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Д.А. Киржниц, Г.И. Колеров

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SPACE, TIME, CAUSALITY IN MICROWORLD

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О г л а в л е н и е

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Предисловие

В этом сборнике помещены статьи, посвященные проблеме пространства, времени и причинности в микромире. Их объединяет идея о том, что геометрические соотношения в малых масштабах пространства - времени могут оказаться принципиально отличными от тех, которые известны в макромире.

Эти работы выполнены в последнее время в Объединенном институте ядерных исследований в Дубне и в Физическом институте АН СССР им. П.Н. Лебедева в Москве.

Для удобства читателя в дополнении приведены некоторые более ранние работы, которые существенны для понимания идей, изложенных в первых статьях этого сборника.

D.I.Blokhintsev

ON CAUSALITY IN MODERN FIELD THEORY

'Causality, as it is usually meant, is only a small part of the world relationship (materialistic addition), a part of not subjective but objectively real relationship'.

V.I.Lenin ('Philosophical Notes')

Introduction

In spite of the modest estimate which V.I.Lenin gave to the causality principle, it is, nevertheless, of fundamental importance in science as the simplest form of relationship between phenomena.

Of special significance is the causality principle for physics not only from the general philosophical point of view, but also as far as the mathematical form for expressing causality is concerned.

In contemporary physics the mathematical form of causality is based on two physical ideas: a) on the idea of the homogeneous and isotropic space-time of Einstein-Minkowski and b) on the idea that the interactions are transferred by the physical fields (electromagnetic fields, meson and neutron fields and the like).

At the same time it is well-known that the application of these principles to extremely small distances and short time intervals leads to the conclusions meaningless from the physical standpoint: the energy of particle interaction at small distances and the proper energy of particles turn out to be infinitely high.

This unsatisfactory result takes place both in quantum and classical physics, and is likely to indicate the same origin of the difficulties* encounted in these two concepts.

2 Causality in Classical Physics.

In classical physics the propagation of a weak (linear) signal from the world point $\mathcal{P}_1(x_1, t_1)$ to the world point $\mathcal{P}_2(x_2, t_2)$ is determined by the Green function $\mathcal{G}$, which is a function of the difference between the coordinates of the points $\mathcal{P}_1$ and $\mathcal{P}_2$: $x = x_1 - x_2$, $t = t_1 - t_2$.

This is the result of the homogeneity of space-time. The requirement that the space-time should be isotropic leads to the fact that the Green function must depend not simply upon the differences x and t, but on the four-dimensional interval $s^2 = x^2 - t^2$. Finally, it appears possible to introduce the time direction ϵ and the direction along the spatial ray η: $\epsilon = t/|t| = \pm 1$, $\eta = 0$ for $s^2 < 0$ and $\eta = x/|x| = \pm 1$, $\epsilon = 0$ for $s^2 > 0$. So, the Green function can be put as

$$\mathcal{G} = \mathcal{G}(s^2, \epsilon, \eta). \qquad /1/$$

This function is an invariant of the Lorentz transformation. Now the requirements of causality are additionally imposed;

a) the signal cannot propagate with a velocity greater than that of light C.

b) the signal is propagated only from the past to the future. These requirements lead to a further specification of the function $\mathcal{G}$:

---

* Some philosophic problems of causality and field theory were treated in /1/ .

$$\mathcal{G} = \mathcal{G}(s^2, +1, 0) \quad \text{for} \quad \epsilon = +1, \ \eta = 0$$

$$\mathcal{G} = 0 \quad \text{otherwise} . \qquad /2/$$

Fig. 1 shows the space-time domain, where the function $\mathcal{G}$ is different from zero. Note that the Fourier component of $\mathcal{G}(s^2, +1,0)$, denoted by $\mathcal{G}(\omega, k)$, depends only upon the invariant $m^2 = \omega^2 - k^2$: $\mathcal{G}(\omega, k) = F(\omega^2 - k^2)$ and different from zero only if $m^2 > 0$, for $m^2 < 0$ we would obtain the function $\mathcal{G}(s^2, 0, \pm 1)$ different from zero in the space-like domain and leading, therefore, to the signals propagating with a velocity greater than that of light.

The experiment shows that for large x and t the (asymptotical) wave field can be always interpreted from the corpuscular point of view. This implies that in the infinity we have a set of waves with the descrete values $m^2 = m_1^2, m_2^2, \dots m_s^2, \dots \geq 0$ *.

The Fourier-component $F(\omega^2 - k^2)$ has the poles at $\omega^2 = k^2 + m_s^2$, while the function $\mathcal{G}(s^2, +1,0)$ possesses the singularities of the form $\delta(s^2)$. In virtue of the properties of the interval s^2 this singularity will also hold in the domain of small x, t (if only $s^2 = 0$) and will lead there to undesirable infinities.

Thus, the reasonable suppositions about the isotropy of the space-time, justifiable for large x and t are transferred automatically to the domain of infinitely small x and t .

3. Causality in Quantum Physics.

The quantum theory, strangely enough, retains, in principle, the classical concept of causality. In other words, in the quantum theory the signal (or interaction) is also transferred by the Green function $D_e(s^2)$ (which is refferred to as the causal function as well). This function connects the quantum transition in the vicinity of the point $\mathcal{P}_1$ with that in the vicinity of the point $\mathcal{P}_2$ **.

Unlike the classical Green function, it is not equal to zero for $s^2 > 0$, as well. However, this is true only for the scales $\sim h/mc$ (of the Compton particle length). To be able to fix the fact of the emission of a signal (quantum) with a positive energy from the vicinity $\mathcal{P}_1$ and the fact of its absorption in the vicinity $\mathcal{P}_2$, it is necessary that these 'vicinities' would be large enough. Namely, in accordance with the uncertainty relation, for the quantum-signal with the energy ϵ and the momentum p the dimensions of the 'vicinities' $\mathcal{P}_1$ and $\mathcal{P}_2$ must be $T \gg h/\epsilon$ with respect to time, and $L > h/p$ with respect to space.

Further these vicinities should not be overlapped (the distance between them is $|x| \gg L$ and the time interval is $|t| \gg T$). As has been shown by M.Firtz for the point particles[3], the properties of the function $D_e(s^2)$ under these conditions provide a purely classical causal relationship between the vicinities of the points $\mathcal{P}_1$ and $\mathcal{P}_2$ (i.e., the relationship equivalent to that given by the Green function $\mathcal{G}(s^2, +1,0)$. When the above inequalities are not fulfilled, the uncertainty relations do not permit to judge about the nature of the causal relationship at all (what happened later, what earlier?). The causal function $D_e(x)$ in the spatial domain is not equal to zero, what leads to an existence of the spatial form-factors of elementary particles $F(q)$ (q is the momentum transferred to a particle).

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* In the quantum field theory the quantity m determines the mass of the particle corresponding to the field under consideration.

** The causality principle in its conventional form was used by N.N.Bogolubov for a new concept of the modern field theory

In consistence with such a form-factor the hard spatial distribution of charges and currents of type $\rho(x) = \int F(q) e^{iqx} d^3q$ may be attributed to a particle. Such a hard distribution allows the signal to be propagated with an infinitely high velocity from the particle periphery to its centre.

It has been shown in[4], however, that in this case the uncertainty relation also does not make it possible to 'accuse' the particle that it propagates signal with a velocity greater than that of light.

In spite of the mentioned difference of the causal Green function $D_e(s^2)$ from the classical one $\mathcal{G}(s^2, +1,0)$, the situation with the singularities in the quantum field theory remains essentially the same as in the classical theory; the singularities of the propagation functions, quite natural for large x and t are transferred inexorably to the domains of small space and time scales.

4 Some Possible Generalizations of the Causal Relationship

The specific features of the propagation functions point out that it is necessary to give up the idea of transferring the macroscopic laws of signal (effect) propagation to the domain of especially small scales of space-time and to try to change them.

What has been said about the meaning of the uncertainty relations enables us to have in mind a possibility of reconciling the conventional form of the macrocausality with other forms of microcausality in small space-time domains.

Consider now some possible generalizations of the theory.

a) The Non-linear Theory.

The Green functions having the above singularities are connected with the propagation of weak fields, obeying the linear equations.

M.Born was the first to notice[5] that strong fields may obey other non-linear equations. In this case the speed of propagating the signal V depends on the strength and the form of the signal (see[6,7,8]).

Indeed, the characteristics of the non-linear equation are different from the straight lines $\frac{dx}{dt} = \pm c$ specific for the linear theory. Therefore the velocity of the non-linear signal V turns out to be a function of the field intensity ϕ and its derivatives $\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial t}$:

$$\frac{dx}{dt} = \pm V\left(\phi, \frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial t}\right). \qquad /3/$$

As has been shown in[5], in some versions of the non-linear theory the quantity V may become imaginary, and the hyperbolic equation for the field will turn into an equation of the elliptic type. Far from the source and the receiver of the signal (we can say – far from the particles) the field will obey the linear equation as earlier, while the Green function will have usual singularities of the type $\delta(s^2)$. However, in the vicinity of the particles, where the fields are strong, the character of the singularities will change. For instance, when the equation turns into the elliptic one, the singularity of the Green function at $x \to 0$, $t \to 0$ will have the form $1/R^2$ where $R^2 = x^2 + t^2$.

This possible change of the type of a field equation near the particles resembles the situation occuring on the wing of an aeroplane flying with a velocity close to that of sound. As is well-known, in the place where the local speed of a current flowing around the wing, exceeds that of sound, the elliptic type of an equation turns into a hyperbolic one.

Fig. 1b shows the domain where the causality may become anomalous. Note, that the violation of the relativistic invariance near $x = 0$, $t = 0$ is only seeming and is due to the fact that the space-time point where the field source is situated is a special point. In the vicinity of this point the non-linear field changes the medium for its own propagation.

The possibility of changing the type of equations for the field propagation in the vicinity of the particles together with a change in the form of the causal relationship is very attractive.

However, nobody has so far succeeded in finding a quantum analog for this model of the field theory.

The question is still open as to what changes in the definitions of the length and of the time interval may be caused by the non-linearity in the signal propagation. It goes without saying, that Einstein's definitions suggest the linearity of the signal.

b) Variation of Causality for Small Space-Time Scales.

We have seen that in the homogeneous space-time the law of the causal relationship cannot be violated in the microworld without violating it in the macroworld. A possible way of modifications is prompted by the non-linearity treated in Sec. a). The conventional regularities of the signal propagation are not broken down everywhere, but only in the neighbourhood of the sources and the receivers of the signal, i.e., near the particles. In other words, this happens where the space homogeneity is violated by the particle located there. This indicates a possibility of violating the conventional laws of the signal propagation near the particles[/9,10/].

From a mathematical point of view this possibility arises because of the appearance of new invariants, besides s^2, ϵ, η. Indeed, the total energy-momentum vector $\mathcal{P}(E, \vec{p})$ which commutes with relative coordinates and with other internal dynamical variables* is associated with a particle or with a system of interacting particles. Besides the invariant $I_1 = s^2$, there appear new invariants such as $I_2 = \mathcal{P}^2 = -m^2$ (where m is the rest mass of the whole system) $I_3 = (\mathcal{P}, s)$ and others. This makes possible to form new invariant combinations such as

$$R^2 = I_1 + I_3^2/I_2$$
$$T^2 = I_3^2/I_2 , \qquad /4/$$

which in the centre-of-mass system pass into r^2 and t^2 ,respectively. Further they are transformed by (4) and (4')[/12/].

In virtue of this, the Green function connected with the system of particles can be written as

$$\mathcal{G} = \mathcal{G}(I_1, I_2, I_3) . \qquad /5/$$

The presence of the invariants I_2, I_3 allows to change the behaviour of $\mathcal{G}$ in the vicinity of $r, t = 0$.

Fig. 1b can be used again to illustrate the behaviour of the function $\mathcal{G}$, which has at $R^2 < a^2$ an elliptic structure, while at $R^2 > a^2$ it turns into the usual Green function with the singularities on the cone $s^2 = 0$.

Just in a similar manner the causal function $D_c(s^2)$ can be changed if it is associated not with the vacuum, but with the particles put into the vacuum and having relative coordinates $x = x_1 - x_2$ and the total momentum $p = p_1 + p_2$:

$$D_c = D_c(I_1, I_2, I_3) . \qquad /6/$$

A complete scheme of such a type has not yet been worked out. It is still obscure what model of the field theory it corresponds to.

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* This made possible for Yu.M.Shirokov to solve correctly the problem of the relativistic rotator,[/11/].

In particular, it has not so far been investigated whether the unitarity of the S-matrix would be kept or not.

c) Modification of the Physical Vacuum Metric

Other possibilities of varying the form of causality may lie in the modifications of our space-time geometry for small space-time domains.

One of such possibilities is the fluctuations of the metric tensor $g_{\mu\nu}$, which can, in principle, be due to the fluctuations of the zero vacuum energy.

The fluctuations of such a kind will lead to the fluctuations of the space-time interval

$$s^2 = \int_{\mathcal{P}_1}^{\mathcal{P}_2} g_{\mu\nu}\, dx_\mu\, dx_\nu \qquad /7/$$

and, therefore, all the functions such as $\mathcal{G}(s^2)$, $D_c(s^2)$ will turn out to be 'diffused'[/13,14/]. If the infinities are eliminated, these fluctuations prove to be essential in the space-time domains of the order of $L_0 = (\frac{h\chi}{c^3})^{1/2} = 0{,}82.10^{-32}$ cm. (Here χ is the gravity constant). These scales seem to be too small to play an essential role in the particle world. The introduction of another scale for the vacuum fluctuation ℓ_0 would mean a new physical hypothesis, whose consequences and internal consistence are far from being investigated.

d) Space-Time 'Quantization'

The old idea of space-time 'quantization'[/15/] was revived several times[/16-18/].

Contemporary tendencies in developing this idea start from the assumption about the non-Euclidian character of metric in the momentum space[/19/]. It is the interval in the space of the momenta $p_1\ p_2\ p_3\ p_4$ which is supposed to be

$$d\sigma^2 = a_{\mu\nu}\, dp_\mu\, dp_\nu . \qquad /8/$$

The radius of the curvature of this metric space plays the role of the limiting cut-off momentum $\mathcal{P}_0$. The space-time coordinates x_1, x_2, x_3, x_4 canonically conjugated to these moments turn out to be non-commuting operators

$$[x_\mu , x_\nu] = i b_{\mu\nu}. \qquad /9/$$

The theory is constructed so that for the scales $\ell \gg \frac{h}{\mathcal{P}_0}$ it becomes a conventional one. It is clear that the concept of the conventional causality in this theory turns out to be not valid (at least, in the space-time domains $\approx \frac{h}{\mathcal{P}_0}$). Indeed, one cannot speak about the signal propagation from the point $\mathcal{P}_1(x_1', x_2', x_3', x_4')$ to the point $\mathcal{P}_2(x_1'', x_2'', x_3'', x_4'')$ if the coordinates of these points remain undetermined. In this theory the process of the signal propagation has the physical sense only for $|x_\mu|$ large enough,when the right-hand side in (9) may be neglected. For smaller scales the relationship between phenomena may be described mathematically by means of the momentum space only. The theory of the quantized space-time has not yet been developed consistently.

Conclusion

The form of causality adopted in the modern theory follows from the basic space-time concepts.

It was borrowed from the macroscopic physics and is automatically applied to the infinitely small scales because of the nature of the singularities of the Green functions. This leads to the appearance of the divergencies (infinities) for a number of the most important physical quantities associated with elementary particles.

We have considered here some preliminary theoretical models which modify essentially causality for small space-time scales, retaining at the same time, the macroscopic causality.

We are not aware which of these models leads us nearer to the truth – we are still playing blind man's buff with it.

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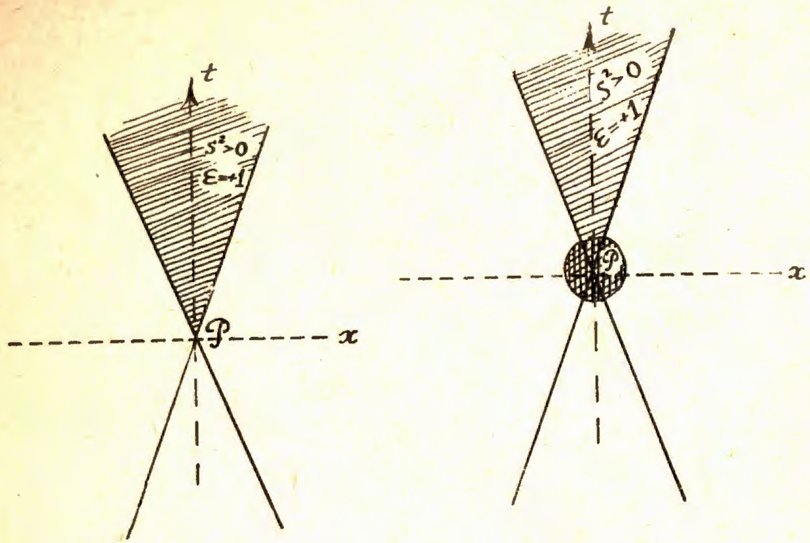

1a 1в

Fig. 1. Fig. 1a shows the shaded space-time domain allowed by the conventional theory of the propagation of the signals going from the point $\mathcal{P}$.

The double shaded area in Fig. 1b shows the domain of anomalous causality (for instance, an elliptical type of the field equations).

D.I.Blokhintsev, G.I.Kolerov

ACAUSALITY AND DISPERSION RELATIONS

1. Introduction

The "ultraviolet catastrophe" in modern field theory is essentially that the vacuum expectation values of the most important physical quantities have singularities on the light cone.

The origin of these singularities may be sought for in the form of causality which is the basis of the physical space-time metric: $s^2 = t^2 - \vec{x}^2$.

On the other hand, there are no grounds to believe that the form of causality in the microworld should coincide with that in the macroworld,as it is adopted in modern theory [1,2,3]. However, in Einstein-Minkowski space there exists no notion of the neighbourhood of the two points $\mathcal{P}(x')$ and $\mathcal{P}(x'')$ since the distance $x^2 = (x' - x'')^2$ is indefinite. For this reason any attempts to introduce the "universal" length which would characterize the small space-time region in Einstein-Minkowski space are doomed.

One may postulate, of course, that not only Einstein-Minkowski metric but also the very notion of space-time continuum are not exact enough, and are not at all valid in the microworld. In this case, since we preserve the notion of space-time continuum the theoretical scheme we are developing will be only a model of reality. In this case also one has to define the notion of the "microworld region"; intuitively we are apt to mean by that elementary particle physics — the region of high energies and small distances.

In the region of large distances and low energies there seem to be no reasons to doubt the validity of the conventional concepts of space-time and causality. At any rate experiment yields no grounds for this.

Thus, whatever possible changes of causality would be on a "small scale" the notion of "smallness" must be defined and so that there would exist a transition to the "large" space-time regions where it is natural to keep old metric relations.

Since the distance $x^2 = t^2 - r^2$ in Einstein-Minkowski space is indefinite, in order to make a transition to large distances it is insufficient to have a scalar universal length a which would provide this transition. It is necessary to have a certain time-like vector n (without restrictions one can consider $n^2 = 1,\ n_0 > 0$)

For the time being we will treat this vector purely formally. The introduction of this vector allows us, besides the invariant $x^2 = t^2 - r^2$ to introduce the invariant $I_x = (x, n) = tn_0 - \vec{r}\vec{n}$.

Using these two invariants it is possible to form a positive-definite quantity

$$R^2 = 2I_x^2 - x^2 \geq 0 , \tag{1}$$

which permits to define the notion of the neighbourhood of two particles in the four-dimensional space-time in a invariant form [3,4].

In the proper coordinate system where $n = (1, 0, 0, 0)$, $R^2 = t^2 + r^2$ we are able,besides R^2, to introduce the invariant

$$L = \frac{1}{\sqrt{2}} [\sqrt{R^2 - I_x^2} \pm I_x] , \tag{1'}$$

which determines the neighbourhood of the point to the light cone (− for the cone of the absolute future and + for the cone of the absolute past); in the proper coordinate system

$$L = \frac{1}{\sqrt{2}} (r \pm t).$$

The second reason which does not make it possible to restrict to the invariant $x^2 = t^2 - r^2$ is that if the signal propagation is allowed in the spatial region ($x^2 < 0$) , as it is supposed in /5,6/, then such a violation of causality is symmetrical with respect to the past and future.

Meanwhile, causality must be violated (apart) independently for advanced and retarded interactions. Indeed, in the conventional theory the general propagation function F may be represented in the form

$$F = a F^{ret} + b F^{adv} , \tag{2}$$

where a and b are arbitrary constants. The violation of causality (which may be weak) must not put a bound on the arbitrariness of the constants a and b .

So, we suppose that there is, besides the invariant $x^2 = t^2 - r^2$ the invariant $I_x = (x, n)$ as well. Further we note that there are two principally different possibilities for the choice of the unit vector n :
a) the vector n is exterior with respect to the system of interacting particles. A similar possibility is treated in papers /7,8/. Under such an assumption concerning the vector n there exists an explicit dependence of the scattering amplitude on the frame of reference (see., e.g. /7/). This means that the scattering amplitude may be different in the laboratory system and in the centre-of-mass system. In other words, a possibility is al-lowed that Mickelson's experiment gives a positive result in the high energy region. This seems to be very attractive, but still very little studied.

Therefore, we will treat another possibility b) when the vector n is connected with the very system of interacting particles (see /3/ and /4/). It is supposed in this case that the violation of the metric relations takes place not in vacuum, but in a medium formed by the matter of colliding particles. As a vector n one may take any unit vector directed along the momenum of one or several particles participating in the collision*. However, it is more reasonable to take the vector n which is more symmetrical with respect to the particles or their states. Such a symmetrical vector in the case of the pairing collision may be, for instance, the centre-of-mass momentum of colliding particles $P = (p + k)$ of the Breit vector $P = (p + p')$:

$$n = \frac{P}{\sqrt{P}} , \qquad P = (p + k) \qquad \text{or} \qquad P = (p + p') \tag{3}$$

(here p is the nucleon momentum , k is the meson momentum before the collision, p', k' are the same quantities after collisions). By such a choice of n the scattering amplitude $\mathfrak{M}$ for the process

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* In this case of many particles each subgroup of the interacting particles may have its internal vector n .

$a + b \to c + d$ will be, as in the conventional theory, a function of only the invariants $s = (p + k)^2$ and $t = (p + p')^2 = (k' - k)^2$ and of some universal length a which characterizes the acausality region: $\mathfrak{M} = \mathfrak{M}(s, t, a)$.

If n is a vector exterior with respect to the system of colliding particles, then in the amplitude $\mathfrak{M}$ there will hold an explicit dependence on the coordinate system so that besides s and t there will be present, at least one invariant $I = (p + k, n)$ which does not reduce to s and t.

2. Retarded and Advanced Amplitudes.

We assume that there exist asymptotic incoming and outgoing waves $\phi_{in}(x)$ and $\phi_{out}(x)$ (see, e.g., /9/), which are related through the unitary matrix S:

$$\phi_{out}(x) = S\,\phi_{in}(x)\,S^{-1}. \tag{4}$$

Then the retarded and advanced matrix elements of the scattering amplitudes $\mathfrak{M}$ for the two-body process $p+k \to p'+k'$ (where p is the nucleon momentum, k is the meson momentum before the collision, p', k' the same quantities after the collision) may be written in the form /9/:

$$\mathfrak{M}^{ret}(p'k';pk) = i\int \exp \frac{i}{2}(k+k', x) < p'| \frac{\delta}{\delta\phi(\frac{x}{2})}\left[\frac{\delta S}{\delta\phi(-\frac{x}{2})} S^{+}\right] |p> , \tag{5}$$

$$\mathfrak{M}^{adv}(p'k';pk) = i\int \exp \frac{i}{2}(k+k', x) <p'| \frac{\delta}{\delta\phi(-\frac{x}{2})}\left[\frac{\delta S}{\delta\phi(\frac{x}{2})} S^{+}\right] |p> , \tag{5'}$$

These processes do not yet imply the causality of the processes.

Denoting the one-particle matrix elements by

$$\Phi^{ret}_{p'p}(x) = i <p'| \frac{\delta}{\delta\phi(\frac{x}{2})}\left[\frac{\delta S}{\delta\phi(-\frac{x}{2})} S^{+}\right] |p> \tag{6}$$

and

$$\Phi^{adv}_{p'p}(x) = i <p'| \frac{\delta}{\delta\phi(-\frac{x}{2})}\left[\frac{\delta S}{\delta\phi(\frac{x}{2})} S^{+}\right] |p> \tag{6'}$$

we notice that

$$\Phi^{ret}_{p'p}(x) = \Phi^{adv}_{pp'}(-x), \qquad \Phi^{*\,ret}_{p'p}(x) = \Phi^{ret}_{pp'}(x) \tag{7}$$

(In the following, for the notational simplicity we shall often omit the indices p and p'. Instead of $\Phi_{p'p}(x)$ we shall write $\Phi(x)$). It follows from (7) that the Fourier transforms of the corresponding functions possess the properties:

$$\tilde{\Phi}^{ret}(Q) = \tilde{\Phi}^{adv}(-Q), \qquad \tilde{\Phi}^{*\,ret}(Q) = \tilde{\Phi}^{ret}(-Q). \tag{8}$$

Now we consider possible types of the causality violation which are compatible with the usual form of the causality for large distances r and large time intervals t.

We will be concerned at first with the usual retarded $F^{ret}(x)$ and advanced $F^{adv}(x)$ propagation functions.

In Fig. 1 the shaded area shows the space-time region where these functions may ba different from zero. At the same time

$$F^{adv}(x) = F^{ret}(-x). \tag{9}$$

The corresponding acausal functions will be designated by $\Phi(x)$. The causality violation is supposed to be that these functiona may be different from zero outside the shaded area as well. However, they must decrease sufficiently rapidly as we go into the "forbidden" region:

$$\Phi^{ret}(x) \to 0 \quad \text{at} \quad L = \frac{1}{\sqrt{2}}(r - t) \to \infty, \tag{10}$$

$$\Phi^{adv}(x) \to 0 \quad \text{at} \quad L = \frac{1}{\sqrt{2}}(r + t) \to \infty. \tag{10'}$$

A more special case would have taken place if causality has been violated only near the vertex of the light cone. Here in (10) and (10^1) we should mean $R \to \infty$ instead of $L \to \infty$.

The remaining functions may he constructed in the usual manner out of $\Phi^{ret}(x)$ and $\Phi^{adv}(x)$. The acausal analogue of the causal commutator $\Phi(x)$ is equal to

$$\Phi(x) = \Phi^{ret}(x) - \Phi^{adv}(x) = \Phi^{+}(x) + \Phi^{-}(x) = \\ = \langle p' | [j(\tfrac{x}{2}), j(-\tfrac{x}{2})] | p \rangle, \tag{11}$$

where $j(x) = \frac{\cdot \delta S}{\delta \phi(x)} S^{+}$ and $\Phi^{\pm}$ mean the positive and negative-frequency parts of the commutator $\Phi(x)$. Similarly, the acausal analogue of the causal function $\mathfrak{D}_c(x)$ is:

$$\Phi_o(x) = \tfrac{1}{2}[\Phi^{ret}(x) + \Phi^{adv}(x)] - \tfrac{1}{2}[\Phi^{+}(x) - \Phi^{-}(x)]. \tag{12}$$

The second requirement which we impose on the acausal propagation functions consists in the conservation of the usual spectrality condition.

It follows naturally from the assumption that the acausality which manifest itself at small distances does not affect the spectrum of free particles. The spectrality condition states that the Fourier transform of the acausal function $\tilde{\Phi}(Q)$:

$$\tilde{\Phi}(Q) = \int \Phi(x)\, e^{iQx}\, d^4x \tag{13}$$

must vanish in some region $\mathfrak{R}(Q)$ which is the same as that for the corresponding causal function $\tilde{F}(Q)$. If we put $\tfrac{1}{2}(p+p') = (\alpha, 0, 0, 0)$ and denote by m_1, m_2 the masses of the lowest intermediate states which may contribute to the terms of the commutator then the region $\mathfrak{R}(Q)$ will be determined by the inequality

$$a - \sqrt{\vec{Q}^2 + m_2^2} < Q_0 < -a + \sqrt{\vec{Q}^2 + m_1^2} \qquad (14)$$

i.e. , this is the region outside two hyperboloids. In the case $a > \frac{m_1 + m_2}{2}$ these hyperboloids intersect. For pion-nucleon scattering , we have:

$$m_1 = 3m , \qquad m_2 = M + m .$$

3. Interaction with an Indefinite Signal Propagation .

In what follows we will consider a model of the acausal theory in which the signal propagates not quite along the light cone.

To start, we take the simplest example which is a direct generalization to the relativiatic region of the acausality case treated in papers /9/ and /10/.

Let $F^{ret}(x)$ be a retarded propagation function of the conventional local theory. We assume that in the acausal theory the interaction may propagate inside the shifted light cone (see Fig.I).

Suppose that the magnitude of the shift is equal to

$$\xi = a n \sigma , \qquad (15)$$

where a is a certain small length, n is a characteristic time vector, σ is the invariant parameter (the "proper time") .

Then the true acausal propagation function will be

$$\Phi^{ret}(x) = F^{ret}(x - \xi) . \qquad (16)$$

Regarding ξ as a function of σ and introducing the propagation function of the shifts $f(\sigma)$, we can write (1.6) in a more general form

$$\Phi^{ret}(x) = \int F^{ret}[x - \xi(\sigma)] \; f_1(\sigma) \, d\sigma . \qquad (17)$$

For the advanced functions we shall have, respectively

$$\Phi^{adv}(x) = \int F^{adv}[x - \xi(\sigma)] \cdot f_2(\sigma) \, d\sigma . \qquad (18)$$

In virtue of condition $f_1(\sigma) = f_2(-\sigma) = f(\sigma)$.

The Fourier-transform of these functions states

$$\tilde{\Phi}^{ret}(Q) = \tilde{F}^{ret}(Q) \, \tilde{f}(Qna) \qquad (19)$$

and

$$\tilde{\Phi}^{adv}(Q) = \tilde{F}^{adv}(Q) \, \tilde{f}(-Qna) , \qquad (19')$$

where

$$\tilde{f}(Qna) = \int e^{iQna\sigma} f(\sigma) \, d\sigma . \qquad (20)$$

It follows from (11), (19) and (19$'$) that

$$\tilde{\Phi}(Q) = \tilde{F}^{ret}(Q)\,\tilde{f}(Qn\alpha) - \tilde{F}^{adv}(Q)\,\tilde{f}(-Qn\alpha)\,. \quad (21)$$

One can just see from here that the spectrality condition is fulfilled if $\tilde{f}(Qn\alpha) = \tilde{f}(-Qn\alpha)$. Besides, since the equality $\tilde{f}(-Qn\alpha) = \tilde{f}^{*}(Qn\alpha)$ must be also fulfilled, the function $\tilde{f}(Qn\alpha)$ must be even and real.

Further, in the proper coordinate system $n = (1,0,0,0)$, $L = \sqrt{2}\,\xi$ (cf. Fig. 2); therefore $L \to \infty$ means that $\sigma \to +\infty$. It follows from here that the condition of the macroscopic causality will be fulfilled (the "anomalous" signal will be whatever small), if $f(\sigma)$ is a sufficiently rapidly decreasing function at $\sigma \to \infty$.

It is seen from the formulae (19) and (19$'$) that for the quantities $\tilde{\Phi}^{ret}(Q)/\tilde{f}(Qn\alpha)$ and $\tilde{\Phi}^{adv}(Q)/\tilde{f}(-Qn\alpha)$ there will hold ordinary dispersion relations. The additional singularities of the acausal functions Φ^{ret} and Φ^{adv} coincide with the singularities of the functions $\tilde{f}(\pm Qn\alpha)$.

Note, that if $f(\sigma)$ falls off very sharply with the growth of σ, then in the Q plane appears a singularity on a circle of infinitely large radius. For example:

$$f(\sigma) = \delta(\sigma - 1), \qquad \tilde{f}(Qn\alpha) \approx e^{iQna}, \quad (22)$$

$$f(\sigma) = e^{-\sigma^2}, \qquad \tilde{f}(Qn\alpha) \approx e^{-a^2(Qn)^2}. \quad (22')$$

For a more smooth, exponential decrease there arises a pole

$$f(\sigma) = e^{-\sigma}, \quad \sigma > 0, \quad \tilde{f}(Qn\alpha) = \frac{1}{1 - iQn\alpha}. \quad (22'')$$

However, in virtue of what has been said above the spectrality conditions are satisfied by the function (22$'$) only.

A. Consider now a more general case of the acausal propagation function

$$\Phi^{ret}(x) = \int F^{ret}(x - \xi)\,\rho_1(\xi, n)\,d^4\xi\,. \quad (23)$$

Here the propagation function $F^{ret}(x)$ is again taken over from the conventional causal theory, while the weight function $\rho_1(\xi, n)$ vanishes at $R \to \infty$. Note that condition (9) requires that $\rho_1(\xi, n) = \rho_2(-\xi, n)$. Therefore, further we omit indices 1 and 2. Due to the vanishing of ρ at $R \to \infty$ macroscopic causality is fulfilled.

Indeed, the signal Φ^{ret} may be regarded as the one from a certain source $\rho(x)$ extended near the coordinate origin $r, t \sim 0$ (see Fig.2). Further the Fourier transform states

$$\tilde{\Phi}^{ret}(Q) = \tilde{F}^{ret}(Q)\,\tilde{\rho}(Q, n), \quad (24)$$

$$\tilde{\Phi}^{adv}(Q) = \tilde{F}^{adv}(Q)\,\tilde{\rho}(-Q, n), \quad (24')$$

where $\tilde{\rho}(Q,n)$ is the Fourier-transform of the function $\rho(\xi,n)$. The symmetry conditions (8) require that

$$\tilde{\rho}(-Q,n) = \tilde{\rho}(Q,n), \quad \tilde{\rho}(-Q,n) = \tilde{\rho}^{*}(Q,n) \qquad (24'')$$

Then

$$\tilde{\Phi}(Q) = [\tilde{F}^{ret}(Q) - \tilde{F}^{adv}(Q)] \cdot \tilde{\rho}(Q,n) \qquad (24''')$$

These functions evidently vanish in the region $\Re(Q)$ and, hence, the spectral condition is fulfilled.

Note, that the analytic properties of the functions $\tilde{\Phi}(Q)/\tilde{\rho}(Q,n)$ coincide with the analytic properties of these functions in causal field theory.

As we have pointed out above the appearance of essential singularities (at infinity) of the function $\tilde{\rho}(I_Q, Q^2)$ is rather an anomaly than a usual situation.

Indeed, for this the space-time region of acausality should be sharply bounded (sharper than by an exponent). In particular, by a sharp cut off $\rho(x,n) = \frac{4}{\pi^2 a^4} \int_0^{a^2} d\xi\, \delta(\xi - R^2)$ we shall have

$$\tilde{\rho}(I_Q, Q^2) = \frac{4}{\pi^2 a^4} \int d^4x\, e^{iQx} \int_0^{a^2} d\xi\, \delta(\xi - R^2) = \qquad (25)$$

$$\frac{8}{a^2 [2(Qn)^2 - Q^2]} J_2(a\sqrt{2(Qn)^2 - Q^2}),$$

where $J_2(z)$ is the Bessel function. Since asymptotically $J_2(z) \approx \sqrt{\frac{2}{\pi z}} \operatorname{Cos}(z - \frac{5}{4}\pi)$, then $\tilde{\rho}(I_Q, Q^2)$ will have a singularity at infinity. This is clearly seen in the proper coordinate system $n = (1,0,0,0)$, where the invariant

$$\sqrt{2(Qn)^2 - Q^2} = \sqrt{Q_0^2 + \vec{Q}^2}$$

In particular, if this is a Breit system then $\sqrt{Q_0^2 + \vec{Q}^2} = \sqrt{2\omega^2 - m^2 - \vec{q}^2}$, where ω is the meson energy, m is its mass, q is the momentum transfer. When $\vec{q} = 0$, $|\omega| \gg m$ the function $\tilde{\rho}$ will contain the factor $\exp(\pm i\sqrt{2}\,\omega a)$. For the Caussian distribution

$$\rho(x) = \frac{1}{a^4} \exp - \frac{R^2}{a^2}, \quad \tilde{\rho}(I_Q, Q^2) = \exp\{-\frac{a^2}{4}[2(Qn)^2 - Q^2]\}$$

and the essential singularity is due to the factor $\exp - \frac{a^2\omega^2}{2}$.

Now we consider in more detail the case when $\rho(x)$ decreases exponentially, or in a more general form

$$\rho(x) \approx R^m \exp(-\frac{R}{a}). \qquad (26)$$

In this case there appear additional poles in the plane ω.

For the sake of definiteness, we will be concerned with the case

$$\rho(x) = \frac{1}{8\pi a^2 R^2} \exp\left(-\frac{R}{a}\right) \qquad (27)$$

(the factor a^n is chosen so that $\tilde{\rho}(Q) \to 1$ when $a \to 0$). Then

$$\tilde{\rho}(Q) = \frac{1}{1 + a^2[2(Qn)^2 - Q^2]}. \qquad (28)$$

Or in the Breit system

$$\tilde{\rho}(Q) = \frac{1}{1 + a^2(2\omega^2 - m^2 - \vec{q}^{\,2})}. \qquad (29)$$

As far as there is no essential singularity at infinity, the dispersion relations with the necessary subtractions may be written for the observed matrix element $\mathfrak{M}(k', p'; k, p)$.

Note that the case (A) treated above is formally obtained from (17), if we put

$$\rho(\xi, n) = \int \delta(\xi - a n \sigma) f(\sigma) d\sigma$$

and integrate over ξ.

B. Now we consider the case when the causality is violated only near the vertex of the light cone. Here one can suppose:

$$\Phi^{ret}(x) = F^{ret}(x) + \phi^{ret}(x, n), \qquad (30)$$

where $\phi^{ret}(x, n)$ is an acausal addition to the causal function $F^{ret}(x)$ vanishing as we go away from the coordinate origin. We assume that $\phi^{ret}(x, n) = \phi^{ret}(R^2, xn)$ and that

$$\phi^{ret}(R^2, xn) \underset{R \to \infty}{\to} 0. \qquad (31)$$

Analogously one can introduce

$$\Phi^{adv}(x) = F^{adv}(x) + \phi^{adv}(x, n)$$

and hence

$$\Phi(x) = \Phi^{ret}(x) - \Phi^{adv}(x) = F(x) + \phi(R^2, xn).$$

At the same time

$$\phi(R^2, xn) = \phi^+(R^2, xn) - \phi^-(R^2, xn).$$

Then

$$\tilde{\phi}^{\pm}(Q) = \int \phi^{\pm}(R^2, xn) \exp iQx\, d^4x =$$

$$= -\tfrac{1}{4} \int \phi^{\pm}(a^2, \beta)\, e^{i\beta\xi} \exp i\left[\eta a^2 + \frac{R^2(Q-\xi n)}{4\eta}\right] da^2 d\beta\, d\eta\, d\xi \qquad (32)$$

$$= \int \phi^{\pm} (a^2 , \beta) e^{i\beta\xi} \frac{J_1 [a R (Q - \xi n)]}{R (Q - \xi n)} a^2 da\, d\beta\, d\xi .$$

Here $R^2 (Q - \xi n)$ has the same meaning as in (1) with the substitution of x by $(Q - \xi n)$; in the system where $n = (1, 0, 0, 0)$, $R^2 = (Q_0 - \xi)^2 + \vec{Q}^2$. In this system our expression is of the form

$$\tilde{\phi}^{\pm} (Q) = \int \phi^{\pm} (a^2 , \beta) e^{i\beta\xi} \frac{J_1 [a\sqrt{(Q_0 - \xi)^2 + \vec{Q}^2}]}{\sqrt{(Q_0 - \xi)^2 + \vec{Q}^2}} a^2 da\, d\beta\, d\xi .$$

In virtue of the spectrality conditions $\tilde{\phi}^{+} (Q) = 0$ for all Q satisfying the inequality $Q_0 > -a + \sqrt{\vec{Q}^2 + m_1^2}$ Similarly $\tilde{\phi}^{-} (Q) = 0$ for all Q satisfying the inequality $Q_0 > a - \sqrt{\vec{Q}^2 + m_2^2}$

The expression

$$(Q_0 - \xi)^2 + \vec{Q}^2 = R^2(\xi) \qquad (33)$$

is a family of the circumferences of radius R and the coordinates of the centre $(\xi, 0, 0, 0)$. We choose $R (\xi) = R_0 (\xi)$ so that the hyperballs (14) would be envelopes for our family of the circumferences. Then for the upper hyperboloid

$$R_0^{+} (\xi) = \frac{(\xi + a)^2}{2} - m_1^2 \qquad (34)$$

and ξ must change within the interval $[+\infty , 2 m_1 - a]$. Here the lower boundary is found from the requirement that $Q = \pm \sqrt{\frac{(\xi + a)^2}{4} - m_1^2}$ be a real value) . Similarly for the lower hyperboloid

$$R_0^{-} (\xi) = \frac{(\xi - a)^2}{2} - m_2^2 \qquad (35)$$

and ξ must change within the interval $[-(2 m_2 - a) , -\infty]$ Therefore, in order to satisfy the causality conditions it is necessary that the integrands would vanish outside the given intervals. Thus, the spectrality conditions are written down in the form

$$\iint \phi^{\pm} (a^2 , \beta) e^{i\beta\xi} d\beta\, J_1 (a \sqrt{ (Q_0 - \xi)^2 + \vec{Q}^2 }) a^2 da$$

$$= \begin{cases} R f^{\pm} (R , \xi) & \text{for} \quad R \le R_0^{\pm} (\xi) \\ 0 & \text{for} \quad R > R_0^{\pm} (\xi) \end{cases} \qquad (36)$$

where ξ changes within the above-mentioned intervals. It follows from theorem /11/ that if

$$\int a \phi^{\pm} (a^2 , \beta) e^{i\beta\xi} d\beta = \int_0^{R_0^{\pm}(\xi)} R' f^{\pm} (R', \xi) J (a R') R' dR' \qquad (37)$$

and $f^{\pm}(R',\xi)$ is a holomorphic function of R' on the segment from 0 up to $R^{\pm}(\xi)$, then the spectrality conditions (36) will be fulfilled x/. Substituting (37) into (32) we get:

$$\tilde{\phi}(Q) = \int_{2m_1-a}^{\infty} d\xi \, \frac{\varphi}{\pi i} \int_0^{R_0^+(\xi)} \frac{f^+(z,\xi)\,dz^2}{z^2-[(Q_0-\xi)^2+\vec{Q}^2]} - \tag{38}$$

$$- \int_{-\infty}^{-(2m_2-a)} d\xi \, \frac{\varphi}{\pi i} \int_0^{R_0^-(\xi)} \frac{f^-(z,\xi)\,dz^2}{z^2-[(Q_0-\xi)^2+\vec{Q}^2]},$$

where $f^{\pm}(z,\xi)$ are the holomorphic functions of the variable z on the half-axis from 0 to ∞. As to the analytic properties of ϕ^{ret} in the continuation in ω to the upper half-plane or in the continuation of ϕ^{adv} to the lower half-plane, they are determined by the properties $f^{ret}_{adv}(z,\xi)$ which is a Bessel transform of index 1 in the first argument and the Fourier transform in the second argument of the function $\phi^{ret}_{adv}(\alpha,\beta)$ One can see by examples that the above-formulated causality condition (31) allows a wide class of analyticity violations involving the appearance of poles, cuts, and singularities.

4. Dispersion Relations

At first we consider the case A) when the scattering amplitude may be represented in the form

$$\mathfrak{M}(p',k';p,k) = N(p',k';p\,k)\,\tilde{\rho}(p',k';p,k) \tag{39}$$

or $N(p',k';p\,k) = \mathfrak{M}(p',k';p,k)\,\tilde{\rho}^{-1}(p',k';p,k)$, where $N(p',k';p,k)$ is the scattering amplitude which possesses all the usual analytic properties of the causal scattering amplitude, and $\tilde{\rho}(p',k';p,k)$ is the real function determined in §3B. To go on with the construction of dispersion relations we choose a special coordinate system - the Breit system in which the expression (3) will be rewritten as

$$N(\omega,\lambda\vec{e}) = \mathfrak{M}(\omega,\lambda\vec{e})\,\tilde{\rho}^{-1}(\omega,\lambda\vec{e}), \tag{40}$$

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x/ This theorem states: If the real part exceeds 1 and if

$$f(\lambda) = \int_p^q \phi(\rho)\,J_n(\lambda\rho)\,\rho\,d\rho \qquad 0 \le p < q \le \infty$$

then

$$\int_0^{\infty} f(\lambda)\,J_n(\lambda r)\,\lambda\,d\lambda = \begin{cases} \phi(r), & p<r<q \\ 0, & 0<r<p,\; q<r<\infty. \end{cases}$$

where ω is the meson energy, e is the unit ort $\perp$p and $\lambda = \sqrt{\omega^2 - p^2 - m^2}$. The concrete form of dispersion relations will depend on the order of the growth of $\tilde{\rho}^{-1}$. Indeed, the dispersion relations in energy for N in case of forward scattering ($\vec{p} = 0$) without subtractions, provided that $\mathfrak{M}^*(\omega) = \mathfrak{M}(-\omega)$ x/ are as follows

$$\mathrm{Re}\,\mathfrak{M}(\omega)\tilde{\rho}^{-1}(\omega) = \frac{2\omega_t\,\mathrm{Re}[\,\mathrm{Res}\,\mathfrak{M}(\omega_t)\,]}{(\omega^2 - \omega_t^2)\tilde{\rho}(\omega_t)} + \tag{41}$$

$$+ \frac{2}{\pi}\mathcal{P}\int_m^\infty \frac{\mathrm{Im}\,\mathfrak{M}(\omega')\omega'd\omega'}{(\omega'^2 - \omega^2)\tilde{\rho}(\omega')}$$

or

$$\mathrm{Re}\,\mathfrak{M}(\omega) = \frac{2\omega_t\,\mathrm{Re}[\,\mathrm{Res}\,\mathfrak{M}(\omega_t)]}{(\omega^2 - \omega_t^2)}\,\frac{\tilde{\rho}(\omega)}{\tilde{\rho}(\omega_t)} + \tag{41'}$$

$$+ \frac{2}{\pi}\tilde{\rho}(\omega)\mathcal{P}\int_m^\infty \frac{\mathrm{Im}\,\mathfrak{M}(\omega')\omega'd\omega'}{(\omega' - \omega)\tilde{\rho}(\omega')} .$$

In the frequency region where $\rho(\omega), \rho(\omega_t) \approx 1$ and if the factor $(\omega'^2 - \omega^2)$ cuts off the integrand stronger than $\tilde{\rho}^{-1}(\omega)$ grows, one obtains ordinary dispersion relations. For the real dispersion relations $\tilde{\rho}^{-1}(\omega')$ must not grow faster than ω'. If $\tilde{\rho}^{-1}(\omega')$ grows faster than ω' then it is neccesary to increase the number of subtractions, and the ordinary dispersion relations will no longer hold.

If the growth of $\mathfrak{M}(\omega)$ remains bounded $\sim \omega$ what corresponds at present to the experimental data, then the acausal dispersion relations may be written down for $\mathfrak{M}(\omega)$ directly. They have the form

$$\mathrm{Re}\,\mathfrak{M}(\omega) = \frac{2\omega_t\,\mathrm{Re}[\,\mathrm{Res}\,\mathfrak{M}(\omega_t)]}{(\omega^2 - \omega_t^2)} + \frac{2}{\pi}\mathcal{P}\int_m^\infty \frac{\mathrm{Im}\,\mathfrak{M}(\omega')\omega'd\omega'}{(\omega'^2 - \omega^2)} + \Psi(\omega), \tag{42}$$

where

$$\Psi(\omega) = \mathrm{Re}\sum_i \frac{1}{2\pi i}\oint_{c_i} \frac{\mathfrak{M}(\nu)d\nu}{\nu - \omega} \tag{43}$$

---

x/ The condition for the field being real is $\phi(x)$.

means the integration over the contours C_i which rule out the singularities of the function $\tilde{\rho}(\nu)$. In particular, if $\tilde{\rho}(\nu)$ has only the poles then for a pair of the conjugated poles we get (cf.(28))

$$\Psi(\omega) = \frac{A + B\omega}{(a-\omega)^2 + b^2} \tag{44}$$

i.e., a relation different from the ordinary dispersion relations not only in the high energy region, but also at low energies (if $A \neq 0$).

In the case B) one cannot write so generally the dispersion relations as it is done in the case A). However, it is possible to apply the following recipe. We divide the total scattering amplitude $\mathfrak{M}(\omega)$ into two parts

$$\mathfrak{M}(\omega) = \mathfrak{M}_0(\omega) + \mathfrak{M}_a(\omega) \, . \tag{45}$$

where $\mathfrak{M}_0(\omega)$ is the scattering amplitude satisfying the 'normal' dispersion relations $\mathfrak{M}(\omega)$ is the acausal part of the amplitude appearing as a result of the causality violation in the vicinity of the vertex of the light cone. In this case the dispersion relations may be written down for the difference

$$\mathfrak{M}_0(\omega) = \mathfrak{M}(\omega) - \mathfrak{M}_a(\omega) \, . \tag{46}$$

We get

$$\mathrm{Re}\,\mathfrak{M}(\omega) = \frac{2\omega_f \, \mathrm{Re}[\mathrm{Res}\,\mathfrak{M}(\omega_f)]}{(\omega^2 - \omega_f^2)} + \tag{47}$$

$$- \frac{2}{\pi} \mathcal{P} \int_m^\infty \frac{\mathrm{Im}\,\mathfrak{M}(\omega')\omega' d\omega'}{(\omega'^2 - \omega^2)} + \Psi(\omega),$$

where

$$\Psi(\omega) = \mathrm{Re}\,\mathfrak{M}_a(\omega) - \frac{2\omega_f \, \mathrm{Re}[\mathrm{Res}\,\mathfrak{M}_a(\omega_f)]}{(\omega^2 - \omega_f^2)} - \tag{48}$$

$$- \frac{2}{\pi} \mathcal{P} \int_m^\infty \frac{\mathrm{Im}\,\mathfrak{M}_a(\omega')\omega' d\omega'}{\omega'^2 - \omega^2}$$

Since the functions $\phi^{ret}_{adv}(R^2, xn)$ are concentrated near the vertex of the light cone, the function $\mathfrak{M}_a(\omega)$ which is the Fourier transform of $\phi^{ret}_{adv}(R^2, xn)$ vanishes at $\omega \to \infty$. If it is not equal to zero everywhere, it is different from zero also at small frequences. Therefore the function $\Psi(\omega)$ is different from zero over the whole frequency interval. In virtue of this, the ordinary dispersion relations will not be fulfilled both at high energies and at low ones.

In conclusion we write down the dispersion relations for $\pi - N$ scattering with two subtractions under the assumption that the scattering amplitude $\mathfrak{M}(\omega)$ is at infinity $\sim \omega$ and has singularities on the imaginary axis:

$$\tilde{\rho}(\omega) = \frac{\Omega^2}{\Omega^2 + \omega^2} \qquad \text{where} \quad \Omega = \frac{1}{a},$$

and a is a universal length. Thus, the amplitude $\mathfrak{M}(\omega)$ has additional poles at the points $\omega = \pm i\Omega$. For charged pions we obtain in this case:

$$\overset{(0)}{D}_+(\omega) + \overset{(0)}{D}_-(\omega) - \overset{(0)}{D}_+(\omega_0) - \overset{(0)}{D}_-(\omega_0) =$$

$$\frac{2}{\pi}(\omega^2 - \omega_0^2)\mathcal{P}\int_m^\infty \frac{[\overset{(0)}{A}_+(\omega') + \overset{(0)}{A}_-(\omega')]\,\omega' d\omega'}{(\omega'^2 - \omega^2)(\omega'^2 - \omega_0^2)} + \tag{49}$$

$$\frac{2g^2}{M}\left(\frac{m^2}{2M}\right)^2 \frac{\omega^2 - \omega_0^2}{[\omega^2 - (\frac{m^2}{2M})^2][\omega_0^2 - (\frac{m}{2M})^2]} + \overset{(0)}{\Psi}_+(\omega),$$

$$\overset{(0)}{D}_+(\omega) - \overset{(0)}{D}_-(\omega) - \frac{\omega}{\omega_0}\{\overset{(0)}{D}_+(\omega_0) - \overset{(0)}{D}_-(\omega_0)\} =$$

$$\frac{2}{\pi}\omega(\omega^2 - \omega_0^2)\mathcal{P}\int_m^\infty \frac{[\overset{(0)}{A}_+(\omega') - \overset{(0)}{A}_-(\omega')]\,d\omega'}{(\omega'^2 - \omega^2)(\omega'^2 - \omega_0^2)} + \tag{49'}$$

$$\frac{4g^2 m^2}{M^2} \frac{\omega(\omega^2 - \omega_0^2)}{[\omega^2 - (\frac{m}{2M})^2][\omega_0^2 - (\frac{m^2}{2M})^2]} + \overset{(0)}{\Psi}_-(\omega), \tag{49''}$$

$$\overset{(1)}{D}_+(\omega) + \overset{(1)}{D}_-(\omega) - \frac{\omega}{\omega_0}\{\overset{(1)}{D}_+(\omega) + \overset{(1)}{D}_+(\omega_0)\} =$$

$$= \frac{2}{\pi}\omega(\omega^2 - \omega_0^2)\mathcal{P}\int_m^\infty \frac{[\overset{(1)}{A}_+(\omega') + \overset{(1)}{A}_-(\omega')]\,d\omega'}{(\omega'^2 - \omega^2)(\omega'^2 - \omega_0^2)} + \tag{49'''}$$

$$+ \frac{2g^2}{M^2} \frac{\omega(\omega^2 - \omega_0^2)}{[\omega^2 - (\frac{m^2}{2M})^2][\omega_0^2 - (\frac{m^2}{2M})^2]} + \overset{(1)}{\Psi}_+(\omega),$$

$$\overset{(1)}{D}_+(\omega) - \overset{(1)}{D}_-(\omega) - \overset{(1)}{D}_+(\omega_0) + \overset{(1)}{D}_-(\omega_0)$$

$$= \frac{2}{\pi}(\omega^2 - \omega_0^2)\mathcal{P}\int_m^\infty \frac{[\overset{(1)}{A}_+(\omega') - \overset{(1)}{A}_-(\omega')]\,\omega' d\omega'}{(\omega'^2 - \omega^2)(\omega'^2 - \omega_0^2)} + \tag{49''''}$$

$$\frac{2g^2}{M}\left(\frac{m^2}{2M}\right)^2 \frac{\omega^2 - \omega_0^2}{[\omega^2 - (\frac{m^2}{2M})^2][\omega_0^2 - (\frac{m}{2M})^2]} + \overset{(1)}{\Psi}_-(\omega);$$

for neutral

$$\overset{(0)}{D}_0(\omega) - \overset{(0)}{D}_0(\omega_0) =$$

$$= \frac{2}{\pi}(\omega^2 - \omega_0^2)\mathcal{P}\int_m^\infty \frac{\overset{(0)}{A}_0(\omega')d\omega'}{(\omega'^2-\omega^2)(\omega'^2-\omega_0^2)} +$$

$$\frac{g^2}{M}\left(\frac{m^2}{2M}\right)^2 \frac{\omega^2-\omega_0^2}{\left[\omega^2-\left(\frac{m^2}{2M}\right)^2\right]\left[\omega_0^2-\left(\frac{m^2}{2M}\right)^2\right]} + \overset{(0)}{\Psi}_0(\omega), \tag{50}$$

$$\overset{(1)}{D}_0(\omega) - \frac{\omega}{\omega_0}\overset{(1)}{D}_0(\omega_0) =$$

$$\frac{2}{\pi}\omega(\omega^2-\omega_0^2)\mathcal{P}\int_m^\infty \frac{\overset{(1)}{A}_0(\omega')d\omega'}{(\omega'^2-\omega^2)(\omega'^2-\omega_0^2)} + \tag{50'}$$

$$\frac{g^2}{M^2}\frac{\omega(\omega^2-\omega_0^2)}{\left[\omega^2-\left(\frac{m^2}{2M}\right)^2\right]\left[\omega_0^2-\left(\frac{m^2}{2M}\right)^2\right]} + \overset{(1)}{\Psi}_0(\omega).$$

Additional terms

$$\overset{(0)}{\Psi}_+(\omega),\ \overset{(0)}{\Psi}_-(\omega),\ \overset{(1)}{\Psi}_+(\omega),\ \overset{(1)}{\Psi}_-(\omega),\ \overset{(0)}{\Psi}_0(\omega),\ \overset{(1)}{\Psi}_0(\omega),$$

may be written out in the form

$$\overset{(0)}{\Psi}_+(\omega) = \frac{\omega^2-\omega_0^2}{\Omega^2+\omega_0^2}\tilde{\rho}(\omega)[\overset{(0)}{d}_+(i\Omega)+\overset{(0)}{d}_-(i\Omega)], \tag{51}$$

$$\overset{(0)}{\Psi}_-(\omega) = \frac{\omega^2-\omega_0^2}{\Omega^2+\omega_0^2}\tilde{\rho}(\omega)\frac{\omega}{\Omega}[\overset{(0)}{a}_+(i\Omega)-\overset{(0)}{a}_-(i\Omega)], \tag{51'}$$

$$\overset{(1)}{\Psi}_+(\omega) = \frac{\omega^2-\omega_0^2}{\Omega^2+\omega_0^2}\tilde{\rho}(\omega)\frac{\omega}{\Omega}[\overset{(1)}{a}_+(i\Omega)+\overset{(1)}{a}_-(i\Omega)], \tag{51''}$$

$$\overset{(1)}{\Psi}_-(\omega) = \frac{\omega^2-\omega_0^2}{\Omega^2+\omega_0^2}\tilde{\rho}(\omega)[\overset{(1)}{d}_+(i\Omega)-\overset{(1)}{d}_-(i\Omega)]. \tag{51'''}$$

$$\overset{(0)}{\Psi}_0(\omega) = \frac{\omega^2-\omega_0^2}{\Omega^2+\omega_0^2}\tilde{\rho}(\omega)\overset{(0)}{d}_0(i\Omega), \tag{52}$$

$$\overset{(1)}{\Psi}_0(\omega) = \frac{\omega^2-\omega_0^2}{\Omega^2+\omega_0^2}\frac{\omega}{\Omega}\tilde{\rho}(\omega)\overset{(1)}{a}_0(i\Omega), \tag{52'}$$

where

$$d(z) = \operatorname{Re} N(z), \quad a(z) = \operatorname{Im} N(z).$$

Suppose that

$$d(i\Omega) \underset{\Omega\to\infty}{\approx} \alpha\, \Omega^{m}, \qquad a(i\Omega) \underset{\Omega\to\infty}{\approx} \beta\, \Omega^{n},$$

where $m \le 0 \quad n \le 1$,

then the additional terms $\Psi(\omega)$ will be of the order

$$\frac{\omega^2 - \omega_0^2}{\Omega^2 + \omega_0^2} \quad \frac{\Omega^2}{\Omega^2 + \omega^2} \quad \alpha\, \Omega^{m},$$

$$\frac{\omega^2 - \omega_0^2}{\Omega^2 + \omega_0^2} \quad \frac{\Omega^2}{\Omega^2 + \omega^2} \quad \frac{\omega}{\Omega}\, \beta\, \Omega^{n}. \tag{53}$$

It is seen from here that at $\omega \ll \Omega$ the additional terms are small. But they become essential at $\omega \ge \Omega$. If the length $a = \frac{h}{MC} \approx 10^{-14}$ cm, then already in the region ω of several GeV essential deviations from the normal dispersion relations will take place. The analysis made in /12, 13, 14/ shows that with the presently available accuracy the dispersion relations for πN scattering are fulfilled with an accuracy of 5-10% in the region of 0.1 – 0.5 GeV and in the region of 10-20 GeV – with an accuracy of 10-20%. This points out that the universal length is probably less than 10^{14} cm.

5. Conclusion

We have considered two types of acausality: the acausality concentrated near the surface of the light cone (the case A) and the acausality concentrated near its vertex (the case B).

A measure of concentration of acausality is a certain universal length a . As such we can take, for example, the Compton nucleon length $a_M = \frac{h}{MC} = 2.\,10^{-14}$ cm . or a characteristic length of weak interaction $a_F = \sqrt{\frac{g_F}{hc}} = 6.\,10^{-17}$ cm. Both these possibilities do not contradict the presently available experimental data.

In the cases A) and B) the conditions of microscopic causality and spectrality were fulfilled.

It turned out that the appearance, due to the acausality of the interaction, of the singularities at infinity in the complex plane ω is rather an exception than a rule: for this it is necessary to bound sufficiently sharply the space-time region in which the usual causality is violated.

Besides, one should borne in mind that the appearance of the factor $e^{ia\omega}$ in the scattering amplitude will lead, in virtue of the optical theorem, to the oscillations of the total cross sections, while the appearance of the factor $e^{-a^2\omega^2}$ to an essential decrease of the total cross section with the increasing ω

Both these possibilities are likely to be in contradiction with the well-known experimental facts. One can draw a conlclusion that the space-time region of acausality must have a diffuse boundary (the decrease is not faster than the exponential one) .

In this case no singularities appear at infinity in the complex plane ω . However, there appear other additional singularities coinciding with those of the Fourier transforms of the functions $\rho(x,)$ (cf. (23)) or $\phi^{ret}(x,n)$ (cf. (30)). These function do not vanish in the spatial region of the variable $x \sim a$

and therefore the singularities of their Fourier transfoms differ from the usual singularities characteristic of causal theory.

In view of these new singularities the dispersion relations for the scattering amplitude suffer this or that change, what depends on the nature of the singularities of the function $\tilde{\rho}(Q,n)$ or $\tilde{\phi}(Q,n)$

This change is displayed in the appearance in the dispersion relation of additional terms of the type $\Psi(\omega)$, (51) and (52) in the general case are essential not only in the high energy region $(\omega >> \frac{1}{a})$ but also over the whole energy interval involving low energies.

The example given in the previous Section shows that the universal length a is probably less than 10^{14}cm. If this is so, then in order to find acausality it is necessary to make the verification of dispersion relations more precise. In particular, when $a \approx 10^{16}$ cm, for the pions of 10 GeV energy, the accuracy should be higher than 3 %, for 20 GeV pions it should be more than 10%.

Therefore, the experimental verification of the dispersion relations for πN scattering (in this case, the non-physical region ω is known to play no role) seems to be extrimely important and apparently quite a real problem of today's experiments.

Although we carried out the calculations in the explicit form for the case when the vector n is an internal vector of a system of interacting particles, all our conclusions hold true for the case when this vector is external, i.e. when the homogeneity of space-time is violated. Here it seems to be more important to check up a possible violation of this homogeneity rather than to verify dispersion relations. This can be accomplished by comparing the results of scattering experiments (of electrons) in the laboratory system and in the centre-of-mass system. When the vector n is external both these systems are equivalent: the system in which space inhomogeneties are at rest is singled out if compared with the others. The validity of this singling out will be treated in another publication.

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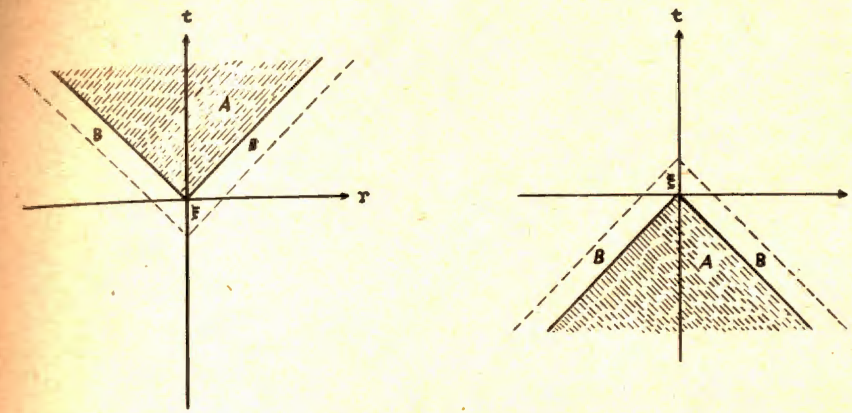

Fig 1. (a) retarded, (b) advanced interaction.
A is the region of usual causality. B is the region of acausality.

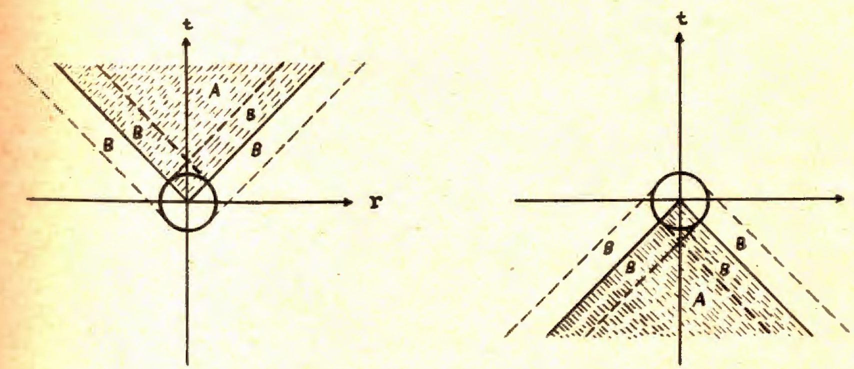

Fig. 2 (a) retarded (b) advanced interaction. A is the region of usual causality. B is the region of acausality.
The circle at the centre is the region where $\rho(R) \neq 0$.

D.I.Blokhintsev

ON EXPERIMENTAL VERIFICATION OF HOMOGENEITY AND ISOTROPY OF SPACE

It is well-known that, due to gravitation, space and time become inhomogeneous in the region of large scales. However, nothing is known about the behaviour of space and time in the region of extremely small scales. A priopi there are no grounds to believe that the metric relations in the small scale region must be the same as those known from macroscopic physics. In particular, space-time may be inhomogeneous and non-isotropic /1/.

The situation may turn out to be similar to that which takes place in a crystal; for long waves the crystal is a homogeneous and isotropic medium. Therefore the laws which govern the propagation of these waves are invariant under group of the translations by an arbitrary displacement and under the group of rotations by any angle. For short waves the admissible shifts and rotations are discrete.

The crystal inhomogeneity causes the known scattering of the short waves A similar situation may arise in the "empty" space either: for long waves space may be homogeneous and isotropic so that Lorentz transformations will be valid, and for short waves its inhomogeneity may be displayed.

It was shown in /1/ that in order to distinguish between small distances and large ones in Einstein-Minkowski space it is not sufficient to have a universal length a which could be used to define the notion of smallness for the distance, it is also necessary to have a certain time-like vector n (which we can choose to be a unit vector: $n^2 = 1$)

With the help of such a vector one can define a positive-definite quantity $R^2 = 2l^2 - x^2 \geq 0$ where $x = t^2 - r^2$ is the invariant interval, while $l = (nx)$ is the scalar product of the vectors n and x. The notion of smallness is then determined by comparing R with a.

It is clear that the introduction of such a vector into the theory singles out the coordinate system in which the space-time inhomogeneities are at rest. Such a singling out of the coordinate system contradicts the conventional concepts based on the theory of relativity. However, as we can see later on, this does not disagree with the well-known experimental facts.

For the sake of definiteness, we consider the elastic scattering of two particles. This scattering may be fully characterized by means of the scattering amplitude A which in the conventional theory depends upon two invariants: $s = \mathcal{P}^2, t = q^2$; here $\mathcal{P}^2$ is the square of the total energy of the system of particles, and q^2 is the square of the momentum transfer. If there exist inhomogeneities of space-time in the region a, then the amplitude A must, generally speaking, depend on two more invariants: $\alpha = a(\mathcal{P}n)$ and $\beta = a(qn)$, where a is the universal length, $\mathcal{P}$ is the total momentum of the system, q is the momentum transfer.

Thus, we have

$$A = A(s, t, \alpha, \beta). \qquad (1)$$

Now we suppose that the earth is almost at rest with respect to the space-time inhomogeneities so that in the laboratory ysystem tied to the earth the vector $n = (1, 0, 0, 0)$. In this system

$$\alpha = a\mathcal{P}_0 = a(E_0 + m), \qquad \beta = aq_0 = a(E_0 - E) \qquad (2)$$

where E_0 is the particle energy, and m is the mass of the target particle, q_0 is the energy transferred by collision.

Let us now consider another case in the same system - the case when both particles move, and their centre of gravity is at rest (the centre-of-mass system). Here we have

$$\alpha = a2W, \quad \beta = 0, \qquad (3)$$

where $2W$ is the total energy in the centre-of-mass system related to E_0 by means of the well-known formula

$$W = \tfrac{1}{2}m(E_0 + m). \qquad (4)$$

In the conventional theory the scattering amplitude A would have been the same in the corresponding experiments (i.e. in the experiments in which the invariants s and t are equal to each other, respectvely). In our theoretical scheme the parameters α and β turn out to be quite different and one should expect, therefore, that the results of measurements will also turn out to be essentially different.

As an example, we will be concerned with elastic scattering of electrons. If the universal length is $a = \frac{\hbar}{\mu c} = 2 10^{-14}$ cm then for the electrons with the energy of several GeV the parameters α and $\beta \simeq 1$.

In the case of colliding beams the same parameters are $\alpha = 0{,}02$ $\beta = 0$

If the universal length a is $\sqrt{g_F/\hbar c} = 6.10^{-17}$ cm (a characteristic length for weak interaction) then the situation becomes unfavourable for the present day accelerators, since in this case the electron energy must be of the order $E = 10^2$ GeV, , and for the colliding beams - of the order of several GeV.

The assumption that the amplitude A depends not only on the invariants s and t , but also on the parameters a and β contradicts no experiments since all today"s experiments are performed only in the same coordinate system - in the laboratory one - and the dependence of the amplitude A on s and t was never and nowhere verified - this is a purely theoretical assumption.

The second assumption made above consisted in the following. The laboratory system (earth) is singled out by the fact that it is almost at rest with respect to possible space-time inhomogeneities. This assumption is not, at least from the purely logical point of view, an absurd one. The case is that the accelerations of real bodies decrease sharply as the dimensions of the system become larger. For the particles in the atomic nucleus the accelerations are, by the order of a magnitude, 10^{32} cm/sec^2, for macroscopic bodies they are of the order of 10^8 cm/sec^2, for planets of 1 cm/sec^2 and for stars and galaxies they are only 10^{-8} cm/sec :

Therefore, for the atomic system almost any macroscopic system of bodies is a possible inertial frame of reference.

As the dimensions of the systems increase the number of real systems which may be an inertial frame of reference reduces very much. Therefore, one can assume that the frame of reference characteristic of galaxies may be a singled out system which gradually changes in the transition to still far distant galaxies.

It should be emphasized that the comparison of the experimental results on particle scattering in the laboratory system and in the centre-of-mass system is a straightforward verification of the assumption about the space-time homogeneity up to the scales determined by the quantity $a \simeq 1/E_0$. The purpose of this note was to empasize that the negative result of Michelson"s experiment may be due to the fact that the waves used in this experiment were extremely long.

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FIELD THEORY IN QUANTIZED SPACE-TIME

1. Present work is devoted to the problem of construction of a consequent quantum field theory without divergences. We start from the formulation of usual theory in p -representation, based on the equation [1,2]:

$$R(\lambda r) = \tilde{\mathfrak{L}}(\lambda r) + \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{dr'}{r' - i\epsilon} \tilde{\mathfrak{L}}(\lambda r - \lambda r') R(\lambda r') \qquad (1)$$

Here r is the one-dimensional invariant parameter, λ is a four-vector satisfying conditions: $\lambda^2 = \lambda_0^2 - \vec{\lambda}^2 = 1$, $\lambda_0 > 0$, $\tilde{\mathfrak{L}}(\lambda r)$ is the Fourier transform of the interaction Lagrangian $\mathfrak{L}(x)$ in the interaction representation: $\tilde{\mathfrak{L}}(\lambda r) = \int e^{-i\lambda r x}\mathfrak{L}(x)dx$ and the operator $R(\lambda r)$ is connected with the scattering matrix $S(\infty;-\infty)$ by $S(\infty;-\infty) = 1 + iR(0)$. Eq.(1) is equivalent to the Tomonaga-Schwinger one for the S-matrix if in the latter the planes of the form $\lambda x = \sigma$ are used as space-like surfaces. In fact, assuming

$$S(\sigma;-\infty) = 1 + \frac{1}{2\pi}\int \frac{dr}{r' - i\epsilon} R(\lambda r) e^{i\sigma r} \qquad (2)$$

$$L(\sigma) = \int \delta(\sigma - \lambda x)\mathfrak{L}(x)\,dx = \frac{1}{2\pi}\int e^{i\sigma r}\tilde{\mathfrak{L}}(\lambda r)\,dr \qquad (3)$$

we have from (1): $\dfrac{dS(\sigma;-\infty)}{d\sigma} = iL(\sigma)S(\sigma;-\infty)$

In what follows, for simplicity, $\mathfrak{L}(x) = g:\phi^3(x):$ and correspondingly

$$\tilde{\mathfrak{L}}(\lambda r) = \frac{g}{\sqrt{2\pi}}\int \delta^{(4)}[(\lambda r - k_1 - k_2 - k_3) :\phi(k_1)\phi(k_2)\phi(k_3): dk_1\,dk_2\,dk_3 \qquad (4)$$

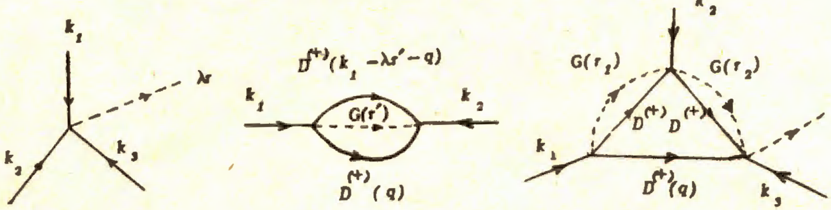
2. Let $R(\lambda r) = \sum_{n=1}^{\infty} R_n(\lambda r)$ be the expansion of the operator $R(\lambda r)$ in powers of g. Then from (1) it follows that $R_n(\lambda r)$ can be written in the form :

$$R_n(\lambda r) = \frac{1}{(2\pi)^{n-1}} \int \tilde{\mathfrak{L}}(\lambda r_1) \frac{dr_1}{r - r_1 - i\epsilon}\, \mathfrak{L}(\lambda r_2 - \lambda r_1) \cdots \frac{dr_{n-1}}{r - r_{n-1} - i\epsilon}\, \tilde{\mathfrak{L}}(\lambda r - \lambda r_{n-1}) \qquad (5)$$

or, afer the N -ordering

$$R_n(\lambda r) = \sum_{m=1}^{3n} \int \delta^{(4)}(\lambda r - k_1 - \dots - k_m)\, f_m^{(n)}(r; k_1, \dots, k_m) : \phi(k_1) \dots \phi(k_m) : dk_1 \dots dk_m \qquad (6)$$

where the coefficient functions $f_m^{(n)}$ are multiple integrals of the products $D^{(+)}(p) = \theta(p^0)\delta(p^2 - m^2)$ and functions $G(r') = \frac{1}{2\pi} \frac{1}{r - r' - i\epsilon}$. For an effective construction of normal products of the type (6) a special diagram technique has been developed whose sense becomes clear from Figs. 1-3 :



1. 2. 3.

We see that the diagrams in the considered formalism topologically differ from the Feynman graphs in the presence of additional dashed lines joining all the vertices. The particles in the intermediate states are real ones since for them the relations $p^2 = m_1^2$ $p^0 > 0$ are fulfilled. Taking into account this fact the dashed lines of the diagrams may be interpreted as the representation of certain "quasiparticles" interacting with real physical particles. Then from diagram 1 representing operator (4) it follows that the wave function of quasiparticle is the plane wave $\psi(x) = e^{-i\lambda r x}$ or in σ -representation $(\sigma = \lambda x)$, $\psi = e^{-i\sigma r}$ The vector λ is obviously the four-velocity of the quasiparticle and the parameters σ and r may be given the sense of its proper time and mass. The function $\psi(\sigma)$ obeys the equation $i \frac{d}{d\sigma} \psi(\sigma) = r\psi(\sigma)$, the propagator $G(r')$ being a "retarded" Green function of this equation.

Thus, the interaction mechanism for real physical particles can be conceived as multiple exchange of the same real particles and, in addition, of some quasiparticles. If the quasiparticles have mass $r \neq 0$ then any process involving real particles occurs with non-conservation of the 4-momentum, since in this case of free quasiparticle with 4-momentum λr is necessarily emitted. If real particles interact with massless quasiparticles $(r = 0)$ the energy and momentum are conserved. In space-time picture to the first case there corresponds a consideration of physical processes at time moments lying on the plane $\lambda x = \sigma$ to the second one- a consideration for $\sigma = \infty$. Thus, the description in terms of quasiparticles is a dynamic equivalent of the space-time description. Of extreme importance for a future consideration is the fact that ultraviolet divergences of the perturbation theory in the considered formalism occurs only in integrals over dashed lines for large $|r|$. Hence, it may be said that the usual field theory is bad because, accoding to this theory, quasiparticles transfer a too large 4-momentum in the interaction with real particles.

3. Now we postulate that in a true theory the quasiparticles four-momentum can not exceed a certain portion:

$$|\lambda r| = |r| \leqslant \frac{\pi\hbar}{2\ell} . \tag{7}$$

where ℓ is a new constant of the dimension of length ("fundamental length"). Divergences will not obviously occur if condition (7) is fulfilled. It is also clear that the existence of this condition requires the introduction of a new translation group on the straight line r with the composition rule

$$r_1 (+) r_2 = r_1 + r_2 - 2\pi[\theta(r_1 + r_2 - \pi) - \theta(-(r_1 + r_2) - \pi)] \tag{8}$$

(here and in what follows the system of units $\hbar = c = 2\ell = 1$ is used). Taking into account (7) and (8) eq. (1) can be written in the form:

$$R(\lambda; r) = \tilde{\mathfrak{L}}(\lambda; r) + \frac{1}{2\pi} \int_{-\pi}^{\pi} \tilde{\mathfrak{L}}(\lambda; r(-)r') \frac{dr'}{r' - i\epsilon} R(\lambda; r') \tag{9}$$

(the arguments of R and $\tilde{\mathfrak{L}}$ are arranged in (15) in a somewhat different way than earlier[x]).

As long as the Fourier transformation is now performed by exponentials like e^{inr} then the carried out transition to the new group of translations means the quantization of the quasiparticle proper time or, in other words, the quantization of the space-time metric along an arbitrary vector λ. Performing in (9) the Fourier transformation and introducing notations similar to (2) and (3) we have[xx]:

$$\frac{d}{dn} S(n; -\infty) = iL(n) S(n; -\infty) \tag{10}$$

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[x] Owing to the identity of the points $r = \pi$ and $r = -\pi$ the operators $R(\lambda; r)$ and $\tilde{\mathfrak{L}}(\lambda; r)$ must be obviously periodic functions r with period 2π.

[xx] It is not difficult to see that $S(\infty; -\infty) = 1 + iR(\lambda; 0)$.

$\frac{d}{dn}$ denotes the "differentiation" with respect to the integer n , which is determined for the functions $f(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{in\tau}\tilde{f}(\tau)\,d\tau$ as follows:

$$\frac{df}{dn} = \frac{1}{2\pi}\int_{-\pi}^{\pi} i\tau e^{in\tau}\tilde{f}(\tau)\,d\tau = \sum_{m=-\infty}^{\infty}\frac{d}{dn}(\delta_{nm})f(m) = \sum_{m\neq n}\frac{(-1)^{n-m}}{n-m}f(m) \tag{11}$$

It is easily seen that operation (11) possesses all properties of the ordinary differentiation. In particular, $\frac{d}{dn}(const) = 0$, $\frac{d(f_1 f_2)}{dn} = \frac{df_1}{dn}f_2 + f_1\frac{df_2}{dn}$ and so on. Therefore from eq. (10) and the equality $L(n) = L^+(n)$ is follows that $\frac{d}{dn}(S(n;-\infty)S^+(n;-\infty)) = 0$ from where we get $S(n;-\infty)S^+(n;-\infty) = 1$ provided $S(-\infty;-\infty) = 1$.

Eq. (10) may be written also in an "integral" form:

$$S(n;-\infty) = 1 + i\sum_{m=-\infty}^{\infty}\theta(n-m)L(m)S(m;-\infty)\,, \tag{12}$$

where $\theta(n-m) = \frac{1}{2\pi i}\int_{-\pi}^{\pi} e^{i\tau(n-m)}\frac{d\tau}{\tau - i\epsilon}$ owing to $\frac{d}{dn}\theta(n-m) = \delta_{nm}$ can be considered as an analog of the θ-function of a continuous argument. From eq. (12) it is seen that because of the smearing of $\theta(n-m)$ the usual causal properties of the S-matrix in the given formalism are sharply distorted in the region of small time intervals n . However for large intervals n the causality condition is approximately fulfilled.

4. In the aforementioned new scheme the Lagrangian $\tilde{\mathfrak{L}}(\lambda;\tau)$ has been considered as a periodic operator function τ disregarding its functional dependence on fields ϕ . To write down explicitly this dependence, first, it is necessary to redetermine the ϕ field itself in conformity with the new formalism. This can be easily done if one introduces into the ordinary Klein-Gordon equation the quantity $\sigma = \lambda x$ as one of the variables and then go over to $\sigma = n$ and $\frac{\partial}{\partial\sigma} = \frac{\partial}{\partial n}$. Having the equation of motion we can develop the second quantization formalism, determine N-product, construct the state vectors. It is important that all the quantities in the theory determined in p-representation become periodic functions of k with period $2\pi\lambda$ or, what is equivalent, of variables $k\lambda$ with period 2π . In particular, e.g. the function $\delta^{(4)}(k_1+k_2+k_3)$ is now written as

$$\delta^{(4)}(k_1(+)k_2(+)k_3) = \delta(k_1\lambda(+)k_2\lambda(+)k_3\lambda)\frac{1}{(2\pi)^3}\int e^{i\xi(k_1+k_2+k_3)}\tilde{\delta}(\xi\lambda)\,d\xi\,. \tag{13}$$

The operation $(+)$ means the summation either over the module of 2π (see (8)), or over that of $2\pi\lambda$. Taking into account (13) $\tilde{\mathfrak{L}}(\lambda;\tau)$ is represented in the form analogous to (14):

$$\tilde{\mathfrak{L}}(\lambda;\tau) = \frac{g}{\sqrt{2\pi}}\int\delta^{(4)}(\lambda\tau(-)k_1(-)k_2(-)k_3)\,d\Omega_{k_1}d\Omega_{k_2}d\Omega_{k_3} :\phi(k_1)\phi(k_2)\phi(k_3): \tag{14}$$

where $d\Omega_k = \theta(\pi - |k\lambda|)dk$. It is obvious that $d\Omega_{k(+)p} = d\Omega_k$ Inserting (14) into (9) we can solve (9) according to perturbation theory employing the earlier diagram technique (§ 2). But, in contrast to the usual theory all the vertex four-momenta are necessarily to be summed over the module of $2\pi\lambda$. For this reason, even if on the mass shell $r = 0$ in the considered scheme there may exist processes involving "real" quasiparticles with four-momentum $2\pi\lambda$ x), i.e. like physical particles, quasiparticles may be created and annihilated. Undoubtedly, because of the large mass $(\sim \frac{\hbar c}{l})$ of the quasiparticle such phenomena will occur only at high energies of physical particles.

The existence of processes involving "real" quasiparticles can be understood as a manifestation of the specific interaction of physical particles with the space-time itself having the discrete structure. In other words the discrete 4-space plays here the role of some new field capable to absorb and give back 4-momentum and quasiparticles are specific quanta of this field.

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x) From the mathematical point of view these processes are similar to the well-known "umklapprozesse" in crystals.

В.Г. Кадышевский

ОБ ОДНОМ ОБОБЩЕНИИ КВАНТОВОЙ ТЕОРИИ ПОЛЯ

§ 1.

Трудности с ультрафиолетовыми расходимостями в квантовой теории поля[1/] вот уже много лет являются стимулом для поисков такой схемы, в которой бы расходимости не возникали ни на одном из этапов вычислений и, наряду с этим, сохранялись основные принципы теории. В настоящей работе сделана попытка найти решение этой проблемы, исходя из геометрических соображений. Теория, которая строится ниже, оказывается буквально изоморфной обычной теории с точки зрения применяемого математического аппарата. Однако, что касается физических следствий и физической интерпретации новой схемы, то уже предварительный анализ показывает, что в области очень больших энергий наше описание физических явлений отличается от обычного описания в чрезвычайно сильной степени.

§ 2.

Прежде чем строить новую теорию, необходимо обычную теорию представить в такой форме, которая наиболее легко обобщается в интересующем нас направлении. Мы будем исходить из формулировки квантовой теории поля в p -представлении, основанной на уравнении[/1,2/]:

$$R(\lambda r) = \tilde{\mathcal{L}}(\lambda r) + \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{\mathcal{L}}(\lambda r - \lambda r')\, \frac{dr'}{r' - i\epsilon}\, R(\lambda r'). \qquad (1)$$

Здесь r –одномерный инвариантный параметр, λ –вектор, определяемый условием:

$$\lambda^2 = \lambda_0^2 - \vec{\lambda}^2 = 1\,, \qquad \lambda^0 > 0\ ; \qquad (2)$$

$\tilde{\mathcal{L}}(\lambda r)$ – фурье-образ лагранжиана взаимодействия $\mathcal{L}(x)$ в представлении взаимодействия:

$$\tilde{\mathcal{L}}(\lambda r) = \int e^{-i\lambda r x}\, \mathcal{L}(x)\, dx \qquad (3)$$

а оператор $R(\lambda r)$ связан с матрицей рассеяния $S(\infty;-\infty)$ соотношением

[1/] Как известно, ультрафиолетовые расходимости, возникающие в теории возмущений, есть проявление дефекта, характерного для всех формулировок квантовой теории поля: неоднозначности T -произведения операторов поля при совпадении их аргументов.

$$S(\infty;-\infty)=1+i\,R(0). \tag{4}$$

Уравнение (1) эквивалентно уравнению Томонага-Швингера для S -матрицы, если в последнем в качестве пространственно-подобных поверхностей использовать плоскости вида $\lambda x=\sigma$. Действительно, полагая

$$S(\sigma;-\infty)=1+\frac{1}{2\pi}\int\frac{dr}{r-i\epsilon}R(\lambda r)\,e^{i\sigma r}, \tag{5}$$

$$L(\sigma)=\int\delta(\sigma-\lambda x)\,\mathfrak{L}(x)\,dx=\frac{1}{2\pi}\int e^{i\sigma r}\,\tilde{\tilde{\mathfrak{L}}}(\lambda r)\,dr \tag{6}$$

будем иметь из (1)

$$\frac{dS(\sigma;-\infty)}{d\sigma}=i\,L(\sigma)\,S(\sigma;-\infty). \tag{7}$$

Если исключить из (1) и эрмитовски сопряженного к нему уравнения лагранжиан $\tilde{\tilde{\mathfrak{L}}}(\lambda r)$, то в результате получится соотношение, являющееся по существу ковариантной операторной записью уравнения Лоу /3/

$$R(\lambda r)-R^{+}(-\lambda r)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{dr'}{r'-i\epsilon}\left[R^{+}(-\lambda r')R(\lambda r-\lambda r')+\right. \tag{8}$$

$$\left.+R^{+}(\lambda r'-\lambda r)R(\lambda r')\right].$$

При $r=0$ (8) превращается в условие унитарности S -матрицы:

$$R(0)-R^{+}(0)=i\,R^{+}(0)\,R(0). \tag{9}$$

В дальнейшем мы для простоты ограничимся рассмотрением самодействующего скалярного поля $\phi(x)$, причем $\mathfrak{L}(x)$ выберем в виде:

$$\mathfrak{L}(x)=g:\phi^{3}(x): \tag{10}$$

Обобщение на случай других взаимодействий не содержит никаких принципиальных моментов.

Из определения (3) и соотношения (10) при условии, что

$$\phi(x)=\frac{1}{(2\pi)^{3/2}}\int e^{ikx}\phi(k)\,dk \tag{11}$$

будем иметь:

$$\tilde{\mathfrak{L}}(\lambda r)=\frac{g}{\sqrt{2\pi}}\int\delta^{(4)}(\lambda r-k_1-k_2-k_3):\phi(k_1)\,\phi(k_2)\,\phi(k_3):\cdot \tag{12}$$

$$\cdot\,dk_1\,dk_2\,dk_3$$

§ 3.

Обратимся теперь к теории возмущений. Пусть

$$R(\lambda r) = \sum_{n=1}^{\infty} R_n(\lambda r)$$

есть разложение оператора $R(\lambda r)$ по степеням g. Тогда из (1) следует, что $R_n(\lambda r)$ можно записать в виде:

$$R_n(\lambda r) = \frac{1}{(2\pi)^{n-1}} \int \tilde{\mathfrak{L}}(\lambda r_1) \frac{dr_1}{r - r_1 - i\epsilon} \tilde{\mathfrak{L}}(\lambda r_2 - \lambda r_1) \ldots \tag{13}$$

$$\ldots \frac{dr_{n-1}}{r - r_{n-1} - i\epsilon} \tilde{\mathfrak{L}}(\lambda r - \lambda r_{n-1})$$

или, после N -упорядочения,

$$R_n(\lambda r) = \sum_{m=1}^{3n} \int \delta^{(4)}(\lambda r - k_1 - \ldots - k_m) f_m^{(n)}(r; k_1, \ldots, k_m) : \phi(k_1) \ldots \phi(k_m) : dk_1 \ldots dk_m, \tag{14}$$

где коэффициентные функции $f_m^{(n)}$ являются многократными интегралами от произведений $D^{(+)}(p) = \theta(p_0)\,\delta(p^2 - m^2)$ и функций вида $G(r') = \frac{1}{2\pi} \frac{1}{r - r' - i\epsilon}$.

Для эффективного построения нормальных произведений вида (14) развита специальная диаграммная техника /2/. Проиллюстрируем ее на нескольких простых примерах.

Оператор $R_1(\lambda r) = \tilde{\mathfrak{L}}(\lambda r)$ графически изображается диаграммой рис. 1., т.е. каждому полю $\phi(k)$ ставится в соответствие сплошная линия с 4-импульсом k, направленным к вершине, а для соблюдения закона сохранения $\lambda r = k_1 + k_2 + k_3$ к этой вершине подсоединяется "выходящая" пунктирная линия с 4-импульсом λr (на массовой поверхности $r = 0$ эта линия отсутствует). Диаграмма собственной энергии частицы, отвечающая функции $f_2^{(2)}(r; k_1, k_2)$, является суммой двух графов а) и б) на рис. 2, причем сплошным внутренним линиям сопоставляются функции $D^{(+)}(p) = \theta(p_0)\ \delta(p^2 - m^2)$ от соответствующих аргументов, пунктирным внутренним линиям-пропагаторы $G(r') = \frac{1}{2\pi} \frac{1}{r - r' - i\epsilon}$, а по независимым переменным q и r' производится интеграция в бесконечных пределах.

В качестве еще одного примера на рис.3 изображена диаграмма, описывающая процесс рассеяния частиц в четвертом порядке теории возмущений. По-прежнему здесь сплошным внутренним линиям отвечают функции $D^{(+)}(p)$, а внутренним пунктирным $-G(r')$.

Мы видим, таким образом, что диаграммы в рассматриваемом формализме топологически отличаются от фейнмановских диаграмм наличием дополнительных пунктирных линий, соединяющих все вершины. При этом частицы в промежуточных состояниях являются реальными частицами, поскольку для них выполняются соотношения $p^2 = m^2$, $p_0 > 0$. Принимая во внимание это обстоятельство, можно интерпретировать пунктирные линии диаграмм как изображение некоторых квазичастиц, взаимодействующих с реальными физическими частицами. Тогда из диаграммы первого порядка, (рис. 1), отвечающей лагранжиану $\mathcal{L}(\lambda r) = \int^{\infty} e^{-i\lambda r x}\,\mathcal{L}(x)\,dx$, следует, что волновой функцией квазичастицы является плоская волна $\psi(x) = e^{-i\lambda r x}$, или в σ -представлении ($\sigma = \lambda x$)

$$\psi(\sigma) = e^{-ir\sigma}. \tag{15}$$

Вектор λ есть, очевидно, 4-скорость квазичастицы, а параметры σ и r имеют смысл ее собственного времени и массы. Функция $\psi(\sigma)$ подчиняется уравнению

$$i\,\frac{d\psi(\sigma)}{d\sigma} = r\,\psi(\sigma) \tag{16}$$

Легко убедиться, что пропагатор $G(r')$, который в диаграммной технике ставится в соответствие внутренней пунктирной линии, является "запаздывающей" функцией Грина уравнения (16) в r' -представлении. Действительно, полагая

$$G(\sigma-\sigma') = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{e^{-i(\sigma-\sigma')r'}}{r-r'-i\epsilon}\,dr' = i\,\theta(\sigma-\sigma')\,e^{-ir(\sigma-\sigma')}, \tag{17}$$

находим:

$$\left(i\,\frac{d}{d\sigma} - r\right) G(\sigma-\sigma') = -\delta(\sigma-\sigma'). \tag{18}$$

Таким образом, механизм взаимодействия реальных физических частиц можно представлять себе как многократный обмен их между собой такими же реальными частицами и, кроме того, некоторыми квазичастицами. Если квазичастицы обладают массой $r \neq 0$, то любой процесс с участием реальных частиц идет с несохранением 4-импульса, так как при этом обязательно излучается свободная квазичастица с 4-импульсом λr. Если же реальные частицы взаимодействуют с безмассовыми квазичастицами ($r=0$), то энергия и импульс сохраняются. В пространственно-временной картине первому случаю соответствует рассмотрение физических процессов в моменты времени, принадлежащие плоскости $\lambda x = \sigma$, второму случаю - рассмотрение при $\sigma = \infty$. Следовательно, описание в терминах квазичастиц является динамическим эквивалентом пространственно-временного описания. Чрезвычайно важным для дальнейшего обстоятельством является тот факт, что свойственные теории возмущений ультрафиолетовые расходимости в рассматриваемом формализме возникают только в интегралах по пунктирным линиям при больших $|r|$; т.е. можно сказать, что существующая теория поля плоха потому, что квазичастицам в процессе взаимодействия с реальными частицами разрешается переносить слишком большой 4-импульс.

§ 4.

Постулируем теперь, что в <u>правильной</u> теории 4-импульс квазичастицы не может превышать определенной порции:

$$|\lambda r| = |r| < \frac{\pi h}{2\ell} \,. \tag{19}$$

Здесь ℓ -новая постоянная размерности длины ("фундаментальная длина"). Очевидно, при соблюдении условия (19) расходимости в теории не возникнут. Однако схема, основанная лишь на допущении (19), еще не является математически последовательной, так как в рассмотренном выше формализме во всех соотношениях в p -представлении мы фактически имели дело с интегралами на группе трансляций на прямой r, относительно которой условие (19) не инвариантно. Поэтому нам необходимо перейти к интегрированию на такой группе, которая была бы согласована с (19). Единственной однопараметрической группой, удовлетворяющей этому условию и принципу соответствия при $\ell \to 0$, является группа трансляций на окружности радиуса h/ℓ с отождествленными диаметрально противоположными точками [4]. В пределе $\ell \to 0$ такая окружность перестает отличаться от прямой, что непосредственно следует и из явного вида новой операции сдвига (здесь и в дальнейшем принимается система единиц $h = c = 2\ell = 1$):

$$r_1 (+) r_2 = r_1 + r_2 - 2\pi [\theta(r_1 + r_2 - \pi) - \theta(-(r_1 + r_2) - \pi)] . \tag{20}$$

Уравнение (1) теперь можно записать в виде:

$$R(\lambda;r) = \tilde{\mathfrak{L}}(\lambda;r) + \frac{1}{2\pi} \int_{-\pi}^{\pi} \tilde{\mathfrak{L}}(\lambda;r(-)r') \frac{dr'}{r' - i\epsilon} R(\lambda;r') \tag{21}$$

(аргументы в R и $\tilde{\mathfrak{L}}$ расположены в (21) несколько иначе, чем раньше [2/]).

Из уравнения (21) получаются аналоги всех соотношений для оператора R, которые следовали ранее из (1). Так, например, вместо (8) теперь будем иметь:

$$R(\lambda;r) - R^{+}(\lambda;-r) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{dr'}{r' - i\epsilon} [R^{+}(\lambda;-r') R(\lambda;r(-)r') + R^{+}(\lambda;r'(-)r) R(\lambda;r')] , \tag{22}$$

откуда, в полной аналогии с (9),

$$R(\lambda;0) - R^{+}(\lambda;0) = i R^{+}(\lambda;0) R(\lambda;0) , \tag{23}$$

т.е. обычное условие унитарности для матрицы $S = 1 + i R(\lambda;0)$.

В отличие от обычного формализма преобразование Фурье в новой схеме должно производиться с экспонентами вида e^{irn}, где n -целое. Следовательно, пере-

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2/ Из-за тождественности точек $r = \pi$ и $r = -\pi$ операторы $R(\lambda,r)$ и $\tilde{\mathfrak{L}}(\lambda;r)$, очевидно, должны быть периодическими функциями r с периодом 2π.

ход к новой группе трансляций означает квантование собственного времени σ квазичастицы, или, другими словами, квантование метрики пространства-времени вдоль направления λ .

Пусть

$$f(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{in\tau} \tilde{f}(\tau)\, d\tau \qquad (24)$$

есть разложение Фурье некоторой функции $f(n)$. Введем понятие производной этой функции по целочисленному аргументу n , полагая, по определению,

$$\frac{df(n)}{dn} = \frac{1}{2\pi} \int_{-\pi}^{\pi} i\tau e^{in\tau} \tilde{f}(\tau)\, d\tau \,. \qquad (25)$$

Легко убедиться, что операция (25) обладает всеми свойствами обычного дифференцирования. Например,

$$\frac{d\,\mathrm{const}}{dn} \equiv 0 \,, \qquad (26a)$$

$$\frac{d(f_1(n) f_2(n))}{dn} = \frac{df_1(n)}{dn} f_2(n) + f_1(n) \frac{df_2(n)}{dn} \qquad (26б)$$

и т.д. Функцию $\frac{df(n)}{dn}$ можно выразить непосредственно и через $f(n)$, если подставить в (25) вместо $\tilde{f}(\tau)$ ее разложение в ряд Фурье:

$$\frac{df(n)}{dn} = \sum_{m=-\infty}^{m=\infty} \frac{d}{dn} (\delta_{nm}) f(m) = \sum_{m \neq n} \frac{(-1)^{n-m}}{n-m} f(m) \,. \qquad (27)$$

Из этой записи особенно отчетливо видно, что дифференцирование $\frac{d}{dn}$ является "нелокальной" операцией.

Рассмотрим далее функцию

$$\theta(n-m) = \frac{1}{2\pi i} \int_{-\pi}^{\pi} e^{i(n-m)\tau} \frac{d\tau}{\tau - i\epsilon} \,. \qquad (28)$$

Согласно (25)

$$\frac{d\theta(n-m)}{dn} = \delta_{nm} \,. \qquad (29)$$

Поэтому $\theta(n-m)$ можно рассматривать как аналог θ -функции от непрерывного аргумента. Однако в отличие от последней $\theta(n-m)$ "размазана" в окрестности точки $n = m$.

С помощью функции $\theta(n-m)$ можно ввести понятие "неопределенного интеграла" по целочисленному аргументу. Именно, положим

$$F(n) = \int_{-\infty}^{n} f(n')\,dn' = \sum_{m=-\infty}^{m=\infty} \theta(n-m)\, f(m). \qquad (30)$$

Очевидно (см. (29)),

$$\frac{dF(n)}{dn} = f(n).$$

Совершая преобразование Фурье в уравнении (21) и вводя обозначения, аналогичные (5) и (6)[3/],

$$S(n;-\infty) = 1 + \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{in\tau}\frac{d\tau}{\tau - i\epsilon} R(\lambda;\tau) \qquad (31)$$

$$L(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{in\tau}\, \overset{\infty}{\mathfrak{L}}(\lambda;\tau)\,d\tau \qquad (32)$$

будем иметь:

$$\frac{d\,S(n;-\infty)}{dn} = i\,L(n)\,S(n;-\infty). \qquad (33)$$

Уравнение (33) является уравнением Шредингера для S -матрицы, записанным в терминах дискретного собственного времени квазичастицы. Считая, что по-прежнему $L(n) = L^{+}(n)$, и принимая во внимание (26б), находим из (33):

$$\frac{d}{dn}\left(S(n;-\infty)\,S^{+}(n;-\infty)\right) = 0$$

откуда, с учетом (26а) и начального условия $S(-\infty;-\infty) = 1$, следует:

$$S(n;-\infty)\,S^{+}(n;-\infty) = 1. \qquad (34)$$

Уравнение (33) может быть представлено и в "интегральной" форме, если воспользоваться определением (30):

$$S(n;-\infty) = 1 + i\sum_{m=-\infty}^{\infty} \theta(n-m)L(m)\,S(m;-\infty). \qquad (35)$$

Из уравнения (35) видно, что обычные причинные свойства S -матрицы в данном формализме из-за "размазанности" функции $\theta(n-m)$ резко искажены в области малых интервалов времени n. Однако для больших интервалов n условие причинности приближенно выполняется.

3/ Из (31) следует, что $S(\infty;-\infty) = 1 + i\,R(\lambda;0)$.

С помощью (35) нетрудно установить, что матрица $S(\infty;-\infty)$ может быть записана в виде:

$$S(\infty;-\infty)=T\exp\{i\sum_{m=-\infty}^{\infty} L(n)\}, \qquad (36)$$

где символ T означает "упорядочение" по n с помощью функций $\theta(n-n')$. Интересно отметить, что у оператора $S(\infty;-\infty)$ сохраняется групповое свойство

$$S(\infty;-\infty)=S(\infty;n)\,S(n;-\infty), \qquad (37)$$

являющееся отражением условия причинности.

В заключение этого параграфа укажем, каким образом модифицируется формализм, описывающий квазичастицу (см. § 3), если перейти к $\sigma=n$. Очевидно, волновая функция квазичастицы теперь есть $\psi(n)=e^{-inr}$, причем

$$i\frac{d}{dn}\psi(n)=r\psi(n). \qquad (38)$$

Далее, в прямой аналогии с (18), имеем:

$$\left(i\frac{d}{dn}-r\right)G(n-n')=-\delta_{nn'},$$

откуда

$$G(n-n')=i\theta(n-n')\,e^{-ir(n-n')}=\int_{-\pi}^{\pi}G(r')\,e^{-ir'(n-n')}dr'=$$

$$=\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{dr'}{r(-)r'-i\epsilon}\,e^{-ir'(n-n')}.$$

§ 5.

В предыдущих построениях, относящихся к новой схеме, мы рассматривали лагранжиан $\tilde{\tilde{\mathfrak{L}}}(\lambda;r)$ лишь как периодическую операторную функцию r, игнорируя его функциональную зависимость от полей ϕ. Чтобы выписать указанную зависимость в явной форме, необходимо сначала переопределить само поле ϕ применительно к новому аппарату. Для этого представим обычное уравнение Клейна-Гордона для оператора $\phi(x)$ в виде системы уравнений в переменных[4/] $\sigma=x\lambda$ и $\xi=x-\lambda\sigma$:

[4/] В системе (39) дифференцирование по σ и ξ производится как по независимым переменным, однако, решение $\phi(\sigma,\xi)$, в силу уравнения (39а), фактически определено лишь на поверхности $\xi\lambda=0$ (ср. с (41)).

$$\begin{cases} \left(\Box_\xi - \dfrac{\partial^2}{\partial\sigma^2} - m^2\right)\phi(\xi,\sigma) = 0 & \text{(39а)} \\ i\lambda \dfrac{\partial\phi(\xi,\sigma)}{\partial\xi} = 0\ , & \text{(39б)} \end{cases}$$

а затем перейдем к $\sigma = n$, полагая $\dfrac{\partial}{\partial\sigma} = \dfrac{\partial}{\partial n}$. В результате будем иметь:

$$\begin{cases} \left(\Box_\xi - \dfrac{\partial^2}{\partial n^2} - m^2\right)\phi(\xi, n) = 0 & \text{(40а)} \\ i\lambda \dfrac{\partial\phi(\xi,n)}{\partial\xi} = 0\ . & \text{(40б)} \end{cases}$$

Решением системы (40а) является оператор

$$\phi(\xi,n) = \frac{1}{(2\pi)^{3/2}} \int_{-\pi}^{\pi} e^{-iEn}\, dE \int d^4k_\perp\, \delta(k_\perp\lambda)\, \delta(E^2 + k_\perp^2 - m^2)\, e^{ik_\perp\xi}\, \phi(k_\perp, E) \qquad (41)$$

где величину $\phi(k_\perp, E)$ нужно рассматривать как оператор поля в импульсном представлении (в системе отсчета, где $\vec{\lambda} = 0$, очевидно, $\phi(k_\perp, E) = \phi(\vec{k}, k_0)$, поскольку $E = k\lambda$, $k_\perp = k - \lambda E$.

Из (41) следует, что поле $\phi(k_\perp, E)$ периодично по переменной $E = k\lambda$ с периодом 2π, или, что эквивалентно, по 4-импульсу k с периодом $2\pi\lambda$. Очевидно, этим свойством должны обладать все величины, определяемые в p -представлении. Так, например, функция

$$\delta^{(4)}(k_1 + k_2 + k_3) = \delta(k_1\lambda + k_2\lambda + k_3\lambda)\, \frac{1}{(2\pi)^3} \int e^{i\xi(k_{1\perp} + k_{2\perp} + k_{3\perp})}\, \delta(\xi\lambda)\, d^4\xi \qquad (42)$$

теперь заменяется "периодической" δ -функцией

$$\delta^{(4)}(k_1 (+) k_2 (+) k_3) = \sum_{N=-\infty}^{\infty} \delta\,(k_1 + k_2 + k_3 - 2\pi N\lambda) =$$

$$= \delta\,(k_1\lambda\, (+)\, k_2\lambda\, (+)\, k_3\lambda)\, \frac{1}{(2\pi)^3} \int e^{i\xi(k_{1\perp} + k_{2\perp} + k_{3\perp})}\, \delta(\xi\lambda)\, d^4\xi = \qquad (43)$$

$$= \frac{1}{(2\pi)^4} \sum_{n=-\infty}^{\infty} \int \delta(\xi\lambda)\, d^4\xi\, e^{in\sum_{i=1}^{3} k_i\lambda + i\xi\sum_{i=1}^{3} k_{i\perp}}$$

Операция (+) здесь означает либо сложение по модулю $2\pi\lambda$, либо по модулю 2π (см. (20)).

Имея уравнение движения (40) и его решение (41), нетрудно развить формализм вторичного квантования, построить векторы состояний $|k_1, \ldots, k_n>$ и динамические переменные, определить N -произведение и т.д. В частности, лагранжиан взаимодействия $\overset{=}{\mathfrak{L}}(\lambda;r)$ при этом запишется в виде (ср. с (12)):

$$\overset{=}{\mathfrak{L}}(\lambda;r) = \frac{g}{\sqrt{2\pi}} \int \delta^{(4)}(\lambda r (-) k_1 (-) k_2 (-) k_3) : \phi(k_1)\phi(k_2)\phi(k_3) : d\Omega_{k_1}\, d\Omega_{k_2}\, d\Omega_{k_3} = \quad (44)$$

$$= \frac{g}{\sqrt{2\pi}} \sum_{N=-\infty}^{\infty} \int \delta^4(\lambda r - k_1 - k_2 - k_3 - 2\pi N\lambda) : \phi(k_1)\phi(k_2)\phi(k_3) : \prod_{i=1}^{3} d\Omega_{k_i},$$

где $d\Omega_k = \theta(\pi - |k\lambda|)\, d^4k$, $\qquad d\Omega_{k(+)p} = d\Omega_k$.

Подставляя лагранжиан (44) в уравнение (21), мы можем решать его по теории возмущений, применяя прежнюю диаграммную технику (§ 3). Но в отличие от обычной теории, все 4-импульсы в вершинах теперь необходимо складывать по модулю $2\pi\lambda$. По этой причине даже на массовой поверхности $r=0$ в рассматриваемой схеме могут существовать процессы с участием реальных квазичастиц с 4-импульсом $2\pi\lambda$ [5/], или, другими словами, в общую совокупность графов будут входить диаграммы с внешними пунктирными линиями. Например, в первом порядке по g появятся диаграммы, изображенные на рис. 4 и 5.

Диаграмму рис. 4 можно считать графическим изображением процесса распада квазичастицы с массой 2π на физические частицы, а диаграмму рис. 5 - изображением обратного процесса. Аналогичную интерпретацию допускают графы с внешними пунктирными линиями и в более высоких приближениях теории возмущений [6/].

Таким образом, квазичастицы с массой 2π становятся в какой-то степени равноправными с физическими частицами. Однако это обстоятельство никак не отражено в записи векторов состояний и лагранжиана $\overset{=}{\mathfrak{L}}(\lambda;r)$, поскольку до сих пор квазичастице не ставился в соответствие оператор поля. Ниже мы частично ликвидируем эту асимметрию между описанием физических частиц и квазичастиц.

[5/] Эти процессы в математическом отношении эквивалентны известному явлению "переброса" в кристаллах.

[6/] Из-за большой массы квазичастицы ($= \frac{1}{l}$) рассматриваемые явления, очевидно, могут происходить лишь при высоких энергиях физических частиц.

Именно, введем в рассмотрение оператор $a(N)$, полагая по определению

$$< N_1 \mid a(N) \mid N_2 > = \delta_{N_1+N,\, N_2}, \tag{45}$$

где переменная N имеет смысл "числа квазичастиц" с 4-импульсом $2\pi\lambda$. Принимая во внимание, что

$$< N_1 \mid N \mid N_2 > = \delta_{N_1,\, N_2} \tag{46}$$

и учитывая (45), легко убедиться в справедливости соотношения:

$$[\, a(N') \,, N \,] = N' \cdot a(N'). \tag{47}$$

Следовательно, $a(N)$ является оператором уничтожения N квазичастиц с 4-импульсом $2\pi\lambda$, и, соответственно, величина $a(N)^+ = a(-N)$ -оператором рождения N квазичастиц.

Теперь лагранжиан (44) можно представить в новом операторном виде:

$$\hat{\tilde{\mathfrak{L}}}(\lambda; r) = \frac{g}{\sqrt{2\pi}} \sum_{N=-\infty}^{\infty} \int \delta^{(4)}(\lambda r - k_1 - k_2 - k_3 - 2\pi N\lambda) \, . \tag{48}$$

$$a(N) : \phi(k_1)\, \phi(k_2)\, \phi(k_3) : d\Omega_{k_1}\, d\Omega_{k_2}\, d\Omega_{k_3},$$

предполагая, что матричные элементы от $\hat{\tilde{\mathfrak{L}}}(\lambda; r)$ должны браться между состояниями вида $|k_1, \ldots, k_n \,; N>$. В силу определения (45) эти матричные элементы будут пропорциональны функции $\delta^{(4)}(k' + 2\pi N'\lambda - k - 2\pi N\lambda)$, где k и k' - суммарные 4-импульсы "входящих" и "уходящих" физических частиц, а $2\pi N'\lambda$ и $2\pi N\lambda$ - аналогичные величины для квазичастиц.

Взятие матричных элементов от произведений лагранжианов $\hat{\tilde{\mathfrak{L}}}$, которые возникают при разложении оператора $\hat{R}(\lambda; r)$ по теории возмущений, облегчается существованием "группового" свойства у величин $a(N)$, вытекающим из (45):

$$a(N_1)\, a(N_2) \ldots a(N_m) = a(N_1 + N_2 + \ldots + N_m). \tag{49}$$

Например, во втором порядке по g, благодаря (49), сразу находим:

$$< k', N' \mid \hat{R}\, (\lambda; 0) \mid k, N > = \delta^{(4)}(k' + 2\pi N'\lambda - k - 2\pi N\lambda) \, .$$

$$\cdot \int_{-\pi}^{\pi} \frac{dr'}{-r' - i\epsilon} \int \delta^{(4)}(\lambda r' (+) q'_1 (+) q'_2 (+) q'_3)\, d\Omega_{q'_1} \ldots d\Omega_{q'_3} \, .$$

$$\langle k'| : \phi(q_1)\phi(q_2)\phi(q_3) : : \phi(q'_1)\phi(q'_2)\phi(q'_3) : |k\rangle .$$

Из (49) с учетом соотношения $a^+(N) = a(-N)$ следует, что

$$[a(N_1), a^+(N_2)]_- = 0. \qquad (50)$$

Поэтому описание квазичастиц в терминах $a(N)$ все же нельзя назвать вторичным квантованием. Эта величина скорее является аналогом c -числовой волновой функции e^{-irn} квазичастицы (см. § 3) в пределе $r=0$.

Как известно, в обычной теории амплитуды состояний $|k_1, ..., k_n\rangle$ являются собственными векторами оператора 4-импульса поля с собственными векторами оператора 4-импульса поля с собственными значениями $k_1 + ... + k_n$. В данной схеме мы имеем аналогичную ситуацию с новыми амплитудами состояний

$$\Phi = |k_1, ..., k_n : N\rangle. \qquad (51)$$

Действительно, из-за дискретности 4-пространства в направлении λ оператор 4-импульса поли теперь определяется лишь с точностью до аддитивной постоянной, кратной $2\pi\lambda$:

$$P_n = \int k_n\, d\Omega_k \delta(k^2 - m^2)\, \theta(k^0)\, \phi^{(+)}(k)\phi^{(-)}(k) + 2\pi N\lambda. \qquad (52)$$

Второе слагаемое в (52) можно интерпретировать как вклад в суммарный 4 импульс от квазичастиц, и поэтому

$$P_n \Phi = (k_1 + ... + k_n + 2\pi N\lambda)\,\Phi. \qquad (53)$$

Так как число N в (52) пробегает все целочисленные значения от $-\infty$ до ∞, то, вообще говоря, нам необходимо рассматривать квазичастицы как с положительной, так и с отрицательной энергией. Однако ввиду существования тождеств

$$\langle N_1 | \hat{R}(\lambda;0) | N_2\rangle \equiv \langle -N_2 | \hat{R}(\lambda;0) | -N_1\rangle \equiv$$
$$\equiv \langle N_1 - N_2 | \hat{R}(\lambda;0) | 0\rangle \equiv \langle 0 | \hat{R}(\lambda;0) | N_2 - N_1\rangle \qquad (54)$$

мы всегда вправе считать, что поглощение квазичастицы с отрицательной энергией в начальном состоянии эквивалентно рождению квазичастицы с положительной энергией в конечном состоянии и т.п. Конечно, здесь следует оговориться, что последовательное рассмотрение процессов поглощения и рождения квазичастиц, а также связанных с этим вопросом об устойчивости вакуума и одночастичного состояния, возможно лишь после проведения вторичного квантования "поля" квазичастиц.

§ 6.

Существование процессов с участием "реальных" квазичастиц может быть понято как проявление специфического взаимодействия физических частиц с самим пространством-временем, имеющим дискретную структуру. Другими словами, дискретное 4-пространство здесь выступает как некое новое поле, способное принимать и отдавать 4-импульс, а квазичастицы являются своеобразными квантами этого поля. Если квазичастицы и физические частицы рассматривать как единую замкнутую систему S, то для этой системы остаются справедливыми закон сохранения 4-импульса и релятивистская инвариантность. При низких энергиях квазичастицы не излучаются, и S фактически состоит лишь из физических частиц. При высоких энергиях игнорировать существование квазичастиц нельзя, а потому отдельно для физических частиц сохранение 4-импульса уже не имеет места, так же как и не имеет места из-за зависимости всех величин от λ релятивистская инвариантность. Подчеркнем особо, что вектор λ в развиваемой теории является не просто вспомогательным параметром, как это, например, имеет место в известном λ -процессе Вентцеля-Дирака[/5/], а может быть интерпретирован как 4-скорость некоторых специфических квазичастиц.

Может оказаться, если предлагаемая теория имеет отношение к действительности, что вектор λ следует считать совпадающим с 4-скоростью той системы отсчета, в которой покоится вся материя во вселенной. Не исключена также интерпретация этого вектора по Инграхаму[/6/]. Согласно[/6/], в качестве λ необходимо выбирать 4-скорость той системы отсчета, в которой производится эксперимент и задаются все 4-импульсы[7/]. При таком подходе описание физических процессов при высоких энергиях должно зависеть от наблюдателя, что может быть непосредственно проверено на опыте путем сравнения результатов экспериментов, произведенных в разных системах отсчета (например, в лабораторной системе и системе центра масс[/6/], ср. с[/7/]).

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7/ Другими словами, для каждого наблюдателя квазичастицы всегда покоятся.

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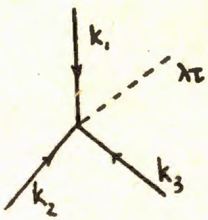
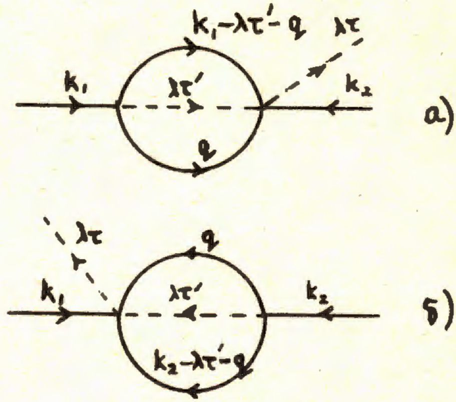


Р и с. 1.



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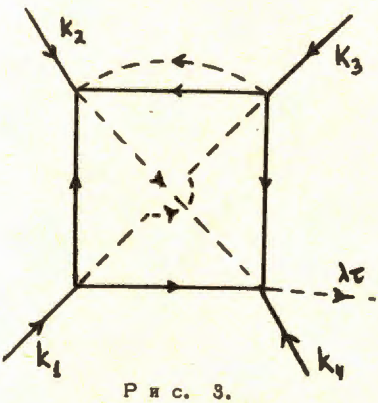

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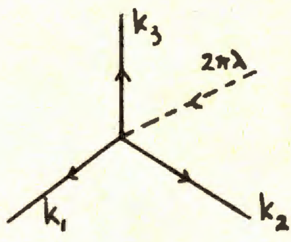

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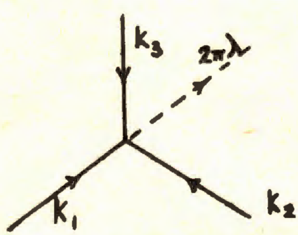

Р и с. 5.

Ju.A.Golfand

GENERALISATION OF QUANTUM MECHANICS TO THEORIES WITH DISCRETE TIME

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We consider one possibility of generalising the quantum mechanical equations for the case of theories with discrete time. In the limit of continuous time the proposed equation reverts to the conventional Schrödinger equation for the density matrix (correspondence principle). However the generalised equation has some peculiarities, the most interesting of which is violation of time reversibility.

We assume that the values of "time" are taken from the discrete set of numbers of the form $n\tau$, where n is an integer number ($-\infty < n < \infty$). and τ is a fundamental constant with the dimension of time. Such a quantisation of time is apparently not relativistically invariant, but in fact can be performed in a completely relativistically invariant way.

The transition from discrete time $n\tau$ to continuous time t is obtained by the substitution $n\tau = t$ and $\tau \to 0$.

As is well known quantum mechanics can be expressed in terms of the ψ -function (state vector) or in terms of the density matrix $\hat{\rho}$. Both forms are equivalent for pure states. On the other hand, the density matrix allows the consideration of pure states and mixed states, on an equal footing.

The first characteristic feature of the proposed scheme as compared with the usual one, is that it describes the states of a quantum mechanical system only by means of the density matrix but not by means of the ψ -function.

The density matrix must have the following properties:

Hermiticity

$$\hat{\rho}^{+} = \hat{\rho} \tag{1}$$

positive definiteness

$$\langle \psi | \hat{\rho} | \psi \rangle > 0 \tag{2}$$

normalisation condition

$$Tr\hat{\rho} = 1 \tag{3}$$

On the other hand, every matrix satisfying requirements (1)-(3) can be a density mantrix of any state of a quantum mechanical system.

If is easy to see that the class of all possible density matrices satisfying conditions (1)-(3) is closed with respect to the following two transformations:

(i) unitary transformations

$$\hat{\rho} \to U \hat{\rho} U^{+}$$

(ii) linear transformations of the form

$$\hat{\rho} = a_1 \hat{\rho}_1 + a_2 \hat{\rho}_2 ,$$

where a_1 , a_2 are positive numbers satisfying

$$a_1 + a_2 = 1$$

The properties (i) and (ii) allow us to construct the equation of motion for the density matrix in the case of quantized time. The simplest case of such an equation is

$$\hat{\rho}[(n+1)\tau] = U \hat{\rho}(n\tau) U^{+} , \qquad (4)$$

where U is an unitary operator. This equation, in an obvious way, satisfies the correspondence principle when $\tau \to 0$. However, by means of properties (i) and (ii), we can construct an equation of a more general form. We introduce two positive constants α and β satisfying $\alpha + \beta = 1$ and two unitary operators U and V . Then we postulate the equantion of motion for the density matrix $\hat{\rho}$ of the form

$$\hat{\rho}[(n+1)\tau] = \alpha U \hat{\rho}(n\tau) U^{+} + \beta V \hat{\rho}(n\tau) V^{+} . \qquad (5)$$

Equation (5), due to the properties (i) and (ii), transforms the density matrix $\hat{\rho}(n\tau)$ into the matrix $\hat{\rho}[(n+1)\tau]$ satisfying conditions (1)-(3).

We consider now the limit of continuous time in equation (5). When $\tau \to 0$ the unitary operators U and V differ from unit operator by infinitely small terms of order τ . We can write

$$U = 1 - iA\tau$$
$$V = 1 - iB\tau \qquad (6)$$

where A and B are Hermitian operators. By means of (6) and (5) we obtain the relation, in which the terms of second order in τ are dropped

$$\hat{\rho}[(n+1)\tau] = \hat{\rho}(n\tau) - i\tau[(\alpha A + \beta B), \hat{\rho}(n\tau)] .$$

If we substitute t instead of $n\tau$ and allow $\tau \to 0$ we obtain the differential equation

$$i \frac{d\hat{\rho}}{dt} = [(\alpha A + \beta B), \hat{\rho}] . \qquad (7)$$

The equation (7) has the form of the conventional quantum mechanical equation for the density matrix. The role of the Hamiltonian H is played by the Hermition operator $\alpha A + \beta B$ $(\hbar = 1)$. It is significant that we do not impose on the constants α and β any condition that they be small.

The equation of motion (5) has some peculiar properties resulting from the discreteness of time. One of them is the violation of time reversibility. The simplest way to demonstrate this irreversibility is to consider "entropy" which we define by the relation.

$$S(\rho) = -Tr\{\hat{\rho} \ln \hat{\rho}\} \tag{8}$$

The "entropy" S is a state function, i.e. depends only on the density matrix $\hat{\rho}$. It can be shown, that the "entropy" (8) increases monotonically with time due to the equation of motion (5). The entropy increase during the time interval τ is of order τ^2 , and the effect of entropy increase vanishes in the limiting case of continuous time. This corresponds to the invariance of the Schrödinger equation with respect to reversal of the sign of time.

One can say that equation of motion (5) possesses a dissipative character. This dissipation appears only as a result of the discretness of time and it vanishes completely in the limiting case of continuous time.

In conclusion, it should be noted that the increase of entropy mentioned here is not identical with the general law of entropy increase for macroscopic systems.

As is known the foundation of the law of entropy increase is an as yet unsolved problem and the attempts to infer this law in the framework of classical or quantum mechanics have been unsuccessful. However, the question of the connection of the law of entropy increase with a possible quantization of space-time should be a matter for further investigation.

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FIELD THEORY WITH NONLOCAL INTERACTION

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1. At the present time, the most widespread approaches in the theory of elementary particles are based on the following three basic postulates:

a) relativistic invariance,

b) the quantum nature of particles

c) locality, which expresses the condition of microscopic causality.

This last condition has to a significant degree a mathematical character, being an extrapolation of the basic classical condition of causality, the impossibility of realizing events occuring at a point, on which is based the condition of microscopic causality, leads to the fact that it is not permissible to consider as completely excluded the possibility of the violation of this condition in small regions of space-time. This most important question could be decided in principle experimentally: for example , by verifying dispersion relations.

On the other hand, the hopes for removing the difficulties from a local theory. (the problems of internal consistency, unrenormalizable interactions) have for a long time been connected with the changing of existing conceptions in small regions of space-time and the appearance of a new fundamental constant, the elementary length ℓ . In as much as the appearance of the fundamental constants c and $\hbar$ is connected with the first two postulates, it is natural to expect, that the constant ℓ appears because of a violation of the postulate of locality that is, in going over to a non-local field theory (NFT).

The consistent construction of NFT must be based on new physical ideas, possibly related to the replacement of the spacetime structure "in the small", The positive content of these ideas at the present time is to a large extent conjectural. However, even at the present time, it is possible to attempt the construction of a phenomenological NFT, compatible with postulates a) and b) and at large distances reducing to the usual theory. This may be accomplished by introducing into the interaction several given functions (form factors).

The possibility itself of realizing this program is doubtful due to the presence of a large number of difficulties, with which going outside the framework of local concepts is connected: that is, mathematical compatibility, the unitarity of the scattering matrix, macroscopic causality, convergence, etc.

The results show that the basic difficulties of NFT can be eliminated. Their appearance is connected with too direct an application of the apparatus of the usual theory to NFT.

2. It is convenient to put into the basis of the theory the following expression for the S -matrix.

$$S = \tilde{T}_\lambda \exp\left(i\int_0^1 d\lambda\, \sigma(\lambda)\right) \qquad (1)$$

which is unitary, relativistically invariant and which possesses the correct local limit. Here $\sigma(\lambda) = \int d^4x\, \mathfrak{L}_{int}(x)$ is the action, corresponding to the local Lagrangian $\lambda\ \mathfrak{L}_{int}$, in which by some means or other the form factors are introduced. $\tilde{T}_\lambda$ - is the "antichronological" ordering with respect to λ operator.

The usual method of introducing a form factor corresponds to replacing each vertex by some given function. In this case, the matrix element (1) differs from the corresponding local expression, excepting the form factors, only in the condition that we exclude the residues and integrals along the cuts of the form factors. The analysis of the convergence of such expressions shows, that the method considered ensures the convergence only of the matrix elements which are logarithmically divergent in the local theory and therefore is not appropriate, in particular, for the description of unrenormalizable interactions. Moreover, additional singularities for small values of the kinematic invariants appear in the matrix elements, which actually destroy causality.

There exists a whole series of other methods of introducing form factors, compatible with postulates a) and b) and free of the above difficulties[x]. The simplest possibility consists in replacing the field operators by unphysical operators, whose contractions are regular on the light cone,

$$\tilde{D}_F = D_F - \int_{\Lambda^2}^{\infty} d\kappa^2 \rho(\kappa^2) D_F(\kappa^2)$$

(~ signifies this replacement). In doing this, the action has the form

$$\sigma(\lambda) = P\,\tilde{\sigma}(\lambda)\,P\,, \qquad (2)$$

where P is the projection operator onto the space of physical states. In so doing, retarded functions appear in the theory only in a regularized form, which guarantees convergence. The S -matrix (1), (2) differ from the causal (but

[x] I.E.Tamm (private communication) suggested a method based on the use of a momentum space of non-zero curvature, which removes the divergences with respect to angles.

non unitary) Pauli-Villars regularized S -matrix

$$P\,T_{\lambda}\exp\left(i\int_0^1 d\lambda\,\tilde{\sigma}(\lambda)\right)P$$

only by threshold terms, different from zero for large values of the kinematic invariants ($s, t, \ldots \gtrsim \Lambda^2$).

For this reason the investigated variant is completely causal, as long as the values of s, t etc. do not exceed the threshold value. This is somewhat (but not completely) equivalent to fulfilling the condition of macroscopic causality. The investigation of the condition of causality in its space-time aspect is made difficult by the absence of an exact formulation of the physical condition of causality. In any case, the assertion of Strucklberg and Wanders concerning the incompatibility of unitarity and causality is refuted by the straightforward construction of a macrocausal vertex function in the third order of perturbation theory (A.N.Lesnov, personal communication).

3. In NFT it is also possible to introduce a spacetime description. To this end, we may take the incomplete matrix $S(\tau)$ up to the surface τ , in which the quantity σ is replaced by $\int^{\tau} d^4x\,\mathcal{L}_{int}(x)$. The evolution of the state from τ' to τ is determined by the matrix $S(\tau,\tau') = S(\tau)\,S^{+}(\tau')$.

The effective Hamiltonian of NFT has the form

$$\mathcal{H}(x,\tau) = -\int_0^1 d\lambda\,S(\tau)\,\mathcal{L}_{int}(x)\,S^{+}(\tau). \qquad (3)$$

It in fact depends on the surface τ and automatically satisfies Bloch"s condition of compatibility. The latter coincides with the condition of microscopic causality $[\mathcal{H}(x),\mathcal{H}(y)] = 0, (x-y)^2 < 0$ only for Hamiltonians not depending on τ . It is impossible to consider such Hamiltonians in NFT.

The field operators in the Heisenberg representation are not connected in a unitary fashion with the "in" operators and thus do not commute outside the light cone. The operators of the integrals of motion (energy-momentum, charge) are obtained just as in a local theory by inserting the "in" -operators in the corresponding free expressions. The wave functions of the stationary states are correspondingly determined; in particular, the vacuum.

The Green"s function in NFT

$$D'_F(x) = -i\,\langle 0|\,T(\phi(x), \phi(y))|0\rangle \qquad (4)$$

is relativistically invariant in consequence of the relation $\langle 0|[\phi(x), \phi(y)]|0\rangle = 0$, $(x-y)^2 < 0$ and satisfies the Lehman representation. It in reality differs from the function

$$D''_F(x) = -i\,\langle 0|\,S^{+}\,T(\phi_{in}(x)\,\phi_{in}(y)\,S)|0\rangle \qquad (5)$$

which is directly connected with the S -matrix and which has acausal singularities.

In NFT the LSZ relation for the scattering matrix element can be obtained in the usual way; the proof of its relativistic invariance exists in the literature.

In conclusion we remark that nonlocal effects can show up not only at high energies, but also at ultra high compression, when the average distance between the particles is of the order of the elementary length. This could tell something about the properties of bodies of large mass and of the universe as a whole.

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ДОПОЛНЕНИЯ

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ФИЗИКА

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О РАСПРОСТРАНЕНИИ СИГНАЛОВ В НЕЛИНЕЙНОЙ ТЕОРИИ ПОЛЯ

(Представлено академиком Д. В. Скобельцыным 3 XII 1951)

§ 1. Введение

Для современной теории поля характерны две основные черты: а) линейность уравнений и б) точечность частиц. Вместе с тем, подобные теории приводят к известным расходимостям ряда физических величин (нулевая энергия поля, собственная энергия частиц). Эти расходимости приходится преодолевать более или менее искусственными приемами (например, метод „перенормировки“ массы и заряда ([1])). Само существование этих расходимостей указывает на несостоятельность теории применительно к малым областям пространства и времени и на необходимость радикального изменения теории для малых промежутков времени и малых расстояний.

Поскольку указанные выше общие черты современной теории формулируются еще до квантования величин, то они являются „классическими“, а не „квантовыми“ особенностями теории. В силу этого возможные физические последствия отказа от этих черт могут быть, в духе принципа соответствия, изучены еще в рамках классического рассмотрения.

Ранее нами ([2-4]) были исследованы последствия отказа от точечности частицы и было показано, что теория, допускающая неточечное взаимодействие, неминуемо ведет к распространению взаимодействия (мы будем в дальнейшем говорить «сигналов») со скоростями, превышающими скорость света*. Теперь мы намерены рассмотреть последствия отказа от линейности теории.

Оказывается, что нелинейные теории поля, так же как и неточечные линейные теории, ведут к поразительному факту распространения сигнала со скоростью, большей скорости света в пустоте. Поэтому кажется весьма вероятным, что такое «аномальное» распространение сигналов в малых областях пространства — времени должно быть характерной чертой будущей теории поля.

§ 2. Нелинейная теория скалярного или псевдоскалярного мезонного поля

Лагранжиан для этого случая поля может быть написан как функция двух инвариантов:

$$K=\frac{1}{2}\left[\left(\frac{\partial\psi}{\partial t}\right)^2-(\nabla\psi)^2\right],\quad I=\frac{1}{2}\psi^2, \tag{1}$$

* Рассмотренная в работах ([2-4]) неточечная теория взаимодействия полей по физической сущности очень близка к квантовой теории М. А. Маркова ([5, 6]) протяженных частиц. Недавно, на много лет позднее наших работ, сходное исследование произведено Юленбеком и Пэй ([7]).

так что

$$L = L(K, I). \qquad (2)$$

Из релятивистски инвариантного вариационного принципа

$$\delta \int L(K, I)\, dt\, dx = 0 \qquad (3)$$

нетрудно получить следующее уравнение для поля ψ:

$$\frac{\partial L}{\partial K}\left[-\frac{\partial^2\psi}{\partial t^2} + \nabla^2\psi\right] - \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial K}\right)\frac{\partial\psi}{\partial t} + \nabla\left(\frac{\partial L}{\partial K}\right)\nabla\psi + \frac{\partial L}{\partial I}\psi = 0, \qquad (4)$$

которое очевидным образом инвариантно относительно преобразования Лорентца.

Имея в виду, что

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial K}\right) = \frac{\partial^2 L}{\partial K^2}\frac{\partial K}{\partial t} + \frac{\partial^2 L}{\partial K\, \partial I}\frac{\partial I}{\partial t}, \qquad (5)$$

$$\nabla\left(\frac{\partial L}{\partial K}\right) = \frac{\partial^2 L}{\partial K^2}\nabla K + \frac{\partial^2 L}{\partial K\, \partial I}\nabla I, \qquad (5')$$

мы легко получим из (4) уравнение для поля ψ в явном виде. Мы выпишем это уравнение для одномерного пространства (t, x). Обозначая первые производные через $p = \frac{\partial\psi}{\partial t}$, $q = \frac{\partial\psi}{\partial x}$, а вторые через $r = \frac{\partial^2\psi}{\partial t^2}$, $s = \frac{\partial^2\psi}{\partial t\, \partial x}$, $t = \frac{\partial^2\psi}{\partial x^2}$ и $\frac{\partial^2 L}{\partial K^2} \Big/ \frac{\partial L}{\partial K} = \alpha$, $\frac{\partial^2 L}{\partial K\, \partial I} \Big/ \frac{\partial L}{\partial K} = \beta$, $\frac{\partial L}{\partial I} \Big/ \frac{\partial L}{\partial K} = \gamma$, найдем:

$$Ar + 2Bs + Ct + R = 0, \qquad (6)$$

где $A = -(1 + \alpha p^2)$, $B = \alpha pq$, $C = (1 - \alpha q^2)$ и $R = [\gamma - 2\beta K]$.

Обозначая далее направление характеристики через $\xi = dx/dt$, получим для ξ уравнение:

$$A\xi^2 - 2B\xi + C = 0, \qquad (7)$$

которое имеет решение

$$\xi = \frac{\pm\sqrt{1 + 2\alpha K} - \alpha pq}{(1 + \alpha p^2)}, \qquad (8)$$

или, при малых $\alpha(K, I)$:

$$\xi = \pm 1 \mp {}^1\!/_2\, \alpha (p \pm q)^2 + \ldots \qquad (9)$$

Если теперь представить себе, что на отрезке $a < x < b$ в момент $t = 0$ задано некоторое начальное состояние $\psi, p, q = \partial\psi/\partial x$, то, как непосредственно видно из (9), при $\alpha < 0$ $|\xi| > 1$, т. е. $|dx/dt| > 1$. Т. е. состояние из точек $a < x < b$ будет распространяться со скоростью dx/dt, большей скорости света в пустоте ($c = 1$), кроме исключительного случая $p = \mp q$.

§ 3. Нелинейная теория электромагнитного поля

Рассмотрим теперь тот же вопрос применительно к нелинейной теории М. Борна ([8]). В этом случае лагранжиан может быть написан в виде:

$$L = L(K, I^2), \qquad (10)$$

где $K = {}^1\!/_2(\mathbf{E}^2 - \mathbf{H}^2)$, $I = (\mathbf{E}, \mathbf{H})$, а $\mathbf{E}$ и $\mathbf{H}$ имеют смысл напряженностей электромагнитного поля. Варьируя электромагнитные потенциалы

A, φ $(\mathbf{E} = -\partial \mathbf{A} / \partial t - \nabla\varphi, \mathbf{H} = \operatorname{rot} \mathbf{A})$, получим

$$\frac{\partial \mathbf{D}}{\partial t} = \operatorname{rot} \mathbf{B}, \quad \operatorname{div} \mathbf{D} = 0, \tag{11}$$

где

$$\mathbf{D} = M\mathbf{E} - N\mathbf{H}, \quad \mathbf{B} = M\mathbf{H} + N\mathbf{E} \tag{12}$$

и $M = \partial L / \partial K$, $N = \partial L / \partial I$.

Вторая группа уравнений

$$\frac{\partial \mathbf{H}}{\partial t} = -\operatorname{rot} \mathbf{E}, \quad \operatorname{div} \mathbf{H} = 0 \tag{13}$$

получается сама собой из определения $\mathbf{E}$, $\mathbf{H}$ через $\mathbf{A}$ и φ. В явном виде уравнения для $\mathbf{E}$ и $\mathbf{H}$ получим, если выразить производные от $\mathbf{D}$ и $\mathbf{B}$ через производные от $\mathbf{E}$, $\mathbf{H}$, K и I.

В дальнейшем рассмотрим одномерный случай, именно $\mathbf{E} = (E_x, 0, 0)$, $\mathbf{H} = (0, H_y, 0)$ и $E_x = E(t, z)$, $H_y = H(t, z)$. Тогда уравнения (11), (13) примут простой вид:

$$(1 + \alpha E^2) \frac{\partial E}{\partial t} + \alpha EH \frac{\partial E}{\partial z} - \alpha EH \frac{\partial H}{\partial t} + (1 - \alpha H^2) \frac{\partial H}{\partial z} = 0, \tag{14}$$

$$\frac{\partial E}{\partial z} + \frac{\partial H}{\partial t} = 0, \tag{14'}$$

где попрежнему $\alpha = \frac{\partial^2 L}{\partial K^2} \Big/ \frac{\partial L}{\partial K}$. Уравнение для характеристического направления $\xi = dz / dt$ будет теперь:

$$(1 + \alpha E^2) \xi^2 + 2\alpha EH\xi + (1 - \alpha H^2) = 0, \tag{15}$$

откуда

$$\xi = \frac{\pm \sqrt{1 + 2\alpha K} - \alpha EH}{(1 + \alpha E^2)}, \tag{16}$$

или, при малых α:

$$\xi = \pm 1 \mp {}^1/_2 \, \alpha (E \pm H)^2 + \ldots, \tag{16'}$$

т. е. и в этом случае мы приходим к возможности сверхсветовых сигналов.

Интересно еще и то обстоятельство, что в подобных нелинейных теориях нельзя заранее исключить и такую ситуацию, когда при определенных значениях E, H или ψ характеристические направления сделаются мнимыми, так что уравнения поля станут в (t, x) уравнениями эллиптического типа, а это будет означать, что понятие причинной последовательности событий потеряет свой смысл, и мы будем иметь дело со связанным «комком» событий, которые взаимно друг друга обусловливают, но не следуют одно за другим. Может ли на самом деле возникнуть нечто подобное, например, «внутри» частиц,— это остается пока открытым вопросом.

Известно, что квантование нелинейных уравнений поля представляет нерешенную математическую задачу. Из изложенного видно, что трудности формулировки квантовых условий для нелинейного поля имеют не только математическую природу. Ввиду возможности появления взаимодействия, распространяющегося со скоростью больше c, метод Гамильтона применительно к нелинейному полю будет столь же несостоятелен, как он несостоятелен по отношению к неточечным взаимодействиям.

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The Non-linear Field Theory and the Theory of Relativity.

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CONTENTS. — **1.** Introduction. – **2.** A classification of non-linear theories. **3.** – Equations of class A. – **4.** Equations of class B.

1. – Introduction.

In recent years, the development of the quantum field theory has been marked by successful application of the method of renormalizing the mass of the particles. However, this method is only a mathematical procedure that makes it possible to circumvent phenomena and processes relating to very high frequencies and to very small scales. In addition, this method is not always applicable.

For this reason, along with the development of methods of renormalization, various theories were developed in which the divergence of the self-energy of the particles is excluded not by additional methods but on the basis of the initial physical content of the theory.

The following two trends are represented by an especially large number of works: 1) the non-local field theory, and 2) the non-linear field theory.

In this report we shall consider the main peculiarities of the second trend, the non-linear field theory.

The introduction of a certain field scale φ_0 is characteristic of this theory: thus, indirectly, there is introduced a certain length $s_0 = \sqrt{g/\varphi_0}$ (where g is the charge of the particle), length which may conditionally be regarded as « the size of the particle ».

The following main problem is then discussed: to what extent is the non-linear field theory capable of eliminating, in principle, the above-mentioned difficulties of the quantum theory?

It is not necessary to analyze the concrete form of the theory in order to investigate this aspect of the problem.

It is sufficient to examine its general features.

2. – A Classification of Non-linear Theories.

Non-linear equations of any physical field (for example, of a meson, electromagnetic, and any other such field) may be obtained from the variational principle which is invariant with respect to Lorentz transformation

$$\delta\int L(K, I, ...)\,dx\,dt = 0\,, \tag{1}$$

where L is the Lagrange function, and $K, I, ...$ are field invariants. We shall limit ourselves to equations not above the second order, and therefore $K, I, ...$ are assumed to be composed of components of the field and its first derivatives. Then the fields arising from equation (1) have the following form (*):

$$A\frac{\partial^2\varphi}{\partial t^2} + 2B\frac{\partial^2\varphi}{\partial t\,\partial x} + C\frac{\partial^2\varphi}{\partial x^2} + D = 0\,. \tag{2}$$

With the aim of correspondence to non-linear theory, the Lagrangian should be selected in such a way that in the region where $|\varphi| \ll \varphi_0$, $|\partial\varphi/x|$, $|\partial\varphi/\partial t| \ll \varphi_0/s_0$, the equations become linear. This is a very slight limitation on the selection of the value L.

Let us now consider the velocity of signal propagation in non-linear theory, that is the velocity of the front of a wave bounded by a weak discontinuity. The value of this velocity ξ is equal to the slope of the characteristics and is determined from the equation:

$$A\xi^2 - 2B\xi + C = 0\,. \tag{3}$$

It is obvious that this velocity will be a function of the field φ and of its derivatives.

With respect to the velocity of signal propagation, all the Lawrence-invariant equations may be divided into two classes: A) equations, the characteristics of which do not differ from the characteristics of linear equations $|\xi| = 1$; B) those where $|\xi|$ is in general not equal to 1.

To the first class belongs for example an equation of the following type:

$$-\frac{\partial^2\varphi}{\partial t^2} + \frac{\partial^2\varphi}{\partial x^2} + D(\varphi) = 0\,, \tag{4}$$

(*) We limit ourselves to the one dimensional case, which is entirely sufficient for our purposes.

considered by SCHIFF in a number of his works [1, 4]. To the second class belong the equations of the electromagnetic field, equations which were suggested long ago by M. BORN [2]. These equations follow from the variational principle (1) when $K = \frac{1}{2}(\varepsilon^2 - H^2)$ and $I = (\varepsilon H)^2$.

For the one dimensional case these equations are:

$$(1 + \alpha\varepsilon^2)\frac{\partial\varepsilon}{\partial t} + \alpha\varepsilon H\,\frac{\partial\varepsilon}{\partial x} - \alpha\varepsilon H\,\frac{\partial H}{\partial t} + (1 - \alpha H^2)\frac{\partial H}{\partial x} = 0\,, \tag{5}$$

$$\frac{\partial\varepsilon}{\partial x} + \frac{\partial H}{\partial t} = 0\,, \tag{5'}$$

$c = 1$ is the velocity of light in the vacuum, and $\alpha = (\partial^2 L/\partial K^2)/(\partial L/\partial K)$. HEISENBERG [3] recently considered a similar equation for the meson field φ. It should be noted that the signal velocity ξ of these equations is obtained in the following form:

$$\xi = \frac{\pm\sqrt{1 + 2\alpha K} - \alpha pq}{(1 + \alpha p^2)}\,. \tag{6}$$

In the case of an electromagnetic field $K = \frac{1}{2}(\varepsilon^2 - H^2)$, $p = \varepsilon$, $q = H$. In the case of a meson field $K = \frac{1}{2}(p^2 - q^2)$, $p = \partial\varphi/\partial t$, $q = \partial\varphi/\partial x$ and ξ may be either less than the velocity of light in the vacuum or it may be greater.

3. - Equations of Class A.

Non-linear equations having characteristics $\xi = \pm 1$ naturally represent a theory entirely compatible with the theory of relativity. There may also be found variants (the selection of the function L) which give the limited self-energy of the particles. Therefore, such equations may serve as a basis for a classical non-linear field theory. These equations are also compatible with the usual rules of quantization:

$$[\dot\varphi(x, t), \varphi(x', t)] = i\delta(x - x')\,. \tag{7}$$

Nevertheless, they do not lead to a consistent quantum field theory that does not contain divergencies. The fact of the matter is that the zero energy E_0 of the field, in the case of non-linear theory, is not only infinite (as is the case also in linear theory), but is also non-additive to the energy of the excited states.

In other words, the energy of a non-linear field cannot be represented as the sum of the energy of excitation ε and the energy of the vacuum E_0.

For this reason SCHIFF introduced a method of quantization of a non-linear field, the essence of which consists in substituting the continuous functions $\varphi(x)$ by the assembly of values φ_s in the nodes of a certain spatial lattice with a period l [4].

In the linear theory, this method would not be of any fundamental importance, since all the results of the theory would be retained.

In the non-linear theory, the self-values of the energy of a field tend to ∞ as $\exp[-3(n-2)/(n+2)]$ for $U = \varkappa\varphi^n$.

Whereas when $l \neq 0$, from the equations of the field

$$\ddot{\varphi}_s + \frac{\partial U}{\partial \varphi_s} = \sum_{r \neq s} A_{sr}\varphi_r \tag{8}$$

where

$$A_{sr} = \frac{1}{N}\sum_k k^2 \exp[ik(x_s - x_r)], \qquad |k| < \frac{\pi}{l}, \tag{9}$$

it follows that, if $\varphi_r = a_r \cdot \delta(t)$, then

$$\dot{\varphi}_s(+0) - \dot{\varphi}_s(-0) = A_{sr}a_r \neq 0, \tag{10}$$

which signifies that the signal propagates through the lattice with an infinitely large velocity (though its intensity diminishes as the distance becomes larger than $l\,|x_s - x_r|$).

Hence it follows that: either the self-values of the field diverge ($\varepsilon \to \infty$) or the theory becomes incompatible with the theory of relativity (the velocity of the signal $|\xi| > 1$). That is, the theory acquires features of the non-local field theory. This, however, does not discredit the non-linear equations in themselves, since we may still count on the application of renormalization. For example, the Hamiltonian:

$$H = \int dx\{\tfrac{1}{2}\pi^2 + \tfrac{1}{2}\nabla\varphi^2 + \tfrac{1}{2}\varkappa^2\varphi^2 + \tfrac{1}{2}\alpha\varkappa^2\varphi^4\}, \tag{11}$$

may be substituted by

$$H^* = \int dx\{\tfrac{1}{2}\pi^2 + \tfrac{1}{2}\nabla\varphi^2 + \tfrac{1}{2}\varkappa^2\varphi^2 + \tfrac{1}{2}\alpha\varkappa^2(\varphi^2 - A)^2\} - E_0, \tag{11'}$$

where A and E_0 are renormalizing constants. Following this course, we may obtain approximate self-values H^*, if we borrow the value φ^2 from linear theory:

we obtain:

$$H^* = \sum_k h\omega_k n_k + \sum_k \frac{h\omega_k}{2} - E_0, \tag{12}$$

$$\omega_k^2 = c^2k^2 + c^2\varkappa^2\left[1 + \tfrac{1}{2}\alpha h \sum_k (n_k^0/\omega_k^0)\right], \quad \omega_k^0 = \sqrt{c^2k^2 + c^2\varkappa^2}. \tag{13}$$

It follows from the above that non-linear field theory does not of itself eliminate quantum divergencies, and requires additional procedures of the renormalization type.

4. – Equations of Class B.

The propagation of the field in this case is similar in several respects to hydrodynamics on the one hand, and to crystal optics on the other. The possible equations in this class may be divided into two groups: 1) $|\xi| \leq 1$; and 2) $|\xi|$ may also be greater than 1 (*).

The equations of the first group do not contradict the theory of relativity, but of course lead to the same difficulties as the equations of class A do. For example, the M. BORN (+) equations belong to this class. It should be noted that the application of these equations to an electromagnetic field is objected to because it is proved that the velocity of propagation of electromagnetic signals close to the charges is not equal to the velocity of light.

Therefore, the Einstein determination of time near the particles becomes invalid. One might expect that it would be possible to construct such a metrics that would preserve the relativistic requirement of the velocity of light being constant.

However, such a metrics actually proves non-unique, since (as is the case in crystal optics) there is not one but several velocities of propagation of luminous signals.

In a general case, there are also directions of propagation for which the characteristics are imaginary [6].

The second group of equations, in which the velocity of signal propagation may be greater than the velocity of light in the vacuum, has some features of the non-local theory and is incompatible with that form of causality on which the physical interpretation of the theory of relativity is essentially based.

Thus, a very curious situation arises: the Lorentz invariancy of the varia-

(*) A curious case is possible where the equation may become elliptic [5].

(+) V. V. ORLOV showed [6] that the Lagrangian given by M. BORN is the only one which does not lead to the formation of field shock waves.

tional principle proves insufficient by itself for the compatibility of the field theory with the theory of relativity.

It is still necessary to require that the propagation velocity of weak discontinuities should always be less than the velocity of light in the vacum.

Thus we see that non-linear field theories of the class B contain fundamental difficulties prior to quantization, and that not any formally invariant non-linear field theory is compatible with the theory of relativity.

Even in his first work dealing with the theory of relativity, A. EINSTEIN left the question of simultaneousness «in a point» for future investigators.

It is hard to say whether the time has come for a critical revision of the conceptions of space, time and causality in the region of small scales. At any rate, if we take non-linear field theories seriously, they bring us right up to these problems.

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Д.И. Блохинцев

РАСПРОСТРАНЕНИЕ СИГНАЛА ВНУТРИ ЭЛЕМЕНТАРНОЙ ЧАСТИЦЫ

§ 1. Введение

Современная квантовая теория поля существенно основывается на микропричинности в той ее форме, которая, в сущности, тождественна классической макропричинности (ср., напр.[1]). С другой стороны известно, что взаимодействие в квантовой теории поля описывается "причинной" функцией $D_c(x_1 - x_2)$, где x_1 и x_2 пространственно-временные координаты двух мировых точек, в одной из которых сигнал излучается, а в другой принимается. Однако эта функция, в отличие от функций Грина классической теории, не исчезает для пространственного интервала $s^2 = (x_1^2 - x_2^2) = (\vec{x}_1 - \vec{x}_2)^2 - (t_1 - t_2)^2 > 0$. Таким образом, кажется, что возможно обнаружить распространение сигнала со скоростью, большей скорости света, в частности, с бесконечно большой скоростью.

Для случая взаимодействия двух точечных частиц этот парадокс был рассмотрен М. Фирцем[2].

В настоящей работе мы намерены рассмотреть этот парадокс для элементарной частицы, имеющей конечные размеры[x]. В этом случае мы встречаемся, в сущности, с тем же парадоксом, поскольку форм-фактор частицы $F(q)$ (q –передаваемый при взаимодействии импульс) не исчезает для пространственно-подобных значений q и, следовательно, частице можно приписать жесткое пространственное распределение зарядов и токов $\rho(\vec{x}) = \int F(q) e^{i\vec{q}\vec{x}} d^3x$, способное передать сигнал с бесконечно большой скоростью.

§ 2. Энергия взаимодействия

Для изучаемого вопроса достаточно ограничиться рассмотрением взаимодействия виртуального фотона со спинорной частицей.

Поле фотона $k(\vec{k}, k_0)$ будем описывать вектор-потенциалом $A_\mu(x)$, а спинорное поле частицы $p(\vec{p}, p_0)$, $p^2 = -M^2$ (M – масса частицы) – оператором $\psi(x)$. Оператор взаимодействия, в первом приближении по заряду e, может быть написан в виде:

$$W = \int \psi(x_3) J_\mu(x_3 x_2 x_1) \psi(x_1) A_\mu(x_2) d^4x_3 d^4x_2 d^4x_1 \quad (1)$$

где $J_\mu(x_3 x_2 x_1)$ – плотность тока, создаваемого спинорной частицей (см. диаграмму на рис. 1). Амплитуда вероятности перехода частицы из состояния i в состояние f, вызванного этим взаимодействием, будет равна:

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[x] Под элементарной частицей мы понимаем частицу, которая не может быть разложена на "составляющие" ее компоненты без затраты энергии D, меньшей, нежели собственная энергия более легкой компоненты. В этом понятии атом водорода неэлементарная частица, так как его масса $m_H = m_p + m_e - D$, при этом $D \ll m_e, m_p$ (m_p – масса протона, m_e –масса электрона); напротив, протон элементарная частица, так как $m_p = m_A + m_B - D$ и $D \geq m_A, m_B$, для любых реальных частиц A и B.

$$S_{fi} = \frac{i}{\hbar} \mathfrak{T}_{fi} = \int (f/J_\mu(x)/i)\, A_\mu(x_2)\, d^4x_2, \qquad (2)$$

где $(f/J_\mu(x_2)/i)$ есть матричный элемент плотности тока $J_\mu(x_3 x_2 x_1)$, отнесенный к переходу $i \to f$:

$$(f/J_\mu(x_2)/i) = \int \bar{\Psi}_f(x_3)\, J_\mu(x_3 x_2 x_1)\, \Psi_i(x_1)\, d^4x_3\, d^4x_1 . \qquad (3)$$

Мы будем считать волновые функции $\bar{\Psi}_f$ и Ψ_i произвольными, с тем лишь ограничением, что они являются суппер-позициями состояний только с положительной энергией:

$$\bar{\Psi}_f(x_3) = \int C_f^*(p_3)\, \bar{u}_f(p_3)\, e^{ip_3x_3}\, d^3p_3 \qquad (4)$$

$$\Psi_i(x_1) = \int C_i(p_1)\, u_i(p_1)\, e^{ip_1x_1}\, d^3p_1 \qquad (4^1)$$

(здесь $\bar{u}_f$, u_i –биспиноры Дирака, C_f^*, C_i –амплитуды состояний с p_{03}, $p_{01} > 0$), Обозначая $p_3 - p_1 = q$ и пользуясь соотношением:

$$(p_3/J_\mu(x_2)/p_1) = \int e^{ip_3x_3} J_\mu(x_3 x_2 x_1)\, e^{-ip_1x_1} d^4x_3\, d^4x_1 = e^{iqx_2} F_\mu(q), \qquad (5)$$

получим из (3), (4) и (4^1):

$$(f/J_\mu(x_2)/i) = \int C_f^*(p_3)\, C_i(p_1)\, \bar{u}_f(p_3)\, u_i(p_1)\, e^{iqx_2} F_\mu(q), \qquad (6)$$

где $F_\mu(q)$ – оператор тока в импульсном представлении. Этот оператор, как известно, может быть записан в виде:

$$F_\mu(q) = e F_1(q^2) + \kappa \sigma_{\mu\nu} q_\nu F_2(q^2), \qquad (7)$$

где e –заряд частицы, κ – ее магнитный момент, и F_1 и F_2 –электрический и магнитный формфакторы частицы. Величины F_1 и F_2 при таком определении безразмерны и должны зависеть лишь от отношения q^2/q_m^2, где q_m – некоторое значение q, определяющее скорость убывания F_1 и F_2 при $/q/ \to \infty$. Размер частицы R связан с q_m соотношением $R \approx \frac{\hbar}{q_m}$.

§ 3. Необходимые волновые функции

Для того, чтобы рассмотреть распространение сигнала внутри частицы, мы будем считать состояния Ψ_f и Ψ_i, локализованными волновыми пакетами.

Фотонное поле $A_\mu(x)$ будем также считать локализованным. Для определенности мы положим:

$$C_f(p) = \frac{b_f^{3/2}}{\pi^{3/4}}\, e^{-\frac{1}{2} b_f^2 (\vec{p} - \vec{p}_f)^2 - i\vec{p}\vec{x}_f} \qquad (8)$$

$$C_i(p) = \frac{b_i^{3/2}}{\pi^{3/4}}\, e^{-\frac{1}{2} b_i^2 (\vec{p} - \vec{p}_i)^2 - i\vec{p}\vec{x}_i} \qquad (8^1)$$

$$A_\mu(x) = a_\mu e^{-i[\vec{k}(\vec{x} - \vec{X}) - k_0(t - T)]} \times e^{-\frac{1}{2a^2}[\vec{x} - \vec{X}] - \frac{1}{2a^2}[(\vec{x} - \vec{u}t) - (\vec{X} - \vec{u}T)]^2} \qquad (9)$$

Волновые функции (8) и (8^1) представляют собою волновые пакеты со средним импульсом $\vec{p}_f$ (или $\vec{p}_i$), локализованные при $t = 0$ около $\vec{x} = \vec{x}_f$ (или $\vec{x}_i$), при средних квадратичных отклонениях:

$$\overline{\Delta(\vec{p} - \vec{p}_f)^2} = \frac{\hbar^2}{b_f^2}, \quad \overline{\Delta(\vec{x} - \vec{x}_f)^2} = a^2 b_f^2 + \beta^2 \frac{\hbar}{MC} b_f, \qquad (10)$$

где $a^2, \beta^2 \approx 1$, M –масса частицы. Те же соотношения имеются и в пакете $C_i(p)$ x/.

Поле $A_\mu(x)$ представляет собою волновой пакет виртуальных фотонов с характерным волновым вектором $k(\vec{k}, k_0)$, "скоростью" распространения $\vec{u}$ и локализованный при $T = 0$ около $\vec{x} = \vec{X}$, в области $\overline{\Delta x^2} = a^2$, $\overline{\Delta t^2} = a^2/c^2$.

Такой выбор формы пакета вирутальных фотонов соответствует предположению, что он мог бы быть излучен реальной физической частицей xx/.

Заметим, что если было бы возможно использовать бесконечно узкие пакеты и в пространстве и во времени, в пределе бесконечно узкие, типа $\Psi_f = \delta^4(x - x_f)$, $\Psi_i = \delta^4(x - x_i)$, $A = \delta^4(x - X)$ то, как видно из (1) и (2), амплитуда перехода оказалась бы равной непосредственно $J_\mu(x_f, \Lambda, x_i)$ и факт распространения сигнала с запрещенной скоростью был бы установлен.

Однако мы не можем использовать волновые пакеты, протяженность которых как угодно мала. Действительно, прием сигнала частицей заключается в <u>устанавливаемом</u> факте изменения ее импульса $p_i \to p_f$. Это изменение должно быть больше разброса импульсов Δp_i, Δp_f, Δk в исходном и конечных пакетах, т.е.

$$/\vec{p}_f - \vec{p}_i/ = /q_{fi}/ \gg \frac{\hbar}{b}, \quad /\vec{k}/ \gg \frac{\hbar}{a}. \qquad (11)$$

Отсюда вытекает также неравенство

$$/p_f/, \ /p_i/ > \frac{\hbar}{b}. \qquad (11^1)$$

С другой стороны, размеры всех трех пакетов должны быть меньше размеров частицы,т.е.

x/ По поводу второго из равенств (10) см. дополнение А.
xx/ См. дополнение В.

$$b,\ a \ll R\ . \tag{12}$$

Условия (11) и (11^1) и (12) не противоречат принципам квантовой механики. Таким образом на первый взгляд создается все же принципиальная возможность уловить сигнал, идущий внутри частицы со скоростью, большей скорости света. Однако, как показывает дальнейший расчет, эта возможность оказывается иллюзорной.

§ 4. Расчет распространения сигнала

Вычислим амплитуду вероятности S_{fi} (2) для волновых пакетов Ψ_f (8), Ψ_i (8^1) и A (9), удовлетворяющих условиям (11), (11^1) и (12). Эта амплитуда определяет интенсивность сигнала, принимаемого в центре частицы из ее периферической области.

Согласно (2) имеем:

$$S_{fi} = \frac{i}{\hbar}\,\frac{b^3}{\pi^{3/2}} \int d^3x_2\,dt_2 \int d^3p_3\,d^3p_1 \times e^{-\frac{1}{2}b^2[(\vec p_3-\vec p_f)^2+(\vec p_1-\vec p_i)^2]+iqx_2} \times$$
$$\times\, u_f^*(p_3)\,u_i(p_1)\,F(q)\,e^{-i(k,x_2-X)-\frac{1}{2a^2}\{(\vec x_2-\vec X)^2+[(\vec x_2-\vec u t)-(\vec X-\vec u T)]^2\}} \tag{13}$$

При этом, чтобы избежать излишних выкладок, мы положили $\vec x_i = \vec x_f = 0$, b_f, $b_i = b$, далее введено обозначение $F(q) = a_\mu \gamma_4 F_\mu(q)$.

Если размеры пакетов a и b во много больше $\frac{\hbar}{p_f}$, $\frac{\hbar}{p_i}$ и $\frac{\hbar}{k}$, то интегрирование в (13) сразу выполняется и дает обычный в теории рассеяния результат:

$$S_{fi} = \frac{2\pi i}{\hbar}\,\delta^4(k-q)\,u_f^*(p_f)\,F(q_{fi})\,u_i(p_i)\,. \tag{14}$$

Однако место посылки сигнала (точка x_2) и место его приема (точки x_1 и x_3) в этом случае полностью не определены.

Обратимся теперь к случаю конечных размеров пакетов a, b, удовлетворяющих указанным ранее условиям.

По предположению $|q_{fi}| \gg \frac{\hbar}{b} \gg q_m$, поэтому множитель $F(q)$ можно вынести из-под знака интеграла. Мы вынесем также $u_f^*(p_3)$, $u_i(p_1)$, поскольку эти величины также медленно меняются.

Далее, заметим [x/], что

$$q_0 = E(p_3) - E(p_1) = E_f - E_i + \vec V_f\vec\xi - \vec V_i\vec\eta \tag{15}$$
$$\vec\xi = \vec p_3 - \vec p_f,\quad \vec\eta = \vec p_1 - \vec p_i,\quad \vec V_f = \nabla E_f,\quad \vec V_i = \nabla E_i\ .$$

Интегрируя теперь по $\vec\xi$ и $\vec\eta$, получим:

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x/ См. дополнение В.

$$S_{fi} = \frac{i}{\hbar}\,u_f^*(p_f)\,F(q_{fi})\,u_i(p_i)\,e^{i(\vec k\vec X-k_0T)}\int d^3x\,dt\,e^{-\Phi(x,t)}, \tag{16}$$

где

$$\Phi(x,t) = i(\vec k-\vec q)\vec x - i(k_0-q_0)t + \frac{1}{2b^2}(\vec x-\vec v_i t)^2 + \frac{1}{2b^2}(\vec x-\vec v_f t)^2 +$$
$$+\frac{1}{2a^2}(\vec x-\vec X)^2 + \frac{1}{2a^2}[(\vec x-\vec u t)-(\vec X-\vec u T)]^2 \tag{17}$$

(причем, для сокращения переобозначено $\vec q_{fi} = \vec q$, $E_f - E_i = q_0$). Таким образом S_{fi} определяется перекрытием в пространстве и времени четырех пакетов.

Эта ситуация изображена на рис. 2. Существенно, что пакеты в точках $\vec x$, $t = 0$ и $\vec x = \vec X$, $t = T$ ограничены не только в пространстве, но и во времени так, что они до некоторой степени иммитируют идеальные δ-образные сигналы типа $\delta^4(x)$.

Вычисление последнего интеграла в (16) приводит к результату:

$$S_{fi} = \frac{i}{\hbar}\,u_f^*(p_f)\,F(q_{fi})\,u_i(p_i)\,e^{i(\vec k\vec X-k_0T)}\cdot e^{-\Psi(\vec X,T)}, \tag{18}$$

где $\Psi(\vec X,T)$ равно: [x/]

$$\Psi(\vec X,T) = \frac{1}{2a^2}\vec X^2 + \frac{1}{2a^2}(\vec X-\vec u T)^2 - \frac{C^2}{a^4\Delta^2}[2\vec X-\vec u T]^2 -$$
$$-\frac{A^2}{a^4\Delta^2}[(\vec X-\vec u T,\vec u)]^2 - \frac{\vec B}{a^4\Delta^2}[(2\vec X-\vec u T)(\vec X-\vec u T,\vec u)] + \tag{19}$$
$$+\frac{C^2}{\Delta^2}(k-q)^2 + \frac{A^2}{\Delta^2}(k_0-q_0)^2 + J_m\Psi\ .$$

Коэффициенты A^2, $\vec B$, C^2, Δ^2 суть функции ширины пакетов a и b и скоростей $\vec V_i$, $\vec V_f$, $\vec u$. Эта квадратичная форма положительно дефинитна и экспоненциальный множитель $e^{-\Psi(\vec x,T)}$ исчезает при больших $/\vec X/$ как $e^{-\frac{\vec X^2}{a^2}}$, где $a^2 \sim a^2$ (или $\sim b^2$).

Это означает, что факт передачи сигнала от периферии к центру частицы может быть замечен лишь в том случае, если расстояние от периферии до центра $/\vec X/$ не превышает ширину пакетов a или b.

В этом случае волновые пакеты перекрываются далеко внутри частицы и передача импульса осуществляется непосредственным контактом.

Воображаемое пространственное распределение токов $J_\mu(x_3x_2x_1)$ которое могло бы передать сигнал с бесконечно большой скоростью, полностью выпадает из игры. Таким обра-

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x/ См. дополнение С.

зом пространственно-протяженную частицу оказывается невозможным "уличить" в передаче сигнала, идущего со скоростью, большей скорости света.

Это можно рассматривать как указание на то, что в схему теории поля можно ввести и более фундаментальные отклонения от принятой формы причинности, которые из-за соотношения неопределенностей окажутся совместимыми с макроскопической причинностью x/.

Дополнение А. Релятивистский волновой пакет

Среднее значение $\overline{\Delta x^2} = \overline{x^2}$ (при $\bar{x} = 0$) можно написать в виде:

$$\overline{x^2} = Sp \int \psi_a^*(x) x^2 \psi_\beta(x)\, dx, \tag{1}$$

где $\psi_a(x)$ — дираковский спинор:

$$\psi_a(x) = \int C(p) u_a(p) \frac{e^{i\frac{px}{h}}}{(2\eta h)^{1/2}}\, dp \tag{2}$$

При этом взято $u_a(p)$ только для $E(p) > 0$:

$$Sp\, u_a^*(p) u_\beta(p) = \delta_{a\beta} \tag{3}$$

$$Sp \frac{d^* u_a}{dp} \frac{du_\beta}{dp} = \begin{cases} \dfrac{1}{4M^2c^2} + \dots \text{ при } & p \ll Mc \\ \dfrac{M^2c^2}{2p^3} + \dots \text{ при } & p \gg Mc \end{cases} \quad x/ \tag{4}$$

Далее

$$\int C^*(p) C(p)\, dp = 1 .$$

Положим (5)

$$C(p) = p_0^{-1/2} f(p/p_0),$$

где p_0 –характеризует ширину пакета в импульсном пространстве. (6)

Подставляя (2) в (1) и заменяя x на $-ih\frac{d}{dp}$, получим:

$$\overline{x^2} = h^2 Sp \iint \frac{d[C^* u_a^*]}{dp} \delta(p-p') \frac{d[C'u']}{dp'}\, dp\,dp' =$$

$$= h^2 \int \left|\frac{dC}{dp}(p)\right|^2 dp + h^2 Sp \int \quad C(p)|^2 \frac{du_a^*}{dp} \frac{du_\beta}{dp}\, dp . \tag{7}$$

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x/ Обсуждение возможных форм причинности, отличных от принятой в современной теории, дано в /3/.

xx/ См. например /4/.

Пользуясь теперь (4) и (6), получаем:

$$\overline{x^2} = \frac{h^2}{p_0^2} \int [f'(\xi)]^2 d\xi + \frac{h^2}{4M^2c^2} \int_{-\frac{Mc}{p_0}}^{\frac{Mc}{p_0}} f^2(0)\, d\xi . \tag{8}$$

Откуда:

$$\overline{x^2} = a^2 \frac{h^2}{p_0^2} + \beta^2 \frac{h}{MC} \frac{h}{p_0}, \tag{9}$$

где $a^2, \beta^2 \sim 1$. Это можно также переписать в виде:

$$\overline{\Delta x^2} \cdot \overline{\Delta p^2} = a^2 h^2 + \beta^2 \frac{h}{MC} \overline{\Delta p^2}^{1/2} \tag{10}$$

(т.к. $\overline{\Delta p^2} \sim p_0^2$). Отсюда видно, что в релятивистском случае $\overline{\Delta x^2}$ медленнее убывает с ростом p_0. Однако пакет, имеющий $\overline{\Delta x^2} \to 0$ и построенный из состояний одного знака энергии, оказывается вполне возможным.

Дополнение В. Пакет виртуальных фотонов

Виртуальные фотоны нельзя рассматривать отдельно от их источника. Допустим, что таким источником является частица, локализованная около точки $\vec{x} = 0$ и совершающая там квантовый переход около момента времени $t = 0$. Ток, создаваемый при этом переходе частицей будет равен:

$$J_\mu \cong \int d^3p_3\, d^3p_1\, e^{-\frac{1}{2}a^2[(\vec{p}_3 - \vec{p}_f)^2 + (\vec{p}_1 - \vec{p}_i)^2] - i(\vec{k}\vec{x} - k_0 t)}\, \bar{u}_f(p_3) F_\mu u_i(p_1) . \tag{1}$$

Поле пакета вирутальных фотонов, возникающих при этом переходе, описывается уравнением:

$$A_\mu(\vec{x},t) = \frac{1}{\Box^2 + m^2} J_\mu = \int d^3p_3\, d^3p_1 \frac{\bar{u}_f F_\mu u_i}{(\vec{k}^2 - k_0^2 + m^2)} \times \tag{2}$$

$$\times\, e^{-\frac{a^2}{2}[(\vec{p}_3 - \vec{p}_f)^2 + (\vec{p}_1 - \vec{p}_i)^2] - i(\vec{k}\vec{x} - k_0 t)} .$$

Для вычисления интегралов в (2), положим:

$$\vec{p}_1 = \vec{\xi} + \vec{p}_i , \quad \vec{p}_3 = \vec{\eta} + \vec{p}_f$$

$$\vec{p}_3 - \vec{p}_1 = (\vec{\eta} - \vec{\xi}) + (\vec{p}_f - \vec{p}_i), \quad \vec{k} \cong (\vec{p}_f - \vec{p}_i)$$

$$p_{03} - p_{01} = E(p_3) - E(p_1) = E_f - E_i + E_f\vec{\eta} - E_i\vec{\xi} + \dots \tag{3}$$

$$K_0 \cong E_f - E_i , \quad \nabla E_f = \vec{u}_f , \quad \nabla E_i = \vec{u}_i .$$

Мы будем считать, что пакет мал, однако, $|\vec{q}_{fi}| = |\vec{p}_f - \vec{p}_i| \gg h/a$. Далее, для простоты примем, что излучающая частица является точечной (т.е. будем считать, что $m^2 \gg q_{fi}^2$, тогда для потенциала пакета получаем из (2)

$$A_\mu(\vec{x},t) = e^{i(\vec{k}\vec{x} - k_0 t) - \frac{1}{2a^2}[(\vec{x} - \vec{u}_f t)^2 + (\vec{x} - \vec{u}_i t)^2]} \tag{4}$$

— выражение, которое и было принято в основном тексте (при этом было положено $\vec{u}_1 = 0$, $\vec{u}_1 = \vec{u}$).

Дополнение С. Вычисление интеграла (16)

Заметим, что интеграл

$$I = \iint_{-\infty}^{+\infty} e^{-\Phi(x,t)}\, dx\, dt, \tag{1}$$

где

$$\Phi(x,t) = A^2 x^2 + Bxt + C^2 t^2 + Dx + \mathcal{E} t + g \tag{2}$$

при $\Delta^2 = 4A^2C^2 - B^2 > 0$, равен

$$I = \frac{2\pi}{\Delta} e^{\Psi} \tag{3}$$

где

$$\Psi = \frac{A^2}{\Delta^2}\mathcal{E}^2 + \frac{C^2}{\Delta^2} D^2 - \frac{B}{\Delta^2}\mathcal{E} D - g\ . \tag{4}$$

В нашем случае:

$$\Delta^2 = 2\left(\frac{1}{a^2} + \frac{1}{b^2}\right)\left[\frac{v_1^2+v^2}{b^2} + \frac{u^2}{a^2}\right] - \left[\frac{\vec{v}_1+\vec{v}_1}{b^2} + \frac{\vec{u}}{a^2}\right]^2 \tag{5}$$

$$A^2 = \frac{1}{a^2} + \frac{1}{b^2} \tag{5^1}$$

$$C^2 = ½\left(\frac{v_1^2+v_1^2}{b^2} + \frac{u^2}{a^2}\right) \tag{5^2}$$

$$\vec{B} = -\left(\frac{\vec{v}_1+\vec{v}_1}{b^2} + \frac{\vec{u}}{a^2}\right) \tag{5^3}$$

$$\vec{D} = i(\vec{k} - \vec{q}) - \frac{1}{a^2}(2\vec{X} - \vec{u}T) \tag{6}$$

$$\mathcal{E} = -i(k_0 - q_0) + \frac{1}{a^2}[(\vec{X} - \vec{u}T, \vec{u})]\ . \tag{6^1}$$

Специально при

$$\Delta^2 = \frac{1}{a^4}\left[2(v_1^2 + v_1^2 + u^2) + (\vec{v}_1 + \vec{v}_1 - \vec{u})^2\right] \tag{7}$$

$$A^2 = 2/a^2 \tag{7^1}$$

$$C^2 = -\frac{1}{a^2}(v_1^2 + v_1^2 + u^2) \tag{7^2}$$

$$\vec{B} = -\frac{1}{a^2}(\vec{v}_1 + \vec{v}_1 + \vec{u}). \tag{7^3}$$

Отсюда получаем:

$$\Psi = -\frac{\overline{X^2}}{2a^2} - \frac{(X - uT)^2}{2a^2} + \frac{C^2}{a^4 \Delta^2} [2X - uT]^2 +$$

$$+ \frac{A^2}{a^4 \Delta^2} [(\vec{X} - \vec{u}T, \vec{u})]^2 + \frac{\vec{B}}{a^4 \Delta^2} [(2\vec{X} - \vec{u}T)(\vec{X} - \vec{u}T, u)] - \qquad (8)$$

$$- \frac{C^2}{\Delta^2} (\vec{k} - \vec{q})^2 - \frac{A^2}{\Delta^2} (k_0 - q_0) + Im\Psi .$$

Так как $Im\Psi = 0$ при $\vec{k} = \vec{q}$, $k_0 = q_0$, то в (8) $Im\Psi$ вообще можно опустить,

Квадратичная форма Ψ положительно-дефинитна. Приведем ее частные значения при $a = b$, $X_\perp = 0$, $T = 0$, $X_\parallel \neq 0$ (здесь $X_\parallel = \frac{1}{u}(\vec{X}\vec{u})$):

$$\Psi = -\frac{X_\parallel^2}{a^2}, \quad a^2 = 2a^2 \left[1 + \tfrac{1}{2} \frac{(\vec{v}_1 + \vec{v}_1 - \vec{u})^2}{(v_1 + v_1 + u^2)}\right] \qquad (9)$$

для $X_\parallel = 0$, $T = 0$

$$\Psi = -\frac{X_\perp^2}{a^2}$$

для $X = 0$, $T \neq 0$

$$\Psi = -\frac{u^2 T^2}{\beta^2}$$

$$\beta^2 = 2a^2 \frac{2(v_1^2 + v_1^2 + u^2) + (\vec{v}_1 + \vec{v}_1 - \vec{u})^2}{2(v_1^2 + v_1^2 + u^2) + u^2} \qquad (9^1)$$

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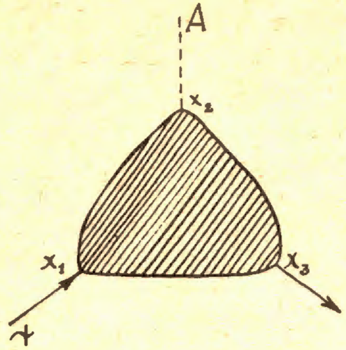
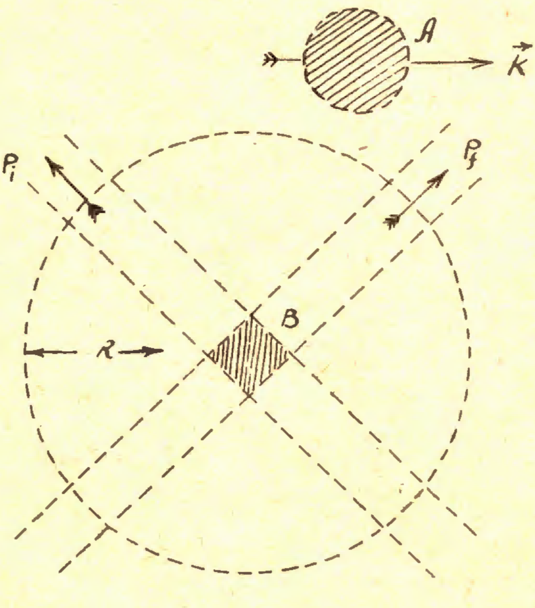


Р и с. 1.

Диаграмма взаимодействия протяженной частицы с фотоном. x_1 – точка входа частицы в область взаимодействия (эта область заштрихована), x_3 –точка выхода, x_2 –точка поглощения фотона A.



Р и с. 2.

В заштрихованной области B сосредоточен ток J_μ, связанный с переходом $\vec{p}_i \to \vec{p}_f$. Большой пунктирный круг ограничивает размеры частицы B. В верхней части рисунка показан пакет виртуальных фотонов A, поглощаемый частицей B.

D. I. BLOHINČEV
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Fluctuations of Space-Time Metric.

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1. – Introduction.

In those regions of space where there are powerful turbulent motions of matter accompanied by considerable changes in the density of matter or having large irregular velocities of motions (v/c is not small!), the metric tensor $\mathfrak{G}^{\mu\nu}$ is a random quantity. This implies that the interval of time t_{AB} and the distance x_{AB} separating two physical world-points A and B also become random quantities. Therefore, one may speak only of the probability that $t_{AB}=t$, $x_{AB}=l$.

In the microworld such a statistical character of the metric may be due to the statistical features of the vacuum, or in other words, to the « zero » oscillations of the quantized fields. However, we face here a very intricate problem and it is probable that the statistical features of the metric which reflect zero vacuum oscillations are essential only in the extremely small volumes which are likely to be beyond the limits of quantum theory.

Nevertheless, it seems interesting to make a theoretical attempt to enter this region. It is sufficient for the time being to restrict oneself to the simplest problem.

2. – Fluctuations of metric.

We will assume that the energy tensor of matter $T^{\mu\nu}$ may be expanded into two terms

$$T_0^{\mu\nu} = T^{\mu\nu} + \delta T^{\mu\nu}, \tag{1}$$

so that $T_0^{\mu\nu}$ describes the global motion of matter characterized by large scales L and periods of time T, whereas the term $\delta T^{\mu\nu}$ is due to the turbulent motion of matter characterized by small scales and short periods τ ($\lambda \ll L$, $\tau \ll T$). The mean value of $\delta T^{\mu\nu}$ by the time intervals comparable with T or by the scales comparable with L is assumed to be zero. Therefore,

$$\langle T^{\mu\nu}\rangle = T_0^{\mu\nu}, \qquad \langle \delta T^{\mu\nu}\rangle = 0, \tag{2}$$

where $\langle ... \rangle$ means the averaging over the turbulent motion. Correspondingly the metric tensor (1) may be decomposed into two parts:

(3)
$$\mathfrak{G}^{\mu\nu} = \mathfrak{G}_0^{\mu\nu} + g^{\mu\nu} + \dots .$$

The magnitude of the turbulent fluctuations of matter $\delta T^{\mu\nu}$ is also assumed to be small, thus the quantities $g^{\mu\nu}$ are also small compared with $\mathfrak{G}_0^{\mu\nu}$ determining the global space-time metric. Under these assumptions the Einstein gravitational equation may be put as

(4)
$$-\tfrac{1}{2}\Box^2 g^{\mu\nu} = \varkappa t^{\mu\nu} .$$

Here:

$$\varkappa = \frac{8\pi\gamma}{c^4}, \qquad \gamma = 6.7\cdot10^{-8}\,\frac{au^3}{2-au^2}\ \text{cm}^3\cdot\text{g}^{-1}\cdot\text{s}^2 ,$$

is the Newtonian gravitational constant;

$$\Box^2 = \mathfrak{G}_0^{\alpha\beta}\frac{\partial}{\partial x_\alpha}\,\frac{\partial}{\partial x_\beta},$$

and the tensor

(5)
$$t^{\mu\nu} = \delta T^{\mu\nu} - \tfrac{1}{2}\mathfrak{G}_0^{\mu\nu}\,\delta T ,$$

where

$$\delta T = \mathfrak{G}_0^{\alpha\beta}\,\delta T_{\alpha\beta} ,$$

is an invariant.

From (4) we find

(6)
$$g^{\mu\nu}(x) = -2\varkappa\Box^{-2}t^{\mu\nu}(x) ,$$

where $\Box^{-2}$ is the operator, reverse to $\Box^2$. According to (6) we can write now for the correlations of the metric tensor components at the two space-time points x and x':

(7)
$$\langle g^{\alpha\beta}(x)\,g^{\mu\nu}(x')\rangle = 4\varkappa^2\,\Box_x^{-2}\Box_{x'}^{-2}\langle t^{\alpha\beta}(x)\,t^{\mu\nu}(x')\rangle .$$

Because of the smallness of $g^{\mu\nu}(x)$, the interval between the two physical world-points A and B

(8)
$$S_{AB} = \int_A^B \sqrt{\mathfrak{G}^{\mu\nu}\,dx_\mu\,dx_\nu} ,$$

may be represented as follows

(9)
$$S_{AB} = S_{AB}^0 + \frac{1}{2}\int_A^B \frac{g^{\mu\nu}\,dx_\mu\,dx_\nu}{\sqrt{\mathfrak{G}_{\mu\nu}^0\,dx_\mu\,dx_\nu}} + \dots .$$

The mean value of $\langle S_{AB}\rangle$, in the linear approximation, is equal to S_{AB}^0, whereas the root-mean-square deviation $\langle (S_{AB} - S_{AB}^0)^2\rangle = \Delta S_{AB}^2$, according to (9), may be written as

(10)
$$\Delta S_{AB}^2 = \frac{1}{4}\int_A^B dx_\mu \int dx'_\mu \langle g_{\mu\mu}(x)\,g_{\mu\mu}(x')\rangle .$$

Here the direction of the interval S_{AB}^0 is taken along the $0x_\mu$ axis.

Making use of (7) we express now ΔS_{AB}^2 in terms of the matter fluctuations

(11)
$$\Delta S_{AB}^2 = \frac{\varkappa^2}{2}\int_A^B dx_\mu \int_A^B dx'_\mu\,\Box_x^{-2}\Box_{x'}^{-2}\langle t^{\mu\mu}(x)\,t^{\mu\mu}(x') + t^{\mu\mu}(x')\,t^{\mu\mu}(x)\rangle .$$

Thus, in the linear approximation, the problem reduces to the calculation of the double correlations of the tensor $t^{\mu\nu}(x)$.

3. – An estimation of metric fluctuations in the macroworld.

The motion of matter will be treated as a motion of a perfect compressible fluid. The tensor of matter for this case reads:

(12)
$$T^{\mu\nu} = \left(\varrho + \frac{P}{c^2}\right)\frac{u^\mu u^\nu}{c^2} - \frac{P}{c^2}\,g^{\mu\nu} .$$

Here ϱ is the rest mass density of the medium, $P = f(\varrho)$ is the pressure, u^μ are the velocity components of the medium. From (5) and (12) we obtain

(13)
$$t^{\mu\nu}(x) = A^{\mu\nu}(x)\,\delta\varrho + B(x)\,\frac{\delta(u^\mu u^\nu)}{c^2} ,$$

where

$$A^{\mu\nu}(x) = \left(1 - \frac{v^2}{c^2}\right)\left(\frac{u^\mu u^\nu}{c^2} - \frac{1}{2}\mathfrak{G}_0^{\mu\nu}\right),$$

and

$$B(x) = \left(\varrho + \frac{P}{c^2}\right), \qquad v^2 = \frac{dP}{d\varrho},$$

is the square of the velocity of sound. Note, that the density fluctuations $\delta\varrho$, by the order of magnitude, are equal to $(\delta u^2/v^2)\varrho$.

Consider now the tensor correlations $g^{\mu\nu}$ outside the volume Ω occupied by the turbulent matter. According to (7), we get

(14)
$$\langle g^{\mu\nu}(x)\,g^{\mu\nu}(x')\rangle = 4\varkappa^2\iint_{\Omega\,\Omega}\frac{d^3y\,d^3z}{R(x,y)\,R(x',z)}\langle t^{\mu\nu}(|t|,y)\,t^{\mu\nu}(|t'|,z)\rangle ,$$

where $R(x, y)$ and $R(x', z)$ are the distances between x and y, x' and z, while

$$[t] = t - \frac{R(x, y)}{c}, \qquad [t'] = t' - \frac{R(x', z)}{c},$$

are the retarded moments of time.

If δu^2 is not small compared with v^2, but still appreciably less than c^2, then among the components $t^{\mu\nu}$ only the term t^{44} is important. At the same time $A^{44} \simeq \frac{1}{2}$, $B^{44} \ll A^{44}$, therefore: $\langle t^{44}(y) t^{44}(z)\rangle \simeq \langle \delta\varrho(y)\, \delta\varrho(z)\rangle$.

When the medium is sufficiently homogeneous this quantity will depend weakly upon $\frac{1}{2}(y+z)$ and essentially depend upon $(y-z)$. Passing now in (14) to the coordinates $\frac{1}{2}(y+z)$ and $(y-z)$, it is not difficult to obtain an estimate of (14) for

$$x = (\boldsymbol{x}, t), \qquad x' = (\boldsymbol{x}, t'),$$

(15)
$$\langle g^{44}(t, \boldsymbol{x})\, g^{44}(t', \boldsymbol{x})\rangle \simeq \frac{\varkappa^2 \Omega \omega(t'-t)\, \delta\varrho^2}{R^2}.$$

Here $\Omega = (4\pi/3)R^3$ is the volume of the medium, $\omega(t'-t)$ is the correlation function, which is equal, at $t=t'$, to the fluctuation volume (λ^3), $\delta\varrho$ is the amplitude of the medium density fluctuation. As is seen from (15), the fluctuations of the metric tensor are proportional to $\sim R^{\frac{1}{2}}\lambda^{\frac{3}{2}}$, where R are the linear dimensions of the medium, λ is the linear scale of turbulentness. Correspondingly:

(16)
$$\Delta S^2_{AB} \simeq \frac{\varkappa^2 \Omega \omega \tau\, \delta\varrho^2}{R^2}\, t \simeq \varkappa^2 \omega\tau \cdot \delta\varrho^2 \cdot R \cdot t,$$

where τ is the time scale of turbulentness.

4. – An estimation of metric fluctuations in the microworld.

Let us now evaluate the correlation between the quantities g^{44} at the points x and x' which is due to the oscillations of the scalar field μ, with the non-zero rest mass.

In this case the Lagrange function is

(17)
$$\mathscr{L} = \frac{1}{2}\sqrt{\mathfrak{G}}\left(\mathfrak{G}^{\alpha\beta}\, \frac{\partial\varphi}{\partial x_\alpha}\, \frac{\partial\varphi}{\partial x_\beta} - \mu^2\varphi^2\right),$$

and the tensor of matter equals

(18)
$$T^{\mu\nu}(x) = \mathfrak{G}_0^{\mu\alpha}\, \mathfrak{G}_0^{\nu\beta}\, \frac{\partial\varphi}{\partial x_\alpha}\, \frac{\partial\varphi}{\partial x_\beta} - \delta T^{\mu\nu}\cdot \mathscr{L}.$$

Due to the nature of the vacuum the quantities $\mathfrak{G}_0^{\mu\nu}$ have now the Galilean values. It is not the tensor $T^{\mu\nu}$ but only its fluctuations $\delta T^{\mu\nu}$ we are interested in. In order to obtain $\delta T^{\mu\nu}$ from (18) it is sufficient to mean by $(\partial\varphi/\partial x_\alpha)(\partial\varphi/\partial x_\beta)$, φ^2 etc. the normal products of these operators. Therefore, according to (5) and (18), we get

$$t^{44}(x) = \frac{1}{2}\,\mathfrak{G}_0^{\alpha\beta}\,\frac{\partial\varphi}{\partial x_\alpha}\,\frac{\partial\varphi}{\partial x_\beta} - \mu^2\varphi^2\,, \tag{19}$$

here the products of the operators are considered already normal.

Expanding, as usual, the field φ into a Fourier series

$$\varphi = \frac{1}{V^{\frac{1}{2}}}\sum_k\sum_{k'}\left(\frac{\hbar}{2\omega_k}\right)^{\frac{1}{2}}\left(a_k \exp\left[i(k, x)\right] + a_k^+ \exp\left[-i(k, x)\right]\right), \tag{20}$$

where V is the normalized volume, $\omega_k = c\sqrt{k^2+\mu^2}$, $k=(\boldsymbol{k}, \omega)$, a_k, a_k^+ are the annihilation and production operators of the field particles. A substitution of (20) into (19) yields

$$t^{44}(x) = \frac{1}{V}\sum_k\sum_{k'}\left(\frac{\hbar^2}{2\omega_k\omega_{k'}}\right)^{\frac{1}{2}}\left\{A_{kk'}(x)a_k a_{k'} + B_{kk'}(x)a_k^+ a_{k'} + B_{kk'}^*(x)a_{k'}^+ a_k + A_{kk'}^*(x)a_k^+ a_{k'}^+\right\}, \tag{21}$$

and

$$A_{kk'}(x) = \tfrac{1}{2}\left[2\mu^2 + (k, k')\right]\exp\left[i(k + k', x)\right], \tag{22}$$

$$B_{kk'}(x) = \tfrac{1}{2}\left[2\mu^2 - (k, k')\right]\exp\left[-i(k - k', x)\right]. \tag{22'}$$

From (21) and (7) and by averaging over the vacuum, we find

$$\begin{aligned}\tfrac{1}{2}\langle g_{44}(x)g_{44}(x') &+ g_{44}(x')g_{44}(x)\rangle = \\ &= \hbar^2\varkappa^2\iint\frac{\mathrm{d}^3k\,\mathrm{d}^3k'}{\omega_k\omega_{k'}}\left[\frac{2\mu^2 + (k, k')}{(k + k', k + k')}\right]^2\cos(k + k', x - x')\,.\end{aligned} \tag{23}$$

For $x = (\boldsymbol{x}, ct)$, $x' = (\boldsymbol{x}, ct')$, $t' - t = T$ we get

$$\begin{aligned}\tfrac{1}{2}\langle g_{44}(x)g_{44}(x') &+ g_{44}(x')g_{44}(x)\rangle = \\ &= \frac{2\pi\hbar^2\varkappa^2}{c^2}\int_0^k\!\!\int_0^k\frac{k^2\,\mathrm{d}k\,k'^2\,\mathrm{d}k'}{\omega\omega'}\left\{\frac{1}{2} + \frac{\mu^2}{2kk'}\lg\frac{\mu^2 + \omega\omega' + kk'}{\mu^2 + \omega\omega' - kk'} + \frac{\mu^2}{\mu^2 + k^2 + k'^2}\right\}\cos(\omega + \omega', T)\,.\end{aligned} \tag{24}$$

This integral is divergent at the upper limit, by $K \to \infty$.

If the rest mass of the field particles is zero ($\mu = 0$), then for $KcT \gg 1$ the integral in (16) is tending to zero like $1/T^2$, whereas for small times $KcT \ll 1$ it behaves like K^4 viz:

$$\tfrac{1}{2}\langle g_{44}(t)g_{44}(t') + g_{44}(t')g_{44}(t)\rangle = \begin{cases}\dfrac{2\pi\hbar^2\varkappa^2}{c^2}\,\dfrac{K^4}{8}\,, & KcT \ll 1\\[2ex] \dfrac{2\pi\hbar^2\varkappa^2}{c^2}\,\dfrac{K^2}{c^2T^2}\cos 2KcT\,, & KcT \gg 1\,.\end{cases} \tag{25}$$

It is seen from here, that the metric fluctuations become essential, if $cT \ll 1/K = L_0$ and the scale L_0 is determined by the formula

$$L_0 = \left(\frac{\hbar \varkappa}{c}\right)^{\frac{1}{2}} = 0.82 \cdot 10^{-32}\ \text{cm}\,. \tag{26}$$

This scale is much larger than the gravitational radii of particles $L_g = \varkappa\mu$ (μ is the mass of particles), which are usually treated as characteristic dimensions of that region of space in which the gravitational effects in the microworld could be essential. However, it is still considerably smaller even than those small scales which are characteristic of weak interactions ($\sim 10^{-16}$ cm).

Note, that the mass of the field particles is of no importance for the metric until the Compton length of the particle $L_c = \hbar/\mu c$ is longer than its gravitational radius L_g since $L_0 = (L_g L_c)^{\frac{1}{2}}$, then the condition $L_c > L_g$ is equivalent to the condition $L_0 > L_g$.