ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ
ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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ON THE DIFFERENCE BETWEEN MUON AND ELECTRON MASSES

(on two types of Dirac fields)

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Abstract

Starting from the Konopinski-Mahmoud hypothesis, the term of the weak interaction through symmetrical neutral currents can be written

$$-G(\bar{\psi}_e \gamma_\rho \psi_e \bar{\psi}_e \gamma_\rho \psi_e - \bar{\psi}_\mu \gamma_\rho \psi_\mu \bar{\psi}_\mu \gamma_\rho \psi_\mu) \tag{1}$$

This interaction yields contributions, equal in quantity but opposite in sign, to the muon and electron bare masses.

The idea of two types of Dirac fields (in particular, electron and muon ones) conjugate in the sense of the generalized Konopinski-Mahmoud hypothesis (e^- , μ^+ - particles, e^+ , μ^--antiparticles, Ξ^- , p^+ particles, etc.) can more conveniently be presented and interpreted with the aid of the second order equation for four-component spinors. It is equivalent to the two Dirac equations $(i\hat{p} + m)\psi_I = 0$ and $(i\hat{p} - m)\psi_{II} = 0$ whose field quanta are automatically connected by the Konopinski-Mahmoud hypothesis. The quanta of the "conjugate" fields ψ_I and ψ_{II} possess equal bare masses.

1. Electron and Muon

Many years ago a second order equation was proposed for the spinor field (of electrons), with a Lagrangian in the form [1]

$$L = \frac{1}{2m} \sum_{\mu\rho} \frac{\partial \bar{\phi}}{\partial x_\rho} \gamma_\rho \gamma_\mu \frac{\partial \phi}{\partial x_\mu} - \frac{m}{2} \bar{\phi} \phi . \tag{1}$$

Both energy and charge in this theory are non-positively definite.

Such a theory can describe the electron as a particle of positive energy and negative charge e^- and the positron as a hole in occupied electron states of negative energies. The theory also allows for another particle of positive charge and positive energy and for its antiparticle. The latter is a hole in the distribution of levels occupied by particles of positive charges and negative energies.

An electron may annihilate with a positron (hole), but not with another , positively charged particle. According to this equation, there should exist a "second" electron. Its properties must be, in a sense, opposite to the electron: the particle is a field quantum charged positively while the antiparticle is charged negatively.

Since there seem to be no "second" electrons in nature a question arises whether the μ^+ meson could be interpreted as the second particle in this theory.

Interest in the old Konopinski-Mahmoud hypothesis concerning the muon and electron [2] has recently revived in connection with experimental evidence in favour of the existence of muon and electron neutrinos.

According to this hypothesis the e^- electron and μ^+ meson are treated as particles and the e^+ positron and μ^- meson as antiparticles. This is the situation described by the above equation.

It appears of interest to investigate which kinds of the known (or experimentally allowable) interactions can remove the degeneracy with respect to the masses of the free equation describing these particles, i.e., lead to $m_\mu \neq m_e$: to what extent the muon and electron neutrinos can naturally be included in this scheme; and in general how universal is such a classification of bare Fermi particles: can this scheme, for example, incorporate baryons ?

The second order equation for the four-component spinor is equivalent to the first order equation for the eight-component function. This equation can be written in the form of two separated equations

$$(i\hat{p} + m)\psi_I = 0, \tag{2}$$

$$\hat{p} = -i\gamma_\mu \frac{\partial}{\partial x_\mu} ;$$

$$(i\hat{p} - m)\psi_{II} = 0, \tag{3}$$

or, introducing the eight row matrices

$$\Gamma_\mu = \begin{vmatrix} \gamma_\mu & 0 \\ 0 & \gamma_\mu \end{vmatrix} \quad ; \qquad \mu = 1, 2, 3, 4 \; , \qquad \Gamma = \begin{vmatrix} I & 0 \\ 0 & -I \end{vmatrix} \; , \tag{4}$$

eqs. (2) and (3) can be written as "one" equation

$$(i\hat{P} + \Gamma m)\,\psi = 0 \; , \qquad \psi = \begin{vmatrix} \psi_I \\ \psi_{II} \end{vmatrix} \; , \qquad \hat{P} = \Gamma_\mu \, p_\mu \; . \tag{5}$$

Dirac arrived at his equation by expanding into factors the operator $P_\mu^2 - m^2 = (i\hat{p} + m)(i\hat{p} - m)$ and taking one of the factors as the operator of his equation.

The eight-component equation (5) possesses special symmetry properties as compared with Dirac's four-component equation.

2. Symmetries of the Generalized Dirac Equation

The Fermi fields satisfying eq. (2) shall be called fields I and the fields obeying eq. (3) fields II.

Lagrangian (1) leads to the expression for the charge density ρ :

$$\rho = -\frac{i}{2mc}\left(\sum_\rho \frac{\partial \bar{\psi}}{\partial x_\rho} \gamma_\rho \gamma_4 \psi - \bar{\psi}\gamma_4 \sum_\rho \gamma_\rho \frac{\partial \psi}{\partial x_\rho} \right).$$

In this respect eqs. (2) and (3) differ in that /1/ eq. (2) selects the functions ψ_I for which ρ is negative (electron solutions). Eq. (3) selects the functions ψ_{II} for which ρ is positive (μ − meson fields). Only in this interrelation of eqs. (2) and (3) it is expedient to differentiate the conjugate fields ψ_I and ψ_{II} .

The symmetries of the Dirac equation for the eight-component ψ − function (5) are in many respects analogous to those for the four-component function if one puts in the latter case $m = 0$.

Indeed, in the transformations

$$\begin{aligned} \psi_I &\to \gamma_5 \psi_{II} \; , \qquad \bar{\psi} \to -\bar{\psi}\gamma_5 \\ \psi_{II} &\to \gamma_5 \psi_I \; , \end{aligned} \tag{6}$$

eq. (2) takes on the form of eq. (3) and vice versa.

The same holds good for the transformations

$$\begin{aligned} \psi_I &\to \gamma_\mu \psi_{II} \; , \qquad \frac{\partial}{\partial x_\mu} \to -\frac{\partial}{\partial x_\mu} \; , \\ \psi_{II} &\to \gamma_\mu \psi_I \; , \qquad \frac{\partial}{\partial x_\mu} \to -\frac{\partial}{\partial x_\mu} \; . \end{aligned} \tag{7}$$

In this sense the four-spinor (bi-bispinor) of eq. (5) is as much single whole as the bispinor of the Dirac equation for $m = 0$; eqs. (2) and (3) are each invariant with respect to the Lorentz transformations, but in the transformations (6) and (7) one bispinor passes into the other.

These new symmetry properties arise because and only because the masses in eqs. (2) and (3) are assumed equal.

Naturally, only such kinds of interactions which are not invariant with respect to these transformations can remove the degeneracy over the bare particle masses.

The set of 8 x 8 matrices corresponding to the generalized Dirac equation (5) possesses several specific properties*. Eq. (5) can be obtained from the Lagrangian

$$L = \tfrac{1}{2} \left(\bar{\psi} \Gamma_\nu \frac{\partial \psi}{\partial x_\nu} - \frac{\partial \bar{\psi}}{\partial x_\nu} \Gamma_\nu \psi \right) - m \bar{\psi} \psi . \qquad (8)$$

According to eq. (8), the ψ field energy-momentum tensor is given by the expression

$$T_{\mu\nu} = \tfrac{1}{2} \left(\bar{\psi} \Gamma_\nu \frac{\partial \psi}{\partial x_\mu} - \frac{\partial \bar{\psi}}{\partial x_\mu} \Gamma_\nu \psi \right) . \qquad (9)$$

or, to be more specific, the energy density of the bare electron-meson field.

$$\omega = \frac{1}{2i} \left[\left(-\psi^*_e \frac{\partial \psi_e}{\partial t} + \frac{\partial \psi^*_e}{\partial t} \psi_e \right) + \left(-\psi^*_\mu \frac{\partial \psi_\mu}{\partial t} + \frac{\partial \psi^*_\mu}{\partial t} \psi_\mu \right) \right] . \qquad (10)$$

Lagrangian (8) is invariant with respect to the transformations

$$\psi \to e^{+i\alpha} \psi \quad , \qquad \psi^* \to \psi^* e^{-i\alpha} \quad , \qquad (11)$$

$$\psi \to e^{-i\beta\Gamma} \quad , \qquad \psi^* \to \psi^* e^{+i\beta\Gamma} \quad . \qquad (12)$$

Accordingly, we have two conservation laws

$$\frac{\partial (\bar{\psi} \Gamma_\nu \psi)}{\partial x_\nu} = 0 \quad , \qquad (13)$$

$$\frac{\partial \bar{\psi} \Gamma \Gamma_\nu \psi}{\partial x_\nu} = 0 \quad . \qquad (14)$$

The fourth component of the vector $I'_\nu = -\bar{\psi} \Gamma \Gamma_\nu \psi$ obeying eq. (14) i.e., the quantity

$$\rho = -(\psi^*_I \psi_I - \psi^*_{II} \psi_{II}) = -(\psi^*_e \psi_e - \psi^*_\mu \psi_\mu) \qquad (15)$$

can be interpreted as a quantity connected with the electric charge density.

The corresponding fourth component of the vector $I''_\nu = \bar{\psi} \Gamma_\nu \psi$, namely $\bar{\psi} \Gamma_4 \psi$

$$\eta = \psi^*_I \psi_I + \psi^*_{II} \psi_{II} = \psi^*_e \psi_e + \psi^*_\mu \psi_\mu \qquad (16)$$

can naturally be interpreted as the lepton number density.

Thus the electron-muon field charge is given by the expression ($e = | e |$).

$$eQ = -e \int \bar{\psi} \Gamma \Gamma_4 \psi \, dv$$
$$= -e \int \rho \, dv = -e \int (\psi^*_e \psi_e - \psi^*_\mu \psi_\mu) \, dv \quad , \qquad (17)$$

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* Watanabe/3/ indicated that the concept of the left-handed and right-handed particles can be introduced in the eight-component spinor theory in the case of a non-zero mass particle as well.

and the number of leptons

$$L = \int \bar{\psi} \, \Gamma_4 \, \psi \, dv$$

$$= \int \eta \, dv = \int (\psi_e^* \psi_e + \psi_\mu^* \psi_\mu) dv \, . \tag{18}$$

After the quantization of the ψ field the meaning of eqs. (17) and (18) is revealed in the most comprehensive and consistent manner.

3. Quantization.

In accordance with eqs. (2) and (3), the permutation relations of the ψ_e and ψ_μ fields can naturally be determined

$$[\psi^e(x), \bar{\psi}^e(y)]_+ = (-\hat{p} - im)\Delta(x-y) \, , \tag{19}$$

$$[\psi^\mu(x), \bar{\psi}^\mu(y)]_+ = (-\hat{p} + im)\Delta(x-y) \, . \tag{20}$$

Let us have

$$\psi^e = \Sigma a_r^- \psi_r^+ + \Sigma b_r^+ \psi_r^- \, , \quad \bar{\psi}^e = \Sigma a_r^+ \bar{\psi}_r^+ + \Sigma b_r^- \bar{\psi}_r^- \, ; \tag{21}$$

$$\psi^\mu = \Sigma A_r^- \psi_r^+ + \Sigma B_r^+ \psi_r^- \, , \quad \bar{\psi}^\mu = \Sigma A_r^+ \bar{\psi}_r^+ + \Sigma B_r^- \bar{\psi}_r^- \, , \tag{22}$$

where a^+ is the operator of the production of e^- and A^+ is that of μ^+.

The other operators are interpreted accordingly. $a^+ a^-$ is the number of electrons (e^-) and A^+A^- that of μ^+-muons.

Thus we have

$$eQ = -e\{[\Sigma a_r^+ a_r^- - \Sigma b_r^+ b_r^-] - [\Sigma A_r^+ A_r^- - \Sigma B_r^+ B_r^-]\} \, , \tag{23}$$

or

$$eQ = -\{[\Sigma n_r(e^-) - \Sigma n_r(e^+)] - [\Sigma n_r(\mu^+) - \Sigma n_r(\mu^-)] \, .$$

It can readily be seen that eq. (13) leads to the conservation of some other number L :

$$L = [\Sigma a_r^+ a_r^- - \Sigma b_r^+ b_r^-] + [\Sigma A_r^+ A_r^- - \Sigma B_r^+ B_r^-] \, ,$$

or (24)

$$L = [\Sigma n_r(e^-) - \Sigma n_r(e^+)] + [\Sigma n_r(\mu^+) - \Sigma n_r(\mu^-)] \, .$$

Eq. (13) gives the conservation of the number of particles (or rather the number of particles minus the number of antiparticles).

Thus the Konopinski-Mahmoud hypothesis (e^-, μ^+-particles, e^+, μ^--antiparticles) finds its natural description in the above formalism.

In this sense it can be supposed that there are Dirac fields different by nature: some of them are described by equation like eq. (2) and others by equations like eq. (3). It is natural to extend such a classification onto baryon fields.

We gradually get used to the idea of considerable affinity between the doublets

$$(p^+, n) \quad \text{and} \quad (\Xi^-, \Xi^0).$$

At present there is no explanation of the difference between the masses of p^+ and Ξ^- particles.

Naturally, analogy between e^-, μ^+ on the one hand and Ξ^-, p^+ on the other suggests itself. In the latter case the corresponding generalisation of the Konopinski-Mahmoud hypothesis is trivial enough: baryons of opposite electric charges Ξ^-, p^+ are actually particles.

It is possible that the interactions involving p^+ and Ξ^- are identical to the same degree as the interactions involving electrons and muons have so far been identical experimentally.

Finally, the existence of Σ^+ and Σ^- particles furnishes an example of the realization of the Konopinski-Mahmoud hypothesis.

4. Electrically-Neutral Fermi Fields

There are no general considerations on the basis if which one of eqs. (2) and (3) could be descriminated for electrically neutral fields.

The existence of neutral components of the doublets p^+, n ; Ξ^-, Ξ^0 and probably e^-, ν_e ; μ^+, ν_μ ; confirms the possibility of such a viewpoint.

The neutral components of the doublets

$$Y = \frac{\Lambda^0 - \Sigma^0}{\sqrt{2}} \quad \text{and} \quad Z = \frac{\Lambda^0 + \Sigma^0}{\sqrt{2}} .$$

can be made to correspond (?) , as is often done, to the particles Σ^+ and Σ^- .

Two conservation laws of the type (13) and (14) must likewise hold for electrically neutral fields.

The interpretation of the conserving number L (24) (the number of particles minus the number of antiparticles) is universal.

The intepretation of the second conversation law (23) is also universal if it is borne in mind that eq. (23) has the meaning of electric charge only if the fourth component of the vector $I'_\nu = -\bar{\psi}\Gamma_\nu\psi$ is multiplied by the electromagnetic interaction constant.

The expression

$$f_\rho = -f(\psi_I^* \psi_I - \psi_{II}^* \psi_{II}), \tag{25}$$

where f is a specific constant of any interaction which has a more general meaning than the particular case (23), In this sense the constant $f = G$ imparts to eq. (25) a more universal meaning of the conservation of weak charge for both electrically charged and electrically neutral Fermi fields*.

$$N = - G\{[n(e^-) - n(e^+)] - [n(\mu^+) - n(\mu^-)]\} . \qquad (26)$$

5. Neutrino

From the viewpoint of the two types of Dirac fields, two types of neutrino fields are only a particular manifestation of the general law.

The baryon doublets

$$(p^+, n) ; \qquad (\Xi^-, \Xi^0)$$

can be assumed to correspond to the lepton doublets[6]

$$(\mu^+, \nu_\mu) ; \quad (e^-, \nu_e) .$$

It is natural to suppose that for the neutrino field as well there can be written two different equations similar to eqs. (2) and (3)

$$(i\hat{p} + m_\nu)\psi_\nu^e = 0 , \qquad (27)$$

$$(i\hat{p} - m_\nu)\psi_\nu^\mu = 0 , \qquad (28)$$

$$(i\hat{P} + \Gamma m_\nu)\psi_\nu = 0 , \qquad (29)$$

where m_ν is the bare mass of the muon and electron neutrino. Neutrino bare masses can in principle have any values. In particular, the bare masses of, for example, all leptons and even fermions might be equal.

As for the real masses of the physical neutrinos, it is only known at present that

$$m_\nu^e \leq 10^{-3} m_e , \qquad \text{and} \qquad m_\nu^\mu \leq 8 m_e .$$

For the time being it is not ruled out, of course, that the physical neutrinos have no proper mass. Nor is it ruled out, generally speaking, that bare masses are also zero for neutrinos. The latter case deserves a special analysis.

(a) Case $m_\nu \neq 0$

The case of non-zero bare masses for electron and muon neutrinos is treated before the introduction of the interaction also as a bare free electron-muon field.

Here in a form analogous to eq. (24) we write the conserving number

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* Different "neutrino charges" for an electron and μ -meson were introduced by Ya.B.Zeldovich[4] and G.Marx[5]. Neutrino charges were introduced to obtain restrictions for effects of the type $\mu \to e + \gamma$ etc.

$$L_{\nu} = [n(\nu_e) - n(\tilde{\nu}_e)] + [n(\nu_\mu) - n(\tilde{\nu}_\mu)]. \tag{30}$$

and in a form analogous to eq. (26) the conserving number of the weak charge

$$N_{\nu} = - G[n(\nu_e) - n(\tilde{\nu}_e)] - [n(\nu_\mu) - n(\tilde{\nu}_\mu)]. \tag{31}$$

If it is assumed that all leptons form an isolated system of fermions, that the Lagrangian is invariant with respect to the transformations (11) having the same form for all lepton functions

$$\psi_L \to e^{ia} \psi_L \quad ; \quad \psi^*_L \to \psi^*_L e^{-ia} , \tag{32}$$

the generalized lepton number conservation law is of the form

$$L = [n(e^-) - n(e^+)] + [n(\mu^+) - n(\mu^-)] +$$
$$+ [n(\nu_e) - n(\tilde{\nu}_e)] + [n(\nu_\mu) - n(\tilde{\nu}_\mu)]. \tag{33}$$

Assuming that the transformation of the type (12) of any lepton function

$$\psi_L \to e^{-i\beta\Gamma} \psi_L \quad ; \quad \psi^*_L \to \psi^*_L e^{+i\beta\Gamma} \tag{34}$$

also leaves the Lagrangian unchanged, we obtain the generalization of the weak charge conservation law

$$N = - G \{ [n(e^-) - n(e^+)] - [n(\mu^+) - n(\mu^-)] +$$
$$+ [n(\nu_e) - n(\tilde{\nu}_e)] - [n(\nu_\mu) - n(\tilde{\nu}_\mu)]\}. \tag{35}$$

Owing to the identity

$$(1 \mp \Gamma) e^{-i\beta\Gamma} = e^{\pm i\beta} (1 \mp \Gamma), \tag{36}$$

this means that the fields e^-, ν_e, μ^- (antimuon), $\tilde{\nu}_\mu$ (anti ν_μ-field) are transformed by the law

$$\psi \to e^{-i\beta} \psi , \tag{37}$$

and e^+ (antielectron one), $\tilde{\nu}_e$ (anti ν_e -field) and ν_μ, μ^+ (muon one) by the law

$$\psi \to e^{i\beta} \psi . \tag{38}$$

If the numbers L and N/G are added and subtracted, two conserving numbers can be obtained

$$L_e = [n(e^-) - n(e^+)] + [n(\nu_e) - n(\tilde{\nu}_e)], \tag{39}$$

$$L_\mu = [n(\mu^+) - n(\mu^-)] + [n(\nu_\mu) - n(\tilde{\nu}_\mu)]. \tag{40}$$

Hence the ν_e neutrino is always connected with electrons and the ν_μ neutrino with muons. By the conservation law (33l and (35) or (39) and (40) processes of the type

$$\pi^+ \to e^+ + \nu_e \,,$$

$$\pi^+ \to \mu^+ + \tilde{\nu}_\mu \,,$$

$$\mu^+ \to e^+ + \nu_e + \nu_\mu \,,$$

$$\nu_\mu + p \to n + \mu^+ \,,$$

$$\tilde{\nu}_\mu + n \to p + \mu^- \qquad \text{etc.}$$

are allowed, and the processes

$$\nu_\mu + p \to n + e^+ \,,$$

$$\mu^+ \to e^+ + \gamma \,,$$

are forbidden. $\qquad \mu^+ \to e^+ + e^- + e^+ \qquad$ etc.

It may happen that the physical masses of muon and electron neutrinos m_ν^e and m_ν^μ will vanish as a result of the interaction. The muon and electron neutrinos will still be physically distinct if the interactions do not violate the gauge-properties of the ψ -function (37), (38) and (11) leading to the conservation laws (33) and (35).

The muon and electron neutrinos differ by the sign of the charge of the weak vector interaction G -constant.

In this sense the generalized Konopinski-Mahmoud hypothesis is formulated similarly for a muon and a muon neutrino, on the one hand, and an electron and an electron neutrino, on the other.

In other words, the sign of the weak charge for μ^+ and ν_μ particles is opposite to that for e^- and ν_e particles. The antiparticles of these particles are $\mu^-, \tilde{\nu}_\mu$ and $e^+, \tilde{\nu}_e$ respectively. The analysis has not touched so far on the polarization of muon and electron neutrinos. The study of these properties of ν_μ and ν_e neutrinos requires a further elaboration of the theory and in particular the introduction of interactions.

6. Interactions

(a) Electromagnetic Interactions

According to the interpretation of the vector I'_ν (14), (17) the interaction between the electron-muon and electromagnetic fields is of the form

$$V^{el} = e A_\rho I'_\rho = -e A_\rho \bar{\psi} \Gamma \Gamma_\rho \psi \tag{41}$$

$$= -e \left[A_\rho \bar{\psi}_e \gamma_\rho \psi_e - A_\rho \bar{\psi}_\mu \gamma_\rho \psi_\mu \right] , \tag{42}$$

The form (42) contains the Konopinski-Mahmoud hypothesis treating μ and e -particles as particles carrying electrical charges of opposite signs.

(b) Four-Fermion Interactions

It can hardly be claimed as yet that there is a complete theory of weak interactions. It is inexpedient, for example, to postulate that the Lagrangian of weak interactions should not contain neutral currents. The Bludman' scheme /7/ allowing neutral currents in the Lagrangian of weak interactions is as valuable heuristically at this stage of the development of the theory as the Sudarshan-Marshak and Gell-Mann-Feynman schemes.

For the following it is sufficient that among the possibilities for four-fermion interactions satisfying the requirements of invariance with respect to the transformations of the ψ -function (37), (38) and (32) we can at any rate write the vector interaction analogous to eq. (42)

$$V^{w} = G I'_{\rho} I''_{\rho} = -G \bar{\psi} \Gamma_{\rho} \psi \bar{\psi} \Gamma \Gamma_{\rho} \psi , \tag{43}$$

or

$$V^{w} = -G (\bar{\psi}_{e} \gamma_{\rho} \psi_{e} \bar{\psi}_{e} \gamma_{\rho} \psi_{e} - \bar{\psi}_{\mu} \gamma_{\rho} \psi_{\mu} \bar{\psi}_{\mu} \gamma_{\rho} \psi_{\mu}). \tag{44}$$

The interaction (44) introduces in the Lagrangian the so-called "neutral currents" i.e., currents neutral with respect to electric charge. In the G -charge these currents are also of opposite signs.

7. Case $m_{\nu} = m_{\nu}^{e} = m_{\nu}^{\mu} = 0$.

Suppose the neutrino field bare masses $m_{\nu} = 0$. Suppose the physical neutrino masses m_{ν}^{μ} and $m_{\nu}^{e} = 0$ too.

Let the gauge transformation (11) as well as (12) hold good for the neutrino field. This means that the muon and electron neutrinos are different in this case. In other words, just as in the case $m_{\nu}^{e} \neq 0$, $m_{\nu}^{\mu} \neq 0$ the conservation laws are formulated in the form (33) and (35) or (39) and (40).

If $m_{\nu}^{e} = 0$ and $m_{\nu}^{\mu} = 0$ the interactions introduced should not violate the additional invariant properties of the Lagrangian

$$\psi_{\nu}^{e} \to e^{i\delta\gamma_5} \psi_{\nu}^{e} ; \qquad \psi_{\nu}^{\mu} \to e^{i\epsilon\gamma_5} \psi_{\nu}^{\mu} . \tag{45}$$

Let us have

$$V' = -G (\bar{\psi}_{\mu} \gamma_{\rho} \psi_{\nu}^{\mu}) (\bar{\psi}_{e} \gamma_{\rho} \psi_{\nu}^{e}), \tag{46}$$

i.e., only a vector interaction is introduced.

The transformation (45) can be re-written as

$$\psi_{\nu}^{e} \to [e^{i\delta} (\frac{1+\gamma_5}{2}) + e^{-i\delta} (\frac{1-\gamma_5}{2})] \psi_{\nu}^{e} ,$$

$$\psi_{\nu}^{\mu} \to [e^{i\epsilon} (\frac{1+\gamma_5}{2}) + e^{-i\epsilon} (\frac{1-\gamma_5}{2})] \psi_{\nu}^{\mu} . \tag{47}$$

The interaction (46) is invariant with respect to the transformations of the neutrino function (45) if the electron and muon functions transform simultaneously by eq. (11) or by eq. (12).

Then δ and ϵ in eq. (45) cannot be arbitrary. But one can choose, for example,

$$\delta = \epsilon = a \tag{48}$$

or

$$\delta = \epsilon = -\beta \;. \tag{49}$$

If one assumes eq. (49) then we have

$$\bar{\psi}_e \gamma_\rho \psi_\nu^e \to \bar{\psi}_e e^{i\beta} \gamma_\rho \left[e^{i\delta} \left(\frac{1+\gamma_5}{2}\right) + e^{-i\delta} \left(\frac{1-\gamma_5}{2}\right) \right] \psi_\nu = \tag{50}$$

$$= \bar{\psi}_e \gamma_\rho \left(\frac{1+\gamma_5}{2}\right) \psi_\nu^e$$

$$\bar{\psi}_\mu \gamma_\rho \psi_\nu^\mu \to \bar{\psi}_\mu \gamma_\rho \left(\frac{1-\gamma_5}{2}\right) \psi_\nu^\mu \;. \tag{51}$$

The requirement of invariance of the interaction with respect to the transformations (45) leaves in the interaction (51) only two component neutrinos:

$$\psi_{\nu L}^e = \left(\frac{1+\gamma_5}{2}\right) \psi_\nu^e \;, \tag{52}$$

$$\psi_{\nu R}^\mu = \left(\frac{1-\gamma_5}{2}\right) \psi_\nu^\mu \;. \tag{53}$$

Two two-component neutrinos arise in the theory presented: left-polarized electron neutrino and right-polarized muon neutrino.

Genetically the $\psi_{\nu L}^e$ and $\psi_{\nu R}^\mu$ neutrinos originate from different Dirac fields ψ_ν^e and ψ_ν^μ with different transformation properties (37) and (38). But for polarized neutrinos the identity

$$(1 \mp \gamma_5)\, e^{-i\gamma_5 \beta} = e^{\pm i\beta} (1 \mp \gamma_5) \;. \tag{54}$$

is essential. Because of eq. (54) the transformations (37) and (38) for polarized neutrinos coincide with the transformations (45) under the condition (49). The latter circumstance makes it possible to unite formally the two two-component spinors into one four-component function with the properties*.

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* On the other hand, the situation can be presented in this way: in the case $m_\nu^e = m_\nu^\mu = 0$ for the two-component functions ψ_I^ν and ψ_{II}^ν two equations are written

$$\frac{\partial \psi_I^\nu}{\partial t} + i\bar{\sigma}\bar{p}\,\psi_I^\nu = 0 \;; \tag{A}$$

$$\frac{\partial \psi_{II}^\nu}{\partial t} - i\bar{\sigma}\bar{p}\,\Psi_{II}^\nu = 0 \;, \tag{B}$$

These ψ_I^ν and ψ_{II}^ν are two fields with different transformation properties ($\psi_I^\nu \to e^{-i\beta} \psi_I^\nu$; $\psi_{II}^\nu \to e^{+i\beta} \psi_{II}^\nu$). These fields differ in the same sense as the fields ψ_e and ψ_μ described by the four-component functions. In the representation in which eqs. (A) and (B) are written the γ_5-matrix is diagonal $\gamma_5 = \begin{vmatrix} I & 0 \\ 0 & -I \end{vmatrix}$. In this case γ_5 plays the same role (12) as the matrix Γ (4) for the electron-muon field.

$$\psi_\nu \to e^{-i\beta\gamma_5}\psi_\nu \, . \qquad (55)$$

In this form the theory coincides with the theory of two types of neutrinos as presented by Kawakami[8].

In this interpretation[8] the ideas of two neutrinos are presented most distinctly for the case $m_\nu^e = m_\nu^\mu = 0$. As a matter of fact these ideas are contained in earlier works by Schwinger[9] and Nishijima[10] and can be traced to the old papers by Fierz[11].

It is noteworthy that starting in eqs. (50) and (51) from the vector variant of the interaction of fermion fields under the condition (45) i.e., under the condition $m_\nu^e = m_\nu^\mu = 0$ one can arrive at the interaction in the form containing vectorial and axial variants.

The view making the peculiar properties of the neutrino entirely responsible for the non-conservation of parity in weak interactions remains quite attractive.

It is not ruled out that the existence of weak non-lepton decays is not an unsurmountable obstacle for such a view[12].

8. $m_\mu \neq m_e$ Problem

If the bare electron and bare muon are described by the same equation, then with the identity of all muon and electron interactions it is impossible to understand the origin of the difference in the masses of these particles in terms of field theory.

In the Konopinski-Mahmoud hypothesis the muon is for the first time interpreted not as just a heavy electron. A question suggests itself whether it is possible to understand on the basis of the Konopinski-Mahmoud hypothesis, establishing new differences in the properties of the muon and electron, the differences in the masses of these particles as well.

In the form (2) and (3) of the equations for the bare electron and muon ($m^0_\mu = m^0_e = m$) it is desirable, for example to indicate an interaction the inclusion of which in the generalized Dirac equation (5) would give rise to contributions to the mass of the particles of the same sign. Then we would have

$$m_e = m - \delta m \; ; \quad m_\mu = -m - \delta m \, . \qquad (56)$$

With a bare mass of particles equal to, say, half of the real mass of the muon and δm close to this value we could in principle explain the differences in the muon and electron masses.

In the search for the interaction removing the degeneracy with respect to the masses of the generalized Dirac equation (5), a specific property of such interactions viz. violation of the invariance of the Lagrangian with respect to the transformation (6) must be the clue.

The interaction of electrons and muons with electromagnetic field

$$-e\, A_\rho (\bar\psi_e \gamma_\rho \psi_e - \bar\psi_\mu \gamma_\rho \psi_\mu)$$

is non-invariant with respect to the transformation (6). This expression changes its sign in the transformation (6).

True, by "accidental" circumstances the electromagnetic interaction in simple Feynman graphs does not actually violate the symmetry under discussion since all contributions to the mass from the terms with the odd powers of electric charge vanish*. An entirely different situation is created when interactions of the type (44) are introduced.

In the vector interaction (44) as well the graph corresponding to the form

$$\bar\psi_\mu \gamma_\rho \psi_\mu \bar\psi_\mu \gamma_\rho \psi_\mu , \tag{57}$$

yields no contribution to the proper energy of the particles.

On the other hand, the graph corresponding to the form

$$\bar\psi_\mu \gamma_\rho \psi_\mu \psi_\mu \gamma_\rho \psi_\mu , \tag{58}$$

leads just as in the scalar variant of interaction to non-zero expressions, viz.,

$$= - G\bar\psi_e \gamma_\rho S^c_e(0)\gamma_\rho \psi_e = - G\bar\psi_e \gamma_\rho \int i \frac{i\hat p - m}{p^2 + m^2} d^4p\, \gamma_\rho \psi_e , \tag{59}$$

$$= + i\bar\psi_e G m \int \frac{d^4 p}{p^2 + m^2} \psi_e ; \quad = \psi_e \frac{m}{(2\pi)^2} G K^2_{max} \psi_e , \tag{60}$$

or

$$\delta m^e = - \frac{m}{(2\pi)^2} G K^2_{max} , \tag{61}$$

where K_{max} is the upper value of the momentum in the integral (60).

Taking into account another form of the meson field S^c_μ function

$$i \frac{i\hat p_e - m}{p_e^2 + m^2} \rightarrow i \frac{i\hat p_\mu + m}{p_\mu^2 + m^2} , \tag{62}$$

one obtains for δm_μ in accordance with eq. (56) an expression of the same sign as in the case of electron field.

$$\delta m^\mu = - \frac{m}{(2\pi)^2} G K^2_{max} .$$

The pseudo-vector interaction of the type (e,e) (• e,e) does not lead, any more than the scalar or pseudo-scalar one, to the removal of the degeneracy under discussion; they all equally change the initial values m°_μ and m°_e**

It is the identity of all known manifestations of the interactions for the electron and muon that is enigmatic in the problem $m_\mu - m_e$.

The possibility in principle of solving the $m_\mu - m_e$ riddle takes shape in the interactions (44). It appears impossible to obtain the corresponding numerical values at the present stage of the theory.

The fact is that along with the graph of the type (I)

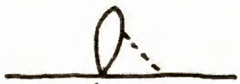
I

(I)

* Evidently, an electrically neutral vector field cannot yield an essential contribution to the mass of the elementary particles: the mass terms originating from these fields depend only logarithmically on the upper limit of the intermediate momentum – this requires the introduction of lengths smaller than the gravitation radius of the particles.

** Provided, of course, the signs of the specific constants in these interactions are the same for the electron and muon.

discussed above there may be indicated graphs of the type (II)



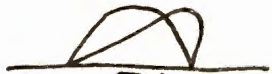
II

(II)

where the intermittent line corresponds to the photon; or a graph of the form (III)

III

or



III'

(III')

The interaction can be taken in such a form of "normal" product which excludes, for example graph 1 from consideration.

The graph of the type (II) yields corresponding additions to the masses of bare particles in the form*:

$$\delta m \approx -\frac{m\,G\,a}{(2\pi)^3}\,K^2_{max}. \tag{63}$$

But the main thing is that if $GK^2_{max} \approx 1$ infinite chains of widely different graphs have to be summed up. At the current stage of the theory (divergence, lack of rational method of estimating higher graphs) it does not appear possible to perform correctly such calculations.

An unsatisfactory feature of all such theories is the introduction of the bare masses of the particles of unknown origins.

The situation in the four-fermion interaction theory is essentially different: the theory incorporates a priori a length constant ℓ which could in particular act the role of a bare mass (see eq. (64)).

It can readily be seen that only the graphs of odd powers of G can lead to $m_\mu \neq m_e$.

The graphs of the form IV

IV

(IV)

even in the powers of G yield the same contribution to the masses of bare particles, decreasing, as is shown by calculations, their bare mass.

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* Graph (II) for the $V-A$ interaction is evaluated

$$m^{\circ}_{\mu} = \frac{m-a}{(1+a/3)\sqrt{1-\left(\frac{a/3}{1+a/3}\right)^2}},$$

where

$$a = \pm\frac{G\,a}{(2\pi)^3}\,K^2_{max}\ln\frac{K^2_{max}}{m^2}.$$

The graphs even in the powers of G may decrease considerably the absolute value of the bare mass even if it exceeds a great deal the real mass of the muon* and is directly connected with, for example, fundamental length

$$m = \frac{A}{\ell} \qquad \text{when} \qquad A = 1, \qquad m = 300\,\text{Gev}.$$

In the attempts to solve the $m_{\mu} - m_{e}$ problem by the introduction, along with known interactions, unknown ones specific only for the muon, the estimation of the corrections to the anomalous magnetic moment of the muon is a rather rigid criterion. From this point of view it is difficult to draw unambiguous conclusions for the possibility under discussion.

The interaction (44) is included as an interaction between bare particles. Query: in which effective form should this interaction be written for non-bare particles?

Essentially, the Dirac equation written for the interaction (44) is the generalized Heisenberg equation (64)-(65). Is it necessary in this equation to write along with the term (44) the electromagnetic interaction** with bare particles? Or the electromagnetic interaction may arise by itself in the equation for non-bare particles as a peculiar manifestation of the four-fermion interactions of bare particles in the sense of Heisenberg's well-known ideas. In the latter case there can be no specific corrections at all to the anomalous magnetic moment of the particles caused by the interaction (44)/12/.

*

* *

The idea of the existence of two types of Dirac fields as a generalization of the Konopinski-Mahmoud hypothesis is adequately reflected mathematically in the second order equation for spinors or, which is the same, in the Dirac equation for eight-component functions. Analysis of these equations may logically lead to the Konopinski-Mahmoud hypothesis. But once the idea of two Dirac fields originated it can be and is actually described in terms of the same type of the Dirac equation for four-component functions. The physical differences of the fields: the intepretation of the particles and antiparticles of different fields and the signs of the vector constants of the interactions (44) must in such a description be introduced by additional propositions.

The contemplated existence of the two types of Dirac fields may also prove useful for the generalization of Heisenberg's equations.

Essentially, the two equations

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* The consideration of charged currents has been omitted for the sake of simplicity. Some of them may lead to $m_{\mu} \neq m_{e}$ in the graphs of the odd powers in G beginning from G^3 (graphs III'). Graphs IV also decrease the bare mass common for μ and e . There arises a considerable number of graphs IV if the interactions with bare baryons are taken into account.

** In this case a graph of the type

could make a contribution to the anomalous magnetic moment of the electron and muon $\delta\mu \approx \frac{m^{e}_{\mu}}{m} \, \frac{a}{2\pi}$

When $m = \frac{1}{\ell} \approx 300$ GeV, $\delta\mu \approx 10^{-7}$. The correction (graph II) leads to the difference in the muon and electron masses

$$m_{\mu} - m_{e} \approx 40\, m_{e}$$

These rough estimates are purely illustrative, of course.

$$\gamma_\nu \frac{\partial \psi_I}{\partial x_\nu} - \ell^2 \gamma_\nu \psi_I (\bar{\psi}_I \gamma_\nu \psi_I) + \frac{A}{\ell} \psi_I = 0 \qquad (64)$$

and

$$\gamma_\nu \frac{\partial \psi_{II}}{\partial x_\nu} + \ell^2 \gamma_\nu \psi_{II} (\bar{\psi}_{II} \gamma_\nu \psi_{II}) - \frac{A}{\ell} \psi_{II} , \qquad (65)$$

where A is a numerical constant have been implied throughout the above discussion.

Apart from other difficulties of Heisenberg's theory, it should be noted thet one fundamental ψ -field seems to have too poor possibilities for the description of a large variety of particles.

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