JOINT INSTITUTE FOR NUCLEAR RESEARCH

Laboratory of Nuclear Problems

Laboratory of Theoretical Physics

D-489

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ЖЭТФ, 1960, т 39, в2, с364-372.

Dubna 1960

D-489

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602/8 пр.

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Submitted to JETP

Abstract

It is shown that in the energy spectrum of the particle a from reaction $A + B \to a + C + D$ there may be some anomalies near the threshold of the reaction $C + D \to E + F$.

As an example, the spectrum of K-meson obtained from the reaction $N + N \to \Lambda + N + K$ is analysed in the region where the energy of the $\Lambda - N$ pairs is near to the threshold of the process $\Lambda + N \to \Sigma + N$. The well known formula for final states interaction are derived in a simple way. The polarization vectors of the baryons in the final state are calculated when the incident nucleons are polarized.

The energy anomalies in the spectrum of final state particle are also discussed for some other cases.

In appendix the production of K-meson by n–p collisions is treated. The case of scalar K-meson is also considered there.

Introduction

It is well known that in the particle production processes interaction of two of the final particles may change appreciably the energy spectrum and the angular distribution of the third particle. In some cases the effect of the final state interaction may be isolated from the primary production mechanism. It happens when the radius of the primary interaction is much smaller than the radius of interaction between the pair of particles in the final state. Furthermore, if the interaction between this pair of particles with other outgoing particles is weak, the final state interaction between this pair can be characterized by their two body scattering length.

The theory of final state interaction has been applied to the production of π -meson in N-N - collision by Migdal/1/, Bruckner and Watson/2/ and Parynzeva/3/. Recently Henley/4/, Feldman and Matthews/5/ applied this theory to analyse the reaction

$$N + N \to Y + N + K.$$

They have shown that the interaction between $Y - N$ pair may cause rather large change in the energy spectrum of K-meson.

Karplus and Rodberg/6/ have generalized the theory of final state interaction to those cases where strong interaction in final states may lead to inelastic processes.

In the present paper we shall show that near the production of threshold of the Σ hyperon in the reaction $\Lambda + N \to \Sigma + N$ there will be some energy anomalies in the energy spectrum of K-mesons which are produced together with Λ -particles. They are new examples of those threshold anomalies which are intensively studied in recent years/7/.

The form and the magnitude of the threshold anomalies, besides the cross section of the new production process, depend on the spin and the parity of the particle. Analysis with sufficient accuracy of these anomalies may help us to determine the properties of the produced particles.

It is assumed that in the final state of the reaction (1) the transition matrix element is described only by triplet and singlet S -state of the Y-N system. In the next section the kinematics are analysed and the expression for the energy spectrum of the K-meson, the polarization vector of the Λ - particle and nucleon with polarized incident particles are obtained.

In the section 3 the general formulation of the theory of inelastic interaction in final states is given with the help of the unitarity and analyticity properties of the S-matrix.

In the section 4 the local threshold anomalies in the energy spectrum of K-meson produced in the reaction $N+N \to \Lambda+N+K$ is studied near the production threshold of the Σ -hyperon (in the reaction $\Lambda+N \to \Sigma+N$).

In conclusion some similar processes are indicated and possible generalization of the present method to these processes are discussed.

2. Kinematics. Phenomenological Analysis.

It is convenient to introduce the Jacobi coordinates for the three particles system in final states

$$\vec{R} = \frac{M_N \vec{z}_N + M_Y \vec{z}_Y + M_K \vec{z}_K}{M_N + M_Y + M_K} \; ; \; \vec{\rho} = \vec{z}_K - \frac{M_N \vec{z}_N + M_Y \vec{z}_Y}{M_N + M_Y} \; ; \; \vec{z} = \vec{z}_N - \vec{z}_Y \tag{2}$$

where M_N , M_Y and M_K are the masses of nucleon, hyperon and K-meson respectively; $\vec{z}_N$, $\vec{z}_Y$ and $\vec{z}_K$ are their coordinates. Momentums conjugate to $\vec{R}$, $\vec{\rho}$ and $\vec{z}$ will be denoted by $\vec{P}$, $\vec{p}_Y$ and $\vec{q}$ respectively. Total energy E in the center of mass system can be expressed in terms of these new variables in a simple form

$$E = \frac{p_Y^2}{2m_Y} + \frac{q^2}{2\mu} + M_K + M_Y - M_N \tag{3}$$

where

$$m_Y = \frac{M_N M_Y}{M_N + M_Y}$$

$$\mu = \frac{M_K (M_N + M_Y)}{M_K + M_N + M_Y} \tag{4}$$

are the corresponding reduced mass respectively.

Phase volume in the final states has the form

$$dJ = m_Y P_Y \, d\Omega_{P_Y} \, q^2 \, dq \, d\Omega_q \tag{5}$$

where $d\Omega_{P_Y}$ and $d\Omega_q$ are the differential solid angle for the momentum $\vec{P}_Y$ and $\vec{q}$ respectively.

Consider for concreteness the reaction

$$p + p \rightarrow \Lambda + p + K^+ \tag{6}$$

below the threshold for the reaction

$$p + p \rightarrow \Sigma^0 + p + K^+ . \tag{7}$$

Since the available kinetic energy in final state of reaction (6) is less than 80 MeV in center of mass system, it is reasonable to assume that the final particles are produced in S-state.

Let us express the element of S-matrix in the form

$$\langle p \Lambda K^+ | S | pp \rangle = -2\pi i \, \delta(E_i - E_f) \langle \Lambda p K^+ | T | pp \rangle \tag{8}$$

If K-meson is a pseudoscalar particle, (intrinsic parity of $\Lambda K P$ opposite to PP) the spin structure of the T -matrix has the form

$$\langle \Lambda p K | T | pp \rangle = A_\Lambda (\vec{\sigma}_1 + \vec{\sigma}_2, \vec{k}) + B_\Lambda \{ (\vec{\sigma}_1 - \vec{\sigma}_2, \vec{k}) + i ([\vec{\sigma}_1 \vec{\sigma}_2] \cdot \vec{k}) \} +$$

$$+ C_\Lambda \{ (\vec{\sigma}_1 - \vec{\sigma}_2, \vec{k}) - i ([\vec{\sigma}_1 \cdot \vec{\sigma}_2] \cdot \vec{k}) \} \tag{9}$$

where $\vec{\sigma}$ is the Pauli matrix; $\vec{k}$ is an unit vector along the direction of the incident particle; A_Λ , B_Λ and C_Λ are scalar functions of the total energy E and the relative momentum P_Λ of the $\Lambda - N$ pair. Since in the initial state there are two identical particles, the transition matrix element T must be antisymmetrical with respect to the interchange of the two initial nucleons, which leads to the requirement $B_\Lambda = 0$.

Cross section for the reaction (6) with unpolarized incident beam has the form

$$\frac{d\sigma}{d\Omega_{p_\Lambda} d\Omega_q dT} = (2\pi)^4 \frac{E}{2(E^2-4M_N^2)^{1/2}} (2m_\Lambda \mu)^{3/2} \left[T(T_{max}-T)\right]^{1/2}$$

$$\left[|A_\Lambda + C_\Lambda|^2 + |A_\Lambda - C_\Lambda|^2 + 2|C_\Lambda|^2\right] \qquad (10)$$

where $T = q^2/2\mu$ is the kinetic energy of the K-meson with respect to the center of mass of the $\Lambda - N$ system.

If the proton in the initial state is polarized with polarization vector $\vec{\mathcal{P}}$, then the polarization vector P_Λ of the Λ -particle in the final state is equal to

$$\vec{P}_\Lambda \left[|A_\Lambda + C_\Lambda|^2 + |A_\Lambda - C_\Lambda|^2 + 2|C_\Lambda|^2\right] = 2\left[|A_\Lambda + C_\Lambda|^2 - |C_\Lambda|^2\right](\vec{k}\vec{\mathcal{P}})\vec{k} +$$

$$+\left[|A_\Lambda - C_\Lambda|^2 - |A_\Lambda + C_\Lambda|^2\right]\vec{\mathcal{P}} . \qquad (11)$$

The expression for the polarization vector of nucleon in final state can be obtained from (11) by changing the sign before C_Λ.

3. Elastic Final State Interaction

Consider the unitarity condition

$$\langle \Lambda p k | T - T^+ | pp \rangle = 2\pi i \sum_n \langle \Lambda p k | T | n \rangle \langle n | T^+ | pp \rangle \delta(E_i - E_n) \qquad (12)$$

where $|n\rangle$ is a possible intermediate state lying on the same energy hypersurface as the initial state. We shall adopt the assumption that the imaginary part of the T -matrix is caused mainly by the strong interaction between $\Lambda - p$ pair. In this case we may neglect all other intermediate states except the state $\Lambda p k$ and approximate the matrix element

$$\langle \Lambda p k | T | \Lambda' p' k' \rangle \cong \langle \Lambda p | T | \Lambda' p' \rangle \langle k | k' \rangle .$$

This corresponds to the neglection of the interaction between K-mesons and the $\Lambda - p$ pair.

Matrix element $\langle \Lambda p | T | \Lambda' p' \rangle$ in the low energy region has the form

$$\langle \Lambda p | T | \Lambda' p' \rangle = (4\pi^2 p_\Lambda m_\Lambda)^{-1} \left[\frac{1}{4}(3 + \vec{\sigma}_1 \cdot \vec{\sigma}_2)\alpha_3 + \frac{1}{4}(1 - \vec{\sigma}_1 \cdot \vec{\sigma}_2)\alpha_1\right] \qquad (13)$$

where

$$\alpha_3 = e^{i\delta_3} \sin\delta_3 \qquad \alpha_1 = e^{i\delta_1} \sin\delta_1 \tag{14}$$

while δ_1 and δ_3 are the phase shifts of the singlet and triplet states respectively.

Based on all these assumptions and with the help of the time reversal invariance we get from (12)

$$\mathrm{Im}\, A_\Lambda = \frac{\mathrm{Re}\,\alpha_3}{1-\mathrm{Im}\,\alpha_3}\,\mathrm{Re}\,A_\Lambda = \frac{\mathrm{Im}\,\alpha_3}{\mathrm{Re}\,\alpha_3}\,\mathrm{Re}\,A_\Lambda = tg\,\delta_3\,\mathrm{Re}\,A_\Lambda$$

$$\mathrm{Im}\, C_\Lambda = tg\,\delta_1\,\mathrm{Re}\,C_\Lambda\,. \tag{15'}$$

Near the threshold of production where P_Λ is very small

$$A_\Lambda = (1+i\,tg\,\delta_3)\,\mathrm{Re}\,A_\Lambda \cong (1+i\,a_3 P_\Lambda)\,\mathrm{Re}\,A_\Lambda$$

$$C_\Lambda = (1+i\,tg\,\delta_1)\,\mathrm{Re}\,C_\Lambda \cong (1+i\,a_1 P_\Lambda)\,\mathrm{Re}\,C_\Lambda\,. \tag{15''}$$

From (15') it is clear that function A_Λ and C_Λ are approximately real quantities at $\delta \to 0$, i.e. in the absence of final state interaction. In the energy region considered here the matrix element are functions of the total energy E_Λ and the total energy ω of the Λ-P system.

If all the singularities of the amplitude are determined by physical processes, then A_Λ and C_Λ as an analytic function of E and ω can be represented in the form

$$\frac{e^{i\delta(\omega)}\sin\delta(\omega)}{P_\Lambda a}\,f(\omega)\,F_\Lambda(E) \tag{16}$$

where $f(\omega)$ is a entire function, which may be put equal to a constant for small energy.

Therefore A_Λ and C_Λ will be approximated finally by the expressions

$$A_\Lambda = A_\Lambda^\circ\,\frac{e^{i\delta_3}\sin\delta_3}{P_\Lambda a_3}$$

$$C_\Lambda = C_\Lambda^\circ\,\frac{e^{i\delta_1}\sin\delta_1}{P_\Lambda a_1}$$

where a_3 and a_1 are the triplet and singlet scattering length of the Λ-N system in S-state, while A_Λ° and C_Λ° are approximately real functions of total energy E only.

Thus we see that the unitarity condition of the S-matrix and the analyticity properties of the reaction amplitude lead directly to the basic formula of the theory of final state interaction (See e.g. /8/).

With the help of (16) the cross section and the polarization vector of the Λ -particle can be written in the form

$$\frac{d\sigma}{dT} = (2\pi)^4 \frac{E}{2(E^2-4M_N^2)^{1/2}} (4\pi)^2 (2m_\Lambda\mu)^{3/2} \left[T(T_{max}-T)\right]^{1/2} \times$$

$$\left[2 \cdot \frac{\sin^2\delta_3}{(P_\Lambda a_3)^2} |A_\Lambda^\circ|^2 + 4 \frac{\sin^2\delta_1}{(P_\Lambda a_1)^2} |C_\Lambda^\circ|^2\right] \tag{17}$$

$$\vec{P}_\Lambda \left[\frac{\sin^2\delta_3}{(P_\Lambda a_3)^2} |A_\Lambda^\circ|^2 + 2 \frac{\sin^2\delta_1}{(P_\Lambda a_1)^2} |C_\Lambda^\circ|^2 \right] =$$

$$= \left[\frac{\sin^2\delta_3}{(P_\Lambda a_3)^2} |A_\Lambda^\circ|^2 + 2 A_\Lambda^\circ C_\Lambda^\circ \frac{\sin\delta_1 \sin\delta_3 \cos(\delta_1-\delta_3)}{P_\Lambda^2 a_3 a_1} \right] (\vec{k}\vec{\mathcal{P}})\vec{k} - \tag{18}$$

$$- 2 A_\Lambda^\circ C_\Lambda^\circ \frac{\sin\delta_1 \sin\delta_3 \cos(\delta_1-\delta_3)}{P_\Lambda^2 a_3 a_1} \vec{\mathcal{P}}$$

where $P_\Lambda^2 = 2m_\Lambda (T_{max}-T)$. Changing the sign before in (18) we get the polarization vector of the recoiled proton. Expressions (17) and (18) may be regarded as a generalization of the results given by Henley, in which the spin dependence of the matrix element is neglected.

From (17) and (18) it is clear that the study of the energy spectrum of K-meson and expecially the polarization of the Λ -particle and the recoiled nucleon near the threshold is very useful for the determination of the scattering length of Λ-P system.

4. Inelastic Interaction Threshold Anomalies.

When the energy is sufficient for the production of a Σ -hyperon, the energy spectrum of the K-meson and other characteristics in $\Lambda P K$ chanals are expected to have some changes.

In this case it is necessary to consider the state $\Sigma N K$ as one of the important intermediate state. In the following we shall consider only interaction in S-state.

As in the previous section we shall make the assumption

$$\langle \Lambda N K | T | \Sigma N' K' \rangle \cong \langle \Lambda N | T | \Sigma N \rangle \langle K | K' \rangle$$

and write

$$\langle \Lambda N | T | \Sigma N \rangle = \left[4\pi^2 P_\Lambda^{1/2} P_\Sigma^{1/2} m_\Lambda^{1/2} m_\Sigma^{1/2} \right]^{-1} \left[\frac{1}{4} (3 + \vec{\sigma}_1 \cdot \vec{\sigma}_2) \beta_3 + \right.$$
$$\left. + \frac{1}{4} (1 - \vec{\sigma}_1 \cdot \vec{\sigma}_2) \beta_1 \right] \qquad (19)$$

where the indices Λ and Σ denote quantities in corresponding chanals. e.g.

$$P_\Sigma = [2 m_\Sigma (E' - T)]^{1/2}, \quad E' = E - m_\Sigma + m_\Lambda . \qquad (20)$$

Assuming that there are no bound state or resonance state in $p - \Sigma$ system in the low energy region, then the energy dependence of β_3 and β_1 have the form

$$\beta_3 = b_3 P_\Sigma^{1/2}, \quad \beta_1 = b_1 P_\Sigma^{1/2} \qquad (21)$$

in the case when the parity of the Λ and the Σ hyperon are the same.

As a result of the opening of Σ -chanal there appears additional terms proportional to P_Σ in the matrix element for $\Lambda - P$ scattering

$$\alpha_3 = \alpha_3^0 + i C_3 P_\Sigma, \quad \alpha_1 = \alpha_1^0 + i C_1 P_\Sigma \qquad (22)$$

where

$$C_{1,3} = \frac{P_\Lambda}{4\pi} \sigma_{1,3}^{\Sigma \to \Lambda} \qquad (23)$$

$\sigma_{2j}^{\Sigma \to \Lambda}$ is the total cross section for the reaction $\Sigma + N \to \Lambda + N$ in the state j.

Taking into consideration both effects results from the opening of the Σ chanal, we get from (8), (19)–(23)

$$\begin{aligned} Jm\, A_\Lambda &= (Jm\, A_\Lambda)_0 + A'_\Lambda P_\Sigma \\ Jm\, C_\Lambda &= (Jm\, C_\Lambda)_0 + C'_\Lambda P_\Sigma \end{aligned} \qquad (24)$$

where

$$A'_\Lambda = tg^2 \delta_3 \cdot A_\Lambda^0 \frac{P_\Lambda}{4\pi} \sigma_3^{\Sigma \to \Lambda} (P_\Sigma = 0) + \frac{A_\Sigma^0 b_3}{\cos^2 \delta_3} \qquad \left(\delta_{1,3} \neq \frac{\pi}{2}\right)$$

$$C'_\Lambda = tg^2 \delta_1 C_\Lambda^0 \frac{P_\Lambda}{4\pi} \sigma_1^{\Sigma \to \Lambda} (P_\Sigma = 0) + \frac{C_\Sigma^0 b_1}{\cos^2 \delta_1} \qquad (25)$$

Relations (24) are valid when the kinetic energy of the K-meson T is less than E'. In the case $T > E'$ it is impossible to create a real Σ -hyperon and we must put $p_\Sigma \to i k_\Sigma$ where

$$k_\Sigma = \sqrt{2 m_\Sigma (T - E')} \qquad , \quad T > E'$$

Therefore the term linearly depending on k_Σ will now appear in the real part of the reaction amplitude.

The existence of term, which is linearly dependent on $p_\Sigma \, (T < E')$ and $k_\Sigma \, (T > E')$, will lead to infinite energy derivative in the spectrum of K -meson and the energy dependence of the polarization of the Λ particles (and nucleon).

The magnitude of these anomalies is given by (24) and (25). Their form is dependent on the relative sign of A^o_Λ, A^o_Σ, $\theta_{3,1}$ and $\delta_{3,1}$. All four kinds of anomalies that have been studied in literature for a binary reaction can also take place in the present case.

All formula in section 2, 3 and 4 are given for the case of $p-p$ collision. It is not difficult to generalize it to the case of $n-p$ collision. This is done in the appendix. The case of scalar K-meson is also treated there.

The results obtained in the present section is restricted to interaction in S-state. The relative great mass difference between Σ and Λ hyperon makes it difficult to give a complete analysis of the inelastic interaction in final state, but the basic results and the reason about the existence of energy anomalies in the spectrum of K-meson will not be changed in a more detailed analysis of the problem.

It has been shown before /9/ , that the direct analytic continuation $p_\Sigma \to i k_\Sigma$ is impossible when there are resonance near the threshold. In this case it is more convenient to use the method of dispersion relation. Since the analytic property of the reaction amplitude as a function of ω is not yet clear, we shall not try such an analysis. However, even if a resonance exists near the threshold, it is still reasonable to expect the appearance of the energy anomalies in the spectrum of K-meson.

If Σ and Λ hyperon have different parities, the first term in the expansion will be proportional to p_Σ^3 and only the second energy derivative will turn out to infinity.

Thus, the study of the threshold anomalies in the energy spectrum of K-meson with sufficient accuracy will be useful for the determination of the relative parity of Σ and Λ particles.

5. Discussion

Therefore endothermic inelastic process $C + \mathcal{D} \rightarrow E + F$ in the final state of the reaction $A + B \rightarrow a + C + \mathcal{D}$ will lead by means of the analiticity and unitarity conditions of the S-matrix to the appearance of the energy anomalies in the spectra of particle a.

Experimental search of these anomalies requires, of course, good accuracy and high energy resolving power. However, the discovery of such anomalies will give very useful information about the interaction between unstable particles, their spins and parities.

It is interesting to note some other similar processes, which may have such energy anomalies in the spectra of final state particles.

In the spectra of $\pi^{\pm}$ -meson from the reaction

$$K^- + p \rightarrow \Lambda + \pi^- + \pi^+ \tag{26}$$

near the threshold of

$$\pi^{\mp} + \Lambda \rightarrow \Sigma^{\mp} + \pi^0 \tag{27}$$

there may have energy anomalies, whose magnitude and form are connected through the amplitude of reaction (27) with the $k^- - p$ scattering amplitude in low energy region.

In the spectra of proton from the process of production of π meson by K-meson

$$K^- + p \rightarrow K^- + \pi^0 + p \tag{28}$$

the energy anomalies may appear at the threshold of

$$K^- + \pi^0 \rightarrow \overline{K}^0 + \pi^- \tag{29}$$

if such a reaction is allowed.

The Lagrangian for $\pi - K$ interaction has the form

$$L_{int} = g \, (\varphi^i_\pi \cdot \varphi^i_\pi)(\varphi^\lambda_\theta \cdot \varphi^\lambda_\theta)$$

in which terms with derivatives are not considered. This Lagrangian is invariant under the rotation of the isotopic spin of each particle separately, which forbids the charge exchange reaction (29).

The above selection rule is dediced under rather restrictive conditions. Therefore discover of the anomalie in proton spectra will be very interesting for our understanding of the symmetry properties of the $\pi - K$ interaction.

In reactions with two π^0 -mesons in final state

$$\gamma + p \to p + \pi^0 + \pi^0$$
$$\pi^- + p \to n + \pi^0 + \pi^0 \qquad (30)$$

it is interesting to note that the energy spectra of nucleon may have similar anomalies near the threshold of the reaction

$$\pi^0 + \pi^0 \to \pi^- + \pi^+ \qquad (31)$$

where the relative energy of the π^0 -meson is greater than 9 MeV.

The threshold anomalies may lead to noticible effects in low energy $\pi - \pi$ scattering.

Existence of the threshold for the reaction (31) may lead to energy anomalies in the spectra of the charged π -meson in the decay of τ meson.

$$\tau^{\pm} \to \pi^{\pm} + \pi^0 + \pi^0$$

Similarly in the spectra of nucleon from the reaction

$$\gamma + p \to p + K^- + K^+$$
$$\pi^- + p \to n + K^- + K^+ \qquad (32)$$

near the threshold of the reaction

$$K^- + K^+ \to \overline{K^0} + K^0 \qquad (33)$$

there may have anomalies, which are connected with $K - K$ interaction. Furthermore in the final state of (33) the Coulomb interaction is absent, which may smooth out the anomalies. (see /10/).

In the spectra of proton in reaction $p + p \to p + p + \pi^0$ near the threshold

$$\pi^0 + p \to n + \pi^+$$

and in the spectra of π^+ -meson in reaction

$$p + p \to n + p + \pi^+$$

near the threshold

$$n + p \to d + \pi^0$$

it is also possible to notice the energy anomalies*.

APPENDIX

A. Production of K-meson in n–p collisions

In n-p collisions there are two channels for the production of Λ -particle

$$n + p \to \Lambda + p + K^{o} \tag{A1}$$

$$n + p \to \Lambda + n + K^{+} \tag{A2}$$

Let us denote the reaction amplitudes in triplet and singlet isotopic spin state by T_1 and T_0 respectively. Reaction (A1) is then described by amplitude $\frac{1}{2}(T_1 + T_0)$, while reaction (A2) by $\frac{1}{2}(T_1 - T_0)$. The spin dependence of the isotriplet amplitude T_1 is given by (9) with $B_\Lambda = 0$, while at the same time

$$\langle \Lambda N K | T_0 | N N \rangle = B_\Lambda \left\{ (\vec{\sigma}_1 - \vec{\sigma}_2, \vec{K}) + i(\vec{K}[\vec{\sigma}_1 \cdot \vec{\sigma}_2]) \right\}. \tag{A3}$$

Under the same assumptions we can calculate the final state interaction for B_Λ

$$B_\Lambda = B_\Lambda^{o} \frac{e^{i\delta_3} \sin\delta_3}{P_1 a_3} \tag{A4}$$

where B_Λ^{o} is a function of the total energy E.

The cross section for the production and the polarization of the Λ particle have the form

$$d\sigma = (2\pi)^4 \frac{E}{2(E^2 - 4M_N^2)^{1/2}} \frac{1}{4} (4\pi)^2 (2m_\Lambda \mu)^{3/2} \left[T(T_{max} - T) \right]^{1/2} dT$$
$$\left[|A_\Lambda + C_\Lambda \pm B_\Lambda|^2 + |A_\Lambda - C_\Lambda \mp B_\Lambda|^2 + 2|C_\Lambda \mp B_\Lambda|^2 \right] \tag{A5}$$

and

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* The scattering length for low energy $\pi^{o} - p$ scattering is in general different from that obtained by isotopic invariant theory. This is due to the existence of energy anomalies near the threshold of $\pi^{o} + p \to n + \pi^{+}$ which destroy the isotopic spin invariance.
An estimation of this effect with the help of dispersion relation leads to 5% correction.

$$\vec{P}_\Lambda \left[|A_\Lambda + C_\Lambda \pm B_\Lambda|^2 + |A_\Lambda - C_\Lambda \mp B_\Lambda|^2 + 2\,|C_\Lambda \mp B_\Lambda|^2 \right]$$

$$= 2\left[|A_\Lambda + C_\Lambda \pm B_\Lambda|^2 - |C_\Lambda \mp B_\Lambda|^2 \right] (\vec{k}\vec{\mathcal{P}})\vec{k}$$

$$+ \left[|A_\Lambda - C_\Lambda \mp B_\Lambda|^2 - |A_\Lambda + C_\Lambda \pm B_\Lambda|^2 \right] \vec{\mathcal{P}} . \qquad (A6)$$

The plus sign before B_Λ holds for the reaction (A1), while minus sign for (A2). From (A5) and (A6) it is easy to find the relations

$$d\sigma(np \to \Lambda p K^0) = d\sigma(np \to \Lambda n K^+) \qquad (A7)$$

and

$$\vec{P}_\Lambda (np \to \Lambda p K^0) = \vec{P}_\Lambda (np \to \Lambda n K^+) \qquad (A8)$$

if $\vec{\mathcal{P}} \parallel \vec{k}$.

These relations are obtained under the assumption that only S-wave is important in final state. They may be used as an experimental test for the stated assumption.

B. Production of scalar K-meson in N–N collisions

In this case we have

$$\langle \Lambda N K | T_1 | N N \rangle = A_\Lambda \qquad (B1)$$

$$\langle \Lambda N K | T_0 | N N \rangle = B_\Lambda (\vec{\sigma}_1 \cdot \vec{\sigma}_2)$$

where

$$A_\Lambda = A_\Lambda^0 \frac{e^{i\delta_1} \sin\delta_1}{p_\Lambda a_1} , \quad B_\Lambda = B_\Lambda^0 \frac{e^{i\delta_3} \sin\delta_3}{p_\Lambda a_3} \qquad (B2)$$

If we introduce the quantity

$$f\,(NN \to \Lambda N K) = \frac{d\sigma(NN\to\Lambda NK)\;\; 8\,(E^2-4M_N^2)^{1/2}}{dT\,\left[T\,(T_{max}-T)\right]^{1/2}(2\pi)^4\,E\,(4\pi)^2\,(2m_\Lambda\kappa)^{3/2}}$$

then for the these reactions we have

$$f\,(pp \to \Lambda p K^+) = |A_\Lambda^0|^2\,\frac{\sin^2\delta_1}{(p_\Lambda a_1)^2} \tag{B3}$$

$$\vec{P}_\Lambda\,(pp \to \Lambda p K) = \vec{\xi} \tag{B4}$$

and

$$f\,(np \to \Lambda N K) = f\,(pp \to \Lambda p K^+) + 3\,|B_0|^2\,\frac{\sin^2\delta_3}{(p_\Lambda a_3)^2} \tag{B5}$$

$$\vec{P}_\Lambda\,(np \to \Lambda N K) = \frac{|A_\Lambda|^2 - |B_\Lambda|^2}{|A_\Lambda|^2 + 3\,|B_\Lambda|^2}\,\vec{\xi}\,. \tag{B6}$$

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Received by Publishing Department
on February 17, 1960.