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ON THE AXIAL VECTOR CURRENT IN THE LEPTONIC DECAY OF BARYONS AND MESONS

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Abstract

The formula of Goldberger and Treiman for the rate of $\pi \to \mu+\nu$ decay is derived in a more convincing manner using the analytic property of some matrix element. It is shown that their formula is correct for a wide class of strong interactions including the usual pseudoscalar coupling theory of pions. A formula is obtained which can be used as an experimental test for the adopted assumptions. The leptonic decay of hyperons and K-mesons are treated in a similar way.

1. INTRODUCTION

The V–A theory of Feynman and Gell-Mann, Sudarshan and Marshak appears to be well established at the present time for β -decay and μ -decay[1]. The observed ratio for the two decay modes of π -meson $\frac{R(\pi \to e+\nu)}{R(\pi \to \mu+\nu)}$ is also in good accordance with the theoretical value. This indicates strongly that the universal V–A interaction extends also to the processes of μ -capture.

A quantitative study of $\pi \longrightarrow \mu + \nu$ decay has been given by Goldberger and Treiman[2] using the technique of dispersion theory. Although many approximations are involved and not justified, the numerical result of their model agrees very well with experiment.

Recently Feynman, Gell-Mann and Levy[3] in a very interesting paper have reconsidered this problem. They have shown that the result of Goldberger and Treiman can be obtained in a quite general way.

Let us write the Hamiltonian for the β -decay and the μ -capture in the form

$$H = \frac{g}{\sqrt{2}}(P_\alpha + V_\alpha)L_\alpha + \text{herm. conj.} \qquad (1)$$

where

$$L_\alpha = \bar{\nu}\gamma_\alpha(1+\gamma_5)e + \bar{\nu}\gamma_\alpha(1+\gamma_5)\mu$$

P_α and V_α are the pseudovector and vector current for the weak interaction. F.G.L. have found three models for the strong interaction Lagrangian, for which the following equation holds

$$\partial_\alpha P_\alpha(x) = i\frac{a}{\sqrt{2}}\pi(x) \qquad (2)$$

where a is a constant parameter and $\pi(x)$ is the field operator for pions. Starting from (2) F.G.L. have deduced the result of G.T. in a simple and elegant way.

It has been mentioned in [3] that in a subsequent paper by Bernstein, Gell-Mann and Thirring this result can be generalized to any theory of strong interaction, in which the decay of pion is permitted. A general form factor $\varphi(S)$ appears in their new theory. The central question is whether this general form factor is likely to be slowly varying. They concluded that in their models, in which equation (2) holds, it is not unreasonable that $\varphi(S)$ be slowly varying, while in the conventional theory, where $\varphi(S)$ is something much more complicated, the conclusion is much less plausible.

In the present paper the formula of Goldberger and Treiman are obtained under the following assumptions.

1. The matrix element $\langle n|\partial_\alpha P_\alpha(0)|p\rangle$ is an analytic function of the variable $S=-(p_p-p_n)^2$.

2. If the matrix element for the equal time commutator vanishes, we can write the dispersion relation without subtractions.

3. In the dispersion relation without subtractions the main contribution is given by the nereby singularities.

With these assumptions it is shown that the general form factor is indeed likely to be slowly varying for a wide class of strong interactions including the conventional pseudoscalar coupling theory of pions.

In $ 2 and $ 3 a general derivation of the formula of Goldberger and Treiman is given. A relation between the axial vector coupling constant g_A for μ -capture, g_A for β -decay and the pseudoscalar coupling constant f for μ -capture, is established. Since these constants can be measured separately, this relation can be used as a test for the assumptions adopted..

In $ 4 the leptonic decay modes of hyperons and K-mesons are treated in a similar way. Comparing the rate of the decay $K \longrightarrow \mu+\nu$ with that of $\pi\to\mu+\nu$, it is shown that the pseudovector coupling constant g_{AY} in the β -decay of hyperons is likely to be an order of magnitude smaller than the pseudovector coupling constant in the g_A -decay of neutron.

II. The result of Goldberger and Treiman for $\pi\to\mu+\nu$ decay

Let us denote the operator $i\partial_\alpha P_\alpha$ by $O(x)$, which is evidently a pseudoscalar

$$i\,\partial_\alpha P_\alpha(x) \equiv O(x) \tag{3}$$

Applying this identity to the decay of pion, we get

$$\langle 0|O(x)|\pi\rangle = -q_\alpha \langle 0|P_\alpha(x)|\pi\rangle \qquad (4)$$

where q_α is the momentum vector of the pion. The matrix element $\langle 0|P_\alpha(0)|\pi\rangle$ can be represented in the form

$$\langle 0|P_\alpha(0)|\pi\rangle = -q_\alpha F(m^2)/\sqrt{2q_0} \qquad (5)$$

where m is the mass of the pion, F is a constant parameter which can be determined by the rate ω of the decay $\pi \longrightarrow \mu+\nu$

$$\omega = \frac{1}{4\pi} m^2 \left(\frac{m_\mu}{m}\right)^2 \left(1-\frac{m_\mu^2}{m^2}\right)^2 m^2 F^2 \qquad (6)$$

Substituting (5) into (4) we get

$$\langle 0|O(0)|\pi\rangle = -m^2 F/\sqrt{2q_0} \qquad (7)$$

Now turn to consider the matrix element for the β decay and the μ -capture.

$$\langle n|O(0)|p\rangle = -(p_p - p_n)_\alpha \langle n|P_\alpha(0)|p\rangle \qquad (8)$$

From invariance consideration the nuclear matrix element for the pseudovector current has the form

$$\langle n|P_\alpha(0)|p\rangle = \bar{u}_n \{ g_A \gamma_\alpha \gamma_5 + i f (P_p - P_n)_\alpha \gamma_5 \} u_p \qquad (9)$$

where g_A and f are functions of the invariant variable $S = -(P_p - P_n)^2$ Substituting (9) into (8) and applying Dirac equation on the nucleon spinor, we get

$$\langle n|O(0)|p\rangle = i(2Mg_A + fS)\,\bar{u}_n \gamma_5 u_p \qquad (10)$$

The central problem is to find the relation between the matrix elements $\langle 0|O(0)|\pi\rangle$ and $\langle n|O(0)|p\rangle$ This is achieved in next section using the technique of dispersion theory. The result is

$$\langle n|O(0)|p\rangle = i\,\bar{u}_n \gamma_5 u_p\, T(S) \qquad (11)$$

with

$$T(S) = -\frac{\sqrt{2}GFm^2}{-S+m^2} + T'(S) \qquad (12)$$

where G is the renormalized π - N coupling constant, T'(S) is function analytic in the region

$$|S| < 9m^2 \qquad (13)$$

It will be shown in the next section that $T'(s)$ can be approximately put equal to a single constant both for the μ -capture and for the β -decay. Rewrite (12) in the form

$$T(S) = -\frac{\sqrt{2}GFm^2}{-S+m^2}\varphi(S) \qquad (14)$$

where

$$\varphi(S) = 1 + \alpha\frac{S-m^2}{m^2} \qquad (15)$$

Comparing (10) with (11) and using (14), we get

$$2Mg_A + fS = -\frac{\sqrt{2}GFm^2}{-S+m^2}\varphi(S) \qquad (16)$$

An important point is that equation (16) holds for all S. Putting S=0, we get

$$F = -\frac{2Mg_{A\beta}}{\sqrt{2}G\varphi(0)}, \quad g_{A\beta} = g_A(0) \qquad (17)$$

This is the formula of Goldberger and Treiman recently obtained by Feynman, Gell-mann and Levy.

For μ -capture $S = -\frac{Mm_\mu^2}{M+m_\mu} = -0.9m_\mu^2$ From (15), (16) and (17) we get the relation

$$2Mg_{A\mu} + S_\mu f_\mu = \frac{m^2}{-S_\mu+m^2}2Mg_{A\beta} \qquad (18)$$

which can be used as a test for the adopted assumptions of universality and analyticity*

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* This relation is implicitly contained in the formula of Goldberger and Treiman on the form factor in μ capture and β -decay/4/.

We shall emphasize here that the relation (18) holds even when α is a large number. However, unlike equation (18) the formula (17) of Goldberger and Treiman is valid only for small α. Using the experimental value of the lifetime of charged pion, the pion-nucleon coupling constant and the axial vector coupling constant in β -decay, we get from (17)

$$\varphi(0) = 0.8 \qquad (19)$$

or

$$\alpha = 0.2 \qquad (20)$$

As has been stressed by Feynman, Gell-Mann and Levy that formula of Goldberger and Treiman is valid only for those theories, for which the constant α can be proved to be small. The difference between one theory and another merely lies in the fact whether $\varphi(S)$ is likely to be slowly varying. We shall discuss this problem in the next section.

III. Analytic Properties of the Matrix Element $\langle n | O(0) | p \rangle$

Now return to the evalution of the matrix element $\langle n | O(0) | p \rangle$ Following standard procedures, we write/5/.

$$\begin{aligned} \langle n | O(0) | p \rangle = & -i \bar{u}_n \int d^4 z \, e^{-i P_n \cdot z} \langle 0 | T(\eta(z) O(0)) | p \rangle \\ & - u_n^* \int d^4 z \, e^{-i P_n \cdot z} \delta(z_0) \langle 0 | [\psi_n(z) \, O(0)] | p \rangle \end{aligned} \qquad (23)$$

where $\eta(z) = i S^+ \delta S / \delta \bar{\psi}_n(z)$ is the source current for the neutron field. We shall drop in the following the equal-time commutator term, which would have ultimately yielded a constant addition to our expression. Since we are now interested only in the analytic structure of the matrix element, i.e. the places for the poles, their residues and the places for the cuts, e.t.c. it is unimportant for dropping this additional constant.

We now observe that

$$T(\eta(z) O(0)) = \theta(-z) [O(0), \eta(z)] + \eta(z) O(0) \qquad (24)$$

the second term makes no contribution to the matrix element. Therefore we have

$$\langle n | O(0) | p \rangle = -i \bar{u}_n \int d^4 z \, e^{-i P_n \cdot z} \theta(-z) \langle 0 | [O(0), \eta(z)] | p \rangle \qquad (25)$$

It is easy to prove by the standard procedure of Bogoliubov[6] that in the system of coordinates $\vec{P}_n = 0$, the function T (S) has a pole at $s = m^2$ and a continuous cut begining at $s = 9\,m^2$. In other points T (S) is an analytic function provided that

$$|\,\mathrm{Im}\, P_{no}\,| > |\,\mathrm{Im}\sqrt{P_{no}^2 - m^2}\,| \tag{26}$$

Where
$$P_{no} = M - \frac{S}{2M} .$$

Although the condition (26) can not be satisfied for real mass of nucleon, we shall assume that the analytic properties can be established by analytic continuation in the mass variable.

The residue on the pole at $s = m^2$ can easily be calculated. The result is

$$\begin{aligned} T(S) &= \frac{\sqrt{2}\,G}{-S+m^2} \langle 0|O(0)|\pi\rangle \sqrt{2q_0} + T'(S) \\ &= -\frac{\sqrt{2}\,G F m^2}{-S+m^2} + T'(S) \end{aligned} \tag{27}$$

The Feynman diagram for the pole term is shown in Fig. I.

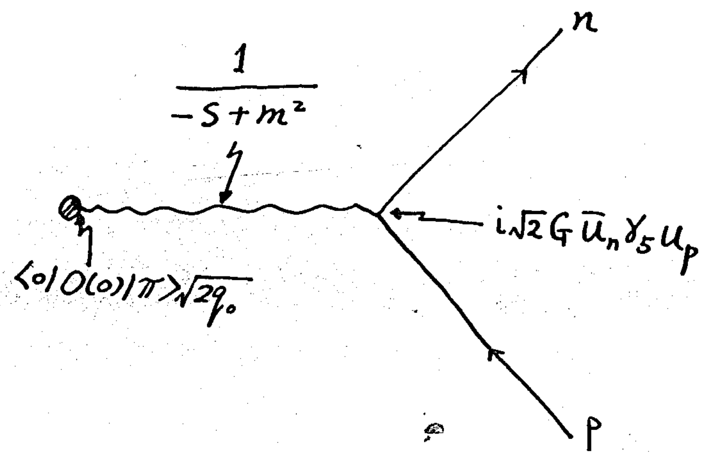

Fig. 1.

In (27) $T'(S)$ is an analytic function with a cut begining at $s = 9\,m^2$.

For small S $T'(S)$ can be expanded as a power series of S

$$T'(S) = \sum a_n S^n \tag{28}$$

which has a radius of convergence $9\,m^2$.

It can be shown for large n that

$$\lim_{n\to\infty} \left| \frac{a_{n+1} S}{a_n} \right| \leq \frac{|S|}{9m^2} \tag{29}$$

In the general case, it is reasonable to assume that one subtraction in the dispersion relation is enough for $T'(S)$. In this case we can write

$$T'(S) = a_0 + \frac{S}{\pi} \int_{9m^2}^{\infty} \frac{\rho(S')}{S'(S'-S)} dS' \tag{30}$$

where $\rho(S')$ is the spectral function.

If the sign of the spectral function does not change in the region where the main contribution of the integral is obtained, the inequality (29) holds also for small n. Expand the second term in (30) we get

$$a_n = \frac{1}{\pi} \int_{9m^2}^{\infty} \frac{\rho(S')}{S'^{n+1}} dS' \tag{31}$$

It is easily verified that

$$|a_{n+1}| = \frac{1}{\pi} \left| \int_{9m^2}^{\infty} \frac{\rho(S')}{S'^{n+2}} dS' \right|$$
$$\leq \frac{1}{9m^2 \pi} \left| \int_{9m^2}^{\infty} \frac{\rho(S')}{S'^{n+1}} dS' \right| = \frac{|a_n|}{9m^2} \tag{32}$$

It is noted that only the values of $T'(S)$ at $S = 0$ and $S = -0.9\,m^2$ are needed, which correspond to the physically interested cases for the β-decay and μ-capture. The distance between these two points are very small in comparison with their distance from the cut. Therefore, it is reasonable to approximate $T'(S)$ by a constant, i.e. $T'(0) \cong T'(-0.9 m_\mu^2)$ The error is expected to be of the order $0.9\,m^2 / 9\,m^2 = 1/10$

With these considerations we have

$$T(S) = -\frac{\sqrt{2}\, G F m^2}{-S+m^2} \varphi(S) \tag{33}$$

where

$$\varphi(S) = 1 + \alpha \frac{S-m^2}{m^2} \tag{34}$$

Now turn to the consideration of the magnitude α. In the general case α is proportional to the constant a_0, which as a subtraction constant may be very large. Therefore, we can not prove the validity of the formula of Goldberger and Treiman in the most general case (see appendix).

In the case when the matrix element for the equal time commutator vanishes, subtractions in the dispersion relation are not necessary. It is resonable to expect that the nearby singularities give the main contribution. Since the two points $S=0$ and $S=-\ 0.9\ m^2$ are very far from the cut, it is reasonable to conclude that the pole term will be dominant in the present case and thus the coefficient α will be small.

Consider now the conventional pseudoscalar theory with Lagrangian density

$$L_{p.s.} = -\bar{N}(\hat{\partial} + M_0 - i G_0 \vec{\tau}\cdot\vec{\pi}\gamma_5) N - \frac{m_0^2 \pi^2}{2} - \frac{(\partial_\alpha \pi)^2}{2} - \lambda_0 \pi^4 . \tag{35}$$

Following the method developed by F.G.L. in /3/, we take for our gauge transformation the following.

$$\begin{aligned} N &\longrightarrow (1 + i\vec{\tau}\cdot\vec{V}\gamma_5) N \\ \vec{\pi} &\longrightarrow \vec{\pi} + \frac{4M_0 + 2M}{3G_0}\vec{V} \end{aligned} \tag{36}$$

where $\vec{V}$ is the gauge function. We then obtain

$$\vec{P}_\alpha(x) = \bar{N}\vec{\tau}\gamma_\alpha\gamma_5 N - i\frac{4M_0 + 2M}{3G_0}\partial_\alpha \pi \tag{37}$$

and

$$\begin{aligned} \vec{O}(x) \equiv i\partial_\alpha P_\alpha(x) &= 2G_0 \bar{N} N \vec{\pi} + i\frac{2(M_0 - M)}{3}\bar{N}\vec{\tau}\gamma_5 N \\ &+ \frac{4M_0 + 2M}{3G_0}(m_0^2 \vec{\pi} + 4\lambda_0 \vec{\pi}^2 \vec{\pi}) \end{aligned} \tag{38}$$

The matrix element for the equal time commutator has the form

$$\vec{I} = \langle 0 |\, 2G_0 N \vec{\pi} + i\frac{2(M_0 - M)}{3}\vec{\tau}\gamma_5 N \,| N \rangle$$

From the consideration of symmetry we can write

$$\vec{I} = i A\, \vec{\tau}\gamma_5 u_N \tag{39}$$

Multiplying both sides of (39) from left side by $\vec{\tau}\gamma_5$, we get

$$i\, 3A\, u_N = -2i \langle 0 | \eta(0) | N \rangle$$

where $\eta(0) = i G_0 \vec{\tau}\cdot\vec{\pi}\gamma_5 N + (M-M_0)N$

is the source current for the nucleon field. It is well known that the matrix element $\langle 0|\eta(0)|N\rangle$ vanishes, we get, therefore, A=0.

Thus, we have shown that the matrix element for the equal time commutator vanishes in the present case and the dispersion relation without subtraction is expected to hold for the matrix element $\langle n|O(0)|p\rangle$ in the conventional pseudoscalar theory of meson nucleon interaction.

In the conventional theory of weak interaction P_α has the simple form $\bar{N}\vec{\tau}\gamma_\alpha\gamma_5 N$ A similar analysis can be carried out for this case and is discussed in the appendix. Although the matrix element for the equal time commutator does not vanish in this case and the dispersion relation should be written with subtractions, it is still possible that the form factor $\varphi(s)$ will be slowly varying (see appendix).

IV. The Leptonic Decay of K-Mesons and Hyperons

The upper limit on the leptonic decay rate of Λ and Σ hyperons is experimentally an order of magnitude below the theoretical value expected if the effective coupling constants are the same as in the neutron β -decay/7/. Many authors have expressed the idea that there may be no universality between strange particle decays and the decay of ordinary particles/8/. We shall assume in this section only a restricted universality between ($e\nu$) and ($\mu\nu$) pairs in strange particle decay, (leptonic current in the form (1)) which seems to us to be very probable. Furthermore, we shall assume that the interaction is of the V, A type and the K-meson is a pseudoscalar meson.

The weak Hamiltonian still has the form (1). We can repite step by step the procedures outlined in II and III. For the conventional theory of strong interaction it is always possible to find suitable pseudovector current such that the matrix element for the equal time commutator vanishes.

In the case of hyperon decay the matrix element $\langle N|P_\alpha(0)|Y\rangle$ has three terms

$$\langle N|P_\alpha(0)|Y\rangle = \bar{u}_N \{ g_{AY}\gamma_\alpha\gamma_5 + i\xi_Y[(\hat{P}_N-\hat{P}_Y)\gamma_\alpha - \gamma_\alpha(\hat{P}_N-\hat{P}_Y)]\gamma_5 + i f_Y(P_Y-P_N)_\alpha\gamma_5 \} u_Y \tag{36}$$

Using (36), we get

$$\langle N|O(0)|Y\rangle = i\,[(M_N+M_Y)g_{AY} + f_Y S]\,\bar{u}_N\gamma_5 u_Y \tag{37}$$

where $\quad S = -(P_Y - P_N)^2$

The remaining steps are straight forward. The final result is

$$(M_N + M_Y)\, g_{AY} + f_Y S = -\frac{G_{KY} F_K m_K^2}{-S + m_K^2} + T_Y'(S) \qquad (41)$$

where G_{KY} is the renormalized coupling constant for the KYN interaction; F_k is a constant parameter related to the matrix element $\langle o | O(o) / K \rangle$ in the following way

$$\langle o | O(o) / K \rangle = -m_K^2 F_K / \sqrt{2q_o} \qquad (42)$$

In (41) $T_Y'(S)$ is an analytic function with a cut begining at $S = (M_K - 2m)^2$

Let us denote the kinetic energy of the recoiled nucleon in the rest system of the hyperon by T_N, the quantity S for the hyperon decay is equal to

$$S = (M_k - 2m)^2 - 2m\, T_N \qquad (43)$$

In the present case the physical points are lying very near to each other and their common distance from the cut is very far. Therefore we can approximate $T_Y'(S)$ by a single constant a_Y. Formula (41) can be regarded as a generalized formula of Goldberger and Treiman in the case of strange particle decay. It is very interesting to verify (41) experimentally, which may serve as a test for the universality between ($e\nu$) and ($\mu\nu$) pairs in the strange particle decay.

Neglecting the term proportional to S and the constant term a_Y, we get

$$(M_N + M_Y)\, g_{AY} = -G_{KY} F_K \qquad (44)$$

Comparing (44) with (17) we obtain approximentely

$$\left(\frac{g_{AY}}{g_{A\beta}}\right)^2 \simeq \frac{G_{KY}^2}{G_\pi^2} \frac{F_K^2}{F_\pi^2} \qquad (45)$$

Since the lifetimes of $K \longrightarrow \mu + \nu$ and $\pi \rightarrow \mu + \nu$ have the same order of magnitude, while the phase volume for the K-decay is much greater than that for the π -decay, the matrix element F_K^2 will be much less than F_π^2. Therefore, even if G_{KY}^2 has the same order of magnitude as G_π^2, we can still conclude that

$$g_{AY}^2 << g_{A\beta}^2 \qquad (46)$$

Therefore, the observed rate of $K \rightarrow \mu + \nu$ decay seems to require a smaller axial vector coupling constant for hyperon decay. This fact has been pointed out first by Sakita [9] using the dispersion relation of Goldberger and Treiman.

Our treatenent can easily be extended to the case of scalar K-meson. In this case we must consider the divergence of the vector current. The matrix element $\langle N | V_\alpha(0) | Y \rangle$ has in general the following form

$$\langle N | V_\alpha(0) | Y \rangle = \bar{u}_N \{ g_{VY} \gamma_\alpha + i \zeta_Y [(\hat{P}_N - \hat{P}_Y)\gamma_\alpha - \gamma_\alpha(\hat{P}_N - \hat{P}_Y)] + i d_Y (P_N - P_Y)_\alpha \} u_Y \qquad (47)$$

It is easy to calculate from (47)

$$\langle N | O(0) | Y \rangle = i [(M_N - M_Y) g_{VY} + d_Y S] \bar{u}_N u_Y \qquad (48)$$

The remaining steps are similar and the final result is

$$(M_N - M_Y) g_{VY} + d_Y S = - \frac{G_{KY} F_K m_K^2}{-S + m_K^2} + a_Y \qquad (49)$$

where a_Y is a constant. Neglecting terms proportional to S and a_Y we get

$$(M_N - M_Y) g_{VY} = - G_{KY} F_K \qquad (50)$$

Comparing (50) with (17), we obtain

$$\frac{g_{VY}}{g_{A\beta}} = \frac{2M_N}{M_N - M_Y} \frac{G_{KY} F_K}{G_\pi F_\pi} \qquad (51)$$

Since in this case $2M_N >> |M_N - M_Y|$, therefore the small coupling constant g_{VY} observed in the hyperon β -decay requires that

$$G_{KY}^2 << G_\pi^2$$

This is quite reasonable since the coupling constant for a scalar interaction is expected to be smaller than the coupling constant for the pseudoscalar interaction. The above considerations can be easily generalized to the case where Λ and Σ have opposite parities.

It is noted that (41) and (49) can be used for the determination of the renormalized coupling constant G_{KY} if the coupling constants g_{AY} and f_Y (g_{VY} and d_Y) are measured for all physical values of S.

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Appendix

In the conventional theory of weak interaction the axial vector current has the simple form

$$\vec{P}_\alpha = \bar{N}\vec{\tau}\gamma_\alpha\gamma_5 N \tag{A.1}$$

The divergence of the axial vector current has been calculated in [3] and is equal to

$$\partial_\alpha P_\alpha(x) = 2M_0\bar{N}\vec{\tau}\gamma_5 N - 2iG_0\bar{N}N\vec{\pi} \tag{A.2}$$

Applying (A.2) to the decay of meson, we get

$$\vec{O}(x) = i\,\partial_\alpha\vec{P}_\alpha(x) = \frac{2M_0}{G_0}\vec{j}(x) + \frac{2M_0}{G_0}\left[(m^2-m_0^2)\vec{\pi} + 4\lambda_0\pi^2\vec{\pi}\right] + 2G_0\bar{N}N\vec{\pi} \tag{A.3}$$

It is wellknown that

$$\langle 0|\vec{j}(0)|\pi\rangle = 0 \tag{A.4}$$

where

$$\vec{j}(x) = iG_0\bar{N}\vec{\tau}\gamma_5 N + (m^2-m_0^2)\vec{\pi} + 4\lambda_0\pi^2\vec{\pi} \tag{A.5}$$

is the source current of π -meson field.

From (A.4) and (A.5) we get

$$\langle 0|O(0)|\pi\rangle = \frac{2M_0}{G_0}(m^2-m_0^2)\sqrt{Z_3}/\sqrt{2q_0} + \frac{8M_0}{G_0}\lambda_0\langle 0|\pi^2\pi|\pi\rangle + 2G_0\langle 0|\bar{N}N\pi|\pi\rangle \tag{A.6}$$

where Z_3 is the wave function renormalization constant for π -meson. If the first term in (A.6) gives the main contribution, we get approximately

$$\langle 0|O(0)|\pi\rangle \cong \frac{2M_0}{G_0}(m^2 - m_0^2)\sqrt{Z_3}\,/\sqrt{2q_0} \tag{A.7}$$

or

$$F \cong \frac{2M_0}{G_0}(m_0^2 - m^2)\sqrt{Z_3}\,/m^2 \tag{A.8}$$

Now turn to the evaluation of $\langle n|O(0)|p\rangle$. Rewrite (A.2) in the following form

$$\vec{O}(x) = i\,\partial_\alpha P_\alpha(x) = \frac{4M_0+2M}{3G_0}\vec{j}(x) + \vec{O}'(x) \tag{A.9}$$

where $\vec{O}'(x)$ is given in (38). From (A.9) we get

$$\begin{aligned}\langle n|O(0)|p\rangle = {} & \frac{4M_0+2M}{3G_0}\sqrt{2}G\,d(s)\Gamma(s)\sqrt{Z_3}\; i\bar{u}_n\gamma_5 u_p \\ & + \langle n|O'(0)|p\rangle\end{aligned} \tag{A.10}$$

where $d(s)$ and $\Gamma(s)$ are form factors for the propagation function of π meson and the pion-nucleon vertex part respectively.

If the first term in (A.10) is small comparing with the pole term in $\langle n|O'(0)|p\rangle$ we again get the result of Goldberger and Treiman.

The coefficient in the first term can be expressed with the help of (A.8) in the form

$$\frac{2M_0+M}{3M_0}\,\frac{m^2}{m_0^2-m^2}\,d(s)\Gamma(s)\left(\sqrt{2}GF\,i\bar{u}_n\gamma_5 u_p\right) \tag{A.11}$$

In the theory of perturbation $\Delta m^2 = m_0^2 - m^2$ is quadratic divergent. It is not unreasonable that

$$\frac{2M_0+M}{3M_0}\,\frac{m^2}{m_0^2-m^2}\,d(s)\Gamma(s) \ll 1$$

Therefore, it is quite plausible to expect that the first term in (A.10) is indeed small and the pole term gives the main contribution.

It is interesting to note a second example, for which the axial vector current is taken to be

$$i P_\alpha(x) = \partial_\alpha \pi(x) \tag{A.12}$$

It follows directly from (A.12) that the matrix element $\langle n | P_\alpha(0) | p \rangle$ for the β-decay vanishes. This means that in this case the pole term and the subtraction constant have the same order of magnitude and cancel each other.

The matrix element $\langle 0 | O(0) | \pi \rangle$ can be calculated and is equal to

$$\langle 0 | O(0) | \pi \rangle = m^2 \sqrt{Z_3} / \sqrt{2q_0} \tag{A.13}$$

From the Lagranging (35) for the pseudoscalar theory we have

$$\begin{aligned} \vec{O}(x) &= i \partial_\alpha \vec{P}_\alpha(x) = m^2 \vec{\pi}(x) - \vec{j}(x) \\ &= \vec{O}'(x) - \vec{j}(x) \end{aligned}$$

Again the matrix element $\langle n | O(0) | p \rangle$ is easily calculated

$$\begin{aligned} \langle n | O(0) | p \rangle = \langle n | O'(0) | p \rangle - \sqrt{2} G d(S) \Gamma(S) \cdot \\ \cdot \sqrt{Z_3}\, i \bar{u}_n \gamma_5 u_p \end{aligned} \tag{A.14}$$

The pole term in $\langle n | O'(0) | p \rangle$ has the form

$$\frac{m^2}{-S+m^2} \sqrt{2} G \sqrt{Z_3}\, i \bar{u}_n \gamma_5 u_p \tag{A.15}$$

Comparing the first term in (A.14) and the pole term (A.15), it is easily seen that they have the same order of magnitude. Therefore in the present case the subtraction constant will be large and is comparable with the pole term.

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