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ON CAUSALITY IN MODERN FIELD THEORY

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'Causality, as it is usually meant, is only a small part of the world relationship (materialistic addition), a part of not subjective but objectively real relationship'.

V.I.Lenin ('Philosophical Notes')

Introduction

In spite of the modest estimate which V.I.Lenin gave to the causality principle, it is, nevertheless, of fundamental importance in science as the simplest form of relationship between phenomena.

Of special significance is the causality principle for physics not only from the general philosophical point of view, but also as far as the mathematical form for expressing causality is concerned.

In contemporary physics the mathematical form of causality is based on two physical ideas: a) on the idea of the homogeneous and isotropic space-time of Einstein-Minkowski and b) on the idea that the interactions are transferred by the physical fields (electromagnetic fields, meson and neutron fields and the like).

At the same time it is well-known that the application of these principles to extremely small distances and short time intervals leads to the conclusions meaningless from the physical standpoint: the energy of particle interaction at small distances and the proper energy of particles turn out to be infinitely high.

This unsatisfactory result takes place both in quantum and classical physics, and is likely to indicate the same origin of the difficulties* encounted in these two concepts.

2 Causality in Classical Physics.

In classical physics the propagation of a weak (linear) signal from the world point $\mathcal{P}_1(x_1 t_1)$ to the world point $\mathcal{P}_2(x_2 t_2)$ is determined by the Green function $\mathcal{G}$, which is a function of the difference between the coordinates of the points $\mathcal{P}_1$ and $\mathcal{P}_2$: $x = x_1 - x_2$, $t = t_1 - t_2$.

This is the result of the homogeneity of space-time. The requirement that the space-time should be isotropic leads to the fact that the Green function must depend not simply upon the differences x and t, but on the four-dimensional interval $s^2 = x^2 - t^2$. Finally, it appears possible to introduce the time direction ϵ and the direction along the spatial ray η: $\epsilon = t/|t| = \pm 1$, $\eta = 0$ for $s^2 < 0$ and $\eta = x/|x| = \pm 1$, $\epsilon = 0$ for $s^2 > 0$. So, the Green function can be put as

$$\mathcal{G} = \mathcal{G}(s^2, \epsilon, \eta). \qquad /1/$$

This function is an invariant of the Lorentz transformation. Now the requirements of causality are additionally imposed;

a) the signal cannot propagate with a velocity greater than that of light C.

b) the signal is propagated only from the past to the future. These requirements lead to a further specification of the function $\mathcal{G}$:

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* Some philosophic problems of causality and field theory were treated in /1/ .

$$\mathcal{G} = \mathcal{G}(s^2, +1, 0) \qquad \text{for} \qquad \epsilon = +1,\ \eta = 0$$
$$\mathcal{G} = 0 \qquad \text{otherwise}. \qquad /2/$$

Fig. 1 shows the space-time domain, where the function $\mathcal{G}$ is different from zero. Note that the Fourier component of $\mathcal{G}(s^2, +1,0)$, denoted by $\mathcal{G}(\omega, k)$, depends only upon the invariant $m^2 = \omega^2 - k^2$: $\mathcal{G}(\omega, k) = F(\omega^2 - k^2)$ and different from zero only if $m^2 > 0$, for $m^2 < 0$ we would obtain the function $\mathcal{G}(s^2, 0, \pm 1)$ different from zero in the space-like domain and leading, therefore, to the signals propagating with a velocity greater than that of light.

The experiment shows that for large x and t the (asymptotical) wave field can be always interpreted from the corpuscular point of view. This implies that in the infinity we have a set of waves with the descrete values $m^2 = m_1^2,\ m_2^2, \dots m_s^2, \dots \geq 0$ *.

The Fourier-component $F(\omega^2 - k^2)$ has the poles at $\omega^2 = k^2 + m_s^2$, while the function $\mathcal{G}(s^2, +1,0)$ possesses the singularities of the form $\delta(s^2)$. In virtue of the properties of the interval s^2 this singularity will also hold in the domain of small x, t (if only $s^2 = 0$) and will lead there to undesirable infinities.

Thus, the reasonable suppositions about the isotropy of the space-time, justifiable for large x and t are transferred automatically to the domain of infinitely small x and t.

3. Causality in Quantum Physics.

The quantum theory, strangely enough, retains, in principle, the classical concept of causality. In other words, in the quantum theory the signal (or interaction) is also transferred by the Green function $D_c(s^2)$ (which is refferred to as the causal function as well). This function connects the quantum transition in the vicinity of the point $\mathcal{P}_1$ with that in the vicinity of the point $\mathcal{P}_2$ **.

Unlike the classical Green function, it is not equal to zero for $s^2 > 0$, as well. However, this is true only for the scales $\approx h / mc$ (of the Compton particle length). To be able to fix the fact of the emission of a signal (quantum) with a positive energy from the vicinity $\mathcal{P}_1$ and the fact of its absorption in the vicinity $\mathcal{P}_2$, it is necessary that these 'vicinities' would be large enough. Namely, in accordance with the uncertainty relation, for the quantum-signal with the energy ϵ and the momentum p the dimensions of the 'vicinities' $\mathcal{P}_1$ and $\mathcal{P}_2$ must be $T \gg h/\epsilon$ with respect to time, and $L > h/p$ with respect to space.

Further these vicinities should not be overlapped (the distance between them is $|x| \gg L$ and the time interval is $|t| \gg T$). As has been shown by M.Firtz for the point particles/3/, the properties of the function $D_c(s^2)$ under these conditions provide a purely classical causal relationship between the vicinities of the points $\mathcal{P}_1$ and $\mathcal{P}_2$ (i.e., the relationship equivalent to that given by the Green function $\mathcal{G}(s^2, +1,0)$. When the above inequalities are not fulfilled, the uncertainty relations do not permit to judge about the nature of the causal relationship at all (what happened later, what earlier?). The causal function $D_c(x)$ in the spatial domain is not equal to zero, what leads to an existence of the spatial form-factors of elementary particles $F(q)$ (q is the momentum transferred to a particle).

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* In the quantum field theory the quantity m determines the mass of the particle corresponding to the field under consideration.

** The causality principle in its conventional form was used by N.N.Bogolubov for a new concept of the modern field theory

In consistence with such a form-factor the hard spatial distribution of charges and currents of type $\rho(x) = \int F(q)\, e^{iqx}\, d^3q$ may be attributed to a particle. Such a hard distribution allows the signal to be propagated with an infinitely high velocity from the particle periphery to its centre.

It has been shown in /4/, however, that in this case the uncertainty relation also does not make it possible to 'accuse' the particle that it propagates signal with a velocity greater than that of light.

In spite of the mentioned difference of the causal Green function $D_c(s^2)$ from the classical one $\mathcal{G}(s^2, +1, 0)$, the situation with the singularities in the quantum field theory remains essentially the same as in the classical theory; the singularities of the propagation functions, quite natural for large x and t are transferred inexorably to the domains of small space and time scales.

4 Some Possible Generalizations of the Causal Relationship

The specific features of the propagation functions point out that it is necessary to give up the idea of transferring the macroscopic laws of signal (effect) propagation to the domain of especially small scales of space-time and to try to change them.

What has been said about the meaning of the uncertainty relations enables us to have in mind a possibility of reconciling the conventional form of the macrocausality with other forms of microcausality in small space-time domains.

Consider now some possible generalizations of the theory.

a) The Non-linear Theory.

The Green functions having the above singularities are connected with the propagation of weak fields, obeying the linear equations.

M.Born was the first to notice /5/ that strong fields may obey other non-linear equations. In this case the speed of propagating the signal V depends on the strength and the form of the signal (see /6,7,8/).

Indeed, the characteristics of the non-linear equation are different from the straight lines $\frac{dx}{dt} = \pm c$ specific for the linear theory. Therefore the velocity of the non-linear signal V turns out to be a function of the field intensity ϕ and its derivatives $\frac{\partial\phi}{\partial x}$, $\frac{\partial\phi}{\partial t}$:

$$\frac{dx}{dt} = \pm V\left(\phi, \frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial t}\right). \qquad /3/$$

As has been shown in /5/, in some versions of the non-linear theory the quantity V may become imaginary, and the hyperbolic equation for the field will turn into an equation of the elliptic type. Far from the source and the receiver of the signal (we can say – far from the particles) the field will obey the linear equation as earlier, while the Green function will have usual singularities of the type $\delta(s^2)$. However, in the vicinity of the particles, where the fields are strong, the character of the singularities will change. For instance, when the equation turns into the elliptic one, the singularity of the Green function at $x \to 0$, $t \to 0$ will have the form $1/R^2$ where $R^2 = x^2 + t^2$.

This possible change of the type of a field equation near the particles resembles the situation occuring on the wing of an aeroplane flying with a velocity close to that of sound. As is well-known, in the place where the local speed of a current flowing around the wing, exceeds that of sound, the elliptic type of an equation turns into a hyperbolic one.

Fig. 1b shows the domain where the causality may become anomalous. Note, that the violation of the relativistic invariance near $x = 0$, $t = 0$ is only seeming and is due to the fact that the space-time point where the field source is situated is a special point. In the vicinity of this point the non-linear field changes the medium for its own propagation.

The possibility of changing the type of equations for the field propagation in the vicinity of the particles together with a change in the form of the causal relationship is very attractive.

However, nobody has so far succeeded in finding a quantum analog for this model of the field theory.

The question is still open as to what changes in the definitions of the length and of the time interval may be caused by the non-linearity in the signal propagation. It goes without saying, that Einstein's definitions suggest the linearity of the signal.

b) Variation of Causality for Small Space-Time Scales.

We have seen that in the homogeneous space-time the law of the causal relationship cannot be violated in the microworld without violating it in the macroworld. A possible way of modifications is prompted by the non-linearity treated in Sec. a). The conventional regularities of the signal propagation are not broken down everywhere, but only in the neighbourhood of the sources and the receivers of the signal, i.e., near the particles. In other words, this happens where the space homogeneity is violated by the particle located there. This indicates a possibility of violating the conventional laws of the signal propagation near the particles/9,10/.

From a mathematical point of view this possibility arises because of the appearance of new invariants, besides s^2, ϵ, η. Indeed, the total energy-momentum vector $\mathcal{P}(E, \vec{p})$ which commutes with relative coordinates and with other internal dynamical variables* is associated with a particle or with a system of interacting particles. Besides the invariant $I_1 = s^2$, there appear new invariants such as $I_2 = \mathcal{P}^2 = -m^2$ (where m is the rest mass of the whole system) $I_3 = (\mathcal{P}, s)$ and others. This makes possible to form new invariant combinations such as

$$R^2 = I_1 + I_3^2/I_2$$
$$T^2 = I_3^2/I_2, \qquad /4/$$

which in the centre-of-mass system pass into r^2 and t^2, respectively. Further they are transformed by (4) and (4')/12/.

In virtue of this, the Green function connected with the system of particles can be written as

$$\mathcal{G} = \mathcal{G}(I_1, I_2, I_3). \qquad /5/$$

The presence of the invariants I_2, I_3 allows to change the behaviour of $\mathcal{G}$ in the vicinity of $r, t = 0$.

Fig. 1b can be used again to illustrate the behaviour of the function $\mathcal{G}$, which has at $R^2 < a^2$ an elliptic structure, while at $R^2 > a^2$ it turns into the usual Green function with the singularities on the cone $s^2 = 0$.

Just in a similar manner the causal function $D_c(s^2)$ can be changed if it is associated not with the vacuum, but with the particles put into the vacuum and having relative coordinates $x = x_1 - x_2$ and the total momentum $p = p_1 + p_2$:

$$D_c = D_c(I_1, I_2, I_3). \qquad /6/$$

A complete scheme of such a type has not yet been worked out. It is still obscure what model of the field theory it corresponds to.

* This made possible for Yu.M.Shirokov to solve correctly the problem of the relativistic rotator,/11/.

In particular, it has not so far been investigated whether the unitarity of the S-matrix would be kept or not.

c) Modification of the Physical Vacuum Metric

Other possibilities of varying the form of causality may lie in the modifications of our space-time geometry for small space-time domains.

One of such possibilities is the fluctuations of the metric tensor $g_{\mu\nu}$, which can, in principle, be due to the fluctuations of the zero vacuum energy.

The fluctuations of such a kind will lead to the fluctuations of the space-time interval

$$s^2 = \int_{\mathcal{P}_1}^{\mathcal{P}_2} g_{\mu\nu}\, dx_\mu\, dx_\nu \qquad /7/$$

and, therefore, all the functions such as $\mathcal{G}(s^2)$, $D_c(s^2)$ will turn out to be 'diffused'/13,14/. If the infinities are eliminated, these fluctuations prove to be essential in the space-time domains of the order of $L_0 = (\frac{h\chi}{c^3})^{1/2} = 0{,}82.10^{-32}$ cm. (Here χ is the gravity constant). These scales seem to be too small to play an essential role in the particle world. The introduction of another scale for the vacuum fluctuation ℓ_0 would mean a new physical hypothesis, whose consequences and internal consistence are far from being investigated.

d) Space-Time 'Quantization'

The old idea of space-time 'quantization'/15/ was revived several times/16-18/.

Contemporary tendencies in developing this idea start from the assumption about the non-Euclidian character of metric in the momentum space/19/. It is the interval in the space of the momenta p_1 p_2 p_3 p_4 which is supposed to be

$$d\sigma^2 = a_{\mu\nu}\, dp_\mu\, dp_\nu \ . \qquad /8/$$

The radius of the curvature of this metric space plays the role of the limiting cut-off momentum $\mathcal{P}_0$. The space-time coordinates x_1 , x_2 , x_3 , x_4 canonically conjugated to these momenta turn out to be non-commuting operators

$$[\, x_\mu \, , \, x_\nu \,] = i\, b_{\mu\nu}. \qquad /9/$$

The theory is constructed so that for the scales $\ell \gg \frac{h}{\mathcal{P}_0}$ it becomes a conventional one. It is clear that the concept of the conventional causality in this theory turns out to be not valid (at least, in the space-time domains $\approx \frac{h}{P_0}$). Indeed, one cannot speak about the signal propagation from the point $\mathcal{P}_1(\, x'_1 \;\; x'_2 \;\; x'_3 \;\; x'_4 \,)$ to the point $\mathcal{P}_2(\, x''_1 \; x''_2 \; x''_3 \; x''_4 \,)$ if the coordinates of these points remain undetermined. In this theory the process of the signal propagation has the physical sense only for $|x_\mu|$ large enough,when the right-hand side in (9) may be neglected. For smaller scales the relationship between phenomena may be described mathematically by means of the momentum space only. The theory of the quantized space-time has not yet been developed consistently.

Conclusion

The form of causality adopted in the modern theory follows from the basic space-time concepts.

It was borrowed from the macroscopic physics and is automatically applied to the infinitely small scales because of the nature of the singularities of the Green functions. This leads to the appearance of the divergencies (infinities) for a number of the most important physical quantities associated with elementary particles.

We have considered here some preliminary theoretical models which modify essentially cansality for small space-time scales, retaining at the same time, the macroscopic causality.

We are not aware which of these models leads us nearer to the truth – we are still playing blind man's buff with it.

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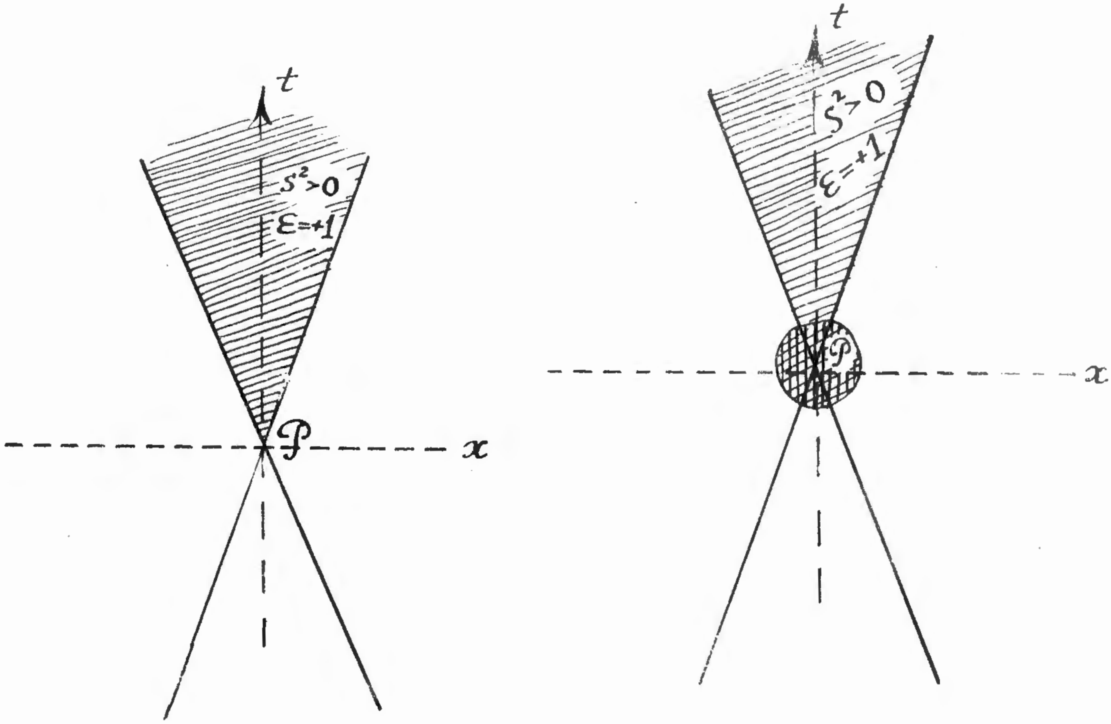

1a   1B

Fig. 1. Fig. 1a shows the shaded space-time domain allowed by the conventional theory of the propagation of the signals going from the point $\mathcal{P}$.

The double shaded area in Fig. 1b shows the domain of anomalous causality (for instance, an elliptical type of the field equations).