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DISPERSION RELATION AND THE ANALYSIS OF THE ENERGY DEPENDENCE OF THE CROSS SECTION NEAR THE THRESHOLD FOR NEW REACTION

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ABSTRACT

Dispersion relations are applied to the analysis of the energy dependence of the scattering (and reaction) amplitude near the threshold for a new reaction.

A general expression which characterizes the nonmonotonic energy dependence for the forward scattering amplitude is obtained.

The energy dependence of one of the $\gamma - d$ *elastic scattering amplitude near the threshold for photodisintegration of deuteron is discussed.*

I

The study of the scattering of photon by nucleon near the threshold for the photoproduction of pion shows that dispersion relations automatically lead to the appearance of the infinite energy derivative at the threshold in the real amplitude if the energy dependence of the reaction is taken into account*.

In the framework of dispersion relation the appearance of infinite energy derivative in the forward scattering amplitude is related to the study of the integral

$$\frac{K_0^2}{4\pi^2} P \int \frac{d\omega}{K} \frac{\sigma(\omega)}{\omega \mp \omega_0} \tag{1}$$

where usual notations are adopted. The total cross section contains both the elastic scattering cross section $\sigma_s(\omega)$ and the inelastic one $\sigma_c(\omega)$.

The energy dependence of $\sigma_c(\omega)$ near the threshold for a binary reaction

$$a + b \rightarrow c + d \tag{2}$$

where the particles have masses μ (incident particle), M (target), m and M, is given by the expression

$$\sigma_c(\omega) = B \frac{q_c}{K_c} \tag{3}$$

where q_c and Kc are the relative momenta before and after the collision in the center-of-mass system respectively; B is a constant.

It is easy to show that

$$\left(\frac{q_c}{K_c}\right)^2 = \frac{(\omega-\omega_t)(\omega+\omega_t-\delta)}{\omega^2-\mu^2} \tag{4}$$

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* The nonmonotonic dependence near the threshold for pion production has been studied phenomenologically by G. Ustinova and in [2].

where $\omega = (K^2 + \mu^2)^{1/2}$ is the total energy of the incident particle in the laboratory system;

$$\omega_t - \mu = \frac{(M+m)^2 - (M+\mu)^2}{2M}$$

is the threshold energy for reaction (2); and

$$\delta = \frac{1}{M}\left\{ m^2 + m^2 - \mu^2 - M^2 \right\}.$$

Kinematic parameters $\omega_t - \mu$ and δ for a series of processes are given in Table 1.

TABLE 1

Process	$\omega_t - \mu$ in MeV	δ in MeV	$\delta/2\omega_t$ %
$\gamma N \to N\pi$	150	20,8	7
$\pi^0 P \to \pi^+ n$	6,1	3,88	1,33
$K^- P \to \bar{K}^0 n$	8,1	6,8	0,8
$K^+ n \to K^0 p$	4,1	1,6	0,16
$\gamma P \to \Lambda K$	907	642	35,5
$\pi P \to \Lambda K$	755	620	34,6
$p\bar{p} \to \bar{\Sigma}^0 \Sigma^0$	1180	1175	27,8

Use of dispersion relation opens the possibility of studying both the local effects near the threshold itself, which in some cases lead to sharp cusps, and the general influence of the inelastic process proceeding in a certain energy interval on the given process at a fixed energy.

II

A study of the γN scattering has shown that from the six scalar amplitudes, which describe the transition matrix in this case, only two of them have local effects. The presence of the other fast varying amplitudes makes the analysis difficult.

For a detailed analysis of the influence of the inelastic processes we must consider dispersion relations with momentum exchange $Q^2 \neq 0$ or, possibly, the double dispersion relation.

In the present paper we consider only the forward dispersion relations for scalar amplitude function A(w), which is equal to the trace of the scattering matrix

$$A(\omega) = Sp\, M(\omega, Q^2 = 0), \tag{7}$$

whose imaginary part is connected with the total cross section.

The contribution of the inelastic process to $D = Re\, A(\omega)$ is characterized by the following two integrals

$$\frac{K_0^2}{4\pi^2} P \int \frac{d\omega}{K} \frac{\sigma_c^-(\omega)}{\omega - \omega_0} \tag{8}$$

and

$$\frac{K_0^2}{4\pi^2} P \int \frac{d\omega}{K} \frac{\sigma_c^+(\omega)}{\omega + \omega_0} \tag{9}$$

where $\sigma_c^-(\omega)$ is the total cross section for reaction (2), while $\sigma_c^+(\omega)$ is that of the crossing reaction.

The energy dependence of the real part of $A(\omega)$ in (7) can be calculated

$$\frac{K_0^2}{4\pi^2} P \int_{\omega_t}^{\omega_1} \frac{d\omega}{K} \frac{\sigma_c^-}{\omega - \omega_0} = \frac{B^- K_0^2}{4\pi^2} P \int_{\omega_t}^{\omega_1} \frac{d\omega}{K^2} \frac{[(\omega - \omega_t)(\omega + \omega_t - \delta)]^{1/2}}{\omega - \omega_0} = \frac{B^- K_0^2}{4\pi^2} \Pi(\omega_0) \tag{10}$$

and

$$\frac{K_0^2}{4\pi^2} P \int_{\omega_t}^{\omega_1} \frac{d\omega}{K} \frac{\sigma_c^+}{\omega + \omega_0} = \frac{B^+ K_0^2}{4\pi^2} \Pi(-\omega_0) \tag{11}$$

where ω_1 is the boundary value of the S-state in σ_c in which formula (3) is still valid.

It is not difficult to obtain

$$-K_0^2 \Pi(\omega_0) = \psi(\omega_0) - \frac{1}{2}\left(1 + \frac{\omega_0}{\mu}\right)\psi(\mu) - \frac{1}{2}\left(1 - \frac{\omega_0}{\mu}\right)\psi(-\mu) \tag{12}$$

where

$$\psi(\omega_0) = \sqrt{-a(\omega_0)} \left[\frac{\pi}{2} + \mathrm{arctg} \frac{(2\omega_0 - \delta)(\omega_1 - \omega_0) + 2a(\omega_0)}{2\sqrt{-a(\omega_0) R}}\right]; \omega_0 \leq \omega_t \tag{13}$$

and

$$\psi(\omega_0) = \sqrt{a(\omega_0)}\ \ln\left|\frac{2a(\omega_0) + (2\omega_0 - \delta)(\omega_1 - \omega_0) + 2\sqrt{a(\omega_0)R}}{(\omega_1 - \omega_0)(2\omega_t - \delta)}\right| \qquad (13')$$

when $\omega_0 \geq \omega_t$

$$a(\omega_0) = (\omega_0 - \omega_t)(\omega_0 + \omega_t - \delta)$$
$$R = (\omega_1 - \omega_t)(\omega_1 + \omega_t - \delta). \qquad (14)$$

The discontinuity in the first energy derivative of $\mathcal{D}_-(\omega_0)$ is caused by $\psi(\omega_0)$, which is characterized by an infinite energy derivative from the side $\omega_0 < \omega_t$ and a finite value from the other side.

One may think that $K_0^2\,\Pi(-\omega_0)$ also contains discontinuity in the derivative at $\omega_0 = \omega_t - \delta$. But this is not correct. The derivative of $K_0^2\,\Pi(-\omega_0)$ is continuous at $\omega_0 = \omega_t - \delta$.

Therefore, the crossing inelastic processes do not lead to local cusps near the threshold, although they give a contribution to the real part of the scattering amplitudes.

With the use of the dispersion relation a detailed knowledge of the magnitude and the half-width of the threshold anomalies can be obtained.

The half-width ε in the side $\omega_0 < \omega_t$ can be estimated roughly in the following way.

Near ω_t $(\omega_0 < \omega_t)$, we have

$$\operatorname{arctg} x = \frac{\pi}{2} - \frac{1}{x} \qquad (15)$$

and

$$\psi = \sqrt{-a}\,\pi. \qquad (16)$$

Define the half-width $\varepsilon = \omega_t - \omega_0$ as that energy value, in which

$$\mathcal{D}(\varepsilon) = \frac{1}{2}\,\mathcal{D}(\omega_t). \qquad (17)$$

From (16) and (17) we obtain

$$\mathcal{D}(\omega_t) - \frac{B}{4\pi}\sqrt{(\omega_t - \omega_0)(\omega_0 + \omega_t - \delta)} = \frac{1}{2}\,\mathcal{D}(\omega_t) \qquad (18)$$

or

$$\mathcal{E} \cong \frac{1}{8(\omega_t - \delta/2)} \left[\frac{\mathcal{D}(\omega_t)}{B/4\pi} \right]^2 . \tag{19}$$

In the limiting case where those parts of $\mathcal{D}(\omega_t)$ which are not connected with (10) and (11) may be neglected, we have

$$\mathcal{D}(\omega_t) = \frac{B}{4\pi^2} J(\omega_t)$$

and

$$\frac{\mathcal{D}(\omega_t)}{B/4\pi} = \frac{1}{\pi} J(\omega_t) \tag{20}$$

where from (13)

$$J(\omega_t) = \frac{1}{2} \left[\psi(\mu)\left(1 + \frac{\omega_t}{\mu}\right) + \psi(-\mu)\left(1 - \frac{\omega_t}{\mu}\right) \right] . \tag{21}$$

III

Consider the photoproduction of a neutral pion

$$\gamma + p \rightarrow p + \pi^0 \tag{22}$$

near the threshold for the reaction

$$\gamma + p \rightarrow n + \pi^+ . \tag{23}$$

In this energy region it is sufficient to consider only the dipole transition. Let us denote the transition matrix element of (22) and (23) by E^0 and E^+, respectively.

From the unitarity condition and the experimental facts $Re\, E^0 \ll Re\, E^+$ we have

$$Im\, E^0 \cong \frac{\sqrt{2}}{3} (\alpha_3 - \alpha_1)\, Re\, E^+ \tag{24}$$

where α_3 and α_1 are the corresponding π-N scattering phase shifts. Substituting the experimental values of α_3, α_1 and E^+ into (24) we obtain

$$\mathrm{Im}\, E^0 = \frac{\sqrt{2}}{3}(a_3 - a_1)\, q_0^{1/2} q_+^{1/2} \sqrt{\frac{q_+}{V}}\; 3{,}3 \cdot 10^{-15}\, cm$$

$$= \frac{\sqrt{2}}{3} \varepsilon_1 (a_3 - a_1)\, q_0^{1/2} q_+ \, \nu^{-1/2} \qquad (25)$$

where

$$q_0^{1/2} \cong (\nu^2 - V_{0t}^2)^{1/4} \qquad , \; q_+ \cong (\nu^2 - V_{+t}^2)^{1/2} .$$

Anomalies near the threshold are determined by the integral

$$\frac{\nu_0^2}{4\pi^2} P \int \frac{(\nu^2 - V_{0t}^2)^{1/4} (\nu^2 - V_{+t}^2)^{1/2}}{\nu^{1/2} (\nu - \nu_0)} d\nu \qquad (26)$$

in which the cusp phenomena certainly occur.

In the general case consider the cross section for the reaction

$$a + b \rightarrow c + d \qquad (27)$$

near the threshold for the reaction

$$a + b \rightarrow e + f . \qquad (28)$$

If the threshold for (28) lies far from that for (27), it is always possible to find an energy region in which

$$\mathrm{Im}\, M_{ab \to cd} = M_{ab \to ef}\, M^+_{ef \to cd} + \cdots = Aq + \cdots$$

where A is a slowly varying function of energy; q is the relative momentum of the e-f system. Other terms in $\mathrm{Im}\, M_{ab \to cd}$ also are slowly varying functions of energy, if there are no other thresholds in the near neighbourhood.

In this case dispersion integral has an ordinary form and we may determine the magnitude and the half-width of the cusp in the same way as the scattering.

For those processes as the photoproduction of pion

$$\mathrm{Im}\, M_{ab \to cd} = A q q_0^{1/2} + \cdots$$

i.e. the threshold for the reaction $ab \to ef$ is near that for $ab \to cd$.

In this case the dispersion integral is quite complicated and we are not able to carry out the integration.

IV

Therefore, causality, unitarity together with (3) and (4) lead to the discontinuity in energy derivative of the real part of the scattering amplitude with infinite derivative from the side $\omega_0 < \omega_t$.

The appearance of the infinity just in the first energy derivative is connected with the form of the energy dependence of (3) and (4). The energy dependence of the cross section for reaction (2), when its product particles have angular momentum equal to ℓ is given by the expression

$$\sigma_\ell^c = B_\ell \frac{\left[(\omega-\omega_t)(\omega+\omega_t-\delta)\right]^{\ell+1/2}}{(\omega^2-\mu^2)^{1/2}} . \qquad (31)$$

Substituting (31) into (8) leads to the appearance of infinity in the ℓ^{th} derivative of the scattering amplitude.

It is of interest to note, that the assumption about the analiticity of partial waves is unnecessary in the dispersion relation approach, which is inherent for the usual approach. Analiticity of the amplitude with the limited momentum exchange $Q^2 < Q^2_{max}$ is sufficient for this purpose.

As has been pointed out by Baz' based on the unitarity of S -matrix the effect in each channel decreases with the increase of the number of the channels. An analysis of $\gamma - N$ scattering near the threshold for photoproduction, when only two of the six amplitudes show the local cusp phenomena, has shown that the effect is also masked with the increase of the spin of the particle.

In the theory of dispersion relation it is important to guarantee the convergence of the dispersion integral by making enough numbers of subtraction. The most of our results are based upon the dispersion relations with a single subtraction. In the case of dispersion relation without subtraction

$$K_0^2 \int \frac{d\omega\, \sigma}{K(\omega-\omega_0)} \rightarrow \int \frac{K\sigma\, d\omega}{\omega-\omega_0} . \qquad (32)$$

With sufficiently high accurrate data the difference between (8) and (32) may serve as information about the number of subtractions required.

We shall note briefly what characteristics one may expect near the threshold for the reaction

$$a + b \rightarrow c + d + f . \qquad (33)$$

Substituting into (8) the cross section of (33)

$$\sigma^c = B' \frac{P^4_{c\,max}}{K_c} \cong \Gamma(\omega-\omega_t)^2 ,$$

where the reaction is assumed to proceed in the S-wave state, we obtain

$$\int \frac{\sigma\, d\omega}{\omega-\omega_0} \cong \Gamma \int \frac{d\omega\,(\omega-\omega_t)^2}{\omega-\omega_0} = \Gamma \Big\{ \frac{1}{2}\Big[(\omega_1-\omega_0)^2 - (\omega_t-\omega_0)^2\Big] +$$

$$+ 2(\omega_0-\omega_t)(\omega_1-\omega_t) + (\omega_0-\omega_t)^2 \ln\left|\frac{\omega_1-\omega_0}{\omega_t-\omega_0}\right| \Big\}$$

which has logarithmic infinity in the second energy derivative of $\mathcal{D}(\omega)$.

For reaction with four particles in the final S-wave state $(\omega_0-\omega_t)^2 \ln|\omega_t-\omega_0|$ is replaced by $(W_0 - W_t)^5 \ln|W_0 - W_t|$. Similar energy dependence appears in the real part of the scattering amplitude near the threshold for all reactions.

A well known example in literature with the application of dispersion relations is the analysis of the coherent scattering of photon in the Coulomb field of a nucleus /3,4/.

It is easy to show that the real part of the scattering amplitude near the threshold for the electron-positron pair creation $\omega_t = 2m$ $\left(\gamma \equiv \frac{\omega}{2m} = 1\right)$ contains terms of the type $x^k \ln x$ $(x = \gamma - 1)$. In order to see it, it is sufficient to consider the real part of the amplitude

$$\mathcal{D}(\omega) = \frac{Z^2}{m}\left(\frac{e^2}{4\pi}\right)^3 \Big\{ \frac{1}{\gamma\pi}\left[2C_1(\gamma) - \mathcal{D}_1(\gamma)\right] +$$

$$+ \frac{\gamma}{27\pi}\left[\left(109 + \frac{64}{\gamma^2}\right)E_1(\gamma) - \left(67 - \frac{6}{\gamma^2}\right)\left(1 - \frac{1}{\gamma^2}\right)F_1(\gamma)\right] -$$

$$- \frac{1}{9\gamma^2} - \frac{9}{4} \Big\} \qquad (35)$$

where

$$C_1(\gamma) = \mathrm{Re}\int_0^\gamma \frac{\arcsin x}{x}\, \mathrm{arch}\left(\frac{\gamma}{x}\right) dx \qquad C_1(1) = 1.62876$$

$$\mathcal{D}_1(\gamma) = \mathrm{Re}\int_0^\gamma \frac{\mathrm{arch}\left(\frac{\gamma}{x}\right)}{(1-x^2)^{1/2}}\, dx \qquad \mathcal{D}_1(1) = 1.83193$$

$$E_1(\gamma) = \begin{cases} E(1/\gamma) & \gamma \geq 1 \\ \frac{1}{\gamma}E(\gamma) + \left(\gamma - \frac{1}{\gamma}\right)K(\gamma) & \gamma \leq 1 \end{cases}$$

$$F_1(\gamma) = \begin{cases} K(1/\gamma) & \gamma \geq 1 \\ \gamma K(\gamma) & \gamma \leq 1 \end{cases}$$

$K(\gamma)$ and $E(\gamma)$ are elliptical integrals of the first and second kind, respectively. As is well known, at $1-\gamma^2 \ll 1$

$$K(\gamma) = \Lambda + \frac{\Lambda-1}{4}(1-\gamma^2) + \frac{9}{64}\left(\Lambda-\frac{7}{6}\right)(1-\gamma^2)^2 + \frac{25}{256}\left(\Lambda-\frac{37}{30}\right)(1-\gamma^2)^3 + \cdots$$

$$E(\gamma) = 1 + \frac{1}{2}\left(\Lambda-\frac{1}{2}\right)(1-\gamma^2) + \frac{3}{64}\left(\Lambda-\frac{13}{12}\right)(1-\gamma^2)^2 + \cdots$$

$$\Lambda = \ln\frac{4}{\sqrt{1-\gamma^2}} \qquad (36)$$

The dependence $x^k \ln x$ is snown to exist. It is not difficult to believe that the scattering of light by light near the threshold for $\gamma+\gamma \to e^- + e^+$ is a well known example in quantum electrodynamic processes, whose amplitudes are characterized by local anomalies (See Fig. 2–4 in /5/)*, while the amplitude for Compton effect contains terms of the type $x^2 \ln x$ near the threshold for the reaction $\gamma + e \to e + e^- + e^+$.

V

Elastic scattering of γ quanta by deuteron near the threshold for photodisintegration is one of the examples of processes, where the use of dispersion relations in the analysis of the threshold anomalies is necessary.

The nonmonotonic energy dependence near the threshold is caused by the magnetic dipole transition. The electric dipole disintegration leads to a marked change in the energy dependence of the γ-d scattering amplitudes in a relatively large energy region.

The amplitude for the forward γ-d scattering can be expressed as

$$\vec{e}' M \vec{e} = A(\vec{e}'\vec{e}) + iB(\vec{S}[\vec{e}\vec{e}']) + \\ + \frac{1}{2}C\left[(\vec{S}\vec{e})(\vec{S}\vec{e}') + (\vec{S}\vec{e}')(\vec{S}\vec{e})\right] + \\ + \frac{1}{2}\mathcal{D}\left[(\vec{S}[\vec{k}\vec{e}])(\vec{S}[\vec{k}\vec{e}']) + (\vec{S}[\vec{k}\vec{e}'])(\vec{S}[\vec{k}\vec{e}])\right] \qquad (37)$$

The cross section for unpolarized γ- quanta and deuteron has the form

$$\sigma_s(0^\circ) = \left|A + \frac{2}{3}C + \frac{2}{3}\mathcal{D}\right|^2 + \frac{1}{18}|C+\mathcal{D}|^2 + \frac{2}{3}|B|^2 + \frac{1}{3}|\mathcal{D}-C|^2. \qquad (38)$$

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* Coulomb interaction has been taken into consideration in /6,7/.

Furthermore

$$K\sigma_t = 4\pi\, \mathrm{Im}\left[A + \tfrac{2}{3}\mathcal{B} + \tfrac{2}{3}C\right].$$

With the help of the dispersion relation for the quantity $L = A + \frac{2}{3}C + \frac{2}{3}\mathcal{D}$

$$\mathrm{Re}\, L(\omega) = -\frac{e^2}{M_d} + \frac{2\omega^2}{\pi} P\int_{\omega_d}^{\infty} \frac{\mathrm{Im}\, L(\omega')}{\omega'(\omega'^2-\omega^2)}\, d\omega' \qquad (39)$$

where ω_d is the threshold for the photodisintegration of deuteron, we shall consider the influence of the inelastic process on the energy dependence of the real part of L.

In calculation of the dispersion integral it is convenient to use the theoretical expression for the cross section of photodisintegration of deuteron, (for example see /8/).

Let us begin with the consideration of the local effects. Expression for the cross section for magnetic dipole disintegration has the form

$$\sigma_c^{(m)} = \frac{2\pi}{3}\frac{e^2}{\hbar c}\left(\frac{\hbar}{Mc}\right)^2(\mu_p-\mu_n)^2\, \frac{(\gamma-1)^{1/2}\left(1+\sqrt{\frac{\varepsilon'}{|\varepsilon|}}\right)^2}{\gamma\left(\gamma-1+\varepsilon'/|\varepsilon|\right)} \qquad (40)$$

where $\gamma = \frac{\nu}{|\varepsilon|}$; ν is the energy of photon; $|\varepsilon| = 2.22$ MeV and $\varepsilon' \cong 70$ KeV are the binding energy of n – p system in $^3S_1$ and $'S_0$ states. Other notations are the usually adopted ones. The presence of the factor $[\gamma-1+\varepsilon'/|\varepsilon|]^{-1}$ makes the direct analytic continuation $K = \sqrt{\gamma-1} \rightarrow i|K|$ impossible. In this case analytical continuation $\gamma\,\sigma_c^{(m)}$ turns out to infinity below the threshold for the photodisintegration at $|K|^2 = \frac{\varepsilon'}{|\varepsilon|}$.

Substituting (40) into dispersion integral

$$Z_L = \varepsilon\, \frac{\nu_0^2}{2\pi^2} P\int_1^{\infty} \frac{d\nu\, \sigma_c^{(m)}(\gamma)}{\nu^2-\nu_0^2}$$

it gives at $\nu_0 \neq \delta'$ $\quad (\delta' = 1 - \varepsilon'/|\varepsilon|)$

$$Z_L(\gamma_0) = \frac{2}{3}\frac{e^2}{Mc^2}\frac{\varepsilon}{2Mc^2}(\mu_p-\mu_n)^2\left(1+\sqrt{\frac{\varepsilon'}{|\varepsilon|}}\right)^2 \times$$

$$\left\{\frac{\sqrt{1-\gamma_0}}{\delta'-\gamma_0}\,\theta(1-\gamma_0) + \frac{\sqrt{1+\gamma_0}}{\delta'+\gamma_0} - \frac{2}{\delta'} - \frac{2\gamma_0^2\sqrt{\varepsilon'/|\varepsilon|}}{\delta'(\delta'^2-\gamma_0^2)}\right\} \qquad (41)$$

where

$$\theta(x) = \begin{cases} 1 & \text{at} \quad x \geq 0 \\ 0 & \text{at} \quad x < 0 \end{cases}$$

and

$$Z_L(\delta') = \frac{2}{3}\frac{e^2}{2Mc^2}\frac{\mathcal{E}}{Mc^2}(\mu_p-\mu_n)^2\left(1+\sqrt{\frac{\mathcal{E}'}{|\mathcal{E}|}}\right)^2 \times$$
$$\left\{\frac{(1+\delta')^{1/2}}{2\delta'} - \frac{2}{\delta'} + \frac{2}{3\delta'}\sqrt{\frac{\mathcal{E}'}{|\mathcal{E}|}} + \frac{1}{2}\sqrt{\frac{|\mathcal{E}|}{\mathcal{E}'}}\right\} \tag{41'}$$

at $\gamma_0 = \delta'$

The energy dependence of $\Delta_L(\gamma_0) = Z_L(\gamma_0)\Big/\frac{e^2}{2Mc^2}$ is shown in Fig. (curve I).

For limiting value of γ_0 we obtain from (41)

$$\Delta_L(\gamma_0) = \frac{2}{3}\frac{\mathcal{E}}{Mc^2}(\mu_p-\mu_n)^2\left(1+\sqrt{\frac{\mathcal{E}'}{|\mathcal{E}|}}\right)^2\left\{\frac{3}{8}+2\frac{\mathcal{E}'}{|\mathcal{E}|}-\sqrt{\frac{\mathcal{E}'}{|\mathcal{E}|}}\left(1+2\frac{\mathcal{E}'}{|\mathcal{E}|}\right)\right\}\gamma_0^2 =$$
$$\cong \frac{2}{3}\frac{\mathcal{E}}{Mc^2}(\mu_p-\mu_n)^2\left(1+\sqrt{\frac{\mathcal{E}'}{|\mathcal{E}|}}\right)^2\frac{1}{4}\gamma_0^2 \tag{42}$$

for $\gamma_0 \ll 1$ and

$$\Delta_L(\gamma_0) = -\frac{4}{3}\frac{\mathcal{E}}{Mc^2}(\mu_p-\mu_n)^2\left(1+\sqrt{\frac{\mathcal{E}'}{|\mathcal{E}|}}\right)^2 \tag{43}$$

for $\gamma_0 \gg 1$

At the threshold for the photodisintegration with $1-\delta' = 1/30$

$$Z_L(1) = 0,24\,\frac{e^2}{2Mc^2}.$$

The half-width of the cusp from the side below the threshold is smaller than $\mathcal{E}'$ i.e. about 50–60 KeV.

The contribution of electric dipole disintegration to $D = \mathrm{Re}\,L$ with

$$\sigma_c^{(d)} = 4\pi\frac{e^2}{Mc^2}\frac{\hbar c}{\mathcal{E}}\frac{(\gamma-1)^{3/2}}{\gamma^3} \tag{44}$$

is of the form

$$Z_p(\gamma_0) = \Delta_p(\gamma_0)\frac{e^2}{2Mc^2} =$$
$$= 2\frac{e^2}{M_d c^2}\left\{\frac{1}{\gamma_0^2}\left[(1-\gamma_0)^{3/2}\theta(1-\gamma_0)+(1+\gamma_0)^{3/2}-2\right]-\frac{3}{4}\right\}. \tag{45}$$

Energy dependence of $\Delta_p(\gamma_0)$ is given in the Fig. (Curve II).

It is easily seen that

$$\Delta_p(\gamma_0) = \frac{2}{32}\gamma_0^2 \quad \text{at} \quad \gamma_0 \ll 1 \tag{46}$$

and

$$\Delta_p(\gamma_0) = -\frac{3}{2} \qquad at \qquad \gamma_0 \gg 1 \tag{47}$$

At the threshold for the photodisintegration

$$\Delta_p(1) \simeq 0.156 .$$

The sum of the electric and magnetic dipole contribution is given in Fig. (curve III). At the threshold the scattering amplitude has changed about 40% when the effect of photodisintegration is taken into consideration.

From (42) and (46) we can see the contribution of the photodisintegration to the polarizability of deuteron.

As the total cross section for the photodisintegration of deuteron is greater than the sum of (40) and (44) in higher energy region, the estimation obtained may be regarded as the lowest limit, although the contribution from higher energy region is expected to be small.

The present analysis of one of the forward γ-d scattering amplitudes may serve as an indication that the inelastic processes, and especially the photodisintegration of deuteron, have a great influence on the elastic γ-d scattering in a wide energy region.

Similar effects should also take place for the scattering of γ -quanta by heavy nuclei.

It has been shown in /9/ that the cross sections for the elastic scattering of γ-quanta with a series of nuclear elements are characterized by noticible picks with an energy width of about 2 MeV, near the threshold of (γn) reaction, which are probably connected with the threshold anomalies.

In order to have a more reliable analysis of this phenomena, it is desirable to increase the accuracy of the experimental data on the energy dependence of the (γn) cross section and that for the elastic scattering of γ -quanta near the threshold of (γn) reaction.

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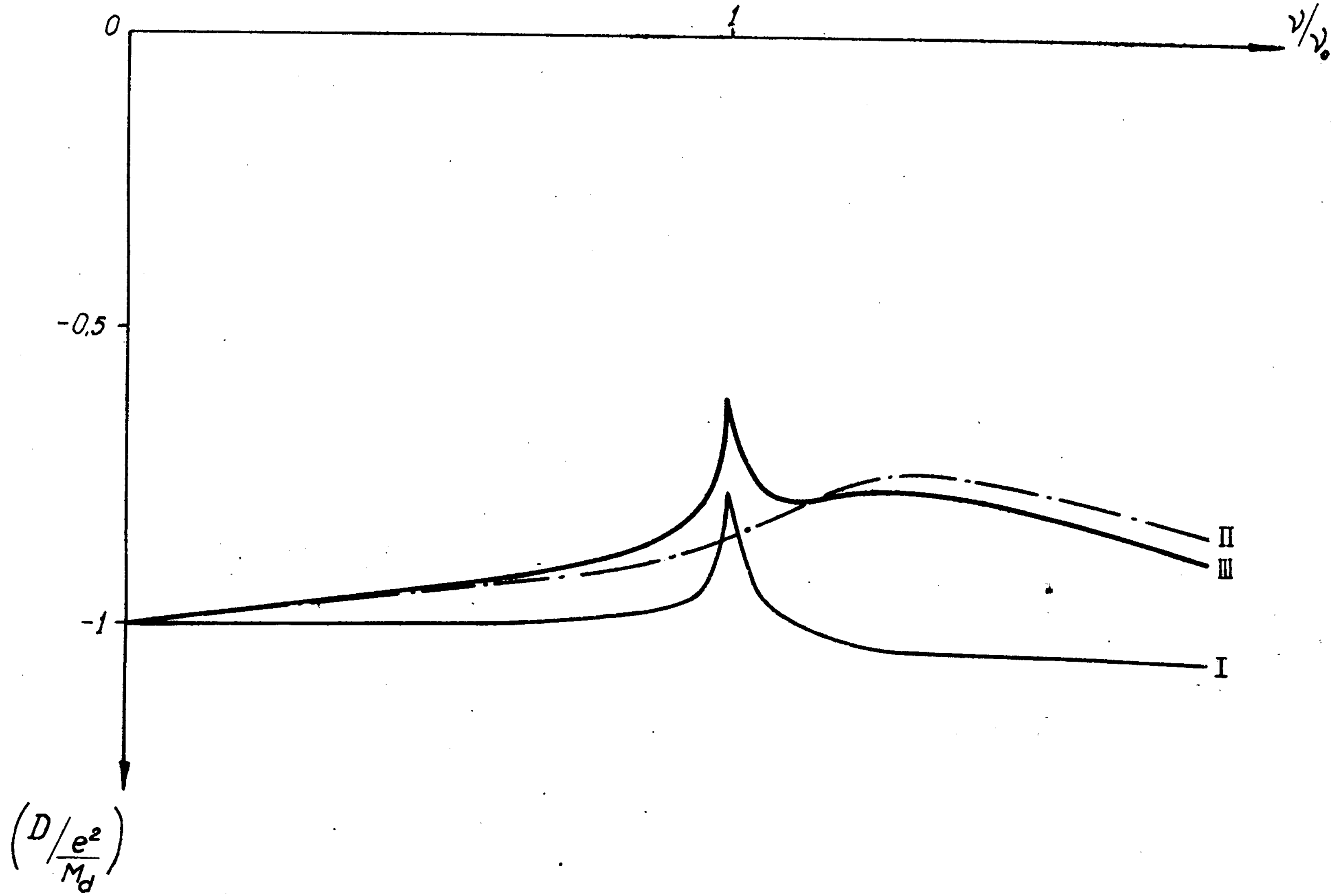
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1
ν/ν₀
-0,5
-1
II
III
I
(D/(e²/M_d))