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ELASTIC SCATTERING OF γ -RAYS BY DEUTERONS BELOW THE THRESHOLD FOR PION PRODUCTION

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The dispersion relations and the unitarity conditions of the S-matrix are used to analyse the elastic γ -quanta scattering by deuterons below the threshold of pion production. The limit of low energies for γ -quanta scattering by nuclei with arbitrary spin is considered. The dependence of the elastic $\gamma - d$ scattering on energy is obtained with the help of experimental data on deuteron photodisintegration. The result greatly differs from the impulse approximation in a wide energy region. The account of pion photoproduction by deuterons turns out to be negligible in the treated energy range.

Introduction

The scattering of γ -quanta by deuterons is one of the processes in which such inelastic reactions as photodisintegration of a deuteron and photoproduction of a π -meson have a great influence on the amplitude of the scattering. The effect caused by the photoproduction of a π -meson is important in the region near and above the photoproduction threshold. The study of this effect requires a detailed analysis of the process $\gamma + d \longrightarrow NN\pi$, $\gamma d \longrightarrow d\pi^0$ and will not be considered here. The influence of photodisintegration on the γ - d elastic scattering has been treated in an earlier paper in the low energy region /1/. The process of photodisintegration causes a sharp decrease in the cross section of elastic scattering near the threshold of photodisintegration. The aim of the present paper is to study the γ - d scattering in a wider energy region where the effect of photoproduction of mesons is still small. The technique of the dispersion theory is adopted.

The experimental data /2/ for the scattering of γ -quanta by deuterons in the region below 50–100 MeV cannot fit the theoretical result based on the impulse approximation /3/. This forced us to carry out the analysis which is not connected with the impulse approximation. On the other hand the contribution to the scattering amplitude by meson production can be neglected since it is quickly damped below the threshold of photoproduction.

We limited our analysis to the forward scattering. Only electric and magnetic dipole amplitudes for photodisintegration have been taken into consideration in the calculation of the dispersion integrals. We begin with the phase shift analysis in order to express the imaginary part of the scattering amplitude in terms of the quantities characterizing deuteron photodisintegration. Then we shall consider the forward dispersion relations and the low energy theorem. Dispersion integrals are estimated in the energy region of γ -quanta below $\approx$ 100 MeV. The real and imaginary parts of the scattering amplitude are obtained and the polarizability of the deuteron and the nucleon are discussed.

2. Phenomenological Analysis

It is well known that the electric and magnetic wave functions of a photon are $\mathcal{Y}^{(\lambda)}_{\ell m}(K)$, $\lambda=0,1$

$$\vec{\mathcal{Y}}^{(0)}_{\ell m} = \sum_{\mu} C^{\ell\, m}_{\ell\, m-\mu\; 1\, \mu}\; Y_{\ell\, m-\mu}(K)\, \vec{\xi}_{\mu} \tag{1a}$$

and

$$\vec{\mathcal{Y}}^{(1)}_{\ell m} = -i\, [\vec{K}, \vec{\mathcal{Y}}^{(0)}_{\ell m}] \tag{1b}$$

where k is the unit vector along the impulse of a photon in the centre of mass system, $Y_{\ell m}$ are spherical harmonic functions and

$$\vec{\xi}_1 = -\frac{1}{\sqrt{2}}(\vec{i}+i\vec{j})\,, \quad \vec{\xi}_0 = \vec{K}\,, \quad \vec{\xi}_{-1} = \frac{1}{\sqrt{2}}(\vec{i}-i\vec{j}) \tag{1}$$

are the eigenfunctions of the photon spin. The photon wave function satisfy the transversal condition

$$\vec{K}\cdot \vec{\mathcal{Y}}^{(\lambda)}_{\ell m}(K) = 0.$$

If we denote the spin function for a deuteron by

$$\eta_1 = -\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\\ 0\end{pmatrix}, \quad \eta_0 = \begin{pmatrix}0\\ 0\\ 1\end{pmatrix}, \quad \eta_{-1} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -i\\ 0\end{pmatrix} \tag{3}$$

the eigenfunctions of the total angular momentum J^2, J_z and the parity for the $\gamma - d$ system can be constructed from (1) and (3).

$$\mathcal{Y}^{(\lambda)}_{j\ell M} = \sum_{z} C^{j\, M}_{\ell\, M-z\; 1\, z}\; \mathcal{Y}^{(\lambda)}_{\ell\, M-z}(K)\, \eta_z. \tag{4}$$

In the centre of mass system all the quantities in the final state will be denoted by symbols with a prime, e.g. k' will denote the direction of the photon impulse in the final state.

The scattering matrix T may be written, with the help of (4)

$$T = \sum_{\substack{jM\ell\ell' \\ \lambda\lambda'}} \mathcal{Y}^{(\lambda')}_{j\ell' M}(\vec{K}')\, \mathcal{Y}^{(\lambda)}_{j\ell M}(\vec{K})\, a^{\lambda\lambda'}_{j\ell\ell'}. \tag{5}$$

According to parity conservation

$$a_{j\ell\ell'}^{\lambda\lambda'} = 0 \qquad \text{if} \qquad (-1)^{\ell+\lambda} \neq (-1)^{\ell'+\lambda'}. \tag{6}$$

The invariance under time reversal leads to the symmetry condition

$$a_{j\ell\ell'}^{\lambda\lambda'} = a_{j\ell'\ell}^{\lambda'\lambda}. \tag{7}$$

It is easy to show that the forward scattering amplitude has the following form:

$$\begin{aligned} e'\cdot T\cdot e = {} & A(\vec{e}'\vec{e}) + iB(\vec{S}\cdot[\vec{e}',\vec{e}]) + \tfrac{1}{2}C\,[(\vec{S}\vec{e})(\vec{S}\vec{e}') \\ & + (\vec{S}\vec{e}')(\vec{S}\vec{e})] + \tfrac{1}{2}\mathcal{D}\,[(\vec{S}\cdot[\vec{K},\vec{e}])(\vec{S}\cdot[\vec{K},\vec{e}']) + (\vec{S}\cdot[\vec{K},\vec{e}'])(\vec{S}\cdot[\vec{K},\vec{e}])] \end{aligned} \tag{8}$$

where $\vec{e}$ and $\vec{e}'$ are polarization vectors of the photon in the initial and final states, respectively, S is the operator for the spin of the deuteron whose components satisfy the following commutation relations

$$\begin{aligned} [S_i, S_j] &= i\,\varepsilon_{ijk} S_k \\ S_i S_j S_k + S_k S_j S_i &= \delta_{ij} S_k + \delta_{jk} S_i . \end{aligned} \tag{9}$$

Using the Stokes parameters for the description of the photon polarization as we have done for $\gamma - N$ scattering/4/; it is easy to obtain

$$\begin{aligned} \zeta_{\pm 1}^{*}\cdot T\cdot \zeta_{\pm 1} &= A \mp B(\vec{S}\vec{K}) + \tfrac{1}{2}(C+\mathcal{D})[2-(\vec{S}\vec{K})^2] \\ \zeta_{\pm 1}^{*}\cdot T\cdot \zeta_{\mp 1} &= \tfrac{1}{2}(\mathcal{D}-C)\{(\vec{S}\vec{i})^2 - (\vec{S}\vec{j})^2 \mp i[(\vec{S}\vec{i})(\vec{S}\vec{j}) + (\vec{S}\vec{j})(\vec{S}\vec{i})]\}. \end{aligned} \tag{10}$$

From (10) one can construct the density matrix in the final state by the method developed in/4/ and calculate all observable quantities. The cross section in the forward direction for the unpolarized deuterons and photons is equal to

$$\sigma(0^\circ) = |A + \tfrac{2}{3}(C+\mathcal{D})|^2 + \tfrac{1}{18}|C+\mathcal{D}|^2 + \tfrac{2}{3}|B|^2 + \tfrac{1}{3}|C-\mathcal{D}|^2 \tag{11}$$

and

$$4\pi\, \mathrm{Im}\,(A + \tfrac{2}{3}C + \tfrac{2}{3}\mathcal{D}) = q\,\sigma_t \tag{11'}$$

where σ_t is the total cross section including both elastic and inelastic cross sections; $\vec{q} = q\,\vec{K}$.

Let us turn to the phase shift analysis. Consider only the electric and magnetic dipole transitions in photodisintegration.

Magnetic dipole transition is characterized by the matrix

$$F_j^{\circ} = \sum C^{jM}_{1\,M-\tau\,1\,\tau}\, C^{jM}_{1\,M-\tau'\,1\,\tau'}\, C^{1\,M-\tau}_{1\,0\,1\,M-\tau}\, C^{1\,M-\tau'}_{1\,0\,1\,M-\tau'}\, \frac{3}{4\pi}\, \eta_\tau \eta^*_{\tau'}\, \zeta_{M-\tau}\, \zeta^*_{M-\tau'} \tag{12}$$

where we have used the relation $Y_{\ell m} = \delta_{\ell m}\sqrt{\frac{2\ell+1}{4\pi}}$ for the forward scattering.

From (12) it is easy to obtain

$$\begin{aligned}
\zeta_1^* \cdot F \cdot \zeta_1 + \zeta_{-1}^* \cdot F \cdot \zeta_{-1} &= \frac{3}{8\pi}[\alpha_j + \beta_j S_z^2] \\
\zeta_1^* \cdot F \cdot \zeta_1 - \zeta_{-1}^* \cdot F \cdot \zeta_{-1} &= \frac{3}{8\pi}\gamma_j S_z \\
\zeta_1^* \cdot F \cdot \zeta_{-1} &= -\frac{3}{8\pi}\beta_j T_{2,-2} = -\frac{3}{8\pi}\beta_j \frac{1}{2}(S_x - iS_y)^2 \\
\zeta_{-1}^* \cdot F \cdot \zeta_1 &= -\frac{3}{8\pi}\beta_j T_{2,2} = -\frac{3}{8\pi}\beta_j \frac{1}{2}(S_x + iS_y)^2
\end{aligned} \tag{13}$$

where

$$\alpha_j = \begin{cases} 1 \\ 1 \\ 0 \end{cases};\quad \beta_j = \begin{cases} 1/6 \\ -1/2 \\ 1/3 \end{cases};\quad \gamma_j = \begin{cases} 5/6 & j=2 \\ -1/2 & j=1 \\ -1/3 & j=0 \end{cases} \tag{14}$$

In obtaining (13) we have used the following relations

$$\begin{aligned}
\eta_1 \eta_1^* &= \frac{1}{2}(S_z + S_z^2) \\
\eta_{-1}\eta_{-1}^* &= \frac{1}{2}(S_z^2 - S_z) \\
\eta_0 \eta_0^* &= 1 - S_z^2 \\
\eta_1 \eta_{-1}^* &= T_{2,2} = \frac{1}{2}[S_x^2 - S_y^2 + i(S_xS_y + S_yS_x)] \\
\eta_{-1}\eta_1^* &= T_{2,-2} = \frac{1}{2}[S_x^2 - S_y^2 - i(S_xS_y + S_yS_x)]
\end{aligned} \tag{15}$$

The matrix for the electric dipole transition can be obtained from (13) when $\vec{\zeta}_1$ in (13) is replaced by $i[\vec{\zeta}_1, \vec{K}] = -\vec{\zeta}_1$ and $\vec{\zeta}_{-1}$ by $i[\vec{\zeta}_{-1}, \vec{K}] = \vec{\zeta}_{-1}$

Comparing (13) and (14) with (10), we obtain

$$\begin{aligned} 2A &= \frac{3}{8\pi}\sum_j (\alpha_j + 2\beta_j)(a_j^{(m)} + a_j^{(e)}) \\ 2B &= -\frac{3}{8\pi}\sum_j \gamma_j\,(a_j^{(m)} + a_j^{(e)}) \\ C &= -3/8\pi \sum_j \beta_j\, a_j^{(e)} \\ D &= -3/8\pi \sum_j \beta_j\, a_j^{(m)}\,. \end{aligned} \tag{16}$$

The unitarity condition for the S-matrix leads to the relation

$$2\pi i\,[T^{*}(-\vec{\kappa}', -\vec{\kappa}, -\vec{e}', -\vec{e}, -\vec{s})_{\gamma d\to\gamma d} - T(\vec{\kappa}', \vec{\kappa}, \vec{e}', \vec{e}, \vec{s})_{\gamma d\to\gamma d}] = q\int d\Omega_{n+p}\, T^{+*}_{\gamma d\to np}\, T_{\gamma d\to n+p} \tag{17}$$

where q is the relative mo mentum for the $\gamma - d$ system. Express $T_{\gamma d\to np}$ in the form

$$T_{\gamma d\to np} = \sum Y^{S}_{j\ell' M}(n)\, \mathcal{Y}^{(\lambda)*}_{j\ell M}(\kappa)\, d^{S\lambda}_{j\ell'\ell}\,. \tag{18}$$

where j is the total angular momentum, ℓ' is the orbital angular momentum in the final state and S is the total spin of the n–p system.

Conservation of parity leads to

$$(-1)^{\ell+\lambda+1} = (-1)^{\ell'}.$$

The quantities $d^{S\lambda}_{j\ell'\ell}$ are connected with the partial cross section for the photodisintegration

$$\begin{aligned} 24\pi\,\sigma^{S(m)}_{j\ell'} &= (2j+1)\,|d^{S0}_{j\ell'1}|^2\,; \\ 24\pi\,\sigma^{S(e)}_{j\ell'} &= (2j+1)\,|d^{S1}_{j\ell'1}|^2. \end{aligned} \tag{19}$$

The total cross section for photodisintegratión is equal to

$$24\pi\,\sigma_{\gamma d\to np} = \sum_{j\ell' S}(2j+1)\,[\,|d^{S0}_{j\ell'1}|^2 + |d^{S1}_{j\ell'1}|^2] \tag{20}$$

Substituting (18) and (5) in (17), we have

$$4\pi\,\mathrm{Im}\,a^{\lambda\lambda'}_{j\ell\ell'}(q) = q\sum_{\ell'' S}(d^{S\lambda}_{j\ell''\ell})^{*}(d^{S\lambda'}_{j\ell''\ell'}). \tag{21}$$

With the help of (16) and (14) we get

$$\mathrm{Im}\,(A+\tfrac{2}{3}C+\tfrac{2}{3}\mathcal{D}) = \frac{3}{16\pi}\sum_j(\alpha_j+\tfrac{2}{3}\beta_j)\,\mathrm{Im}\,(a_j^{(e)}+a_j^{(m)}) = \frac{q}{4\pi}\,\sigma_{\gamma d\to np}$$

$$\mathrm{Im}\,A = \frac{q}{4\pi}\,(3\sigma_0+\tfrac{6}{5}\sigma_2)$$

$$\mathrm{Im}\,B = \frac{q}{4\pi}\,(\tfrac{3}{2}\sigma_0+\tfrac{3}{4}\sigma_1-\tfrac{3}{4}\sigma_2)$$

$$\mathrm{Im}\,C = \frac{q}{4\pi}\,(-3\sigma_0^{(e)}+\tfrac{3}{2}\sigma_1^{(e)}-\tfrac{3}{10}\sigma_2^{(e)})$$

$$\mathrm{Im}\,\mathcal{D} = \frac{q}{4\pi}\,(-3\sigma_0^{(m)}-\tfrac{3}{10}\sigma_2^{(m)})$$

where the partial cross section for photodisintegration in the state j including the factor ($2j+1$) is denoted by σ_j

3. Crossing Symmetry and Dispersion Relations

The retarded amplitude for the $\gamma-d$ forward scattering may be written

$$\langle\mu'|e'\cdot N^{ret}\cdot e|\mu\rangle = -2\pi^2 i \int d^4z\, e^{-iq\cdot z}\langle\vec{p}\,\mu'|\theta(z_0)[e'\cdot j(\tfrac{z}{2}),\, e\cdot j(-\tfrac{z}{2})]|\vec{p},\mu\rangle \tag{23}$$

where μ and μ' are the spin indexes for the deuteron.

Similarly, the advanced amplitude has the form

$$\langle\mu'|e'\cdot N^{adv}\cdot e|\mu\rangle = 2\pi^2 i \int d^4z\, e^{-iq\cdot z}\langle\vec{p}\,\mu'|\theta(-z_0)[e'\cdot j(\tfrac{z}{2}),\, e\cdot j(-\tfrac{z}{2})]|\vec{p},\mu\rangle. \tag{24}$$

For the forward scattering in the laboratory system the deuteron momentum $\vec{p}$ vanishes.

Consider the complex conjugate for (23) and (24)

$$\langle\mu'|e'\cdot N^{ret}_{adv}(q)\cdot e|\mu\rangle^* = \langle\mu|e'\cdot N^{ret}_{adv}(-q)\cdot e|\mu'\rangle. \tag{25}$$

Changing the order of $e'\cdot j(\frac{z}{2})$ and $e\cdot j(-\frac{z}{2})$ in (23) and (24) and then replacing z by -z in the integral, we have

$$\langle \mu' | e' \cdot N^{\text{ret}}_{\text{adv}}(q) \cdot e | \mu \rangle = \langle \mu' | e \cdot N^{\text{adv}}_{\text{ret}}(-q) \cdot e' | \mu \rangle . \tag{26}$$

Representing $N^{ret/adv}$ in the form of (8), we may express the conditions of (25) and (26) in terms of the symmetric properties between the scalar functions.

$$\begin{aligned} A^{ret/adv}(\nu)^* &= A^{ret/adv}(-\nu) \\ B^{ret/adv}(\nu)^* &= -B^{ret/adv}(-\nu) \\ C^{ret/adv}(\nu)^* &= C^{ret/adv}(-\nu) \\ D^{ret/adv}(\nu)^* &= D^{ret/adv}(-\nu) \end{aligned} \tag{27}$$

and

$$\begin{aligned} A^{adv}(\nu) &= A^{ret}(-\nu) \\ B^{adv}(\nu) &= -B^{ret}(-\nu) \\ C^{adv}(\nu) &= C^{ret}(-\nu) \\ D^{adv}(\nu) &= D^{ret}(-\nu) . \end{aligned} \tag{28}$$

Now let us denote A, C, D and B by L_1, L_2, L_3 and L_4, respectively. The dispersion relations for the scalar functions can be written in the following form (lab. s):

$$Re\, L_{1,2,3}(\nu) - Re\, L_{1,2,3}(0) = \frac{2\nu^2}{\pi} P \int_{\nu_D}^{\infty} \frac{d\nu' \, Im\, L_{1,2,3}(\nu')}{\nu'(\nu'^2 - \nu^2)} \tag{29}$$

and

$$Re\, L_4(\nu) - \nu\, Re\, L_4'(0) = \frac{2\nu^3}{\pi} P \int_{\nu_D}^{\infty} \frac{d\nu' \, Im\, L_4(\nu')}{\nu'^2(\nu'^2 - \nu^2)} \tag{30}$$

where ν_D is the threshold energy for deuteron disintegration which is approximately equal to the binding energy of the deuteron.

The dispersion relations (29) and (30) will be useful only when the values of $L_{1,2,3}$ (0) and L'_4 (0) are known. It is necessary therefore to calculate the amplitude in the low energy region.

This calculation will be carried out in the next section. The result for lab. s. is

$$Re\, L_1(0) = -\frac{e^2}{M_d} \; ; \; Re\, L_{2,3}(0) = 0 \, , \; Re\, L_4'(0) = \left(\mu_0 - \frac{e}{M_d}\right)^2 \tag{31}$$

where μ_0 is the magnetic moment of the deuteron.

4. Low Energy Limit for the $\gamma - d$ Scattering

Thirring, Low, Gell-Mann and Goldberger/5/ have shown that the limiting value of the scattering amplitude and its first derivative with ν at $\nu = 0$ are completely determined by the static properties for the system of spin ½.

Following the method developed by Low we shall show that similar results hold for the system with arbitrary spin.

The S-matrix for the photon scattering from the state (q, e) to the state (q', e') can be expressed as follows:

$$S = -e_i' g_{ij} e_j (4 q_0 q_0')^{-1/2} \tag{32}$$

where

$$g_{ij} = \int dx\, dy\, e^{iq\cdot y - iq'\cdot x} \langle\, | P [j_i(x), j_j(y)] | \,\rangle . \tag{33}$$

Using the method of Low, it is easy to get*

$$g_{ij} = g_{ij}^{(0)} + A\,\delta_{ij} + B\,\varepsilon_{ijk} S_k + \mathcal{D}\,(S_i S_j + S_j S_i) \tag{34}$$

where

$$A(q'q) + B(\vec{S}\cdot[\vec{q}',\vec{q}]) + \mathcal{D}[(\vec{S}\cdot\vec{q}')(\vec{S}\cdot q) + (\vec{S}\cdot q)(\vec{S}\cdot q')] \tag{35}$$
$$= q_0 q_0' C$$

and

$$C = \frac{(2\pi)^4}{i} \delta^4(p' + q' - p - q) \sum_{spin} \Big[\frac{\langle q - q' | j_0 | q \rangle \langle q | j_0 | 0 \rangle}{E(q) - E(0) - q_0} \tag{36}$$
$$+ \frac{\langle q - q' | j_0 | -q' \rangle \langle -q' | j_0 | 0 \rangle}{E(q') - E(0) + q'} \Big]$$

while

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* This is the most general expression for arbitrary S if only the constant terms and the terms linear in the energy are taken into consideration.

$$
g_{ij}^{(0)} = \frac{(2\pi)^4}{i}\delta^4(p'+q'-p-q)\sum_{spin}\Big[\frac{\langle q-q'|j_i|q\rangle\langle q|j_j|0\rangle}{E(q)-E(0)-q_0}
+ \frac{\langle q-q'|j_j|-q'\rangle\langle -q'|j_i|0\rangle}{E(q')-E(0)+q_0'}\Big] \tag{37}
$$

Consider the case where the states $|q\rangle$ etc. are the eigenfunctions of a system with the spin S.

In the calculation of (36) and (37) we must know the current matrix $\langle p_2|\vec{j}|p_1\rangle$ in the low energy region with an accuracy of the order of v/c and $\langle p_2|j_0|p_1\rangle$ of v^2/c^2.

It is shown that these matrix elements with the required accuracy can be determined by some general principles.

Since $\vec{j}$ and j_0 are Hermitian operators and the interaction is invariant under three dimensional rotations, the most general form of the matrix elements for the current to the required approximation are

$$
\langle p_2|\vec{j}|p_1\rangle = \frac{e}{2M}(\vec{p}_1+\vec{p}_2) + i\mu[\vec{S},\vec{p}_2-\vec{p}_1] + c\{\vec{S}(\vec{S}\cdot(\vec{p}_1+\vec{p}_2))+(\vec{S}\cdot(\vec{p}_1+\vec{p}_2))\vec{S}\} \tag{38}
$$

and

$$
\langle p_2|j_0|p_1\rangle = a + b(p_1^2+p_2^2) + d(p_1\cdot p_2) + if(\vec{S}\cdot[\vec{p}_2,\vec{p}_1])
+ h[(\vec{S}\cdot\vec{p}_1)(\vec{S}\cdot\vec{p}_2)+(\vec{S}\cdot\vec{p}_2)(\vec{S}\cdot\vec{p}_1)] + g[(\vec{S}\cdot\vec{p}_1)(\vec{S}\cdot\vec{p}_1)+(\vec{S}\cdot\vec{p}_2)(\vec{S}\cdot\vec{p}_2)] \tag{39}
$$

where e is the total charge, $\mu S = \mu_0$ is the total magnetic moment and a,b,c,d,f,h,g are constant parameters.

The current operator j_i transforms like a 4-vector under the Lorentz transformation. The wave functions $|\vec{p},\mu\rangle$ are the basis for the irreducible representation in the inhomogeneous Lorentz group which transforms in the following way

$$
|\vec{p},\mu\rangle \xrightarrow{L} |\vec{p},\mu\rangle' = R_{\mu\mu'}(L,\vec{p})\,|L^{-1}\vec{p},\mu'\rangle \tag{40}
$$

where $R_{\mu\mu'}(L,\vec{p})$ is the matrix for the rotation of the spin under the Lorentz transformation studied in /6,7/.

Consider two coordinate systems. In the first system $\vec{p}_1 = 0$ and $\vec{p}_2 = p$, while in the second system

$$\vec{p}_1 = \vec{q}\,, \quad p_{10} = E_q = \sqrt{q^2+M^2}\,,$$
$$\vec{p}_2 = \vec{p} + \frac{\vec{q}}{E_q}\left[\frac{(\vec{p}\cdot\vec{q})}{q^2}E_q\left(1-\frac{E_q}{M}\right) + \frac{E_q}{M}E_p\right]$$
$$p_{20} = \frac{E_q}{M}\left[E_p + \frac{\vec{p}\cdot\vec{q}}{E_q}\right]. \qquad (41)$$

The second system moves with a velocity $-\frac{\vec{q}}{E_q}$ relative to the first system.

To the order of v^2/c^2 the matrix for the spin rotation under the above Lorentz transformation has the form

$$R(L,\vec{p}) = 1 + i\,\frac{\vec{S}\cdot[\vec{p}\,\vec{q}]}{2M^2}\,. \qquad (42)$$

Further, to the order of v/c we have

$$\begin{aligned}\langle p+q|\vec{j}|q\rangle' &= \frac{e}{2M}(\vec{p}+2\vec{q}) + i\mu[\vec{S},\vec{p}] + c\{\vec{S}(\vec{S},\vec{p}+2\vec{q}) + (\vec{S},\vec{p}+2\vec{q})\vec{S}\}\\ &= \langle p|\vec{j}|0\rangle + \frac{\vec{q}}{M}\langle p|j_0|0\rangle \\ &= \frac{e}{2M}\vec{p} + i\mu[\vec{S},\vec{p}] + c[\vec{S}(\vec{S}\vec{p}) + (\vec{S}\vec{p})\vec{S}] + a\frac{\vec{q}}{M}\end{aligned} \qquad (43)$$

and to the order of v^2/c^2

$$\begin{aligned}\langle p+q|j_0|q\rangle' &= a + b(p^2+2p\cdot q+2q^2) + d(p\cdot q+q^2) + if\,\vec{S}\cdot[\vec{p}\,\vec{q}]\\ &\quad + g\{(\vec{S},\vec{p}+\vec{q})(\vec{S},\vec{p}+\vec{q}) + (\vec{S}\vec{q})(\vec{S}\vec{q})\} + h\{(\vec{S},\vec{p}+\vec{q})(\vec{S}\vec{q}) + (\vec{S}\vec{q})(\vec{S},\vec{p}+\vec{q})\}\\ &= \left[1 - \frac{i\vec{S}\cdot[\vec{p},\vec{q}]}{2M^2}\right]\frac{E_q}{M}\left[\langle p|j_0|0\rangle + \frac{\vec{q}}{M}\cdot\langle p|\vec{j}|0\rangle\right] \\ &= a - ia\frac{\vec{S}\cdot[\vec{p},\vec{q}]}{2M^2} + \frac{a}{2M^2}q^2 + bp^2 + g(\vec{S}\vec{p})(\vec{S}\vec{p}) + \frac{e}{2M^2}(\vec{p}\cdot\vec{q}) + i\frac{\mu}{M}\vec{S}\cdot[\vec{p},\vec{q}].\end{aligned} \qquad (44)$$

From (43) it follows that $a = e$, $c = 0$ and from (44)

$$d + 2b = \frac{e}{2M^2}\,, \quad f = \frac{\mu}{M} - \frac{e}{2M^2}\,; \quad g = h = 0.$$

Finally we have the covariant expressions

$$\langle p_2|\vec{j}|p_1\rangle = \frac{e}{2M}(\vec{p}_1+\vec{p}_2) + i\mu[\vec{S},\vec{p}_2-\vec{p}_1] \qquad (45a)$$

$$\langle p_2|j_0|p_1\rangle = e + i\left(\frac{\mu}{M} - \frac{e}{2M^2}\right)\vec{S}\cdot[\vec{p}_2,\vec{p}_1] + \frac{e}{2M^2}(\vec{p}_1\cdot\vec{p}_2)\,. \qquad (45b)$$
$$+ 2b(p_1-p_2)^2$$

The expression (45a) coincides with the matrix element for the current of a nonrelativistic particle with spin S (see, e.g.[8]). Fortunately, the unknown parameter b in (45b) does not appear in the final results.

With the help of (45), it is easy to obtain

$$S = -\frac{(2\pi)^4}{i}\delta^4(p'+q'-p-q)\frac{1}{(4q_0q_0')^{1/2}}\Big\{\frac{e^2}{M}(\vec{e}'\cdot\vec{e}) - 2i\frac{e}{M}q_0 \tag{46}$$

$$\cdot\vec{S}\cdot[\vec{e}'\vec{e}](\mu-\frac{e}{2M}) - i\frac{\mu^2}{q_0}\vec{S}\cdot[[\vec{e}\vec{q}],[\vec{e}'\vec{q}']] - i\frac{e\mu}{Mq_0}[(\vec{e}\vec{q}')(\vec{S}\cdot[\vec{q}'\vec{e}']) - (\vec{e}'\vec{q})(\vec{S}[\vec{q}\vec{e}])$$

and for the matrix T

$$-T = \frac{e^2}{M}(\vec{e}'\cdot\vec{e}) - 2i\frac{e}{M}\nu(\mu-\frac{e}{2M})\vec{S}\cdot[\vec{e}'\vec{e}] \tag{47}$$

$$-i\frac{\mu^2}{\nu}(\vec{S}\cdot[[\vec{e}\cdot\vec{q}],[\vec{e}'\vec{q}']]) - \frac{ie\mu}{M\nu}[(\vec{e}\vec{q}')(\vec{S}[\vec{q}'\vec{e}']) - (\vec{e}'\vec{q})(\vec{S}[\vec{q}\vec{e}])].$$

For the forward scattering

$$-T = \frac{e^2}{M}(\vec{e}'\cdot\vec{e}) - i\nu\left(\frac{\mu_0}{S} - \frac{e}{M}\right)^2(\vec{S}\cdot[\vec{e}',\vec{e}]). \tag{48}$$

Putting $S=1$ in (48), we get (31).

In the energy region below the meson photoproduction the spin dependent terms give small contribution to the cross section, since in comparison with the case of the nucleon the mass is increased two times while at the same time the magnetic moment is greatly depressed.

5. Results and Discussions

At present the experimental data on deuteron photodisintegration is available up to an energy $\approx$ 500 MeV[9]. The result for the calculation of $\mathrm{Re}\,(L_1 + \frac{2}{3}L_2 + \frac{2}{3}L_3)$ is shown in Fig. 1, where the real part of the amplitude is expressed in terms of the units e^2/M_p (curve 1). The photon energy is measured in the units $\nu/\nu_D \doteq \gamma_0$, e.g. the threshold energy for deuteron disintegration. The imaginary part of $L_1 + \frac{2}{3}L_2 + \frac{2}{3}L_3$ is also shown there.

The main contribution to the dispersion integrals for $\nu \lesssim 100$ MeV comes from the photodisintegration in the energy region $\nu \lesssim 75$ MeV. The detailed analysis of the photodisintegration process is needed for the calculation of other amplitudes. If the main contribution of photodisintegration is assumed to come from the energy region $\nu \lesssim 80$ MeV and the analysis of de Swart and Marshak[10] is used, the dispersive part of all other amplitudes can be estimated. Thus, the spin dependent amplitudes for the forward $\gamma - d$ scattering is shown to be much smaller than those for the $\gamma - N$ scattering. The energy dependence of the cross section for $\gamma - d$ scattering is shown in Fig. 2. The cross section is decreased at first in comparison with the Thompson amplitude, then it begins to increase very quickly ($\gamma \gtrsim 4$ MeV) and reaches a value 4 times larger than $(e^2/M_d)^2$ in the energy region $20 < \gamma < 80$ MeV.

Magnetic dipole absorption is important near the threshold of photodisintegration which leads to an additional sharp cusp in the cross section with a half-width of about 200 – 300 KeV.

The great influence of the inelastic processes on the deuteron as a whole makes it impossible to apply the impulse approximation to the elastic $\gamma - d$ scattering in a wide energy region*.

The existence of photodisintegration process shows a specially great influence on the polarizability of the deuteron. Although the polarizability of the nucleon, as has been shown by Baldin, is completely connected with the process of meson photoproduction, the polarizability of deuterons and other nucleus are caused in general by the low energy photo-nucleon reaction.

From (29) it follows that the electric polarizability of the deuteron is equal to

$$\alpha_d \equiv \frac{d}{d\gamma^2}\left[Re\left(L_1+\frac{2}{3}L_2+\frac{2}{3}L_3\right)\right]_{\gamma=0} = \frac{1}{2\pi^2}\int_{\gamma_D}^{\infty}\frac{\sigma_t(\gamma')}{\gamma'^2}\,d\gamma'. \qquad (49)$$

The analogous formula holds also for other nucleus. The basic contribution to σ_t is coming from the dipole absorption. With this consideration (49) turns out into the well-known formula of Migdal/11/ (see also12/).

Substituting (41) and (46) from/1/ into (49), we get the sum of the electric and the magnetic polarizability of the deuteron

$$\alpha_d = \frac{e^2}{Mc^2}\left(\frac{\hbar c}{\varepsilon}\right)^2\left\{\frac{3}{64}+\frac{\left(1+\sqrt{\frac{\varepsilon'}{\varepsilon}}\right)^2}{12}\frac{\varepsilon}{Mc^2}(\mu_p-\mu_n)^2\right\} = 0.64\cdot 10^{-39}\,\text{cm}^2 \qquad (50)$$

which agrees with the result obtained by Levinger and Rustgi12/.

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* In a recent preprint of Schult and Capps the correction to the impulse approximation for $\gamma - d$ scattering is reconsidered and similar conclusions are drawn.

Since the effect of photodisintegration is very great for $\gamma - d$ scattering and for the polarizability of the deuteron, it is difficult to get available estimation of the polarizability of the neutron from the experimental data on $\gamma - d$ scattering at low energy. With some apprximation the electric polarizability for the neutron may be obtained with the help of (49). For this purpose the replacement σ_t of the total cross section in the dispersion relation by the difference of σ_t with the sum of the photodisintegration cross section σ_{dis}, the elastic production of a π -meson σ_π and the photoproduction of a π -meson by a proton σ_p .

$$\alpha_n \simeq \frac{1}{2\pi^2} P \int_{\nu_D}^{\infty} \frac{d\nu'}{\nu'^2} \left[\sigma_t(\nu') - \sigma_{dis}(\nu') - \sigma_\pi(\nu') - \sigma_p(\nu') \right]. \qquad (51)$$

More detailed analysis on photodisintegration and photoproduction is needed in the calculation of the magnetic polarizability*.

Strictly speaking, the analysis treated in the present paper is valid only for cross sections in the forward direction. However, in dipole approximation the basic results can be generalized for other angles of scattering. We shall not compare our result directly with the approximated data, since in experiments besides the elastic scattering there is inelastic scattering process

$$\gamma + d \longrightarrow n + p + \gamma .$$

Recently Baldin (private communication) considered the correction to impulse approximation for the inelastic γ -scattering and obtained the conclusion that there are large corrections connected with photodisintegration.

Therefore, we may conclude that the experimental data on the scattering of γ - quanta by deuterons in the energy region 50 – 100 MeV do not contradict the theoretical result, if the influence of photodisintegration (and photoproduction) is taken into consideration.

For more reliable comparison of the theoretical results with the experiment it is necessary to perform the analysis of the inelastic processes in a wider energy region.

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* The magnetic polarizability of the proton is very sensitive to the assumptions which have to be made in the analysis of photoproduction f mesons. According to the analysis of Watson it follows from [4] that the magnetic polarizability of protons is negligible. It seems that this conclusion does not contradict the experimental data [18].

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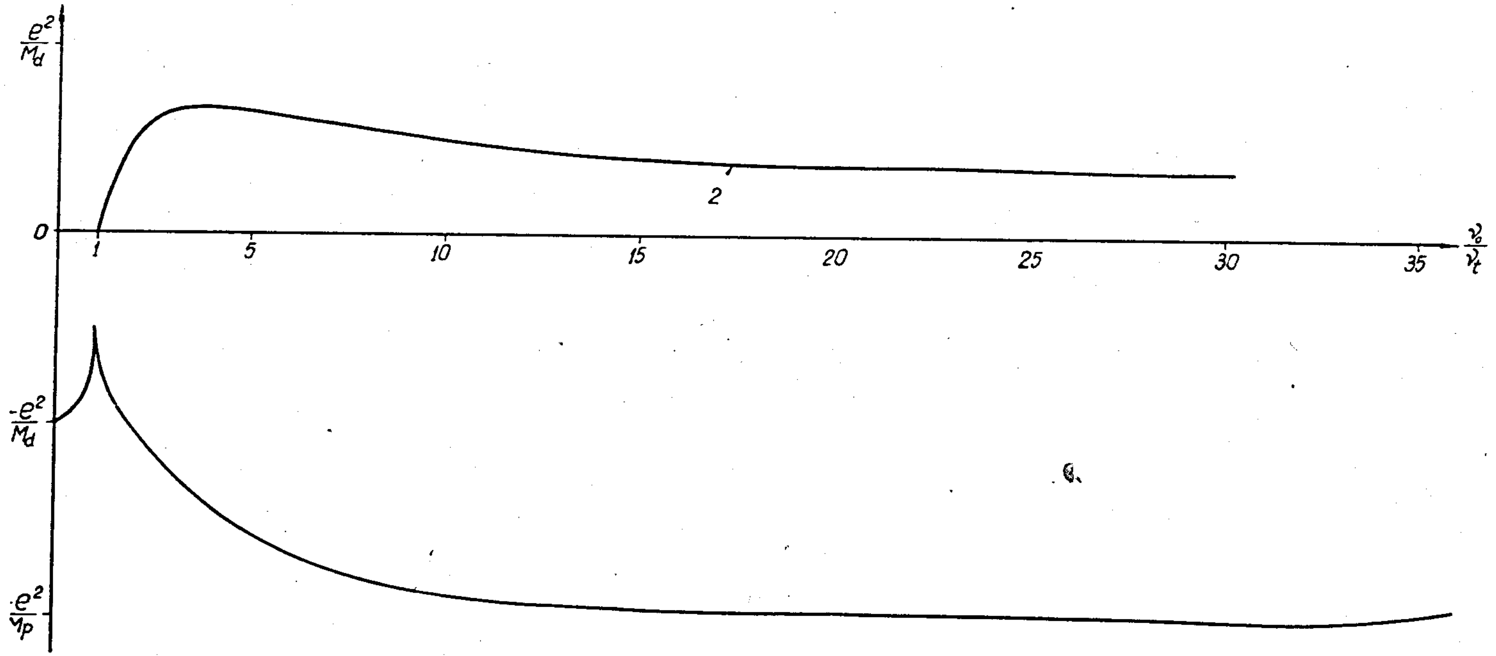

$\frac{e^2}{M_d}$
0
1
5
10
15
20
25
30
35
$\frac{\nu_0}{\nu_t}$
2
$-\frac{e^2}{M_d}$
$-\frac{e^2}{M_p}$

Fig. 1

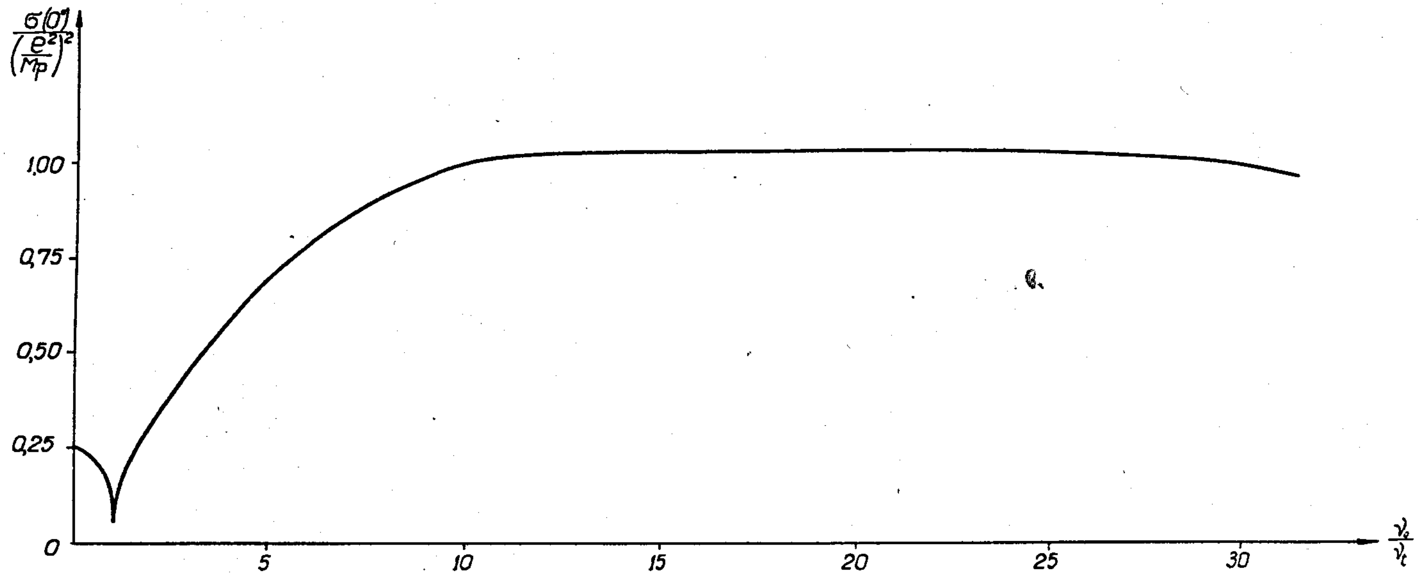

Fig. 2