ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

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INVERSION OF HELICITY IN NUCLEAR REACTIONS

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In the well-known paper of Minami[1](comp.[2]) it has been shown that the cross section of the scattering of a particle with spin 1/2 by a spinless target remains invariant under the transposition of the phase shifts of all pairs of states belonging to the same total angular momentum:

$$\delta(\ell = j - ½) \rightleftarrows \delta(\ell = j + ½). \tag{1}$$

Minami transformation changes, however, the sign of the transversal component of polarization. The resulting double valuedness is therefore eliminated if this sign is known. In the paper of Pusikov, Ryndin and Smorodinsky[3] it has been noted that the additional inversion of sign*:

$$\delta(\ell - ½) \rightleftarrows -\delta(\ell + ½) \tag{2}$$

changes neither the cross section nor the polarization and can lead to an ambiguity of phase shift analysis. Such ambiguity is eliminated, e.g. if polarization is rotated by a magnetic field or by measurements of the energy dependence of the phase shifts at small energies.

In a subsequent paper Zastavenko[4]** discussed the possibility to generalize the transformation (1) to the case of higher spins and relativistic particles. However, the physical meaning of transformation (2) was not made clear enough. In fact, this transformation reflects the symmetry of a simple type. Namely, (2) is nothing that the change of the sign of the helicity of the particles (i.e. the projection of the polarization of a particle on the direction of its velocity). The simple fact that it is impossible to measure the sign of the longitudinal polarization in scattering experiments without the use of external fields (or polarized targets) leads to this doublevaluedness. In such a form this statement may be easily generalized to the case of arbitrary spins. More unexpected is the transition to relativistic systems where "relativistic spin rotation" violates the mentioned symmetry and leads at sufficiently large energies to a "fine structure" of the cross sections of repeated scatterings calculated on the basis of the results of the "complete set of experiments". Transformation (1) may be written in a matrix form:

$$M \rightleftarrows (\sigma n_f)\, M\, (\sigma n_i). \tag{3}$$

Here M is the scattering matrix (in the spin space), n_i and n_f are the unit vectors in the directions of the incident and the scattered particles respectively in c.m.s. Since

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* Recently this substitution has been used by Nauenberg and Pais[5].

** See also[6]. Paper[7] contains an erroneous conclusion.

$\sigma n_{i,f} = \exp i(\sigma n_{i,f}) \frac{1}{2}\pi \equiv U(n_{i,f})$ is the operator of the spin rotation by the angle π around the axis $n_{i,f}$, then (3) is rewritten in the form:

$$M \to U^{+}(n_f)\, M\, U(n_i). \tag{4}$$

It is obvious without calculations that such a transformation changes the sign of the transversal polarization of the particles. Transformation (4) anticommutes with space reflection

$$P(\sigma n) = -(\sigma n) P \tag{5}$$

(n is one of the vectors n_i or n_f . It changes therefore the parity of the state. It is easy to see that the transformation (4) does not violate the condition of unitarity

$$i(M^{+} - M) = \frac{k}{2\pi} M^{+} M. \tag{6}$$

Besides, it is obvious that (6) is invariant under the substitution

$$M \to -M^{+}. \tag{7}$$

This follows from the fact that $MM^{+} = M^{+}M$ due to the unitarity nature of the matrix

$$S = 1 + 2ikM^{+}.$$

Transformation (7) consists of the inversion of the sign, transition to the Hermitian conjugated matrix in the spin space and the transposition of the initial and final momenta. The signs of all spin components are changed. This transformation corresponds to change of the signs of all phase shifts. This is easily seen if we write the scattering matrix in the rM representation in which it is symmetrical (in virtue of the time reflection symmetry).

Two successive transformations: rotation of all spins at π around the respective momenta and transformation (7) lead to the inversion of the helicity of all particles*.

For a paricle with spin 1/2 the inversion of helicity reduces to the substitution (2). The above considerations are immediately generalized to the case of an arbitrary spin, only the form of the operators being changed. In particular, the rotation operator $U(n)$ for a particle with spin S will be equal to exp $i(Sn)\pi$ and its commutation with the space reflection operator is of the form

$$UP = (-1)^{2S} PU. \tag{9}$$

When a system consists of several particles the inversion of helicity is to be made for each particle. The rotation operator decomposes in this case into the product of operator affecting each particle.

* It is worth to mention the similarity of this transformation to the so called G - inversion in the isotopic spin space.

Therefore, for systems with integral spin Minami transformation and inversion of helicity do not change the parity of states, while for systems with half-integral spin parity changes.

In considering any process occuring in a collision of an arbitrary polarized beam with an arbitrary polarized target, we may conclude that if an experiment determines only the absolute value of the components of polarization in the initial and final states, there exist four sets of amplitudes satisfying all results of experiments. If in these experiments the sign of the transversal polarization is determined, then there remain two sets only . To eliminate the remaining double-valuedness it is necessary to measure the sign of the longitudinal polarization. For this it is necessary to measure the pseudoscalar Sn . This can be made by introducing either the magnetic field (measurement of the scalar $(Sn)(Hn)$ of the electric field (scalar $(Sn)(E \cdot n_1 \times n_2)$ or finally, by analysing the energy dependence of the observed effects (Sn is transforms like a scalar under time inversion).

A polarized beam is usually obtained by means of scattering by several target, the scattering planes being different. If particles are nonrelativistic the direction of polarization does not change in transitions from one reference frame to another and repeated scattering does not modify the above conclusion. This is easily seen if we take into account the fact that the cross section of the n-fold scattering started with the scattering of an unpolarized beam is described by the trace of the density matrix

$$\rho = \underbrace{M \ldots M}_{n \text{ times}} \quad \underbrace{M^{+} \ldots M^{+}}_{n \text{ times}} \qquad (10)$$

and that the direction of the incident particle coincides with that of the scattered particle in the previous scattering.

The situation changes if particles are relativistic.

Stapp[8] has pointed out the role of relativistic effects in repeated scattering processes. The polarization of the scattered particle after the first scattering (by a target at rest) rotates at an angle Ω (in the negative direction of the scattering angle) in the transition to the center-of-mass system of the second scattering (See Appendix). Since the rotation at the angle Ω around the axis normal to the scattering plane does not commute with the inversion of helicity, this effect eliminates the above mentioned ambiguity. In this sense Lorentz transformation plays a role similar to that of a magnetic field introduction between successive scatterings.

For spinless particles the second scattering yields no new information compared to that of the first one and does not eliminate the ambiguity of the sign of the phase shifts connected with transformation (7). For particles with spin 1/2 the polarization resulting in the scattering of the unpolarized beam is normal to the scattering plane. Therefore the invariance of the asymmetry of the second scattering under the transformation $M \to -UM^{+}U^{+}$ is not eliminated by the relativistic spin rotation. This invariance is violated starting from the asymmetry of the third scattering*.

For particles with spins higher than 1/2 the vector part of the polarization after the first scattering does not changes by the relativistic rotation around the normal. But the tensor part of this polarization is not invariant under this rotation. In this case the invariance is violated already for the asymmetry of the second scattering**.

Note that the invariance of the observables does not occur at all if condition incompatible with the inversion of helicity of all particles are imposed. If for the scattering of particles with spin 0, 1/2 only an odd number of first $2k+1$ phase shift is assumed to be different from zero, then it is impossible to make the corresponding transformation of phase shifts: all the results of the successive scatterings are different for all of the 2^{2k+1} sets of phase shift ($2k+1$ is the number of phase shifts in the order $s_{1/2}$, $p_{1/2}$ ...) compatible with the cross section of the scattering of an unpolarized beam[8].

APPENDIX 1

We give here a simple derivation of the formula for the relativistic spin rotation based on the formulas of hyperbolic geometry[9]. This formula has been derived by Stapp[10] by multiplying Lorentz transformation matrices.

In order to compare the direction of a four-vector in two different reference frames we need to carry out a parallel translation of the vector from one system of coordinates to another and compare the direction of the transferred vector with the vector given in the second system[11]. In going over from one scattering to another a transition is performed from the rest system R to the c.m.s. C of the first scattering (in which the scattering matrix is given). After this the polarization can be transformed into the system of the target L (Lab.sys.). To calculate the second scattering we must again transfer the

* It is curious that for the system of massless particles (extreme relativistic case) Minami transformation does not change the state of the particles (they are longitudinal) and the state of polarization does not changes by relativistic effects. The only essential transformation will be (7) which in this case is nothing else as charge conjugation.

** This was found by Zastavenko[3] by means of a direct calculations.

polarization to the rest system. Three successive transformations $R \to C \to L \to R$ do not return the vector to the initial value due to the non-euclidean matric of the Lobachevsky velocity space[11].

We consider a space the radius-vector of which (or space point) describes the particle velocity. In this space the hyperboloid metric holds. The segment between the two points determines the relative velocity of the particles:

$$th\, a = \beta ; \quad ch a = (1-\beta^2)^{-1/2} \equiv \gamma ; \quad 2 ch^2 \frac{a}{2} = 1 + \gamma$$

The scalar product of the two velocity vectors can be written in the form

$$p_1 p_2 = m_1 m_2 \gamma_{12}$$

To compare the two vectors one of them is to be displaced parallely along a geodesic line. The rotation under consideration is none other than the hyperbolic defect (the difference betwen π and the sum of the angles of a triangle) of the hyperbolic triangle with vertices defined by velocities of the three systems R, C, L. Several formulas can be written for the hyperbolic defect. If we denote the sides of the triangle by a, b and c then stapp formula coincides with that for the defect :

$$\sin \Omega = \frac{1 + ch a + ch b + ch c}{8 ch^2 \frac{a}{2}\, ch^2 \frac{b}{2}\, ch^2 \frac{c}{2}} (1 + 2\, ch c\ ch b\ ch c - ch^2 a - ch^2 b - ch^2 c)^{1/2}$$

or more briefly,

$$\cos \frac{\Omega}{2} = \frac{1 + ch a + ch b + ch c}{4 ch \frac{c}{2}\, ch \frac{b}{2}\, ch \frac{c}{2}}$$

In annals on Non-Euclidean geometry are usually given other formula, e.g.

$$\sin \frac{\Omega}{2} = \frac{[sh\, p\ sh(p-a)\ sh(p-b)\ sh(p-a)]^{1/2}}{4 ch \frac{a}{2}\, ch \frac{b}{2}\, ch \frac{c}{2}}$$

where $2p = a + b + c$, from which our expressions are obtained by means of simple transformations.

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Note added in proof. After this paper was completed we received the paper by Wick[12], in which he also discusses the interpretation of Ω as a hyperbolic defect .

APPENDIX 2

Transformation of Scattering Matrix Elements and Observables

We have considered the transformations:

$$M \to - M^+ \qquad \text{(a)}$$

$$M \to U^+ M U \qquad \text{(b)}$$

$$M \to - U^+ M^+ U \qquad \text{(c)}$$

1. Spin zero scattering. We put the scattering amplitude into the form

$$M = e^{ia}\sqrt{\sigma}$$

Under the transformation (a) $\sigma \to \sigma$, $a \to \pi - a$ the corresponding phase transformation is $\delta_\ell \to -\delta_\ell$

2. Spin 1/2. $M = a + b(\sigma n)$, where

$$a = \tfrac{1}{2}\sqrt{\sigma}\, e^{ia}(e^{-i\beta}\sqrt{1+P} + e^{i\beta}\sqrt{1-P}$$

$$b = \tfrac{1}{2}\sqrt{\sigma}\, e^{ia}(e^{-i\beta}\sqrt{1+P} - e^{i\beta}\sqrt{1-P}$$

σ is the differential cross-section,

P is the value of the polarization.

Under (a): $a \to -a^*$, $b \to b^*$, $\sigma \to \sigma$, $P \to -P$, $a \to \pi - a$

Under (b): $a \to a\,Cos\theta + ib\,Sin\theta$, $b = -ia\,Sin\theta - b\,Cos\theta$ − $\sigma \to \sigma$, $P \to -P$, $a \to a$, $\beta \to 2\theta - \beta$

Under (c) $a \to -a^*\,Cos\theta + ib^*Sin\theta$, $b = ia^*Sin\theta - b^*Cos\theta$, $\sigma \to \sigma$, $P \to P$, $a \to \pi - a$, $\beta \to 2\theta - \beta$

Under (c) the transversal-longitudinal $(t - \ell)$ components of the tensor which connect the plane components of polarization before and after the scattering, change their signs. The $t - t$ and $\ell - \ell$ components are invariant under (c).

3. Scattering of Nucleons by Nucleons

$$M = \frac{a}{2}[1 + (\sigma_1 n)(\sigma_2 n)] + \frac{b}{2}[1 - (\sigma_1 n)(\sigma_2 n)] + \frac{c}{2}[(\sigma_1 m)(\sigma_2 m) + (\sigma_1 \ell)(\sigma_2 \ell)] +$$
$$+ \frac{d}{2}[(\sigma_1 m)(\sigma_2 m) - (\sigma_1 \ell)(\sigma_2 \ell)] + \frac{e}{2}[(\sigma_1 n) + (\sigma_2 n)]$$

$$a = 2^{-3/2}\sqrt{\sigma}\, e^{ia}(e^{-i\beta}\sqrt{1 + D + K + Q + 4P} + e^{i\beta}\sqrt{1 + D + K + Q - 4P})$$
$$b = 2^{-1/2}\sqrt{\sigma}\, e^{i(a+\gamma)}\sqrt{1 + D - K - Q}$$
$$c = 2^{-1/2}\sqrt{\sigma}\, e^{i(a+\gamma)}\sqrt{1 - D + K - Q}$$
$$d = 2^{-1/2}\sqrt{\sigma}\, e^{i(a+\epsilon)}\sqrt{1 - D - K + Q}$$
$$e = 2^{-3/2}\sqrt{\sigma}\, e^{ia}(e^{-i\beta}\sqrt{1 + D + K + Q + 4P} - e^{i\beta}\sqrt{1 + D + K + Q - 4P})$$

Notation: P -polarization of non-polarized beam

D -depolarization in forward hemisphere

K -polarization in backward hemisphere

Q -normal-normal component of correlation of polarizations.

Under (a): $a \to -a^*$, $b \to -b^*$, $c \to -c^*$, $d \to -d^*$, $e \to e^*$, $a \to \pi - a$

$P, \gamma, \delta, \epsilon$ -change their signs

σ, D, K, Q, β-invariant

Under (b): $a \to a\,Cos2\theta + ie\,Sin2\theta$, $e \to -ia\,Sin2\theta - e\,Cos2\theta$, $b \to b$, $c \to c$, $d \to d$, $2\theta - \beta$, $P \to -P$

others coefficients are invariants.

Under (c): $a \to -a^*Cos2\theta + ie^*Sin2\theta$, $e \to ia^*Sin2\theta - e^*Cos2\theta$, $b \to -b^*$, $c \to -c^*$, $d \to -d^*$, $a \to \pi - a$,

$\beta \to 2\theta - \beta$ $\gamma \to -\gamma$, $\delta \to -\delta$, $\epsilon \to -\epsilon$ others coefficients are invariants.

The corresponding phase transformation in triplet state in terms of phase matrix:

$$S^j_{11} \to \frac{1}{(2j+1)^2}\{S^j_{11} + 4j(j+1)S^j_{22} + 4\sqrt{j(j+1)}\,S^j_{12}\}$$

$$S^j_{12} \to \frac{2\sqrt{j(j+1)}}{(2j+1)^2}(S^j_{11} - S^j_{22}) + \frac{4j(j+1)-1}{(2j+1)^2}\,S^j_{12}$$

$$S^j_{22} \to \frac{1}{(2j+1)^2}\{4j(j+1)\,S^j_{11} + S^j_{22} - 4\sqrt{j(j+1)}\,S^j_{12}\}$$

Using the mixing parameters ϵ :

$$S^j_{11} = Cos^2\epsilon_j \cdot exp\,2i\delta^+_j + Sin^2\epsilon_j \cdot exp\,2i\delta^-_j$$

$$S^j_{22} = Sin^2\epsilon \cdot exp\,2i\delta^+_j + Cos^2\epsilon \cdot exp\,2i\delta^-_j$$

$$S^j_{12} = ½\,Sin\,2\epsilon \cdot (exp\,2i\delta^-_j - exp\,2i\delta^+_j)$$

we get for transformation:

$$\delta^+ \to -\delta^-$$
$$\epsilon \to -\epsilon + arcctg\,2\sqrt{j(j+1)}$$

The phase shifts in singlet states and in states with $j = \ell$ are invariant under (c) and change sign under (a) and (b).

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