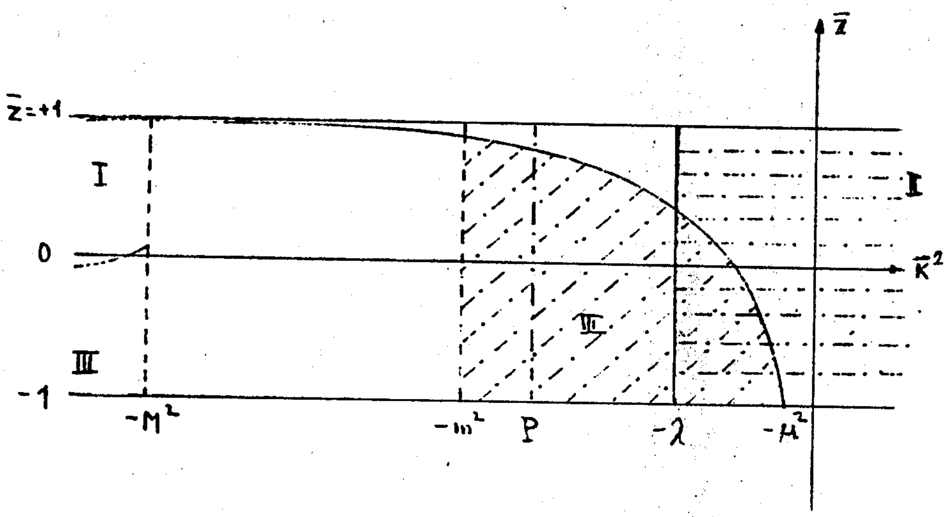

Fig.4