ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

ЛАБОРАТОРИЯ ЯДЕРНЫХ РЕАКЦИЙ

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# RESONANCE MODEL OF NN INTERACTIONS AND ITS APPLICATION TO NUCLEUS

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# РЕЗОНАНСНАЯ МОДЕЛЬ $NN$-ВЗАИМОДЕЙСТВИЙ И ЕЕ ПРИЛОЖЕНИЕ К ЯДРУ



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A b s t r a c t

It is supposed that the account of $\pi\pi$ interaction allows to describe the contribution of higher order approximations of the pion theory to $NN$-potentials by a phenomenological introduction of the one-particle exchange of $\pi\pi$ resonances with definite symmetries. Various types of resonance symmetries which may give the contribution to the nuclear forces are discussed. An important role of the $\omega$-resonance is emphasized. An example of the resonance model is considered in which the exchange of two ( I=0 and I=1 ) vector and one ( I=0 ) scalar resonances occurs. The estimates of the coupling constants of these resonances with the nucleon are made from a comparison with the phenomenological $NN$-potential of Hamada, Johnston (H–J). Good qualitative and satisfactory quantitative agreement with H–J potentials has been obtained. It is shown that, if applied to the nucleus, the resonance model permits to give a microscopic interpretation of the semiempirical formula for the nuclear masses. A comparison with the coefficients of the formula leads to additional restrictions on the parameters of the model.

## 1. $\pi\pi$ Resonances and Nucleon–Nucleon Potentials

It is established at present that the potential model of NN interaction is applicable, at least, to the energy range $\lesssim$ 300 MeV/1/. At the same time no theory yet exists, from which these potentials could be consistently obtained. Numerous attempts to solve this problem according to the pion theory ( see, e.g.,/2/) led, in fact, only to the qualitative conclusion about the important role of two, three etc meson exchange, i.e., about the essential contribution of the higher order approximations of the perturbation theory.

The development of the dispersion methods which are free from the difficulties encountered in the perturbation theory and the recently discovered strong $\pi\pi$ interaction which is demonstrated, first of all, in the so-called pion resonances* make a new approach to the nature of NN-potentials possible. It may be supposed that due to $\pi\pi$ interaction there occurs the exchange of groups of correlated pions, resonances (R) between nucleons, unlike the perturbation theory where the exchange of a number of individual pions takes place. The large $\pi$N coupling constant which in the perturbation theory indicates an essential contribution of the higher order approximations implies in our approach the existence of still large effective coupling constants $g_{RN}$ of the resonances with the nucleon. At the same time very large $g_{RN}$ constans should be considered as an argument for neglecting the contribution from the non-resonance exchange.

In the language of the spectral representations for the exchange of one resonance having a definite mass (we shall neglect the resonance width) with certain quantum numbers there is a pole in the spectral expansion of the matrix element of NN scattering. The one-boson (OBEP) potential corresponding to this pole is well-known for each type of the boson symmetry (see,e.g.,/5/). Thus the total potential may be represented as a superposition of the Ukawa type potentials corresponding to each resonance. It displays, as should be expected, a strong dependence on the quantum numbers of the two-nucleon state.

A question arises as to which kind of resonances give the contribution to the nuclear forces. Evidently, in NN scattering may take place the exchange of bosons only with zero strangeness and isotopic spin $\leq 1$ .

It should be also expected that the presence of the neutral vector resonance ( the $\omega$ meson ) with the large coupling constant $g^2_{\omega}$ must lead to the appearance of a strong repulsive core in the potential, resulting, thereby, in the strong suppression of the effect of the resonances having the mass larger than $\mu_{\omega} = 5.6\mu_{\pi}$ . Speculations of rather general nature suggest also that in the mass region $2\mu_{\pi} < \mu < 5.6\,\mu_{\pi}$ there exist resonances with spin $\leq 1$.** With these limitations there still remain 16 sets of quantum numbers of spin $J = 0,1$; isotopic spin $I = 0,1$; parity $P = \pm 1$ and G-parity $G = \pm 1$, each of which can be referred to, at least, one of the possible resonances. However, as can be shown/7/, the requirement that the interaction of the resonance with the nucleon should be invariant under charge conjugation leaves room only for 10 various types of symmetry, for which the pole contribution of the resonance to the NN amplitude and, hence, to the NN potential, is possible. These types are listed in Table I, where an indication is given of the $\omega, \rho, \eta$ resonances definitely established***, the $\zeta$-resonance not well established and scalar (a) and pseudovector (b,c) resonances which are only assumed to exist. For the sake of completeness the $\pi$- meson is also included.

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* On the $\pi\pi$ resonances see the Proceedings of the XI Rochester Conference/3/, and from later literature see/4/ and the references containing there.

** As is known the f° resonance presently discovered/6/ ( $I=0$, $J=2^{++}$ ) has the mass of about 1250 MeV.

*** The valies for the masses of the $\omega, \rho, \eta$ resonances are taken from /4/. The quantum numbers of the $\zeta$ resonance are analysed in /8/. The value $G_{\zeta} = -1$ follows from the requirement of the charge invariance of the $\zeta$N interaction .

Table 1

| | $\pi$ | $\omega$ | $\rho^+$ | $\rho^0$ | $\eta$ | $\zeta$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|---|---|---|---|---|
| $\mu$ MeV | 140 | 782 | 770 | 750 | 548 | /570/ | | | |
| $I$ | 1 | 0 | 1 | | 0 | 1 | 0 | 0 | 1 |
| $J$ | 0 | 1 | 1 | | 0 | 0 | 0 | 1 | 1 |
| $P$ | – | – | – | | – | + | + | + | + |
| G | – | – | + | | + | – | + | $\pm$ | $\pm$ |

Since the contribution to the NN potential is independent of the G -parity of the resonance, then in the analysis the pseudovector mesons with G=$\pm$1 may be assumed, for simplicity, to be the only one (and not two) resonance.

Thus, at least, in principle, one should take into account eight types of potentials. At the same time, as is known/5/, the vector mesons may have two coupling constants with the nucleon: the vector ( $g_\omega$ , $g_\rho$ ) and tensor ( $f_\omega$ , $f_\rho$ ), the scalar mesons – one scalar ( $g_a$ , $g_\zeta$ ) , the pseudoscalar mesons – one* pseudovector ( $f_\pi$ , $f_\eta$ ), while the pseudovector mesons – either a pseudovector ( $g_b$ , $g_c$ ) or pseudotensor ( $f_b$ , $f_c$ ) coupling constant. After all possible resonances with all possible coupling constants were taken into account, the total N N potential must contain, at least, 12 unknown parameters – three masses and nine coupling constants ( if assume that each type of the symmetry corresponds to not more than one resonance and the pseudovector resonances having G = $\pm$ 1 to be the same).

In view of a large number of the parameters, it is of interest to investigate the problem about the minimum number of resonances necessary for constructing the plausible potentials. In doing this, one should, first of all, take into consideration the known $\omega$, $\rho$, $\eta$ resonances with the definitely established I, J, and P determining the form of the potential. As the analysis already made shows/9/, a semi-quantitative agreement with the phenomenological potentials in the region $x = \mu r > 0.5$ can be obtained only if the neutral scalar resonance is involved. The authors of/9/proceeded from the supposition in the spirit of Sakata's model, that the effect of the one-boson exchange is essential in the region lying outside the repulsive core ( x $\approx$ 0.35/1/ ), while the core itself is due to the interaction of fundamental baryons. However, it is well-known that still earlier** many authors considered heavy mesons to be responsible for the repulsive core. It seems interesting therefore to make an attempt to construct a model of N N potentials with the aid of $\pi\pi$ resonances in which the repulsive core would be obtained quite naturally.

## 2. A Model of Three ($\omega$, $\rho$, a ) Resonances

In constructing the potential we shall proceed from the existence ( see Table I ) of $\omega$, $\rho$, $\eta$ resonances and assume, as it was pointed out above, that there exists a neutral scalar a-resonance ( see also/10/). The total potential is a sum of the contributions from a $\pi$-meson ( OPEP) and the $\omega$, $\rho$, $\eta$, a resonance exchange. A comparison may be made with a rather detailed phenomenological NN potential of Hamada and Johnston (H–J)/1/.

* In the approximation of one-meson exchange the ps and pv types of the ps meson couplings give practically equivalent potentials.

** In/9/ there are references to earlier paper concerned with the role of heavy mesons in the formation of the repulsive core.

The procedure of constructing the potentials was as follows. As the tensor part of the potential contains the least number of the parameters ( the scalar resonance does not give any contribution to $V_T$) we estimated, first of all, the coupling constants $f^2_\omega$ , $f^2_\rho$ , $f^2_\eta$ *, by comparing the $V^+_T$ and $V^-_T$ potentials we obtained with the expressions of H–J. It was assumed in the calculations that $\mu_\omega = \mu_\rho = 5.5\mu_\pi$ , $\mu_\eta = 4.0\mu_\pi$ . It turned out at once that the best agreement is obtained if $f^2_\eta$ is assumed to be zero, since the neutral pseudo scalar $\eta$-meson leads to an inadmissible increase of $/V^-_T / V^+_T /$. Therefore, in further calculations the $\eta$ meson was not taken into account. For vector mesons the values $f^2_\omega = f^2_\rho = 2,2$ were obtained. The tensor potentials with these parameters were calculated

$$\frac{1}{\mu_\pi} V^+_T = - f^2_\pi \frac{e^{-x}}{x}\left(1+\frac{3}{x}+\frac{3}{x^2}\right)\left\{1-18\left[1+\frac{3}{5,5x}+\frac{3}{(5,5x)^2}\right]\left(1+\frac{3}{x}+\frac{3}{x^2}\right)^{-1} e^{-4,5x}\right\} \qquad /1a/$$

$$\frac{1}{\mu_\pi} V^-_T = \frac{1}{3} f^2_\pi \frac{e^{-x}}{x}\left(1+\frac{3}{x}+\frac{3}{x^2}\right)\left\{1-55\left[1+\frac{3}{5,5x}+\frac{3}{(5,5x)^2}\right]\left(1+\frac{3}{x}+\frac{3}{x^2}\right)^{-1} e^{-4,5x}\right\} \qquad /1b/$$

They are shown in Figs. 1,2. The results of H–J are also given there.

In order to calculate the central and LS potentials it is necessary to take into consideration the contribution of the scalar a-resonance. Experiment fails so far to give a strong indication to its existence. So, a-resonance is likely to be a $\pi^+\pi^-$resonance at 395 MeV, whose probable existence was indicated in/11/ and recently established in/12/, ( see, however,/14/). As the situation with the ABC-resonance in the region of 300 MeV is also uncertain (/ 3/ p. 713 ) in the calculations use was made of the mean values out of these two ones: $\mu_a = 2.5\,\mu_\pi$. As a result of the analysis of the $V_c$ and $V_{LS}$ potentials there were obtained the following values of the remaining parameters: $g^2_\omega = 12$, $g^2_\rho = 2.0$ , $g^2_a = 1.2$. Table 2 lists the parameters of the model which were used in the calculations of the potentials. The requirement $g_\omega f_\omega > 0$ , $g_\rho f_\rho > 0$ is necessary to get a correct relation between $V^+_{LS}$ and $V^-_{LS}$. No analysis was made of the quadratic spin-orbital potentials, because of their large ambiguity/1/.

Table 2

| | ω | | ρ | a |
|---|---|---|---|---|
| | $\mu_\omega = 5,5\,\mu_\pi$ | | $\mu_\rho = 5,5\,\mu_\pi$ | $\mu_a = 2,5\,\mu_\pi$ |
| $g^2_\omega = 12$ | $f^2_\omega = 2,2$ | $g^2_\rho = 2,0$ | $f^2_\rho = 2,2$ | $g^2_a = 1,2$ |
| | $g_\omega f_\omega > 0$ | $g_\rho f_\rho > 0$ | | |

The central and LS potentials shown in Figs. 1 –3 are then determined by

$$\frac{1}{\mu_\pi}\, {}^1V^+_c = - f^2_\pi \frac{e^{-x}}{x}\left[1 + 15\, e^{-1,5x} - 65\, e^{-4,5x}\right] \qquad /2a/$$

$$\frac{1}{\mu_\pi}\, {}^3V^+_c = - f^2_\pi \frac{e^{-x}}{x}\left[1 + 15\, e^{-1,5x} - 39\, e^{-4,5x}\right] \qquad /2b/$$

* All the coupling constants are rationalized, so $f^2_\pi = \frac{F^2_\pi}{4\pi} = 0.08$. It is assumed here and further that $\hbar = c = 1$ .

$$\frac{1}{\mu_\pi}\,{}^1V_c^- = 3f_\pi^2\frac{e^{-x}}{x}\left[1 - 5{,}0\,e^{-1{,}5x} + 61\,e^{-4{,}5x}\right] \qquad /2c/$$

$$\frac{1}{\mu_\pi}\,{}^3V_c^- = \frac{1}{3}f_\pi^2\frac{e^{-x}}{x}\left[1 - 45\,e^{-1{,}5x} + 630\,e^{-4{,}5x}\right] \qquad /2d/$$

$$\frac{1}{\mu_\pi}V_{LS}^+ = 0{,}31\left(1+\frac{1}{5{,}5x}\right)\frac{e^{-5{,}5x}}{x^2} - 0{,}034\left(1+\frac{1}{2{,}5x}\right)\frac{e^{-2{,}5x}}{x^2} \qquad /3a/$$

$$\frac{1}{\mu_\pi}V_{LS}^- = -7{,}8\left(1+\frac{1}{5{,}5x}\right)\frac{e^{-5{,}5x}}{x^2} - 0{,}034\left(1+\frac{1}{2{,}5x}\right)\frac{e^{-2{,}5x}}{x^2}\ . \qquad /3b/$$

As is seen from the Figures, the obtained potentials (1) – (3) are in good qualitative and close quantitative agreement with the results of H –J, and especially , if we take into account that deep wells of the phenomenological $V_c^+$ potentials are not, apparently, physical ones and can be eliminated by a certain softening of the repulsive core as is the case in the Signel – Marshak's /13/ potential. They have also the right long-range part which is due to the one-pion exchange and is checked experimentally best of all.

In order to get the repulsive core we are in need of rather a large coupling constant $g_\omega^2 \gg f_\omega^2$ . This relationship between the constants is in agreement with the result obtained in analysing the effect of the vector resonances on the electromagnetic form-factors of the nucleon/14/. Besides, it should be taken into account, that a large $g_\omega^2$ constant implies a great contribution to $f_\omega^2$ of the "normal" $\omega$- moment of the nucleon. It is worthwhile nothing that the value $g_\rho^2 = 2.0$ is close to $g_\rho^2 = 1.7$ obtained in/14/, and to the estimate of Sakurai ( /3/ page 176) $g_\rho^2 \approx 2.0$. The value $f_\rho^2 = 2.2$ is somewhat less than $f_\rho^2 = 3.7$ obtained in /14/.

Of interest is an insignificant role of the $\eta$-resonance ( $f_\eta^2 \approx 0$) which is in evident discrepancy with the large cross section for $\eta$ -meson production in $\pi p$ collisions/4/. This is likely to be a consequence of the existence of the, I=0, $J = 1^+$ resonance whose contribution to the NN potential is close by magnitude, but of the opposite sign/5/. As is well-known the decay of the pseudovector mesons into two $\pi$-mesons is forbidden. Therefore, if the mass of such a meson is somewhat greater or smaller than $3\mu_\pi$ , then it is very difficult to be observed*.

Thus, we think that the question is still open both from the theoretical and experimental point of views as to whether it is necessary in interpreting the nuclear forces to take into account the additional ( to the $\omega$, $\rho$, $a$ ) resonances, particularly the pseudovector ones. It would be very interesting to make a complete analysis of the experimental data in the energy range of $\lesssim$ 300 MeV according to the model including all the possible resonances. It is clear that any criteria and limitations imposed on the coupling constants and on the resonance masses could make this problem easier and lead to a more unique solution. One of the great advantages of the resonance model of **NN** interactions is that the parameters it involves can be independently checked in the course of the analysis of a large number of phenomena, e.g., $\pi$N scattering, electromagnetic form-factors etc. It is also interesting to investigate what information may be obtained from the consideration of the many-nucleon system, i.e., the nucleus.

## 3. Weizsäcker's Formula and the Parameters of the Resonance Model.

It is well-known that the main difficulty in the application of the phenomenological two-nucleon potential to the nucleus is how to take consistently into account the correlations, that is the part of the interaction which cannot be treated in the Hartree-Fock's approximation (H– F) ( see, e.g.,/15/) . The point is that due to the strong singularity the hard repulsive core forbids the consi-

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* The author is grateful to L.L.Nemenov for this comment.

deration by the perturbation theory, though it is well-known that the independent particle model is a good approximation for discribing the nucleus, at low energies , at least. On the other hand, as the repulsive potential of the Ukawa type obtained in the resonance model is not so singular, it can be taken into consideration already in the first order approximation of H–J what simplifies appreciably the consideration of the nuclear ground state on the basis of the nucleon -nucleon interaction.

Here we shall not be concerned with the effect of the principally allowed many-particle forces. Just note, that there are so far no reliable experimental data indicating to a noticeable role of the many-particle forces in the nucleus. Moreover, even if only the pairing interactions are taken into account, this permits to explain the nuclear binding energies /16/. In what will follow therefore we will restrict ourselves to the two-nucleon interaction.

The problem is what limitations on the parameters of the resonance model can be obtained from the analysis of the nuclear binding energy. In doing this, we shall assume that the contribution to the NN potentials is given by all the resonances containing in Table 1.

According to modern concepts about the nucleus based on the success of the shell model the wave functions of individual nucleons are "smeared out" over the whole volume of the nucleus. Thus concerns both the spatial variables of the nucleons and to their spin and isotopic spin variables. Hence, the nucleus can be regarded as an extended system having a definite density of nucleons, of spin and of isotopic spin. Such a system possesses a definite potential energy corresponding to each form of the interaction. Here it is convenient to represent the NN potential as a sum containing different structural elements.*

$$V = V_0 + V_\sigma (\bar{\sigma}_1, \bar{\sigma}_2) + V_\tau (\bar{\tau}_1 \bar{\tau}_2) + V_{\sigma\tau} (\bar{\sigma}_1 \bar{\sigma}_2)(\bar{\tau}_1 \bar{\tau}_2) . \qquad /4/$$

For the extended sources of the meson fields by $\sigma$ and $\tau$ one should mean now the density of the spin and isotopic spin in the nucleus, while expression (4) corresponds to the potential energy of the interaction of two elements in the nuclear volume.** If we assume that the nucleon density inside a spherical nucleus with a sharp*** edge is constant, then for each of the potential energies we shall have ( $R = r_0 A^{1/3}$ )

$$U_i = \frac{3}{2} A \epsilon_i \sum_k \gamma_k^i \frac{g_k^2}{\mu_k^2} I(\mu_k R) , \quad i = 0, \sigma, \tau, \sigma\tau . \qquad /5/$$

Here the following notations are introduced ( N is the number of neutrons, Z is the number of protons, A = N+Z ):

$$\epsilon_0 = 1 , \quad \epsilon_\tau = \left(\frac{N-Z}{A}\right)^2 , \quad \epsilon_{\tau\sigma} = \epsilon_\tau \cdot \epsilon_\sigma , \quad \epsilon_\sigma = \begin{cases} 0 & \text{– even-even nucleus} \\ \frac{1}{A^2} & \text{– odd A} \\ 4/A^2 & \text{– odd-odd nucleus} \end{cases} \qquad /6/$$

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* The role of spin-orbital and tensor forces in a spherical nucleus is likely to be insignificant due to a large averaging.

** This statement implies, in fact, that the potential energy of the nucleus is calculated in the Hartree's approximation. As can be shown /15/ for the nucleon system with N = Z the neglect of the exchange for the most essential part of the potential $V_0$ leads to an error not larger than 25%. This accuracy is sufficient for our purpose – to get preliminary limitations on the parameters of the resonance model. As the main contribution of the repulsive potential is considered in the Hartree's approximation one can also expect that the remaining part of the correlations which was not taken into account is not very appreciable.

*** The assumption about a sharp edge was made for the sake of simplicity and does not affect essentially the quantitative estimates except the term of the surface energy. ( see below).

for the magnitudes determining the density of nucleons, isotopic spin and spin, respectively. At the same time the value $4/A^2$ for odd-odd nucleus corresponds to the assumption on the addition of the spins of odd nucleons what is in agreement with the empirical rule of Nordheim/17/. Since the factors $\left(\frac{N-Z}{A}\right)^2$ and $\frac{1}{A^2}$ are small, we shall neglect further $V_{\sigma\tau}$ as a magnitude of the second order of smallness.

The numerical coefficients $\gamma^i_k$ measure the contribution of the corresponding resonance to the i-structure. For example, $\gamma^o_\omega = 1$ , $\gamma_a = -1$ , $\gamma^\sigma_\omega = 2/3$ , etc.

The function I (x) is of the form

$$I(x) = 1 - \frac{3}{2}\frac{1}{x} + \frac{3}{2}\frac{1}{x^3} - \frac{3}{2}\frac{(x+1)^2}{x^3} e^{-2x} . \qquad /7/$$

In the case of heavy mesons and sufficiently heavy nuclei $\mu_k R \gg 1$ , so that in (7) only two terms should be retained

$$I(\mu R) \approx 1 - \frac{3}{2}\frac{1}{\mu R} \qquad /7a/$$

for the main part of the potential energy $U_0$ and one first term for $U_\sigma$ , $U_\tau$ .

In another limiting case when $\mu R \to 0$ , from (5) and (7) one can get the well-known expression for the Coulomb ( $\epsilon_c = \frac{Z^2}{A^2}$ $\gamma = 1$ ) energy of the nucleus

$$U_c = \frac{3}{5}\frac{Z^2}{A^{1/3}}\frac{e^2}{r_0} .$$

It can be easily seen that the potential energies $U_0$ , $U_\sigma$ , and $U_\tau$ are analogous to different terms of the semiempirical formula of Weizsäcker for the nuclear binding energy/15/ ( all the coefficients are expressed in MeV )

$$-E_b = -15{,}75A + 17{,}8A^{2/3} + 0{,}710\frac{Z^2}{A^{1/3}} + 23{,}7\left(\frac{N-Z}{A}\right)^2 + \frac{34}{A^{3/4}}\delta \qquad /8/$$

$$\delta = \begin{cases} -1 & \text{even-even nucleus} \\ 0 & \text{odd A} \\ +1 & \text{odd-odd nucleus} \end{cases}$$

When expressions (5) are compared with the corresponding terms of (8), we put to be $r_0 = 1.22$ f what is consistent with the coefficient 0.710 for the Coulomb term. Besides the potential energy, one should take into account the kinetic energy of the nucleons. With this aim use is made of the Fermi-gas model. The contribution of the kinetic energy to the volume and surface terms, as well as to the term of the symmetry energy in (8) is 20 MeV, 7.6 MeV, and 11 MeV, respectively. As a result, we get the following relations for the coupling constants and the resonance masses ( $\mu_\pi = 1$ )

It follows from the volume energy that /9/

$$\frac{g_a^2}{\mu_a^2} - \frac{g_\omega^2}{\mu_\omega^2} = 0{,}10 ;$$

from the surface energy*

$$\frac{g_a^2}{\mu_a^3} - \frac{g_\omega^2}{\mu_\omega^3} < 0{,}026 ; \qquad /10/$$

and from the symmetry energy

* The inequality sign is obtained from the account of the fact that the diffusion of the nuclear boundary which actually occurs yields the additional terms $-A^{2/3}$ , decreasing thereby the contribution of the potential energy.

$$\frac{g_\rho^2}{\mu_\rho^2} - \frac{g_\zeta^2}{\mu_\zeta^2} = 0{,}040 \; . \qquad /11/$$

A comparison with the term of the pairing energy yields also a relationship between the constants $f_\omega^2$, $f_\eta^2$, $f_b^2$. However, we are not sure that the term of the pairing energy should be accounted for just in this way, and not for the effect, e.g., the dynamical correlations between nucleons which we neglected.

If, use is made of the value $g_\rho^2 = 2.0$, then from (11) the following estimate of the coupling constant of the scalar $I = 1$ resonance is obtained

$$g_\zeta^2 = 0{,}43 \; . \qquad /12/$$

The account of the $\zeta$ resonance would lead to an increase in the attractive part of the ${}^1V_c^+$ potential which improves agreement with the phenomenological potential ( Fig. 3 ). This has been already pointed out in /10/.

It can be easily seen that the parameters of three resonances listed in Table 2 do not satisfy Eq. (9). For $g_\omega^2 = 12$, $\mu_\omega = 5.5$, and $\mu_a = 2.5$ we get from (9) the value of $g_a^2 = 3.1$. This emphasizes once more that basing only upon the comparison with the phenomenological N N -potential one can hardly get the unambiguous estimates of the parameters of the resonance model. However, it should be kept in mind that the correlations, if taken into account, will give a small probability for NN short distances, reducing thereby the contributions of the $\omega$ and $\rho$ -resonances in the potential energy of the nucleus. Thus, expressions (9), (11) and (12) must be regarded as equations defining the upper limits for $g_a^2$ and $g_\zeta^2$ coupling constants.

Note, finally, that this model allows to write equations for the potentials which the mesons of different symmetry form in the nucleus. As is seen from the above analysis, the main role in the nucleus belongs, apparently to the $\omega$, $a$ resonances. As to the role of the $\pi$ meson potential for example. it is very insignificant, indeed.

Owing to $\mu_a < \mu_\omega$, the nucleus must have the attractive potential of the form $e^{-\mu_a r}/r$ at large distance from the nuclear boundary. If this is compared with the behaviour of the optical potential of Saxon-Wood type $e^{-r/a}$ ( $a = 0.65$ f ) then we get the following estimate for the mass of the neutral scalar a-resonance $\mu_a = 2.2$ which is in good agreement with the value assumed in Sec. 2.

The consideration concerning the nuclear potential energy and the optical model parameters from the point of view of the resonance model provides an additional argu ment for the supposed existence of the neutral scalar a -resonance.

## Conclusion .

There is hardly any doubt that the discovery of the strong $\pi\pi$ interaction implies a new stage in the meson theory of nuclear forces. It is not still clear to what extent the non-resonance exchange of several $\pi$ mesons may be heglected. The consideration given above shows that the account of the resonance exchange only provides a correct qualitative description of the nucleon-nucleon potentials ( Fig. 1–3).

As far as the quantitadve characteristics of the resonance model of the NN interaction are concerned, they should be considered to be still rather tentative. This is due, first of all, to the yet unsolved problem about the number of principally possible resonances ( Table 1) , whose contribution to the nuclear forces is necessary to be taken into consideration. Another reason is a certain ambiguity of the phenomenological potentials, which are used to make the estimates of the model parameters. And, finally, the account of the non-resonance exchange may lead to some change in the quantitative estimates.

Therefore, it is necessary to resort to additional criteria for the coupling constants and resonance masses. The experimental values of the masses of some resonances are known. The limitations on the remaining parameters should be found from an independent analysis of the phenomena different from the NN scattering, e.g., the electromagnetic form-factors. $\pi$N- scattering/14/ and the like. It should be born in mind as well, that the resonance model admits a natural generalization to the potential interaction in the nucleon-antinucleon system that may serve as another source of information about the quantitative characteristics of the model.

Besides, as is shown above, some criteria can be obtained from the consideration of the properties of the many-nucleon systems.

So, the values of the coupling constants of the $\omega$, $\rho$, $a$ resonances with the nucleon should be regarded as preliminary. There are, nevertheless, additional arguments in favour of the large coupling constant of the $\omega$-resonance, different from those of the nuclear forces. Indeed, a small width of the $\omega$-resonance/14/ which is likely to mean that the constant $g_{\omega\pi}$ is small allows to expect a large value of the $g_{\omega N}$ constant because of the large cross section for the $\omega$-meson production in $\pi p$ collisions. This gives additional grounds to believe that it is the $\omega$-meson which is responsible for the appearance of strong repulsion at small distances. Its most important role here is to suppress the influence of possible resonances with the mass $\mu > \mu_{\omega}$. Therefore, in the region of not too high energies, or more accurately, of not too large momentum transfer, one can restrict oneself to the consideration of the resonances with the masses $2\mu_{\pi} < \mu < 5.6\,\mu_{\pi}$. The difference in the estimates of the coupling constants of the $\omega$, $\rho$, $a$ - resonances obtained in this paper and in/9/can be accounted mainly for the role which in our consideration is attributed to the $\omega$-resonance.

In conclusion let us emphasize that the phenomenological resonance approach to the problem of strong interactions appears to be very promising, as it was also noted in the discussions/3/ at Geneva Conference.

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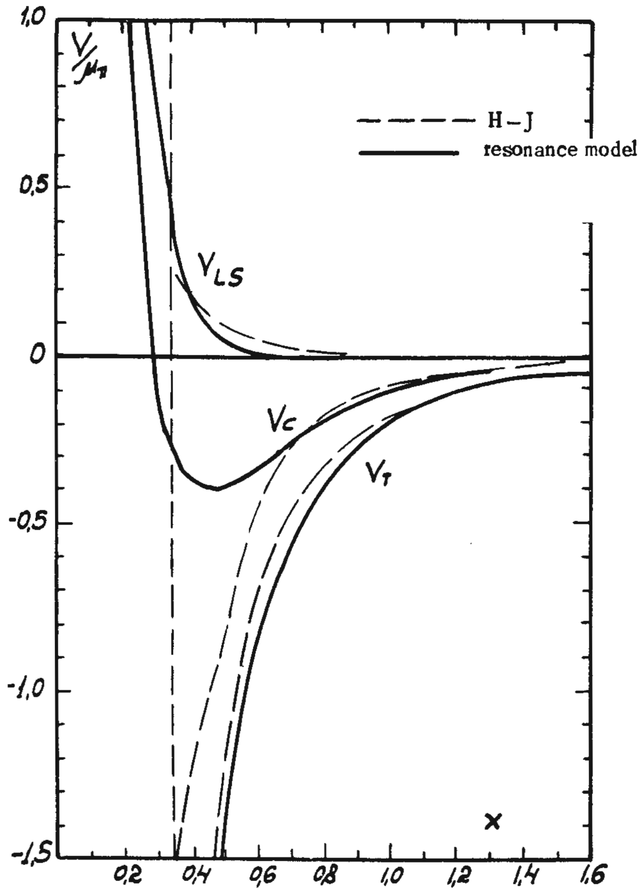


Fig. 1. Triplet even potentials.

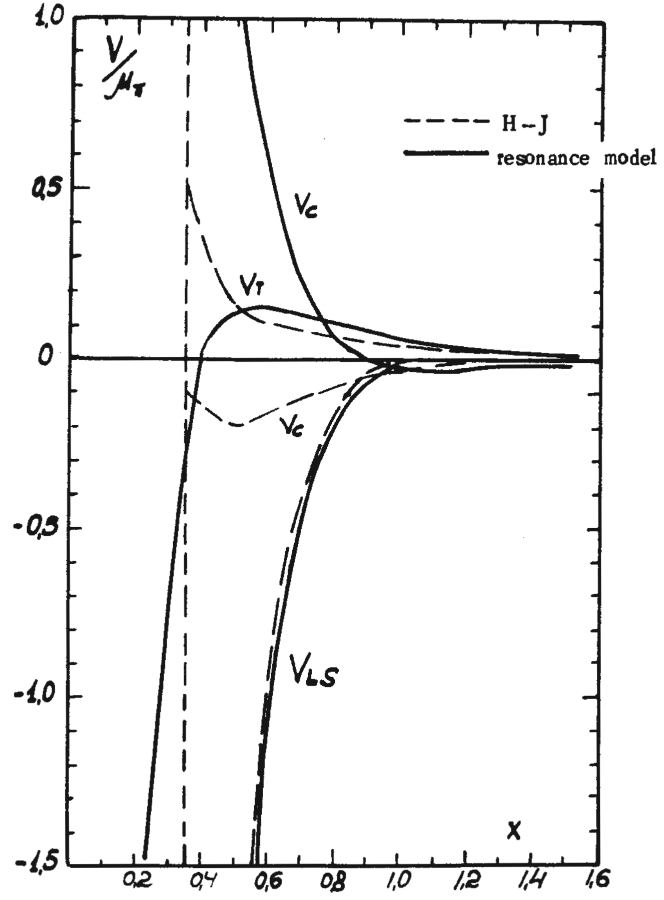


Fig. 2. Triplet odd potentials.

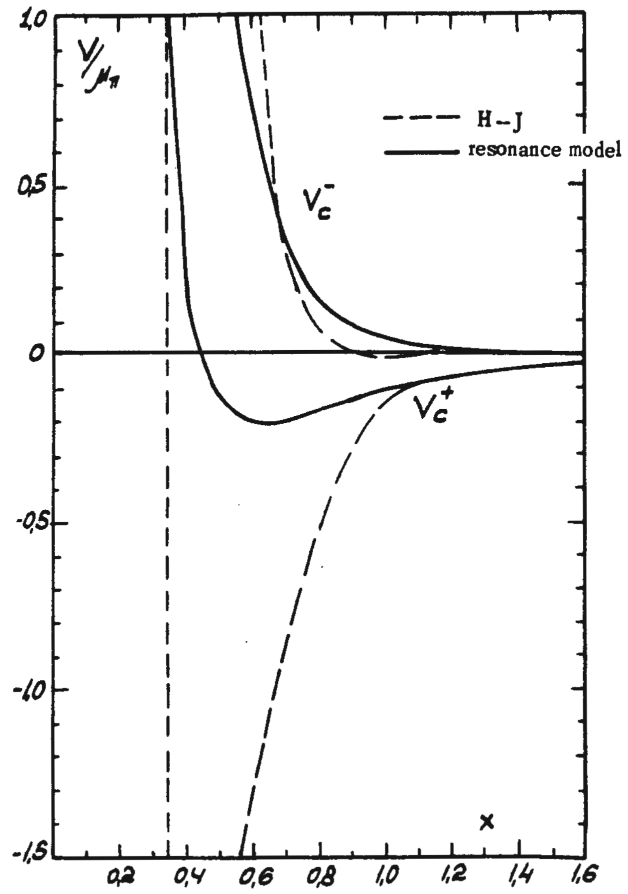


Fig. 3. Singlet potentials.