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# ДЕПОНИРОВАННАЯ ПУБЛИКАЦИЯ

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# Estimation of systematic errors of cross section asymmetries in deep inelastic muon scattering



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ABSTRACT

Formulae are derived to calculate the systematic error of the deep inelastic lepton-nucleon cross section $\sigma(E; E', \theta)$ and of the asymmetry $(\sigma_1 - \sigma_2)/(\sigma_1 + \sigma_2)$ given the error of the beam energy E and of the variables E' and $\theta$. Including smearing, beam and magnetic field uncertainties, numerical results are given for a particularly well-controlled experiment of the NA4 set-up at the CERN SPS.

# ESTIMATION OF SYSTEMATIC ERRORS OF THE CROSS SECTION ASYMMETRIES

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It has been shown that the statistical accuracy of the cross section asymmetry measurements at large $Q^2$ can be better than I%. Here the possible systematic accuracy of those measurements is estimated. As main sources of uncertainties we include the beam angle and energy spread, smearing of the kinematic variables due to the finite resolution and the magnetic field uncertainties. All of them affect the accuracy to which we know E, E', and Θ, i.e. their mean values and standard deviations.

The estimations are performed in 3 steps:

I. calculation of the relative derivatives of the cross section $\sigma$;
2. study of the errors and deviations of E,E' and Θ in order to calculate $\Delta\sigma / \sigma$ as a function of X and Y;
3. estimation of the asymmetry error $\Delta\left[(\sigma_1 - \sigma_2)/(\sigma_1 + \sigma_2)\right]$.

The idea of estimation is to show that the measurement of cross section asymmetries with about a I% accuracy is feasible providing the crucial parameters of NA4 are under control. Of course it can not replace a careful study of systematic effects in the apparatus.

## I. Derivatives of Deep Inelastic Cross Sections

Since we deal with a large number of events, it is reasonable to use the Taylor expansion of $\sigma$(E,E',Θ) and to express the errors of $\alpha \equiv$ (E,E',Θ) by moments $\mu_i^{\alpha}$ of probability distributions $P(\alpha,\tilde{\alpha})$. As variables we use (E',Θ,E) which allow us to discuss different origins of errors separately. Expansion of $\sigma \equiv d\sigma/dE'd\theta$ gives

$$\frac{\Delta\sigma}{\sigma} = (\mathrm{I})\frac{\mu_1^{E'}}{E'} + (\mathrm{II})\mu_1^{\theta}\sqrt{E} + (\mathrm{III})\frac{\mu_1^{E}}{E} + (\mathrm{IV})\frac{\mu_2^{E'}}{E'^2} + (\mathrm{V})\mu_2^{\theta}E + (\mathrm{VI})\frac{\mu_2^{E}}{E^2} + \cdots \qquad (\mathrm{I})$$

where $\mu_i^{\alpha}$ is an i-th moment of the variable $\alpha$, for example,

$$\mu_1^{E'} = \int (\tilde{E}' - E')\, P(E',\tilde{E}')\, d\tilde{E}\ ; \quad (\mathrm{I}) = E'\frac{\partial \ln\sigma}{\partial E'}\ ; \quad (\mathrm{II}) = \frac{1}{\sqrt{E}}\frac{\partial \ln\sigma}{\partial\theta}\ ;$$

$$(\mathrm{III}) = E\frac{\partial \ln\sigma}{\partial E}\ ; \quad (\mathrm{IV}) = \frac{E'^2}{2\sigma}\cdot\frac{\partial^2\sigma}{\partial E'^2}\ ; \quad (\mathrm{V}) = \frac{1}{2E\sigma}\cdot\frac{\partial^2\sigma}{\partial\theta^2}\ ; \quad (\mathrm{VI}) = \frac{E^2}{2\sigma}\frac{\partial^2\sigma}{\partial E^2}\,. \qquad (2)$$

The series of(eq.I) is written in such a way that the expansion coefficients (2) are functions of (x,y) only provided $F_2$ is scale invariant. Crossed and higher order terms are neglected. The coef-ficients (I)-(VI) are given explicitly in the Appendix.

In Fig.I the coefficients (I)- (VI) are given as a function of X at Y=0.5, and in Fig.2 as a function of Y at X=0.4.

Fig.I illustrates that all the coefficients (hence the error $\Delta\sigma/\sigma$ ) become increasingly large for $X \to I$. This behaviour is due to the divergence of the logarithmic derivative of $F_2$ ($F_2 \xrightarrow[x\to 1]{} C\cdot(I-X)^3 \to 0$). It is clear from the Appendix that the derivatives $x \frac{d}{dx} \ln F_2$ and $x^2 \frac{d}{dx^2} F_2$ are characteristic terms in (I)-(VI). They are plotted in Fig.3 versus X for three different parametrizations of the structure function. Scaling violations according to $F_2(X,Q^2) = C(X,Q^2) \cdot F(X)$ yield an additive term, $x \frac{d}{dx} \ln C$ , which is negligible for the Fermilab parametrization [1] of $F_2$. It canbe concluded that the expansion coefficients of eq.(2) depend very weakly on the details of the cross section.

## 2. CROSS SECTION ERROR

Having calculated the expansion coefficients we have to know the moments $\mu_1^\alpha, \mu_2^\alpha$ in order to estimate $\Delta\sigma/\sigma$ . The choice of these moments depends on the extent to which we control the beam, smearing and the magnetic field. Again, $\mu_1^\alpha$ represents the uncertainty of the mean value of $\alpha$ and $\mu_2^\alpha$ is the width of the distribution, e.g. $\mu_2^\alpha = \frac{1}{2}b$ for a Gaussian $e^{-b\alpha^2}$ .

The moments of E depend on the beam, and we use the values given in [2]. The beam determines also $\mu_1^\theta$ if multiple scattering is assumed to be symmetric. The second moment $\mu_2^\theta$ is of the order of $\Theta_{inc}^2 = (\mathrm{I\ mrad})^2$ both for the beam contribution [I] and for smearing [5]. Thus we use $\mu_2^\theta \approx \sqrt{2}\,\Theta_{inc}^2$ and, somewhat arbitrarily, $\mu_1^\theta \approx \frac{1}{20}\Theta_{inc}$. Finally, we have to set the moments of E'. The width of the E' distribution is dominantly caused by energy losses and in [3] it is estimated to be $(.043)^2$. The $\mu_1^{E'}/E'$ measures the extent to which we know the mean energy losss and receives an extra contribution from the magnetic field because of $E' = \frac{\lambda_{1/2}}{8\Delta} C.3B$. We have no idea on $\mu_1^{E'}/E'$ and use a value which gives an effective contribution $(I)\cdot\mu_1^{E'}/E'$ being comparable to $(II)\cdot\mu_1^{\theta}\sqrt{E}$ and $(III)\cdot\mu_1^{E}/E$ (see eq.(I)).

Thus we give estimates for $\frac{\Delta\sigma}{\sigma}$ relying on these moments

TABLE I

| $\mu_1^E/E$ | $\mu_1^\theta\sqrt{E}$ | $\mu_1^{E'}/E'$ | $\mu_2^E/E^2$ | $\mu_2^\theta\cdot E$ | $\mu_2^{E'}/E'^2$ |
|---|---|---|---|---|---|
| 0.25% | $0.05\cdot\theta_{inc}\sqrt{E}$ $\lesssim 0.08\%$ | 0.1% | $0.043^2$ $= 0.19\%$ | $\sqrt{2}\cdot\theta_{inc}\cdot E$ $\lesssim 0.04\%$ | 0.19% |

It is evident from the discussion above that further studies in particular of the moments $\mu_1^\alpha$, have to be performed to estimate the systematical errors more reliably.

In Fig.4 the first (4a) and second order (4b) contributions to $\Delta\sigma/\sigma$ are plotted in % as functions of X at Y=0.5. The first moments have uncertain signs. Thus we give as solid curves in Fig. 4a the upper and lower limit of the first order contribution to $\Delta\sigma/\sigma$ which lies at around I-2% for $X<0.7$. The second moments are definitely positive. Thus the second order contributions can be summed up yielding the solid curve of Fig.4b. As is seen, there is some compensation at $X\sim0.5$ which is possible only because the expansion coefficients (IV)-(VI) have varying signs (cf.dashed-dotted curves in Fig.I).The degree of this compensation relies on the moments $\mu_2^\alpha$. Fig.5 is completely analogous to Fig.4 and gives the y dependence of $\Delta\sigma/\sigma$ at x=0.4.

## 3. ASYMMETRY ERROR

The error of the cross section in eq.(I) receives contributions with uncertain sign from the $\mu_1^\alpha$ terms denoted as $\pm\Delta\sigma'$, and contributions with definitive sign from the $\mu_2^\alpha$ terms denoted as $\Delta\sigma''$.

Assuming that $|\Delta\sigma'|$ and $\Delta\sigma''$ are equal for $\sigma_1$ and $\sigma_2$ and neglecting the error of the sum, we can write for the asymmetry Z and its error the following expressions:

$$Z=\frac{\sigma_1-\sigma_2}{\sigma_1+\sigma_2}\simeq\frac{\sigma_1^\circ-\sigma_2^\circ\pm\Delta\sigma'\sqrt{2}}{\sigma_1^\circ+\sigma_2^\circ}, \qquad (3)$$

($\sigma_\circ$ is a "true" cross section)

$$\Delta Z = \frac{1}{\sqrt{2}} \cdot \frac{\Delta \sigma'}{\sigma} \qquad (4)$$

Within these assumptions the asymmetry error is defined by first moment contributions to the single cross section measurement.

Referring to the discussion of Fig.4,5 in Sect.2 we give in Fig.6 the estimated upper and lower limits of $\Delta Z$ as functions of X and Y. The shaded areas indicate the kinematic region which is recommended for asymmetry measurement where expected systematic errors are less than I%, provided the moments $\mu_1^\alpha$ of Table I are realistic numbers. Note that there is still a possibility of some compensation between the different $\mu_1^\alpha$ terms.

R e f e r e n c e s:


1/ H.Anderson et al. Phys.Rev.Lett.37(I976)4.
2/ R.Clifft and N.Doble, CERN/SPSC/74-I2.
3/ Our proposal and Add.I(SPSC/74-79 and 74-I03).


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## FIGURE CAPTIONS

Fig.I X-dependence of expansion coefficients (I) to (VI) defined explicitly in Appendix and calculated here for $y=0.5$.

Fig.2 y-dependence of expansion coefficients (I) to (VI) calculated for $x=0.4$.

Fig.3 x-dependence of the value $(x \frac{d}{dx} \ln F_2 - 2)$ for three different parametrization of $F_2$. The dashed-dotted curves are used for the estimation of $\Delta\sigma$. The curve $x \frac{d}{dx} \ln C$ indicates the effect of scaling violation.

Fig.4 The first(a) and second(b) order contributions to the cross section error $\Delta\sigma/\sigma$ as functions of x at $y=0.5$.

Fig.5 The first (a) and the second(b) order contributions to the cross section error $\Delta\sigma/\sigma$ as functions of y at $x=0.4$.

Fig.6 The estimated systematic errors in the asymmetry of cross sections measurement as a function of x and y. The shaded areas indicate the kinematical region recommended for the asymmetry measurement.

# A P P E N D I X

Here we list the expansion coefficients of eq.(1),i.e. the first and second derivatives of

$$\frac{d\sigma}{dE'd\Theta} = \frac{4\pi\alpha^2}{E^2E'\Theta^3}\left(y+\frac{2}{y}-2\right)F_2(x) .$$

Differentiation becomes easy using

$$E'\frac{\partial}{\partial E'} = \frac{x}{y}\cdot\frac{\partial}{\partial x} + (y-1)\frac{\partial}{\partial y} , \qquad \frac{\partial}{\partial \theta} = 2\frac{x}{\Theta}\frac{\partial}{\partial x} ,$$

$$E\frac{\partial}{\partial E} = \left(1-\frac{1}{y}\right)\cdot x\frac{\partial}{\partial x} + (1-y)\frac{\partial}{\partial y} .$$

Thus, we get

$$\text{(I)} = E'\frac{\partial \ln\sigma}{\partial E'} = -1 - \frac{(1-y)(y^2-2)}{y^3-2y^2+2y} + \frac{x}{y}\cdot\frac{d}{dx}\ln F_2 ,$$

$$\text{(II)} = \frac{1}{\sqrt{E}}\frac{\partial \ln\sigma}{\partial \theta} = \sqrt{\frac{1-y}{2Mxy}}\left[-3+2x\frac{d}{dx}\ln F_2\right] , \quad \frac{1}{\Theta} = \sqrt{\frac{E(1-y)}{2Mxy}} ,$$

$$\text{(III)} = E\frac{\partial \ln\sigma}{\partial E} = -2 + \frac{(1-y)(y^2-2)}{y^3-2y^2+2y} + \left(1-\frac{1}{y}\right)\cdot x\frac{d}{dx}\ln F_2 .$$

Using the abbreviation $f(y)$ for

$$f(y) = \frac{4-12y+20y^2-20y^3+9y^4-y^5}{\left[y^3-2y^2+2y\right]^2}$$

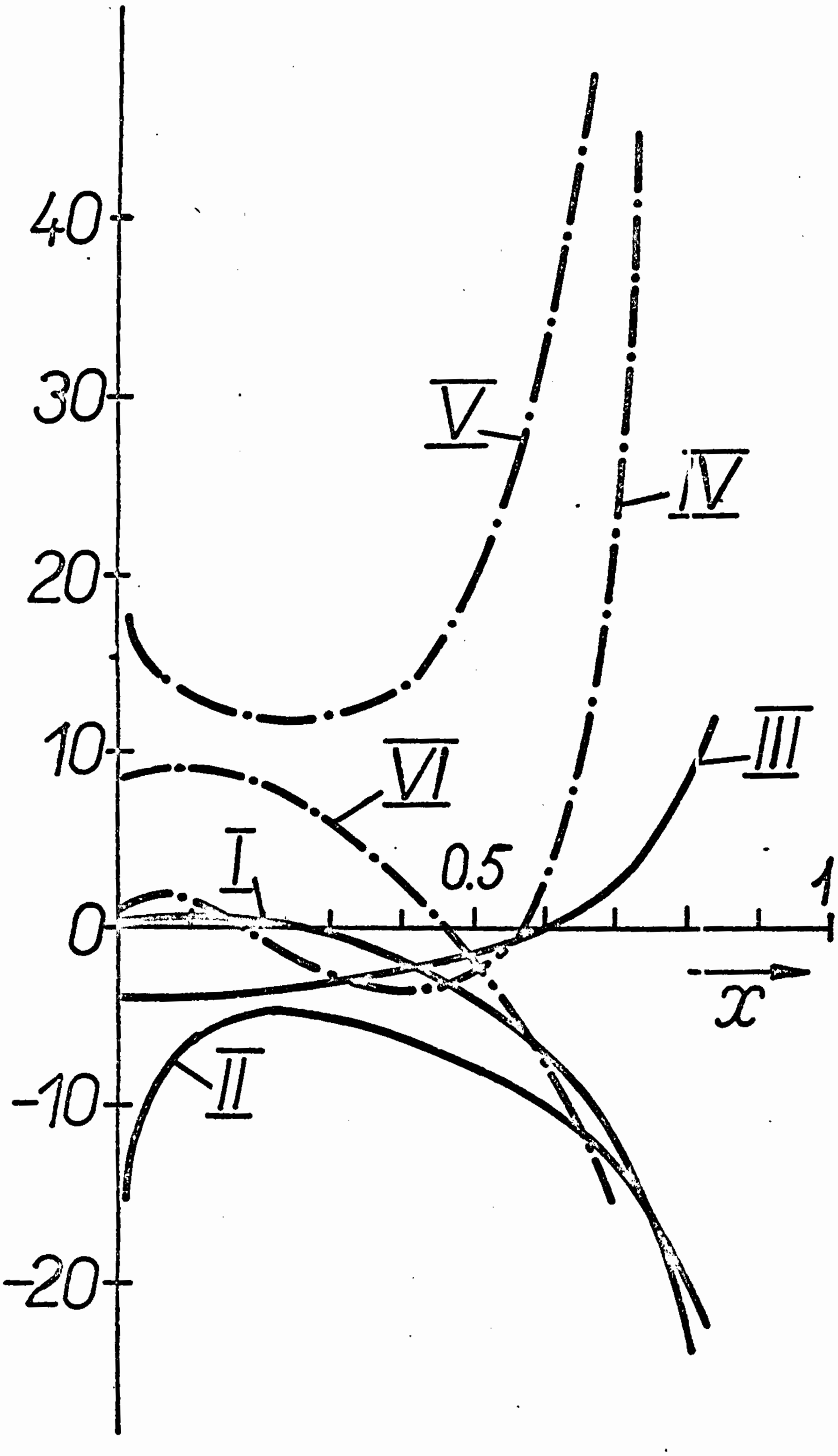
we get the second order terms

$$\text{(IV)} = \frac{1}{2}\cdot\frac{E'^2}{\sigma}\cdot\frac{\partial^2\sigma}{\partial E'^2} = \frac{1}{2}\left\{(\text{I})^2-(\text{I}) + f(y) + \left(\frac{2}{y}-1\right)\frac{x}{y}\cdot\frac{d}{dx}\ln F_2 + \frac{x^2}{y^2}\cdot\frac{d^2}{dx^2}\ln F_2\right\} ,$$

$$\text{(V)} = \frac{1}{2E\sigma}\cdot\frac{\partial^2\sigma}{\partial\theta^2} = \frac{1}{2}\left\{(\text{II})^2 - \sqrt{\frac{1-y}{2Mxy}}\cdot(\text{II}) + \frac{2(1-y)}{Mxy}\cdot x\frac{d}{dx}\ln F_2 + \frac{2(1-y)}{Mxy}x^2\frac{d^2}{dx^2}\ln F_2\right\} ,$$

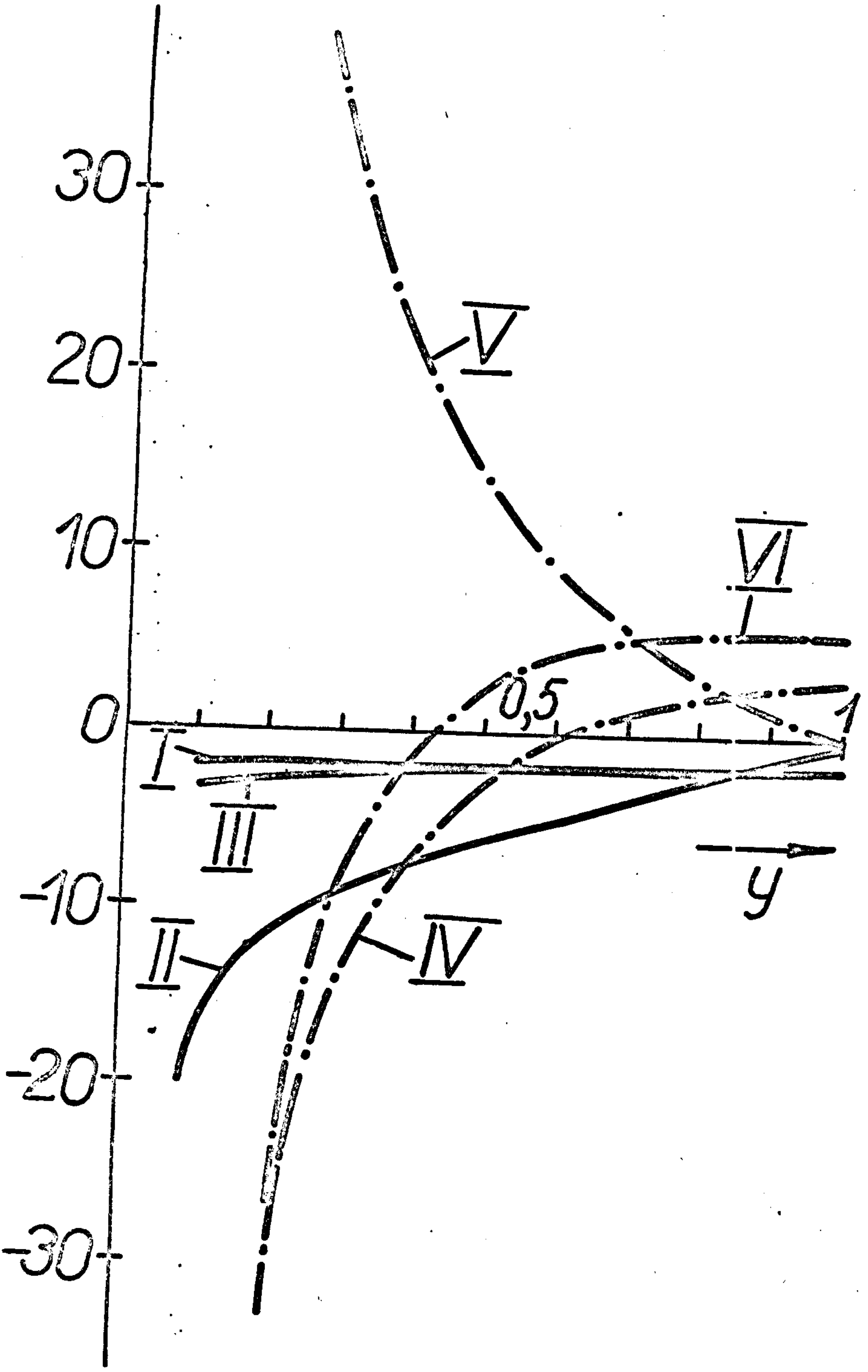
$$\text{(VI)} = \frac{1}{2}\cdot\frac{E^2}{\sigma}\frac{\partial^2\sigma}{\partial E^2} = \frac{1}{2}\left\{(\text{III})^2-(\text{III}) + f(y) + \frac{2-3y+y^2}{y^2}x\frac{d}{dx}\ln F_2 + \left(1-\frac{1}{y}\right)^2\cdot x^2\cdot\frac{d^2}{dx^2}(\ln F_2)\right\} .$$

The coefficients (I-VI) are ullustrated in Fig.1,2. The derivatives of $\ln F_2$ are given in Fig.3



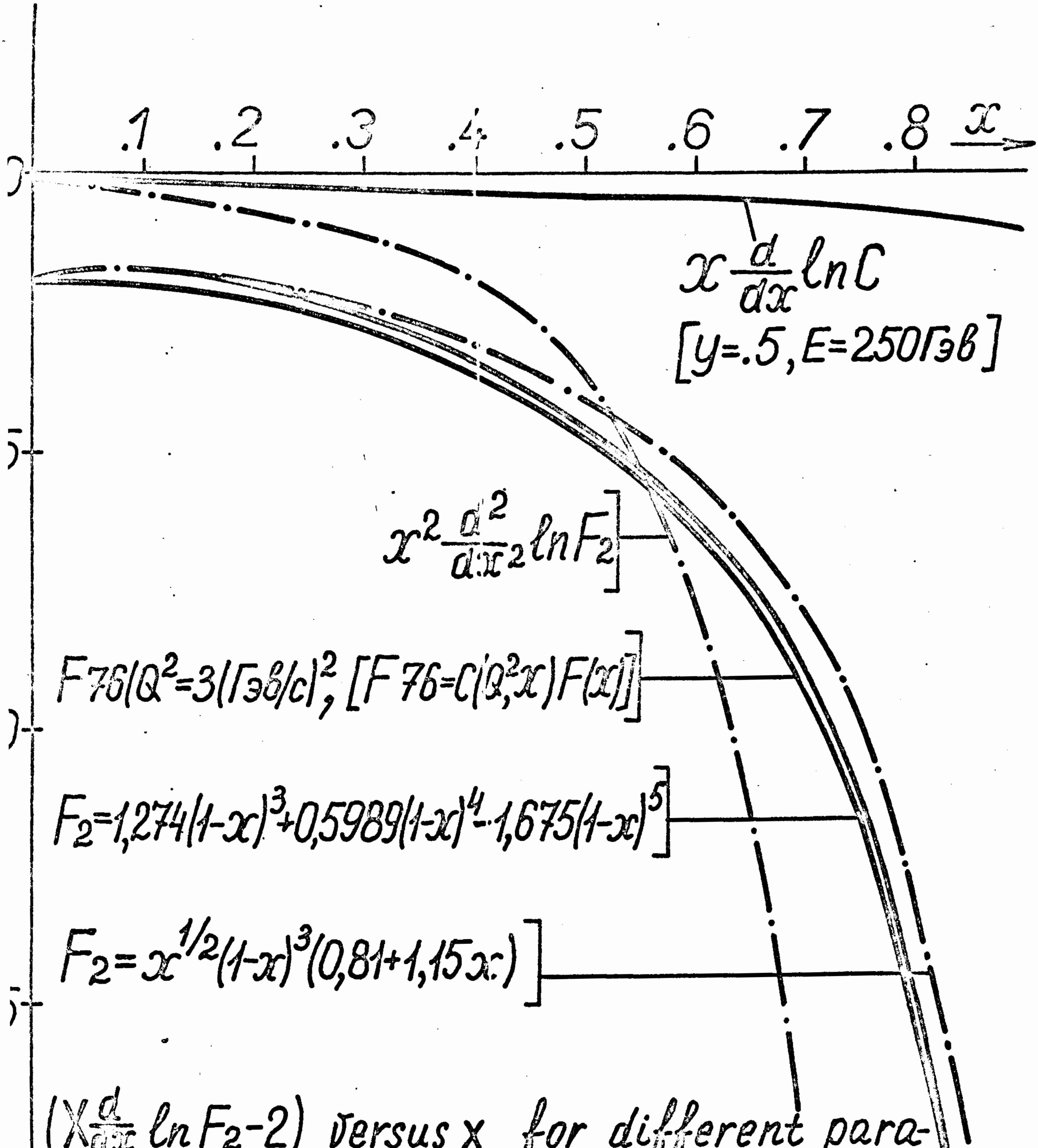

*X-dependence of expansion coefficients*
*I-VI (eq.1) at y = 0.5*

Fig.1



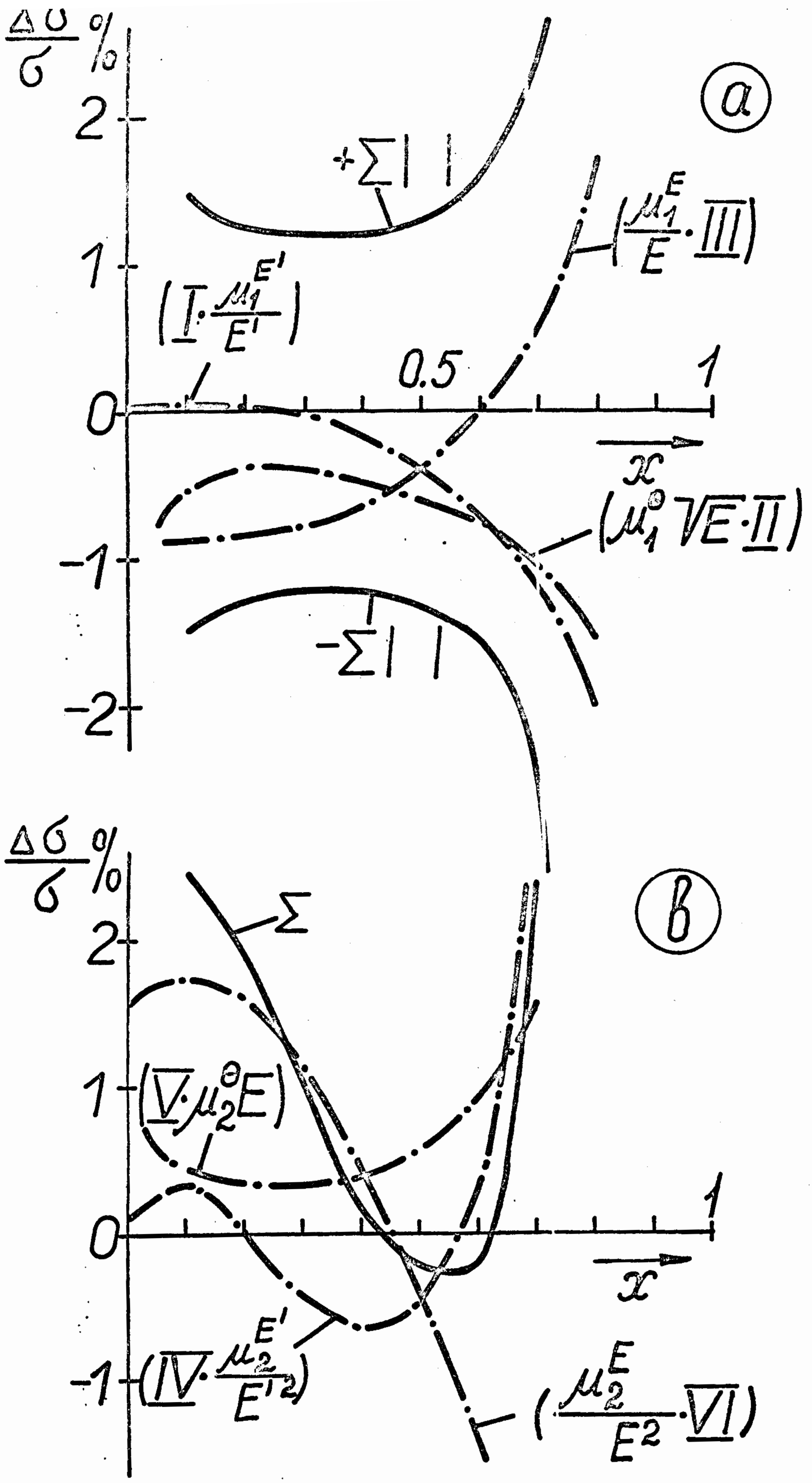

Y-dependence of expansion coefficients I-VI (eg. 1) at X=0.4

Fig. 2




$(x\frac{d}{dx}\ln F_2-2)$ versus x for different parametrizations of $F_2$. The dashed-dotted curves for $x\frac{d}{dx}\ln F_2$ and $x^2\frac{d^2}{dx^2}\ln F_2$ are used for the estimation of $\Delta\sigma$. $x\frac{d}{dx}\ln C$ indicates the effect of scaling violation.

Fig. 3



$\frac{\Delta\sigma}{\sigma}$ %
(a)
2
1
0
-1
-2
0.5
1
$x$
$+\Sigma|\ |$
$-\Sigma|\ |$
$(\mu_1^E/E \cdot \overline{III})$
$(\overline{I} \cdot \mu_1^{E'}/E')$
$(\mu_1^\theta \sqrt{E} \cdot \overline{II})$
$\frac{\Delta\sigma}{\sigma}$ %
(b)
$\Sigma$
$(\overline{V} \cdot \mu_2^\theta E)$
$(\overline{IV} \cdot \mu_2^{E'}/E'^2)$
$(\mu_2^E/E^2 \cdot \overline{VI})$


Fig. 4

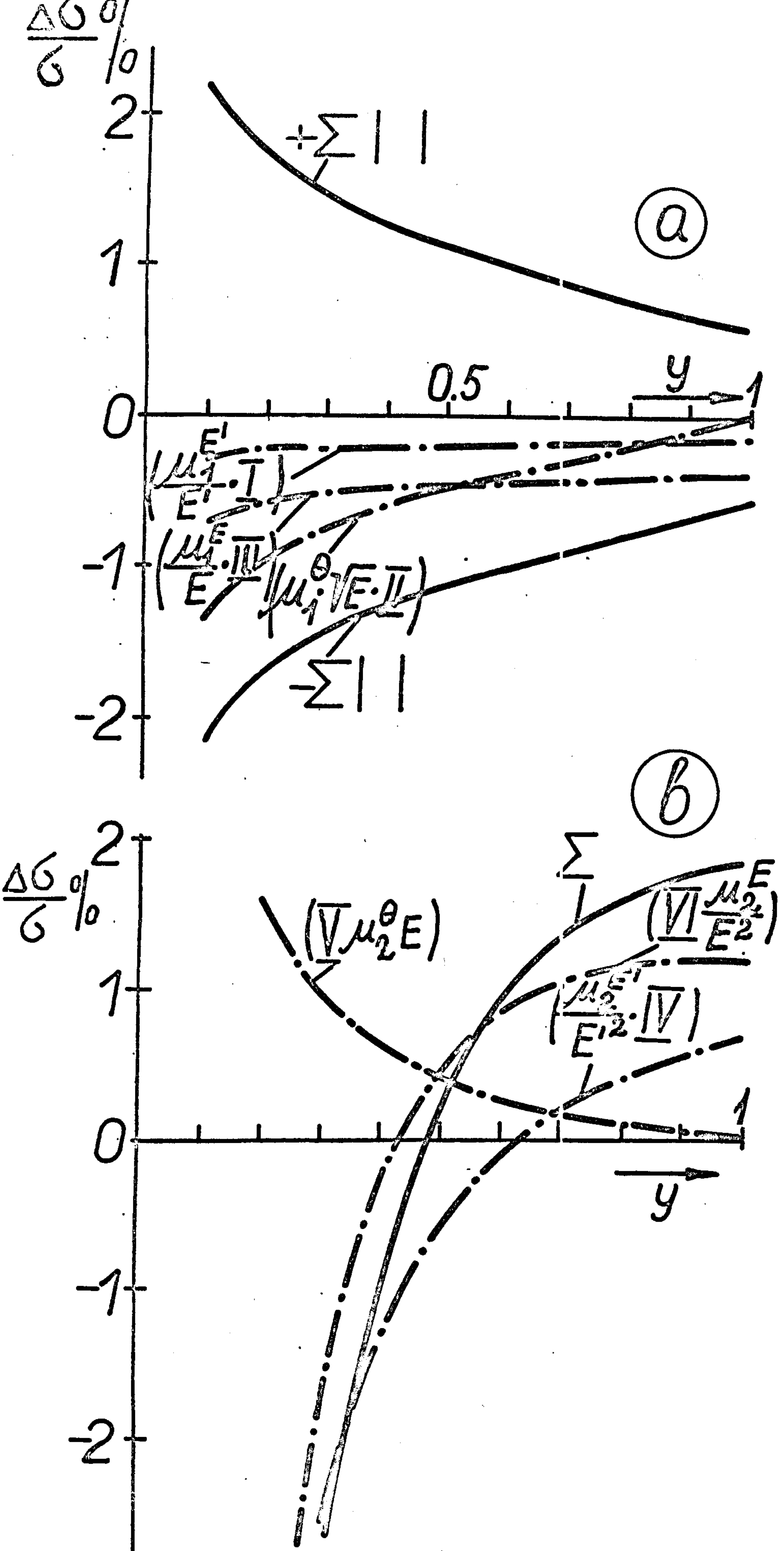



Fig. 5

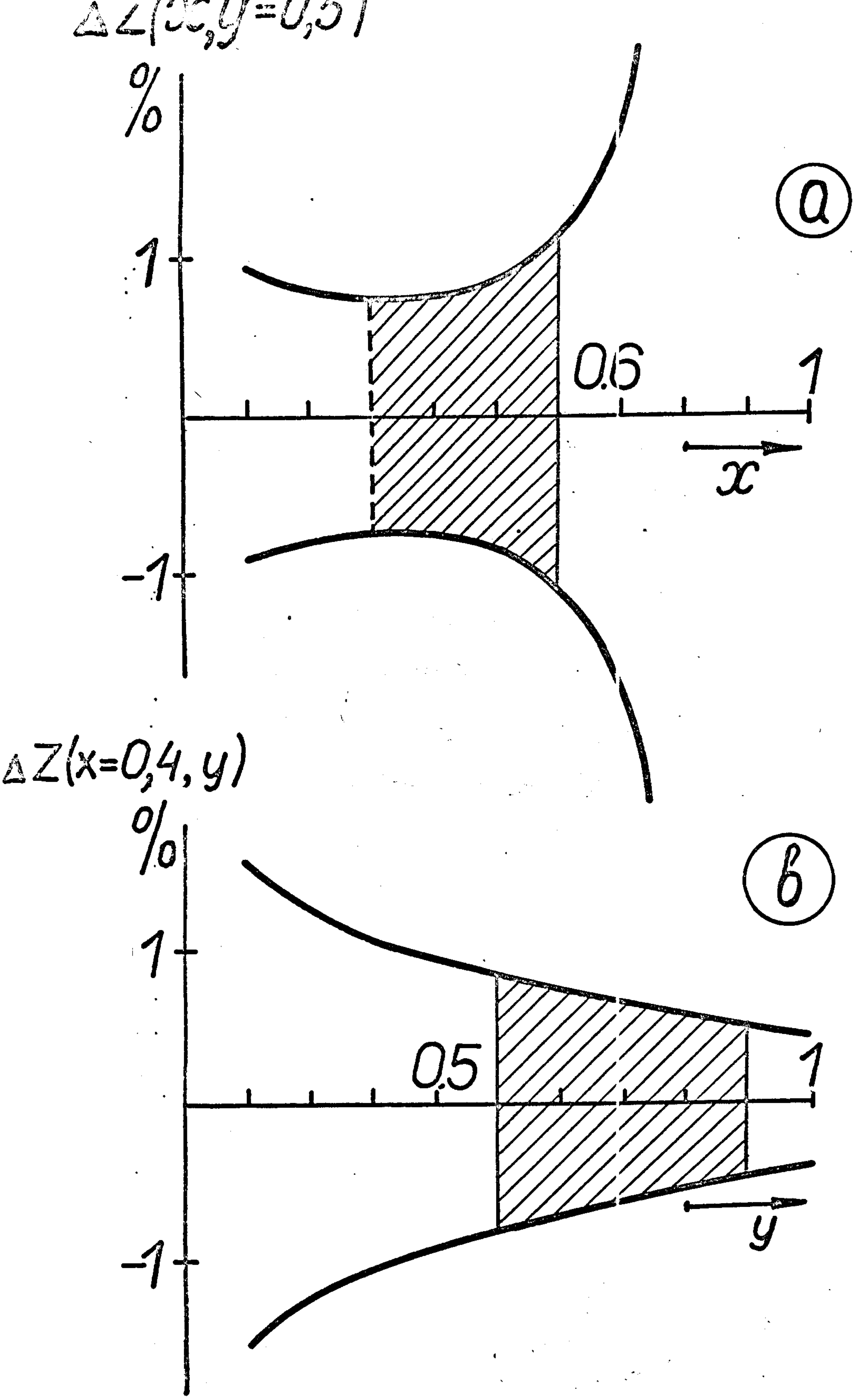



Fig.6. Estimated range of error ΔZ as function of x(y=0,5) and y(x=0,4)