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FOUR-LEPTON DECAYS OF CHARGED PIONS AND KAONS AND POSSIBLE INTERACTIONS OF LEPTONS

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I. Introduction

In this note we discuss the possibility of getting information on the limits of quantum electrodynamics validity from experimental investigations of rare decay modes of kaons and pions.

Four-lepton decays

$$\pi \to \mu \nu e^+ e^- \quad (1) \qquad \pi \to e \nu e^+ e^- \quad (2)$$

$$K \to \mu \nu e^+ e^- \quad (3) \qquad K \to \mu \nu \mu^+ \mu^- \quad (4)$$

$$K \to e \nu e^+ e^- \quad (5) \qquad K \to e \nu \mu^+ \mu^- \quad (6),$$

as it is well known, must take place through virtual electromagnetic interactions (for example/1/, $\pi \to \mu \nu \gamma \to \mu \nu e^+ e^-$).

Below we discuss the possible contributions to these decays of hypothetical interactions:

$$F_{ee} (\bar{e} e)(\bar{e} e) \tag{7}$$

$$F_{e\mu} (\bar{e} e)(\bar{\mu} \mu) \tag{8}$$

$$F_{\mu\mu} (\bar{\mu} \mu)(\bar{\mu} \mu) \tag{9}$$

II. What is Known about Anomalous Interactions of Leptons ?

Upper limits for the values of F 's can be obtained from experiments with colliding $e^- e^-$ and $e^+ e^-$ beams, from $g_e - 2$ and $g_\mu - 2$ measurements, from electron-proton and muon-proton scattering data, from 'trident' study and from other experiments.

In $e^- e^-$ scattering interaction (7) effectively results in changes of the photon propagator. If interaction (7) is a vector one, then

$$\frac{4\pi\alpha}{q^2} \to \frac{4\pi\alpha}{q^2} + F_{ee} . \tag{10}$$

Experiments on electron-electron colliding beams (both with the energy of 300 MeV) showed that the proton form factor/2/ can be written as:

$$(1 - q^2/\Lambda^2)^{-1}, \quad \text{where} \quad |\Lambda| \geq 0.55 \text{ GeV/c (for } \Lambda^2 > 0)$$

and $|\Lambda| \geq 0.98$ GeV/c (for $\Lambda^2 < 0$). From this it is possible to conclude that

$$F_{ee} \approx \frac{4\pi\alpha}{\Lambda^2} \leq (1.6\,\text{GeV/c})^{-2}. \tag{11}$$

We emphasize that the estimate (11) holds for a vector interaction, and generally speaking, will change for a different form of the interaction: S, P, etc.

There are no data for colliding e^+e^- beams.

From experiments on g_e-2 we can also get limits on the effective value F_{ee}. If the interaction is mediated by a vector boson, we have (see later (14))

$$\frac{\delta\mu_e}{\mu_e} \approx \frac{F\, m_e^2}{12\,\pi^2}\,, \tag{12}$$

where μ_e is the magnetic moment of the electron and m_e is the electron mass. This gives together with the experimental value $\frac{\delta\mu_e}{\mu_e} < 3.10^{-8}$,

$$F < (\,0.25\ \mathrm{GeV/c})^{-2}\,. \tag{13}$$

As far as 'tridents' are concerned, for the time being there are no available quantitative data.

Thus, the most stringent limit on F_{ee} comes about from colliding electron experiments (see relation (11)).

As to the $F_{e\mu}$ and $F_{\mu\mu}$ values, the best limits are obtained from the g_μ-2 experiment/3/. If the anomalous interaction in question is due to the exchange of a hypothetical meson χ^0 /4,5/, (see Fig. 1)

$$\frac{\delta\mu_\mu}{\mu_\mu} = \frac{F\, m_\mu^2}{12\,\pi^2}\,. \tag{14}$$

According to ref./3/ $\delta_{\mu_\mu} / \mu_\mu < 5.10^{-6}$, so that

$$F_{\mu\mu}\ ,\ F_{e\mu} < (\,2.5\ \mathrm{GeV/c}\,)^{-2}. \tag{15}$$

About such order of magnitude is obtained also in the case when (8) and (9) are truely four-fermion interactions. In a way analogous to that used in the calculation/6/ of the $\mu \to e\gamma$ rate, it can be shown that according to the diagram of Fig. 2

$$\frac{\delta\mu_\mu}{\mu_\mu} \approx \frac{F^2\Lambda^2 m_\mu^2}{(2\,\pi\,)^4}\,\ell n\,\frac{\Lambda}{m_\mu}\,, \tag{16}$$

where Λ is the cut-off energy.

Assuming a self cut-off for the interaction, we can put $\frac{F\Lambda^2}{2\pi^2} \approx 1$ and then obtain

$$\frac{\delta\mu_\mu}{\mu_\mu} \approx \frac{F\, m_\mu^2}{(2\pi\,)^2}\,\ell n\,\frac{\Lambda}{m_\mu}\,. \tag{17}$$

Since the cut-off is taken into account very roughly, one should not attach importance to the difference in the numerical factors of expressions (14) (17) and even to the logarithm in (17). Thus, it is reasonable to expect that the estimate (15) is stable.

3. Probability of Decays (1)-(6)

Let us see now what information on interactions (7)-(9) might give decays (1)-(6) which are of the order FG, where G is the weak interaction constant ($G = 10^{-5} / m_p^2$).

For an interaction (7) of the vector form the probability of the decay $\pi \to e\,e\,e\,\nu$ (Fig. 3) is:

$$W_{\pi \to ee e\nu} = \frac{G^2 f^2 F^2 m_\pi^7}{2^{13} \cdot 15 \cdot \pi^5}, \qquad (18)$$

where f is the $\pi \to e\nu$ decay amplitude through which the decay rate is expressed as follows:

$$W_{\pi \to e\nu} = \frac{G^2 f^2 m_\pi}{8\pi^2} m_e^2 . \qquad (19)$$

Values close to (8) are obtained also for different forms of interaction (7).

Similar expressions relate the probabilities of the $K \to eee\nu$ and $K \to e\nu$ decays. Emission of muons instead of electrons results in decrease of the phase space volume. In Table I there are presented rough values of the branching ratios R .

Table I

No.	Processes	R_{st}	R_{lim}/R_{st} (%)	$\sqrt{q^2_{max}}$
1.	$\pi \to \mu\,\nu\,e^+e^-$	10^{-11}	2.5	$m_\pi - m_\mu$
2.	$\pi \to e\nu\,e^+e^-$	10^{-9}	17	m_π
3.	$K \to \mu\,\nu\,e^+e^-$	10^{-6} - 10^{-7}	2.5	$m_K - m_\mu$
4.	$K \to \mu\,\nu\,\mu^+\mu^-$	10^{-8} - 10^{-9}	2.5	$m_K - m_\mu$
5.	$K \to e\,\nu\,e^+\,e^-$	10^{-6} - 10^{-7}	17	m_K
6.	$K \to e\,\nu\,\mu^+\mu^-$	10^{-7}	2.5	m_K

In the Table R_{st} is calculated for a standard value of $F = (1\ \mathrm{GeV/c})^{-2}$ according to expression (18) and subsequent remarks; R_{lim} corresponds to the extreme value of the branching ratio, calculated under the assumption that $F_{ee} = (1.6\ \mathrm{GeV/c})^{-2}$, $F_{e\mu}$, $F_{\mu\mu} = (2.5\ \mathrm{GeV/c})^{-2}$; $\sqrt{q^2_{max}}$ is the maximum value of the lepton pair mass.

The small values of R_{lim} presented in the Table show that in order to get by investigations of decays (1)-(6) checks on quantum electrodynamics with the accuracy already obtained in classical experiments , it is necessary to perform extremely difficult experiments. In deciding the feasibility of such experiments, however, one sould take into account that limits on $F_{e\mu}$ and $F_{\mu\mu}$ obtained on the basis of the g_μ -2 experiments may be affected by considerable theoretical uncertainties connected with integration over virtual particle momenta.

As far as decays related to F_{ee} are concerned, one should notice that they give information not identical to that obtained, let us say, by experiments with colliding electron beams because one of the electrons in the diagram of Fig.3 is virtual, whereas in Mooller scattering all four electrons are real.

Consequently it is reasonable to search for the processes under discussion even at lower levels of accuracy than R_{lim}. Anyhow it is of great interest to detect pure electromagnetic processes, to the discussion of which we are presently coming.

IV. Electromagnetic Background

As it was already noticed, decays (1)-(6) can go through the ordinary electromagnetic interaction. In order to 'see' the anomalous interactions over the electromagnetic background, it is necessary to select in the experiments such decays in which q^2, the four-momentum squared of the lepton pairs is close to its maximum value q^2_{max} (see Table I).

As far as the electromagnetic background is concerned, there is an important difference between decays (1), (3), (4), on one hand, and decays (2), (5), (7), on the other hand.

In decays (1), (3), (4) (the pair is here produced by the virtual muon) the anomalous interaction would appear on the electromagnetic background, when $F \approx \frac{4\pi\alpha}{q^2_{max}} \approx \frac{1}{10\, q^2_{max}}$.

Consequently, decay (1) is of no practical interest unless one investigates it with very great accuracy at $q^2 \sim q^2_{max}$. The decays (3) and (4) could, in principle, give information about the values $F \geq 0.1(m_K - m_\mu)^{-2} \approx$ $\approx (1.2\ \mathrm{GeV}/c)^{-2}$ if they were investigated for q^2 close to q^2_{max} with an accuracy of the order of one.

A considerably smaller background of ordinary internal conversion electromagnetic pairs would be expected in decays (2), (5), (6). This is due to the fact that the bremsstrahlung radiation of virtual photons in these decays has small matrix elements. As a matter of fact, bremsstrahlung virtual photons cannot change the 'forced' spirality which characterizes electrons in the $\pi \to e\nu$ and $K \to e\nu$ decays. The amplitude of the 'bremsstrahlung' decays is then proportional to m_e. As far as the e^+e^- or $\mu^+\mu^-$ pair emission contributed by the anomalous interactions (7)-(8) is concerned, it so happens on the contrary that the electron spirality may be 'normal' and the factor m_e / m_π does not appear (this is true if these interactions do not amount only to a modification of the photon propagator). Consequently in decay (2) where the photon emission from virtual adrons is small, it may be expected that the electromagnetic background from wide pairs will be negligible at $q^2 \approx q^2_{max} \approx m^2_\pi$ if $F \geq \frac{4\pi\alpha}{m^2_\pi} \frac{m_e}{m_\pi} \approx (8\ \mathrm{GeV})^{-2}$. In the case of decays (5) and (6) where virtual photon emission by virtual adrons cannot be neglected the estimate $F \geq \frac{4\pi\alpha}{m^2_K} \frac{m_e}{m_K}$ is too stringent; it is reasonable to expect that information may be obtained for, let us say, $F \geq 0.1 \frac{4\pi\alpha}{m^2_K} \approx (5\ \mathrm{GeV})^{-2}$.

V. Final Remarks

Above we have considered interactions (7)-(9) which conserve the muon number. If there exists a non-weak interaction, changing the muon charge by two[7,8], of the form

$$F'_{e\mu} \; (\bar{\mu}\, e)(\bar{\mu}\, e) \tag{20}$$

there must occur decays of the type

$$K^+ \to e^+ e^+ \mu^- \nu \quad , \tag{21}$$

$$K^+ \to \mu^+ \mu^+ e^- \nu \quad , \tag{22}$$

$$\pi^+ \to e^+ e^+ \mu^- \nu \quad . \tag{23}$$

In Fig. 4 there is presented the diagram describing decay (22).

In conclusion we wish to make some considerations of experimental nature.

As it seems, search for process (2) could be made with the help of spark chamber technique in a stopping $\pi^{\pm}$ -beam.

As far as decays (3)-(6) and (21),(22) are concerned, some upper limits for their branching ratios can be already determined on the basis of an analysis of existing bubble chamber pictures of K^+ decays.

Search for the decay $K \to \mu\mu\mu\nu$ could be attempted in an experiment reminding high energy neutrino experiments: a K^+ beam decays in flight, and three muons passing through thick absorbers separated by spark chambers are detected in a magnetic spectrometer. The difficulties of such an experiment are not due to the small rate of the processes but rather to the background

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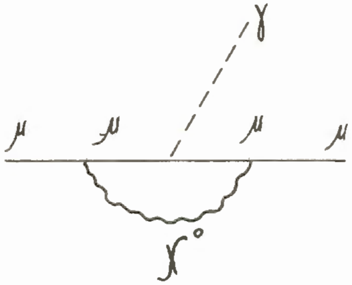

Fig. 1.

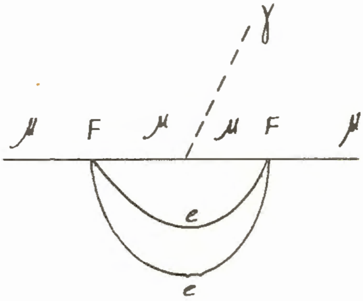

Fig .2.

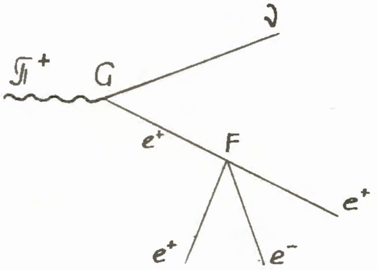

Fig 3.

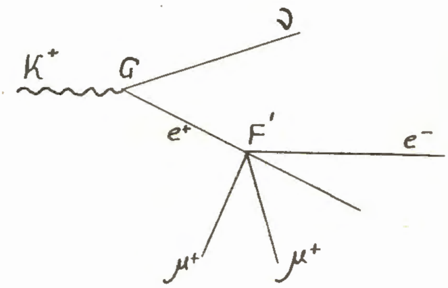

Fig.4 .