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The mathematical procedure developed by N.N. Bogolubov in constructing the theory of superfluidity and superconductivity is very general. It allows to take into account the residual nucleon interactions which lead to the pairing correlations in the shell and unified nuclear models as a many-body problem. It has been proved in a number of investigations that the residual interactions between nucleons near the Fermi surface energy are attractive. Therefore, the ground state of any medium or heavy nucleus is the superfluid nuclear state which is energetically more favourable as compared to the state with subsequently filled levels.

We shall call superfluid such a model which is based upon the shell and unified models and takes into account the residual nucleon interactions near the Fermi surface energy of a nucleus under the following assumptions:

1) the residual interactions between nucleons both in the neutron and the proton shells are described by a Hamiltonian of the form

$$H = \sum_{s,\sigma} \{E(s)-\lambda\} a^+_{s\sigma} a_{s\sigma} - G \sum_{s,s'} a^+_{s+} a^+_{s-} a_{s'-} a_{s'+} \qquad (1)$$

2) the calculations are being made for each definite nucleus neglecting a certain averaging due to the conservation of the number of particles on the average. The nucleon state is described by a set of quantum numbers $(s\sigma)$ determined by the shape of a self-consistent field, $\sigma = \pm 1$ characterizes e.g., the sign of the projection of the angular momentum of each nuclean along the nuclear symmetry axis. A certain simplilification of the physical picture is that the interaction G is assumed to be constant. The chemical potential λ may be determined from the condition

$$n = \sum_{s\sigma} \langle a^+_{s\sigma} a_{s\sigma} \rangle , \qquad (2)$$

where n is the number of nucleons. Hamiltonian (1) should be regarded as a part of a complete Hamiltonian containing, eg., collective interactions. Note, that the effects to which the residual interactions (1) lead cannot be obtained by any modifications of a self-consistent field.

As a first stage in the investigation of a nuclear superfluid model let us consider here the strongly deformed nuclei among the rare-earth elements on the basis of the Nilsson potential under the assumption of the adiabatic approximation.

By means of the Bogolubov variational principle [1] the energy of the ground state of the even shell is obtained as follows

$$\mathcal{E} = \sum_s E(s) \left\{ 1 - \frac{E(s)-\lambda}{\sqrt{C^2 + \{E(s)-\lambda\}^2}} \right\} - C^2/G , \qquad (3)$$

provided C and λ are determined from the Eqs.[2,3]

$$\frac{2}{G} = \sum_{s} \frac{1}{\sqrt{C^2 + \{E(s) - \lambda\}^2}} , \tag{4}$$

$$n = \sum_{s} \left\{ 1 - \frac{E(s) - \lambda}{\sqrt{C^2 + \{E(s) - \lambda\}^2}} \right\} . \tag{5}$$

In the case of the odd shell if the odd nucleon is in the state $s_i \sigma_i$, the energy of the system

$$\mathcal{E}(s_i) = E(s_i) + \sum_{s \neq s_i} E(s) \left\{ 1 - \frac{E(s) - \lambda}{\sqrt{C^2 + \{E(s) - \lambda\}^2}} \right\} , \tag{6}$$

whereas C and λ may be found from the equations

$$\frac{2}{G} = \sum_{s \neq s_i} \frac{1}{\sqrt{C^2 + \{E(s) - \lambda\}^2}} , \tag{7}$$

$$n = 1 + \sum_{s \neq s_i} \left\{ 1 - \frac{E(s) - \lambda}{\sqrt{C^2 + \{E(s) - \lambda\}^2}} \right\} \tag{8}$$

The equations for C and λ excited states of the even shell have a similar form. Thus, to find C and λ it is necessary to solve the corresponding equation system. Note that the values of C and λ change appreciably in the passage both from the even shell to the odd one and from the ground state to the excited one. This may be illustrated in Table 1 for the case of a neutron shell with N = 107, N = 108, the deformation $\delta = 0.26$, provided $E_F = 6.406\, \hbar \mathring{\omega}_0$ ($\hbar \mathring{\omega}_0 = 41\, A^{-1/3}$ MeV). Since the interaction constant G is unknown, the calculations were made for five values of G over the range (0.016 – 0.032) $\hbar \mathring{\omega}_0$ (0.12 MeV – 0.24 MeV) The behaviour of λ characterizes the change in the properties of the many-body system, and, as seen from Table 1, these changes should not be neglected.

It is known that Nilsson scheme[4] does not give an exact order and the difference of energies of the excited states. This is likely to be connected with the shortcomings of the Nilsson scheme itself and with the necessity of taking into account the residual interactions. The calculation of the single-particle levels of the odd nuclei made on the basis of a superfluid model shows that 1) the obtained energy levels describe the excitation spectra more correctly if compared with the levels of the Nilsson scheme what is seen from Table II; 2) the behaviour of the levels is strongly dependent upon G , with increasing G, as is seen from Table III, the excitation energy decreases 3) the residual interactions do not lead, as a rule, to the change in the ground state presented by Nilsson scheme, 4) hole and particle levels behave differently with increasing G . However, the residual interactions fail to lead to the change in the sequence of the hole (particle) levels with respect to each other.

The spectrum of the excited states of the even-even nuclei calculated on the basis of a superfluid nuclear model shows, at least, what values of spins and parities of the excited states are most probable. So, one can see from the calculated levels of the energy Dy^{160} given in Table IV, that among the lower levels there must be states $0+$ and that with increasing G the excitation energies grow.

Let us calculate the pairing energy by the formula

$$P_n(Z,N) = 2\,\mathcal{E}(Z,N-1) - \mathcal{E}(Z,N) - \mathcal{E}(Z,N-2)\,. \qquad (9)$$

The neutron pairing energy Yb^{174} is found to be 0,39 MeV for $G = 0.016\,\hbar\mathring{\omega}_o$; 1.38 MeV for G=0.020, and 3.38 MeV for G=0.028. As for the proton pairing energy Hf^{176} it turned out to be 0.11 MeV for G=0.016; 0.26 MeV for G=0.020; 0.60 MeV for G=0.024; 1.32 MeV for G=0.028 and 2.07 MeV for $G = 0.032\,\hbar\mathring{\omega}_o$ Note, that in calculating the pairing energy it is necessary to take into account the energy due to the change in the deformation of nuclei.

The calculated values for the pairing energy and for the excitation spectra of a number of nuclei are most close to the experimental values at $G = (0.018 - 0.024)\,\hbar\mathring{\omega}_o = (0.13 - 0.18)$ MeV for the neutron ($93 < N < 115$) shell and at $G = (0.022 - 0.028)\,\hbar\mathring{\omega}_o = (0.15 - 0.20)$ MeV for the proton ($63 < Z < 76$) shell.

The obtained values for G and λ make it possible to calculate the corrections to the β and γ transition probabilities which are due to the superfluidity of the ground and the excited states. The corrections enter as factors less than unity. For instance, for a β -decay of the odd nuclei they assume the form

$$\frac{1}{4}\left\{1 \mp \frac{E(s_i)-\lambda_i}{\sqrt{C_i^2+\{E(s_i)-\lambda_i\}^2}}\right\}\cdot\left\{1 \mp \frac{E(s_K)-\lambda_K}{\sqrt{C_K^2+\{E(s_K)-\lambda_K\}^2}}\right\}. \qquad (10)$$

The corrections to a β -decay Er^{163} are equal to 0.45 for $G = 0.020\,\hbar\mathring{\omega}_o$, 0.14 for G = 0.028; for a β decay Dy^{165} 0.3 for G=0.016 and 0.5 for G=0.024; for a $\beta-$ decay Ho^{167}– 0.17 for G=0.016 and 0.02 for G 0.024 $\hbar\mathring{\omega}_o$ etc. The corrections e.g., for the E1-transition with an energy 0.026 MeV in Dy^{161} are equal to 0.36 for G = 0.020; 0.12 for G = 0.024 and 0.04 for $G = 0.028\,\hbar\mathring{\omega}_o$ etc.

The given examples show that there is a real possibility of making the calculations of a number of properties of the strongly-deformed nuclei associated with their intrinsic structure, With this aim it is necessary tc improve the scheme of Nilsson levels so that the single-particle levels of the odd nuclei calculated on the basis of superfluid model satisfy the experimental values. This will enable to calculate the excitation spectrum of the even-even nuclei, the pairing energy, of the inertia moments of the ground and excitation states, the corrections to the probabilities of β and γ -transitions etc.

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Table 1

G ($\hbar\omega_0$)	Even shell N = 108 ground state	Odd shell N = 107					
		Ground state 9/2+ [624]		Excited states			
				Particle ½−[510]		hole 5/2− [512]	
	c	λ	c	λ	c	λ	c	λ

G ($\hbar\omega_0$)	c	λ	c	λ	c	λ	c	λ
0.016	0.075	6.458	0.019	6.430	0.055	6.406	0.063	6.458
0.020	0.137	6.446	0.106	6.423	0.115	6.405	0.118	6.444
0.024	0.204	6.440	0.175	6.415	0.180	6.403	0.181	6.431
0.028	0.271	6.435	0.242	6.409	0.246	6.399	0.246	6.422
0.032	0.338	6.432	0.311	6.405	0.313	6.396	0.312	6.415

Table 11

Energies of excited states in MeV

	Neutron shell		Proton shell	
	N = 101; δ = 0.26 G = 0.020 $\hbar\omega_0$	N = 105; δ = 0.27 G = 0.016 $\hbar\omega_0$	Z = 67, δ = 0.30 G = 0.024 $\hbar\omega_0$	Z = 69 δ = 0.28 G = 0.028 $\hbar\omega_0$
Ground state	0½ −[521]	0½ −[514]	0 7/2 −[523]	0½ + [411]
Particle levels	0.10 5/2 −[512] 0.42 7/2 −[514] 0.67 ½ + [651]	0.32 9/2+ [624] 0.46 ½ + [651] 1,18 ½ − [510]	0.16 ½ + [411] 1.30 7/2+[404] 1.59 5/2+ [402]	0.55 7/2+ [404] 0.73 5/2+ [402] 1.00 9/2− [514]
Hole levels	0.08 7/2+ [633] 0.27 11/2 −[505] 0.84 5/2 −[523]	0.25 5/2 −[512] 0.80 ½ −[521] 0.93 7/2+ [633]	0.45 3/2+ [411] 0.52 5/2+ [413] 1.22 5/2 −[532]	0.13 7/2 −[523] 0.80 3/2+ [411] 0.86 5/2+ [413]

Table III

Energy (in MeV) of the excited states of odd nuclei (N = 107, δ = 0.23); the ground state

9/2 + [624]

Level Characteristics	G ($\hbar\omega_0$)					
	0	0.016	0.020	0.024	0.028	0.032
½ −[510]− particle	0.821	0.652	0.391	0.288	0.225	0.183
3/2 −[512]− -'-	0.930	0.718	0.449	0.326	0.261	0.212
7/2 −[503]− -'-	0.964	0.791	0.507	0.377	0.297	241
7/2 −[514]− hole	0.547	0.484	0.178	0.080	0.043	0.024
5/2 −[512]− -'-	0.907	0.848	0.455	0.275	0.181	0.133

Table IV

Enerby levels $_{66}Dy^{160}_{94}$ in MeV

Proton shell				Neutron shell			
Ω	parity	G=0.024;	0.028 $\hbar\omega_0$;	Ω	parity	G= 0.020 ;	0.024 $\hbar\omega_0$
2−;	5−	0.95	1.35	1+;	4+	0.62 (?)	1.04
1−;	6−	1.03	1.43	0+		0.97	1.28
0+		1.30	1.55	0+		1.00	1.37
1+;	2+	1.33	1.67	0−;	5−	1.07	1.50
0+		1.54	1.83	1−;	4−	1.45	1.77
1+;	4+	1.62	1.90	2−;	5−	1.45	1.83
3−;	4−	1.65	1.86	3+;	6+	1.68	1.99
0+		1.68	1.96	1−;	6−	1.76	2.02