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ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

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AN INVESTIGATION OF PROPERTIES OF TRANSURANIC ELEMENTS BY USING A SUPERFLUID MODEL OF A NUCLEUS

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AN INVESTIGATION OF PROPERTIES OF TRANSURANIC ELEMENTS BY USING A SUPERFLUID MODEL OF A NUCLEUS

The properties of strongly-deformed transuranic elements are studied on the basis of a superfluid model of a nucleus. Insignificant changes in Nilsson schemes have been made, by using experimental data and taking into account the influence of the superfluidity. Pairing energies have been calculated, and the following values have been found for the interaction constants: $G_N = 0.020\hbar\omega_0$, $G_p = 0.024\hbar\omega_0$. The single – particle excitation spectra of odd-mass nuclei have been calculated, the density of the calculated levels is approximately two times greater than that of Nilsson scheme. The single-particle excitation in even-even nuclei have been calculated. It has been obtained, that in all the calculated spectra of the even-even nuclei, Th^{232}, U^{234}, Pu^{238}, Pu^{240}, Pu^{242}, Pu^{244}, Cm^{246}, Cf^{248} the levels 1-, lying below 1 MeV, are the lowest ones. The corrections have been calculated, to β and γ -transitions which are due to the superfluidity of the ground and excited states. The results obtained are mutually- consistent; for the same values of G there are obtained the correct values of the pairing energies and levels of the even and odd nuclei, whereas in changing G by $\lesssim$ (30 – 40)% a sharp discrepancy with experimental data is observed.

Mathematical methods developed by N.N. Bogolubov in constructing the theories of the superfluidity and superconductivity /1/ allowed to solve the problem of taking into account the residual nucleon interactions leading to the pairing correlations in the model of independent particles. On this basis a superfluid model of a nucleus was formulated in /2/.

The present paper is devoted to a study of the properties of the strongly-deformed transuranic elements using a superfluid model of a nucleus. As a self-consistent field we use the Nilsson potential, whose energy levels are slightly corrected in accordance with the experimental data. Under the assumption of the adiabatic approximation the single-particle levels of both odd and even-even nuclei and the pairing energies are calculated. The corrections to β - and γ - transition probabilities are made.

1. Basic Equations

The meaning of a superfluid model formulated in /2/ is as follows: basing upon a self-consistent potential of the shell or unified models, the superfluid model takes into account the nucleon interaction near the Fermi energy surface under the following assumptions:

1) the residual interactions both between neutrons and between protons are described by the Hamiltonian

$$H = \sum_{s,\sigma} \{ E(s) - \lambda \} a^+_{s\sigma} a_{s\sigma} - G \sum_{s,s'} a^+_{s+} a^+_{s-} a_{s'-} a_{s'+} \qquad (1)$$

2) the calculations are being made for each definite nucleus, neglecting a certain averaging associated with the conservation of the number of particles on the average. The nucleon states are characterized by a set of quantum numbers ($s\,\sigma$), which is determined by the shape of the self-consistent field, $\sigma = \pm 1$ characterizes, e.g., the sign of the projection of the angular momentum of each nucleon

along the nuclear symmetry axis, $E(s)$ - the values of the energy levels in the self-consistent field. A certain simplification of the physical picture is that the interaction G is assumed to be constant. The chemical potential λ is determined from the condition

$$n = \sum_{s\sigma} \langle a^+_{s\sigma} a_{s\sigma} \rangle \tag{2}$$

where n is the number of particles, and $\langle \cdots \rangle$ designates the averaging over a certain state. When the variational principle is applied, the chemical potential plays the role of the Lagrange factor. Note that Hamiltonian (1) should be regarded as a part of a complete Hamiltonian containing, e.g., the collective interactions.

To solve the problem we make use of the variational principle suggested by N.N. Bogolubov/3/. Like in/4/, for the ground state of the system consisting of even number of particles we obtain the following equations

$$\frac{2}{G} = \sum_s \frac{1}{\sqrt{c^2 + \{E(s) - \lambda\}^2}} , \tag{3}$$

$$n = \sum_s \left\{ 1 - \frac{E(s) - \lambda}{\sqrt{c^2 + \{E(s) - \lambda\}^2}} \right\} \tag{4}$$

to determine c and λ . The ground state energy is found as

$$\mathcal{E} = \sum_s E(s) \left\{ 1 - \frac{E(s) - \lambda}{\sqrt{c^2 + \{E(s) - \lambda\}^2}} \right\} - \frac{c^2}{G} \tag{5}$$

whereas the wave function

$$\Psi = \prod_s \{ u_s + v_s a^+_{s+} a^+_{s-} \} \Psi_0 \tag{6}$$

$$a_{s\sigma} \Psi_0 = 0$$

$$u_s^2 = \frac{1}{2} \left\{ 1 + \frac{E(s) - \lambda}{\sqrt{c^2 + \{E(s) - \lambda\}^2}} \right\} ; \quad v_s^2 = \frac{1}{2} \left\{ 1 - \frac{E(s) - \lambda}{\sqrt{c^2 + \{E(s) - \lambda\}^2}} \right\} \tag{7}$$

The wave functions of the excited states are written as [5]

$$\Psi(s_1, s_2) = \prod_{\substack{s \neq s_1 \\ s \neq s_2}} (u_s + v_s\, a^+_{s+}\, a^+_{s-})\, a^+_{s_1 \sigma_1}\, a^+_{s_2 \sigma_2}\, \Psi_0 \quad ; \quad s_1 \neq s_2 \qquad /8/$$

$$\Psi(s_1, s_1) = \prod_{s \neq s_1} (u_s + v_s\, a^+_{s+}\, a^+_{s-})(u_{s_1} a^+_{s_1+}\, a^+_{s_1-} - v_{s_1})\, \Psi_0 \qquad /8'/$$

The energy and the basic equations are obtained as follows

$$\mathcal{E}(s_1, s_2) = E(s_1) + E(s_2) + \sum_{\substack{s \neq s_1 \\ s \neq s_2}} E(s) \left\{ 1 - \frac{E(s) - \lambda}{\sqrt{c^2 + \{E(s) - \lambda\}^2}} \right\} - \frac{c^2}{G} + \qquad /9/$$
$$+ \frac{1}{2} G (v_{s_1}^2 + v_{s_2}^2);$$

$$\frac{2}{G} = \sum_{\substack{s \neq s_1 \\ s \neq s_2}} \frac{1}{\sqrt{c^2 + \{E(s) - \lambda\}^2}} \quad ; \quad n = 2 + \sum_{\substack{s \neq s_1 \\ s \neq s_2}} \left\{ 1 - \frac{E(s) - \lambda}{\sqrt{c^2 + \{E(s) - \lambda\}^2}} \right\} \qquad /10/$$

For the odd shell, if the odd nucleon is in the S_i-state, the energy of the system and the equations for C and λ are found as follows

$$\mathcal{E}(s_i) = E(s_i) + \sum_{s \neq s_i} E(s) \left\{ 1 - \frac{E(s) - \lambda}{\sqrt{c^2 + \{E(s) - \lambda\}^2}} \right\} - \frac{c^2}{G} + \frac{G}{2} v_{s_i}^2 \qquad /11/$$

$$\frac{2}{G} = \sum_{s \neq s_i} \frac{1}{\sqrt{c^2 + \{E(s) - \lambda\}^2}} \quad ; \quad n = 1 + \sum_{s \neq s_i} \left\{ 1 - \frac{E(s) - \lambda}{\sqrt{c^2 + \{E(s) - \lambda\}^2}} \right\} \qquad /12/$$

These equations describe both the ground, for the most part when $E(s_i) = E_F$, and the excited states of the system with an odd number of particles; the wave function of this system is put as

$$\Psi(s_i) = \prod_{s \neq s_i} (u_s + v_s\, a^+_{s+}\, a^+_{s-})\, a^+_{s_i \sigma_i}\, \Psi_0 \qquad /13/$$

Thus, to determine C and λ both as ground and excited superfluid states, it is necessary to solve the corresponding equation system. This approach is essentially different from that described in [7]. If in case of a superfluid model, C and λ are found by solving the corresponding equations, and the interaction constant G is determined from the experimental values of the pairing energy then in [7] the values of C are determined from the pairing energy, λ being assumed equal to the Fermi surface energy E_F. The advantage of the approach based on the superfluid model of a nucleus in comparison with that in[7] consists, first, in the possibility of determining C and λ for the excited states, of taking into account the changes of C and λ in changing the deformation of a nucleus and the like, what cannot be made in the case[7]. Second, calculating the change λ in the transition from a nucleus to a nucleus, and from a ground state to an excited one, we, thereby, take into account the change in the properties of a nucleus as a many-body system. Third, our calculations are more inambiguous and reliable since at disposal there is a single interaction constant G which changes slowly and monotonously from a nucleus to a nucleus, while C changes by jumps depending on a concrete behaviour of the energy levels of the self-consistent field.

In a superfluid model of a nucleus arises a question about the orthogonality of the ground and excited states. It is easy to show that all the states of the odd shell are orthogonal, with respect to each other the ground and excited states (8) with $s_1 \neq s_2$ of even shell are also orthogonal relative to each other. However, the ground and excited states (8′) at are not orthogonal relative to each other, viz.,

$$\left(\Psi^*(s_1, s_2)\, \Psi(s_2, s_2) \right) = (u'_{s_1} v''_{s_1} - u''_{s_1} v'_{s_1})(u'_{s_2} v''_{s_2} - u''_{s_2} v'_{s_2}) \prod_{\substack{s \neq s_1 \\ s \neq s_2}} (u'_s u''_s + v'_s v''_s) \qquad (14),$$

where $u'_s\ v'_s$ are referred to the excited state $\Psi(s_1, s_1)$, whereas $u''_s\ v''_s$ - to $\Psi(s_2, s_2)$.

To evaluate the inaccuracy due to the conservation of the number of particles on the average, we calculate the root-mean-square fluctuation Δn of the number of particles n. For the ground state of the even shell, we get

$$(\Delta n)^2 = \sum_s \frac{c^2}{c^2 + \{E(s) - \lambda\}^2}, \qquad (15)$$

For the case of the excited state $\Psi(s_1, s_2)$, in the sum the terms with $s = s_1$ and $s = s_2$ must be absent. For the odd shell state $\Psi(s_i)$ in the sum (15) the term with $s = s_i$ must be absent.

2. Pairing Energy and Single-Particle Levels of Odd Nuclei

It is well-known that the Nilsson potential [8] fails to give either, a necessary sequence of the energy levels or correct values of the energy difference between the levels. This is connected, first, with the shortcomings of the Nilsson scheme itself, for it does not take into account to a sufficient extent all the complicated nuclear phenomena (showing, however, the basic regularities). Secondly, it is connected with the necessity of considering the residual interactions leading to pairing correlations. Making use of the experimental data[9,10] on the single-particle levels of the odd nuclei, we analyse behaviour of the levels in the Nilsson shcemes for $Z > 82$ and $N > 126$ with account of the influence of the superfluidity of the ground and excited states. In/2/, according to the calculations on a superfluid model of a nucleus the influence of the superfluidity was found on the behaviour of the signgle-particle levels of the odd nuclei which reduces to

1) the superfluidity, as a rule, does not lead to a change in the ground state of a nucleus represented by the Nilsson scheme;

2) with increasing the interaction constant G the excitation energies decrease ;

3) hole and par ticle levels behave differently with increasing G. However, the sequence of hole (particle) levels with respect to each other does not change.

By analysing of the experimental data on single-particle levels and equilibrium deformations and taking into account the influence of the superfluidity we can draw a conclusion that it is necessary to make some changes in the Nilsson's schemes presented in[9], viz.,

1) in the scheme of levels for the odd $-Z$ nuclei when $Z > 82$, we decrease the level $11/2-[505]$ by $0.4\,\hbar\omega_0$ which appears in no nucleus,

2) in the scheme of the levels for the even $-N$ nuclei when $N > 126$, we make the following changes:

a) the level $13/2+[606]$ as that appearing in neither of the nuclei decreases by $0.25\,\hbar\omega_0$ what is the same as the decrease of the subshell, $i\ 13/2$.

b) the sub-shell $j\ 15/2$ increases by 0.02 $\hbar\omega_0$, c) the level $1/2+[631]$ decreases by $0.017\,\hbar\omega_0$ what is the same as the decrease of the subshell $d\ 5/2$.

Making use of the corrected Nilsson schemes, we calculate the pairing energies to find G both for proton interaction and neutron one. The numerical solutions of Eqs. (3), (4), (12) are being made by an electronic computer. Having calculated $\mathcal{E}(Z,N)$ by formulas (5), (11) we find the pairing energies

$$P_N(Z,N) = 2\,\mathcal{E}(Z,N-1) - \mathcal{E}(Z,N) - \mathcal{E}(Z,N-2) \qquad (16)$$

for $G = 0.016,\ 0.020,\ 0.024\ \hbar\mathring{\omega}_0$. The results of the calculations are listed in Tables I and II from which it is seen that the pairing energies depend upon G very strongly.

Comparing the calculated pairing energies with the experimental data we obtain that for neutron interaction $G_N \approx 0{,}020\,\hbar\mathring{\omega}_0$ (more exactly ($0{,}018\,\hbar\mathring{\omega}_0 < G_N < 0{,}022\,\hbar\mathring{\omega}_0$)), and for the proton interaction ($G_p \approx 0{,}024\,\hbar\mathring{\omega}_0$ (more exactly $0{,}022\,\hbar\mathring{\omega}_0 < G_p < 0{,}026\,\hbar\mathring{\omega}_0$).

It is seen from the comparison of the magnitudes of the interaction constants G obtained above with the magnitudes in the region $150 < A < 190$ that G decreases insignificantly in passing from the rare-earth region to the transuranic one.

Let us calculate the excitation spectrum of the odd nuclei by solving /12/ and finding the difference in energies (11), for odd N- nuclei from $N = 141$ up to $N = 149$ and for odd Z nuclei from $Z = 91$ up to $Z = 95$ for a number of deformations. In order to characterize the behaviour of the basic magnitudes in Table III, , we give the values C , λ , and Δn for the case $Z = 93$, $\delta = 0.26$ for the ground and two excitation states (and for comparison for the ground state $Z = 94$). It is seen from Table III, that C and λ change strongly both in passing from an even nucleus to an odd one, and in passing from the ground state to the excited one. In the odd Z nuclei when $G_p = 0.016\ \hbar\omega_o$, and in some cases when $G_p = 0{,}020\ \hbar\omega_o$ the pairing correlations are absent in the ground states, when $G_p = 0{,}016\hbar\omega_o$, the superfluidity is observed tc disappear in a number of excited states. In the odd $-N$ nuclei, when $G_N = 0.016\ \hbar\omega_o$ the superfluidity is not observed to disappear, although C is very small for the ground states of some nuclei. It is seen from Table III that the magnitude of the chemical potential λ varies near the energy of the Fermi surface E_F , which for Z 93 is $5.628\ \hbar\overset{o}{\omega}_o$ Note, that these changes of λ are weaker if compared with the changes /2/ in the region $150 < A < 190$.

The second assumption of a superfluid nuclear model on the conservation of the number of particles on the average may strongly restrict the accuracy of the calculations and , therefore, needs a numerical estimation. We calculate the root-mean-square fluctuation Δn of the number of particles and compare it with the double number of levels over which the summation is being made, in our case with the number 48. According to the calculations of Δn , a part of which is given in Table III, we draw a conclusion that the error due to the fluctuation of the number of particles is of the order of 5%.

Comparing the calculated single-particle energy levels of odd nuclei with the experimental data, we, wee, that the agreement is rather rough, the best agreement is obtained when $G_p = 0.024\ \hbar\omega_o$ for the odd Z-nuclei and when $G_N = 0.020\hbar\omega_o$ for the odd N nuclei. It is necessary to note that calculated low-energy levels density is in good agreement with experimental data [10] and approximately two times greater than the density of levels of Nilsson scheme. In Table IV are listed, for example lower the energy levels for a number of nuclei. To obtain as a more detailed agreement of the calculated levels with those measured experimentally it is necessary to improve the scheme of the self-consistent field levels.

Note, that the influence of the superfluidity on the spectra of the odd nuclei in the transuranic region is somewhat weaker compared with the rare-earth region, although G decreases insignificantly. Moreover, for the same values of G the influence of the superfluidity on the odd Z nuclei in the transuranic region is somewhat weakened in comparison with the region $150 < A < 190$, what is connected, very likely, with the change in the density of the levels/10/.

3. *Excitation Spectra of Even - Even Nuclei*

Let us calculate the single-particle levels of the even-even nuclei energies by a superfluid model of a nucleus. To this end we solve Eqs. (3), (4), and (10) and calculate the energy of the ground and excited states. Note, that the excited states of form (8) are double degenerated in contrast to those of (8′). The excitation spectrum calculated on the basis of a superfluid nuclear model shows as minimum, what values of the spins and parities of the lower excited states are the most likely for an analysis of the experimental data i.e., the calculated spectra of the even-even nuclei give, at least, what the Nilsson schemes do for an analysis of the odd nuclei.

The influence of the superfluidity on the excitation spectra of the even-even nuclei is rather strong. At the same time, if in the case of the odd nuclei the single-particle levels condense near the ground state with increasing G , then in the even-even nuclei the single-particle levels move aside from the ground state with increasing G , i.e., the gap is becoming larger. The change in the behaviour of the levels for U^{234} with increasing G is demonstrated in Table VII.

In order to obtain an idea about a behaviour of the basic magnitudes in Tables V and VI, we give the values of C , λ , and Δn for Pu^{238} both for the proton interaction and for the neutron one. We denote by n_F the number of the Fermi surface level, the next level is denoted by $n_F + 1$ etc. The calculations show that C decreases strongly in passing from the ground to the lowest excited states, vanishing in some cases even if $G_N = 0.022\,\hbar\omega.$ and $G_p = 0.024\,\hbar\omega.$ The disappearance of the superfluidity in the excited states with the lowest energies leads to a decrease of the gap in comparison with $2\sqrt{C^2 + \{E_F - \lambda\}^2}$. The values of the chemical potential λ vary about E_F , the deviations of λ from E_F are large compared with those in the odd nuclei and amount to 0.5 MeV.

Let us calculate the root-mean-square fluctuation Δn of the number of particles. Then, comparing it with the number 48 (i.e. with the double number of levels) we obtain that $\Delta n/48$ is of order of 5%. Thus, the change of C and λ in passing from the ground to the excited states both in the even and in the odd nuclei considerably exceeds the errors due to the conservation of the number of particles on the average.

As has been shorn above, the excited states of form (8) are not orthogonal both in respect to each other and in respect to the ground state (we denote it by $|0\rangle$). Let us evaluate the magnitude of the nonorthogonality, e.g., for Pu^{238}, for the proton interactions when $G_p = 0.024\,\hbar\omega.$ we get

$$\langle n_F-1, n_F-1|0\rangle = 0{,}15 \quad ; \quad \langle n_F, n_F|n_F-1, n_F-1\rangle = 0{,}01$$

$$\langle n_F, n_F|0\rangle = 0{,}42 \quad ; \quad \langle n_F+1, n_F+1|n_F-1, n_F-1\rangle = 0{,}07 \qquad (17)$$

$$\langle n_F+1, n_F+1|0\rangle = 0{,}40 \quad ; \quad \langle n_F+1, n_F+1|n_F, n_F\rangle = 0{,}67$$

and for neutron interactions when $G_N = 0.022\ \hbar\omega_0$ we find

$$\langle n_F-1, n_F-1|0\rangle = 0{,}17 \quad ; \quad \langle n_F n_F|n_F-1, n_F-1\rangle = 0{,}001$$

$$\langle n_F, n_F|0\rangle = 0{,}44 \quad ; \quad \langle n_F+1, n_F+1|n_F-1, n_F-1\rangle = 0{,}08 \qquad (17')$$

$$\langle n_F+1, n_F+1|0\rangle = 0{,}31 \quad ; \quad \langle n_F+1, n_F+1|n_F, n_F\rangle = 0{,}22.$$

It is seen from here that the admixture of the excited states of form (8′) in the ground state is great. Therefore, the difference of the energies between the states of form (8) can be calculated with a greater accuracy than their energy with respect to the ground state. The energy of the states 0^+ form (8′) is calculated with a considerably less accuracy. Let us calculate the energies of a number of the excited states U^{234} for the deformation $\delta = 0.24$, Pu^{238}, Pu^{240}, and Cm^{246} when $\delta = 0.26$ and list them in Tables VII, VIII, IX. Among the lower levels there are states 0^+ which allow to treat the states with energies 1.15 MeV and 1.62 MeV U^{234} as single-particle ones. The given values for the energies of the levels make it possible to analyse the experimental data.

The most interesting result of the calculations is that in all the spectra of the even-even nuclei which we have calculated the lowest level is (1^-), since the level $\Omega = 0$ with a negative parity is demonstrated as ($I = 1^-$). This is so because of an accidental combination of the levels in the Nilsson scheme. It follows from our calculations that the single-particle levels 1-, lying below 1 MeV must be observed in the nuclei of Th^{232}, U^{234}, Pu^{238}, Pu^{240}, Pu^{242}, Pu^{244}, Cm^{246}, Cf^{248}. It is likely that analogous calculations may confirm the appearance of the levels 1- in other transuranic elements, particularly, in lighter isotopes Th . Indeed, let us consider the Nilsson scheme for the odd nuclei. We see that beginning with N=130, the following sequence of the levels is observed 1/2-, 1/2+ , 3/2-, 3/2+ , 5/2-, 3/2+ etc,i.e. the levels standing beside each other have opposite parities and $|\Omega_i - \Omega_{i-1}| = 1$ or 0, what is very favourable for the appearance of the levels 1-. A similar picture is observed in the scheme of the levels of odd Z nuclei.

Thus, the above-mentioned calculations point out that the lower levels with $I = 1$ and the negative parity displaying in a number of transuranic elements may be single- particle, but not the collective levels.

4. Corrections to the Probabilities of a β–and γ–Transitions

Let us calculate the corrections to the β and γ transition probabilities. These corrections are due to the superfluidity of the ground and excited states. Being aware of C and λ it is easy to get the wave functions of the ground and excited states (6), (8), (8′), (13). It is not difficult to show that the corrections to the probabilities are indicated as factors less than unity.

Let us calculate the corrections to the probabilities of β-decay, for the β-decay of the odd –nucleus, if the neutron from the state s_1 passes into the proton in the state s_2, i.e. $/ N = 2n_N + 1,\ Z = 2n_z / \longrightarrow / N = 2n_N,\ Z = 2n_z + 1 /$; we have

$$R = (u_{s_1}^{2n_N}\, u_{s_2}^{2n_z})^2 \prod_{s \neq s_1} (u_s^{2n_N}\, u_s^{(s_1)(2n_N+1)} + v_s^{2n_N}\, v^{(s_1)(2n_N+1)})^2 \cdot$$
$$\cdot \prod_{s' \neq s_1} (u_{s'}^{2n_z}\, u_{s'}^{(s_2)(2n_z+1)} + v_{s'}^{2n_z}\, v^{(s_2)(2n_z+1)})^2 \qquad (18)$$

The correction to the probability of a β–decay of the odd–odd nucleus, i.e. $/ N = 2n_N + 1,\ Z = 2n_z - 1 / \longrightarrow / N = 2n_N,\ Z = 2n_z /$

is found as follows

$$R_0 = (u_{s_1}^{2n_N}\, v_{s_2}^{2n_z})^2 \prod_{s \neq s_1} (u_s^{2n_N}\, u_s^{(s_1)(2n_N+1)} + v_s^{2n_N}\, v_s^{(s_1)(2n_N+1)})^2 \cdot$$
$$\prod_{s' \neq s_1} (u_{s'}^{2n_z}\, u_{s'}^{(s_2)(2n_z-1)} + v_{s'}^{2n_z}\, v_{s'}^{(s_2)(2n_z-1)})^2 \qquad (19)$$

where we substitute C and λ belonging to the ground state of the even shell into u_s^{2n}, v_s^{2n} but C and λ belonging to the ground or excited states of the odd shell, if the odd particle is found to in the state S_i into $u_s^{(s_i)(2n\pm1)}$, $v_s^{(s_i)(2n\pm1)}$. In Table X we give the calculated correction to the probabilities of a β–decay of U^{237} Pu^{237} and Np^{239}. It is seen from the Table that the corrections can be very significant in the individual cases. The corrections to the electromagnetic transitions have a more complicated form, for the case of an odd–nucleus we obtain

$$R_\gamma = (u_{s_2}^{(s_1)}\, u_{s_1}^{(s_2)} - \tau\, v_{s_1}^{(s_2)}\, v_{s_2}^{(s_1)})^2 \prod_{s \neq s_1, s_2} (u_s^{(s_1)} u_s^{(s_2)} + v_s^{(s_1)} v_s^{(s_2)})^2 \qquad (20)$$

where $\tau = 1$ for electric and $\tau = -1$ for magnetic transitions.

In Table XI we give the corrections to γ –transitions. Note, that these corrections vary in larger intervals than the corrections to a β–decay.

Thus, our calculations show that in calculating the probabilities of β and γ transitions in strongly-deformed transuranic elements it is necessary to take into acount the superfluidity of the ground and excited states.

5. Conclusion

The superfluid nuclear model is based on the shell and unified models and is their further development. At the first stage of the investigations performed on the basis of a superfluid nuclear model the long-range residual interactions responsible for the main collective properties of a nucleus were not taken into account.

It is worth noting that the results of the calculations by a superfluid nuclear model are inambiguous for the given value of G and the energy levels of the effective potential. In the light of this inambiguity a mutual consistency of the results obtained becomes of special interest, viz., for the same values of the interaction constants G there are obtained reasonable values of the pairing energies and the levels of the even and odd nuclei, whereas in changing G, e.g., by 30%–40% there is a sharp discrepance with the experiment both in the behaviour of the levels and in pairing energies.
the levels and in pairing energies.

It should be noted that on the basis of a superfluid nuclear model it is possible to carry out complex investigations of the properties of the strongly-deformed nuclei. For this purpose the moments of inertia of the ground and excited states should be calculated, as well as the probabilities of β and γ transitions, magnetic moments and the like. To obtain more detailed results it is necessary, first of all, to obtain the energy levels of the self-consistent field more exact.

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T a b l e 1

Neutron pairing energy (MeV)

$\mathcal{N}$	δ	Pairing energy			
		$G=0.016\hbar\omega_0$	$G=0.020\hbar\omega_0$	$G=0.024\hbar\omega_0$	exp. data [11]
150	0.26	0.43	0.94	1.58	0.7–1.1
148	0.26	0.23	0.88	1.57	0.8–1.2
146	0.26	–	*0.82	–	0.7–0.9
144	0.24	0.27	0.97	1.69	0.8–1.0

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Proton pairing energy (MeV)

Z	δ	Pairing energy			
		$G=0.016\hbar\omega_0$	$G=0.020\hbar\omega_0$	$G=0.024\hbar\omega_0$	exp. data [11]
96	0.26	0.14	0.32	0.80	0.4–1.1
96	0.24	–	0.57	–	
94	0.26	0.14	0.36	0.78	0.8–1.1
94	0.24	–	0.57	–	

T a b l e lll

C , λ , Δn as a functions of G

(δ =0.26, E_F = 5.628 $\hbar\omega_0$).

G ($\hbar\omega_0$)	Z =94 ground state			Z = 93 ground state 5/2+ [642]			Z = 93 excited states, particle level 5/2− [523]			Z = 93 excited states, hole level 1/2− [530]		
	C	$\lambda - E_F$	Δn	C	$\lambda - E_F$	Δn	C	$\lambda - E_F$	Δn	C	$\lambda - E_F$	Δn
0.016	0.027	0.005	1.2	0	0.009	0	0	−.038	0	0.020	0.004	1.0
0.020	0.052	0.005	1.6	0	−.005	0	0.030	−.054	1.o	0.041	0.003	1.4
0.024	0.085	0.006	2.0	0.043	−.031	1.1	0.062	−.049	1.6	0.069	0.001	1.7

T a b l e lY

Energy of excited states of odd-A nuclei (MeV)

	odd-N nuclei						odd-Z nucleus	
	$N=143$, $\delta=0.24$ $G=0.020\hbar\omega$.		$N=147$, $\delta=0.26$		$G=0.020\hbar\omega$. $\delta=0.27$		$Z=93$, $\delta=0.25$ $G=0.024\hbar\omega$.	
ground state	0	7/2-	0	5/2+	0	5/2+	0	5/2+
Partioles levels	0.18	1/2+	0.18	7/2+	0.26	7/2+	0.08	5/2-
	0.34	5/2+	0.25	9/2-	0.31	9/2-	0.74	3/2-
	0.71	7/2+	1.07	1/2-	0.96	1/2-	0.77	7/2+
hole levels	0.26	5/2+	0.20	1/2+	0.30	1/2+	0.27	1/2-
	0.56	3/2+	0.47	7/2-	0.56	7/2-	0.51	3/2+
	0.60	5/2-	0.88	5/2+	0.99	5/2+	0.78	3/2-

T a b l e Y

Basic characteristics of the neutron interactions $_{94}Pu^{238}_{144}$ ($\delta = 0.26$)

G	Ground state			Excited states of the form (8′)								
				(n_F, n_F)			(n_F-1, n_F-1)			(n_F+1, n_F+1)		
$\hbar\omega_o$	C	$\lambda-E_F$	Δn	C	$\lambda-E_F$	Δn	C	$\lambda-E_F$	Δn	C	$\lambda-E_F$	Δn
0.018	0.063	0.029	1.87	0.028	0.079	1.07	0.042	0.081	1.44	0.028	-0.023	1.04
0.022	0.108	0.030	2.42	0.057	0.070	1.50	0.076	0.076	1.93	0.059	-0.011	1.53

G	Excited states of the form (8)											
	(n_F, n_F+1)			(n_F, n_F+2)			(n_F-1, n_F)			(n_F+1, n_F+2)		
$\hbar\omega_o$	C	$\lambda-E_F$	Δn	C	$\lambda-E_F$	Δn	C	$\lambda-E_F$	Δn	C	$\lambda-E_F$	Δn
0.018	0	0.084	0	0.0002	0.024	0	0.035	0.080	1.26	0.035	-0.023	1.24
0.022	0	0.051	0	0.051	0.005	1.30	0.067	0.073	1.73	0.069	-0.013	1.76

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Table Yl

Basic characteristics of the proton interactions $_{94}Pu^{238}_{144}$ (δ =0.26)

G $\hbar\omega_0$	Ground state			Excited states of the form (8′)								
				(n_F, n_F)			(n_F-1, n_F-1)			(n_F+1, n_F+1)		
	C	$\lambda-E_F$	Δn	C	$\lambda-E_F$	Δn	C	$\lambda-E_F$	Δn	C	$\lambda-E_F$	Δn
0.020	0.052	0	1.60	0.0004	0.09	10^{-2}	0.001	0.08	0.02	0	-0.06	0
0.024	0.085	0.01	1.96	0.027	0.07	0.78	0.053	0.07	1.37	0.036	-0.05	1.06

G $\hbar\omega_0$	Excited states of the form (8)											
	(n_F, n_F+1)			(n_F, n_F+2)			(n_F-1, n_F)			(n_F+1, n_F+2)		
	C	$\lambda-E_F$	Δn	C	$\lambda-E_F$	Δn	C	$\lambda-E_F$	Δn	C	$\lambda-E_F$	Δn
0.020	0	0.10	0	0.0002	0	10^{-2}	0	0.09	0	0.018	-0.06	0.67
0.024	0	0.08	0	0.01	-0.03	0.25	0.04	0.07	1.11	0.049	-0.05	1.35

Table Y11

Single-particle levels of $_{92}U^{234}_{142}$ in MeV (δ =0.24)

Proton levels			Neutron levels		
Ω , parity	$G=0.020\hbar\omega_0$	$G=0.024\hbar\omega_0$	Ω , parity	$G=0.018\hbar\omega_0$	$G=0.022\hbar\omega_0$
2-, 3-	0.75	1.09	1-, 6-	0.78	1.32
2+, 3+	0.87	1.19	2+, 3+	1.09	1.54
1+, 4+	0.98	1.33	0+	1.10	1.40
0+	1.07	1.29	2-, 5-	1.15	1.57
0+	1.15	1.37	1+, 6+	1.19	1.65
1-, 4-	1.16	1.39	0+	1.20	1.57
0-, 5-	1.19	1.45	0+, 5+	1.31	1.75
0+	1.31	1.51	3-, 4-	1.40	1.68
1-, 4-	1.37	1.69	1-, 2-	1.44	1.80
1-, 2-	1.38	1.59	1+, 4+	1.52	1.84
0+	1.61	1.80	0-, 5-	1.57	1.88
1+, 2+	1.64	1.91	1-, 6-	1.52	1.85
1+, 2+	1.74	1.94	0+	1.74	1.96
1-, 4-	1.94	2.12	0+	1.86	2.13
0-, 3-	1.97	2.15	1-, 4-	1.91	2.18

T a b l e Y111

Single-particle levels of ${}_{94}Pu^{238}_{144}$ and ${}_{94}Pu^{240}_{146}$ in MeV (δ =0.26)

Neutron levels Pu^{238}		Proton levels		Neutron levels Pu^{240}	
Ω , parity	G_N =0.020 $\hbar\omega_0$	Ω , parity	G_P=0.024 $\hbar\omega_0$	Ω , parity	G_N=0.020 $\hbar\omega_0$
3-, 4-	0.89	0-, 5-	0.85	2+, 3+	0.90
0+	1.03	0+	0.97	0+	1.11
0+	1.08	0+	1.13	0+	1.14
1-, 6-	1.18	2+, 3+	1.39	1-, 6-	1.16
2+, 3+	1.32	1-, 4-	1.44	3+, 4+	1.20
2+, 3+	1.40	1+, 6+	1.51	4-, 5-	1.28
1-, 6-	1.55	1+, 4+	1.65	3-, 4-	1.44
0+, 5+	1.56	2-, 3-	1.67	0-, 7-	1.45
0-, 7-	1.60	1-, 4-	1.70	1+, 6+	1.44
1+, 2+	1.64	1-, 6-	1.75	2-, 7-	1.51
0+	1.73	1+, 2+	1.88	0+, 5+	1.56
3+, 4+	1.77	1+, 4+	1.97	0+	1.62

T a b l e 1X

Single-particle levels of $_{96}Cm^{246}_{150}$ in MeV (δ =0.26)

Proton levels		Neutron levels	
Ω , parity	G_p=0.024 $\hbar\omega_0$	Ω , parity	G_N=0.022 $\hbar\omega_0$
1+, 4+	1.00	1-, 8-	0.90
1-, 6-	1.10	0+	0.91
1-, 4-	1.30	0+	0.93
1+, 6+	1.40	2-, 7-	1.25
0+	1.49	1+, 6+	1.34
0+	1.47	4-, 5-	1.62
2-, 5-	1.57	3+, 4-	1.63
0-, 5-	1.62	4+, 5+	1.64
0+	1.80	3+, 4+	1.70
1+, 2+	1.83	0+	1.76
1+, 6+	1.93	2-, 3-	1.80
0+	2.05	0+, 7+	1.96

T a b l e X

Corrections R_β to a β-decay of odd-A nuclei for
$G_p = 0.024\,\hbar\omega_0$ and $G_N = 0.020\,\hbar\omega_0$

Parent nuclei		Daughter nuclei		R_β
nucleus	state	nucleus	state	
$_{94}Pu^{237}_{143}$	7/2 7/2− [743]	$_{93}Np^{237}_{144}$	5/2 5/2+ [642]	0.106
	7/2 7/2− [743]		5/2 5/2− [523]	0.090
$_{93}Np^{239}_{146}$	5/2 5/2+ [642]	$_{94}Pu^{239}_{145}$	1/2 1/2+ [631]	0.195
	5/2 5/2+ [642]		5/2 5/2+ [622]	0.120
	5/2 5/2+ [642]		7/2 7/2 − [743]	0.282
	5/2 5/2+ [642]		5/2 5/2+ [633]	0.326
$_{94}Pu^{243}_{149}$	7/2 7/2+ [624]	$_{95}Am^{243}_{148}$	5/2 5/2+ [642]	0.238
	7/2 7/2+ [624]		7/2 7/2+ [633]	0.613

T a b l e XI

The corrections R_γ to the electromagnetic transitions
for $G_p = 0.024\hbar\omega_0$ and $G_N = 0.020\hbar\omega_0$

Nucleus	Initial state	Final state	Energy (MeV)	Mult.	R_γ
$_{93}Np^{237}_{144}$	5/2 5/2− 523	5/2 5/2+ 642	0.06	E 1	0.464
	3/2 3/2− 521	7/2 5/2− 523	0.16	E 2	0.653
	3/2 3/2− 521	5/2 5/2− 523	0.21	M 1	0.934
$_{94}Pu^{237}_{143}$	1/2 1/2+ 631	7/2 7/2− 743	0.14	E 3	0.263
$_{94}Pu^{239}_{145}$	7/2 7/2− 743	5/2 5/2+ 622	0.11	E 1	$0.58\ 10^{-3}$
	5/2 5/2+ 622	3/2 1/2+ 631	0.29	E 2	0.207

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