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DUAL TREE AND LOOP AMPLITUDES IN THE NEVEU-SCHWARZ-RAMOND MODEL WITH SOME APPLICATIONS

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1. Introduction

The dual-resonance model found in 1968 by Veneziano[1] as a solution of the problem of constructing meromorphic scattering amplitudes with linearly rising Regge trajectories and zero width resonances constitutes a nontrivial approach to the S-matrix theory of strong interactions. It obeys a lot of requirements to be satisfied by any realistic scattering amplitude[2]. This concerns Lorentz invariance, crossing symmetry, analyticity in the Mandelstam variables, Regge behaviour and the possibility to unitarize the model. Up to now two basic dual-resonance models have been constructed. The first one - the generalized Veneziano model - represents the simplest form of a dual model without spin degrees of freedom. The second type of models invented by Neveu, Schwarz and Ramond[3,4,5] takes such spin degrees into account and leads to unified meson-fermion amplitudes containing different families of meson trajectories in two- and three- meson channels and, moreover, fermion trajectories.

This report gives an elementary introduction into the Neveu-Schwarz-Ramond (N.S.R.) model on the tree and one-loop level including some phenomenological applications. As our intention in this report is mostly to give some survey on the physical content of the model we have renounced mathematical rigour prefering, if possible, heuristic argumentations in developing the matter. Repeatedly, the properties of the model are explained using simple examples of explicit N.S.R.-amplitudes. The reader interested in a more systematic representation of the dual theory including its algebraic and group theoretical background as well as its string interpretation is referred to a large amount of review articles[6-13].

The report is organized as follows. In Sect.2 we recapitulate the properties of the dual four-point pion amplitude. In Sect.3 and 4 the Feynman-like operator rules of the Neveu-Schwarz (N.S.) meson model and of the unified N.S.R. fermion-meson model are presented. Sect.5 is devoted to the unitary corrections to Regge trajectories arising from planar one-loop graphs. Finally, Sect.6 reviews some phenomenological applications of the N.S.R. model to processes with vector and tensor meson production in quasi-two-body and quasi-three-body final states.

2. The Dual Pion Model

Assuming that in a first order approximation the only singularities of the scattering amplitude are narrow width resonances lying on linear Regge trajectories, Veneziano/1/ proposed an expression for the scattering amplitude of the process $\pi + \pi \to \pi + \omega$ in terms of Euler's beta function. Soon after this work Lovelace and Shapiro/11/ proposed a similar amplitude for the process $\pi^a + \pi^b \to \pi^c + \pi^d$ (a, b, c and d are isospin indices) shown in fig.1. The Mandelstam variables are defined as

$$
\begin{aligned}
s &= (k_1 + k_2)^2 = (k_3 + k_4)^2 , \qquad s + t + u = 4 m_\pi^2 \\
t &= (k_2 + k_3)^2 = (k_1 + k_4)^2 \qquad\qquad (1) \\
u &= (k_1 + k_3)^2 = (k_2 + k_4)^2
\end{aligned}
$$

The four-pion amplitude of fig.1 reads

$$T_4^{(abcd)}(s,t,u) = \frac{1}{2}\,\mathrm{Tr}(\tau^a\tau^b\tau^c\tau^d)\,F_4(s,t) + \frac{1}{2}\,\mathrm{Tr}(\tau^a\tau^c\tau^b\tau^d)\times F_4(u,t)$$
$$+ \frac{1}{2}\,\mathrm{Tr}(\tau^a\tau^b\tau^d\tau^c)\,F_4(s,u)$$
$$= \delta_{ab}\delta_{cd}\left(F_4(s,t) + F_4(s,u) - F_4(u,t)\right)$$
$$+ \delta_{ac}\delta_{bd}\left(F_4(s,u) + F_4(u,t) - F_4(s,t)\right)$$
$$+ \delta_{ad}\delta_{bc}\left(F_4(s,t) + F_4(u,t) - F_4(s,u)\right) \qquad (2)$$

where the function $F_4(s,t)$ is given (up to a normalization) by[14]

$$F_4(s,t) = g^2\,\frac{\Gamma(1-\alpha_{\rho}(s))\,\Gamma(1-\alpha_{\rho}(t))}{\Gamma(1-\alpha_{\rho}(s)-\alpha_{\rho}(t))} \qquad (3)$$

and $\alpha_{\rho}(s)$ is the exchange-degenerated (linear) $\rho - f$ trajectory

$$\alpha_{\rho}(s) = \alpha_{\rho}^{(0)} + \alpha' s \qquad (4)$$

Experimentally one has the following intercept and slope values of the trajectory $\alpha_{\rho}^{(0)} \sim \frac{1}{2}$, $\alpha' \sim 1\,(GeV)^{-2}$. In eq.(2) the isospin degrees of freedom have been taken into account by treating mesons as quark-antiquark states and associating to them the Pauli matrices $(\tau^{(a)})_{q\bar{q}}$ $(q,\bar{q} = 1,2)$ as Clebsch-Gordan coefficients[15]. This method excludes automatically the appearance of exotic intermediate states with isospin $I = 2$.

The amplitude (2) satisfies the requirement of Lorentz invariance since it depends only on the Mandelstam invariants. Furthermore, it is invariant with respect to any interchange of the external particles. Thus, crossing symmetry is guaranteed by construction because each of the three terms of the first row of eq.(2) is invariant with respect to a cyclic interchange of the external particles and we have, finally, added the contributions of the three noncyclic permutations of external particles. As the Γ-functions have only pole singularities the ansatz (3) fulfills also the requirement of analyticity. Note, that the amplitude (2) exhibits a further property. Each single term, e.g. $F_{\mathfrak{A}}(\bar{s},t)$, may be represented either as a sum of poles (resonances) in the $\bar{s}$-channel or as a sum of poles in the t-channel

$$F_{\mathfrak{A}}(\bar{s},t) = g^2(1-\alpha_{\mathfrak{S}}(s)-\alpha_{\mathfrak{S}}(t)) \sum_{n=0}^{\infty} \frac{\Gamma(n+\alpha_{\mathfrak{S}}(t))}{n!\,\Gamma(\alpha_{\mathfrak{S}}(t))} \frac{1}{n+1-\alpha_{\mathfrak{S}}(s)}$$

$$= g^2(1-\alpha_{\mathfrak{S}}(t)-\alpha_{\mathfrak{S}}(s)) \sum_{n=0}^{\infty} \frac{\Gamma(n+\alpha_{\mathfrak{S}}(s))}{n!\,\Gamma(\alpha_{\mathfrak{S}}(s))} \frac{1}{n+1-\alpha_{\mathfrak{S}}(t)} \quad (5)$$

This important property which is not shared by usual Feynman graphs is called duality. The poles of the s- and t-channels are said to be dual to each other in the sense that either one of the two descriptions contains the other and is, by itself, complete. Fig.2 gives the graphical expression of duality. It follows from eq.(5) that the residuum of a pole at $\alpha_{\mathfrak{S}}(\bar{s}) = J$ is a polynomial of degree J in t or, equivalently, in $\cos\theta_s$ where θ_s is the scattering angle in the s-channel c.m. system. Then, each parent resonance of spin J is accompanied by daughter resonances of spin N, where $0 \leq N \leq J$. Let us now consider the high energy behaviour of the pion amplitude. Introducing the amplitudes of definite t-channel isospin and using the Stirling formula for the asymptotic behaviour of the Γ-functions the following Regge behaviour $(\alpha_{\mathfrak{S}(f)}(\bar{s}) \equiv \alpha(s))$ can easily be derived as $|s| \to \infty$ $(\mathrm{Im}\, s \to \infty)$, t fix

$$T_{(4)}^{(I_t=0)}(t,s,u) = 3\left(F_{\mathfrak{A}}(t,s) + F_{\mathfrak{A}}(t,u) - F_{\mathfrak{A}}(s,u)\right)$$

$$\underset{|s|\to\infty}{\sim} 3\, g^2 [\alpha(s)]^{\alpha(t)} \left(1 + e^{-i\pi\alpha(t)}\right) \Gamma(1-\alpha(t)) \quad (6)$$

$$+ O\left(\bar{s}^{\,\alpha(t)-1}\right)$$

$$\overset{(I_t=1)}{T_{(4)}}(t,s,u) = 2\left(F_{(4)}(t,s) - F_{(4)}(t,u)\right)$$
$$\underset{|s|\to\infty}{\sim} 2g^2\left[\underline{\alpha}(s)\right]^{\underline{\alpha}(t)}\left(-1+e^{-i\pi\underline{\alpha}(t)}\right)\Gamma(1-\underline{\alpha}(t)) + O(s^{\alpha-1})$$
$$\overset{(I_t=2)}{T_{(4)}}(t,s,u) = 2F_{(4)}(s,u) \to 0 \text{ as } O\left(e^{-\pi|\mathrm{Im}\,\underline{\alpha}(s)|}(\underline{\alpha}(s))^{c}\right) \quad (6)$$

Thus, the asymptotic behaviour is dominated by the ρ (P) trajectory of negative (positive) signature. It is worth mentioning that the amplitude (2) satisfies the requirements of PCAC and current algebra which implement the vanishing of the amplitude if the four-momentum of one of the external pions vanishes (Adler zero)/16/. To see this use the experimentally observed half-spacing rule $\alpha_{\rho}(t) = \frac{1}{2} + \alpha_{\pi}(t)$, where $\alpha_{\pi}(t)$ is the pion trajectory with $\alpha_{\pi}(m_{\pi}^2) = 0$, and the fact that $s, t, u \to m_{\pi}^2$ if one of the external pion momenta vanishes, as well as $\alpha_{\rho}(x) \underset{x\to m_{\pi}^2}{\longrightarrow} \frac{1}{2}$. Indeed, the vanishing of the amplitude (3) follows then due to the pole behaviour of the denominator $\lim_{z\to 0}\Gamma(z) = \infty$. It has thus been shown that the pion amplitude (2) satisfies most of the S-matrix requirements quoted in the introduction. Concluding this section, we remark that the model amplitude (2) is a meromorphic amplitude in the Mandelstam variables that does not contain unitary cuts. This obvious lack of unitarity can be, however, cured by including higher order loop contributions into the model (see. Sect.5).

3. Feynman-Like Operator Rules for the Neveu-Schwarz (N.S.) Model

The concept of duality has been generalized to the case of N-particle processes. As a result one obtained for the first time integral representations of N-particle scattering amplitudes with reasonable analytic and asymptotic properties/17/. N-point amplitudes may be introduced in an economic way by using Feynman-like operator rules. Let us first rewrite the function $F_4(s,t)$ of eq.(3) in the form $\left(\alpha_\rho(s) = \frac{1}{2} + \alpha'(s - m_\pi^2)\right)$

$$
\begin{aligned}
F_4(s,t) &= -g^2 \alpha_\rho(t)\, B(1-\alpha_\rho(s), -\alpha_\rho(t)) \\
&= g^2 \int_0^1 dx\, x^{-\alpha_\rho(s)-1} (1-x)^{-\alpha_\rho(t)} \left\{ -2\alpha' k_2 \cdot k_3 \frac{\sqrt{x}}{1-x} \right\}
\end{aligned}
\tag{7}
$$

where $B(\alpha,\beta)$ is Euler's beta function

$$
B(\alpha,\beta) = \frac{\Gamma(\alpha)\,\Gamma(\beta)}{\Gamma(\alpha+\beta)} = \int_0^1 dx\, x^{\alpha-1}(1-x)^{\beta-1} \tag{8}
$$

and we have used*)

$$
\alpha_\rho(t) = 2\alpha' k_2 \cdot k_3 \tag{9}
$$

It is now convenient to introduce a set of "Feynman-like rules" which allows us to rewrite the amplitude (7) as an expectation value of propagator and vertex operators. Following Neveu and Schwarz[3], let us consider a Fock space spanned up by two infinite sets of commuting and anticommuting annihilation and creation operators

$$
\begin{aligned}
&[a_m^\mu, a_n^\nu] = [a_m^{+\mu}, a_n^{+\nu}] = 0 \\
&[a_m^\mu, a_n^{+\nu}] = -g^{\mu\nu}\delta_{mn} \qquad\qquad m,n = 1,2,\ldots,\infty
\end{aligned}
$$

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*) This equality is needed in order to reproduce the Lovelace-Shapiro amplitude (3) by operator rules. Unfortunately, it enforces an (unphysical) pion mass $\alpha' m_\pi^2 = -\frac{1}{2}$. Then we have also $\alpha_\rho(s) = 1 + \alpha' s$ and thus, due to $\alpha_\rho(0) = 1$ the ρ meson of the model is massless. The same mass restrictions follow also independently from the requirement that the N.S. model contains no ghosts (comp. remarks in Sect.4).

$$\{ b_{m}^{(\mu)}, b_{n}^{(\nu)} \} = \{ b_{m}^{+\mu}, b_{n}^{+\nu} \} = 0$$

$$\{ b_{m}^{(\mu)}, b_{n}^{(+\nu)} \} = - g^{\mu\nu} \delta_{mn} \qquad m,n = \tfrac{1}{2}, \tfrac{3}{2}, \dots, \infty$$

$$[a_{m}^{(\mu)}, b_{n}^{(\nu)}] = 0 \tag{10}$$

where μ, ν are Lorentz indices ($\mu, \nu = 1, 2, \dots, d$; d dimension of space-time) and $g^{00} = - g^{ii} = 1$. Here, we admit the possibility $d \neq 4$ for reasons that will become clear later on. Let us furthermore introduce momentum and position operators p^{μ}, x^{ν} satisfying the usual commutation relation

$$[x^{\mu}, p^{\nu}] = - i g^{\mu\nu} \tag{11}$$

as well as a Hamiltonian-like operator

$$H = H_{a} + H_{b} = - \sum_{n=1}^{\infty} n\, a_{n}^{+} \cdot a_{n} - \sum_{m=\frac{1}{2}}^{\infty} m\, b_{m}^{+} \cdot b_{m} \tag{12}$$

There is also a ground state $|0\rangle = |0_a\rangle |0_b\rangle$ characterized by

$$a_{m}^{(\mu)} |0\rangle = b_{n}^{(\mu)} |0\rangle = p^{(\mu)} |0\rangle = 0 \tag{13}$$

Let us, also, introduce the operator function

$$h^{(\mu)}(z) = \sum_{m=-\infty}^{\infty} b_{m}^{(\mu)} z^{-m} \qquad (b_{-m}^{(\mu)} \equiv b_{m}^{+\mu}) \tag{14}$$

The propagator, vertex and one-pion state of the N.S. model may now be defined by $(\alpha' m_{\pi}^2 = -\tfrac{1}{2})$*)

<u>Propagator:</u> $\bullet\text{-----}\!\to$ $D = \dfrac{1}{H - \alpha' p^2 - \frac{1}{2}} = \displaystyle\int_0^1 dx\, x^{H - \alpha' p^2 - \frac{3}{2}}$

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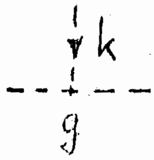
*) These rules correspond to the so-called " F_2-picture"/4/.

vertex: 

$$V_{NS}(k) = g\sqrt{2\alpha'}\,k\cdot h(1)\,V_0(k)$$
$$\equiv g\sqrt{2\alpha'}\,k\cdot h(1)\exp\Big(-\sqrt{2\alpha'}\sum_{n=1}^{\infty} k\cdot\frac{a_n^+}{\sqrt{n}}\Big)\times$$
$$\times\exp\Big(\sum_{n=1}^{\infty}\sqrt{2\alpha'}\,k\cdot\frac{a_n}{\sqrt{n}}\Big)\exp -ikx$$

1-pion state: 

$$|\pi(k)\rangle = |0;k\rangle \equiv e^{-ikx}|0\rangle \qquad (15)$$

The function(7) may be represented by one of the two dual equivalent operator expressions (comp. fig.2)

$$F_4(s,t) = \langle\pi(k_1)|\,V_{NS}(k_2)\,D\,V_{NS}(k_3)\,|\pi(k_4)\rangle$$
$$= \langle\pi(k_2)|\,V_{NS}(k_3)\,D\,V_{NS}(k_4)\,|\pi(k_1)\rangle \qquad (16)$$

This may easily be verified by exploiting the fact that the expectation value (16) factorizes into an a- and b- operator part and by taking into account eqs.(10),(11) as well as the useful formulae

$$f(\alpha_n)\,x^{(H_\alpha)} = x^{(H_\alpha)} f(x^n\alpha_n) \qquad (\alpha_n = a_n \text{ or } b_n) \qquad (17)$$
$$x^{(H_\alpha)} f(\alpha_n^+) = f(x^n\alpha_n^+)\,x^{(H_\alpha)}$$
$$e^A e^B = e^B e^A e^{[A,B]}$$
$$e^{A+B} = e^A e^B e^{-\frac{1}{2}[A,B]} \qquad (18)$$

(Eqs.(18) hold only when $[A,B]$ is a c-number).

Then, the a-operator expectation value can be evaluated as

$$\langle 0_a;k_1|\,e^{-ik_2\cdot x}\,e^{\sum_{n=1}^{\infty}\sqrt{2\alpha'}k_2\cdot\frac{a_n}{\sqrt{n}}}\,x^{(H_a-\alpha' p^2-\frac{3}{2})}\,e^{-\sum_{n=1}^{\infty}\sqrt{2\alpha'}k_3\cdot\frac{a_n^+}{\sqrt{n}}}\,e^{-ik_3\cdot x}\,|0_a;k_4\rangle$$
$$= x^{(-\alpha'(k_1+k_2)^2-3/2)}\langle 0_a|\,e^{\sum_{n=1}^{\infty}\sqrt{2\alpha'}k_2\cdot\frac{a_n}{\sqrt{n}}}\,e^{-\sum_{n=1}^{\infty}\sqrt{2\alpha'}k_3\cdot\frac{x^n a_n^+}{\sqrt{n}}}\,|0_a\rangle$$
$$= x^{(-\alpha'(k_1+k_2)^2-3/2)}\,e^{2\alpha' k_2\cdot k_3\sum_{n=1}^{\infty}\frac{x^n}{n}} = x^{(-\alpha(s)-\frac{1}{2})}(1-x)^{(-2\alpha' k_2\cdot k_3)} \qquad (19)$$

Similarly, the b-operator expectation value reads

$$2\alpha' \langle 0_b | k_2 \cdot h(1)\, x^{H_b}\, k_3 \cdot h(1) | 0_b \rangle = -2\alpha' k_2 \cdot k_3 \frac{\sqrt{x}}{1-x} \tag{20}$$

The generalization of eq.(16) to the N-pion amplitude is now straightforward. Let us consider as the next nontrivial example the six-point function exhibited in fig.3. We set (isospin traces will be suppressed)*)

$$F_6 = \langle \pi(k_1) | V_{N3}(k_2)\, D\, V_{N3}(k_3)\, D\, V_{N3}(k_4)\, D\, V_{N3}(k_5) | \pi(k_6) \rangle \tag{21}$$

To obtain the total crossing symmetric amplitude one has to add to eq.(21) $\frac{1}{2}(6-1)!$ analogous terms corresponding to all the possible noncyclic or nonanticyclic permutations of external particles.

The operator representation is particularly useful for studying the resonance spectrum of the model which arises from the propagator poles. To show this let us diagonalize the propagator by the occupation number states (Lorentz indices are suppressed)

$$|\{\ell^a\}, \{\ell^b\}; k\rangle = \prod_{n,m} K_{n,m} \frac{(a_n^+)^{\ell_n^a}}{\sqrt{\ell_n^a!}} \frac{(b_m^+)^{\ell_m^b}}{\sqrt{\ell_m^b!}} e^{-ik\cdot x} |0\rangle \tag{22}$$

which are eigenstates of H

$$H |\{\ell^a\}, \{\ell^b\}; k\rangle = J |\{\ell^a\}, \{\ell^b\}; k\rangle$$

$$J = \sum_{n=1}^{\infty} n \ell_n^a + \sum_{m=1}^{\infty} m \ell_m^b \tag{23}$$

and $K_{n,m}$ are kinematical factors not to be specified here. By

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*) Because the b-operators of the vertex parts h(1) have to be contracted pairwise in the expectation value of the amplitude, only amplitudes with N even are possible. Thus, the model exhibits some kind of "G-parity" conservation.

inserting the complete set of occupation number states (22) between the vertices of expression (21) the amplitude may be factorized into lower N-point functions (bootstrap condition). Factorizing for example, the above six-point function on the poles between the particles 3 and 4 yields ($k = (k_1+k_2+k_3)^2$)

$$F_6 = \sum_{J} \langle \pi(k_1) | V_{NS}(k_2) D V_{NS}(k_3) | \{\ell^a\}, \{\ell^b\}; k \rangle \frac{1}{J - \alpha_{\pi,\omega}(k^2)} \times \langle \{\ell^a\}, \{\ell^b\}; k | V_{NS}(k_4) D V_{NS}(k_5) | \pi(k_6) \rangle \tag{24}$$

where now the resonances lie on the degenerate π, ω-trajectory $\alpha_{\pi,\omega}(t) = \frac{1}{2} + \alpha' t$ appearing in the 3-pion channels. If we factorized, instead, in a two-pion channel we would again obtain the ρ, f trajectory $\alpha_{\rho}(t) = 1 + \alpha' t$ known already from the four-pion amplitude. The lowest-lying states of the N.S. model can be arranged into the Chew-Frautschi plot shown in fig.4. Here, the states are named according to their quantum numbers spin, parity, isospin and "G-parity" as the corresponding mesons found in nature. Moreover, in table 1 we indicate the explicit form of the occupation number states describing the lowest resonances of positive or negative "G-parity" ($G = -(-1)^{\sum b^+_m \cdot b_m}$)/4/

G=-1	M^2	I	J^P	name	Fock space state (F_2-picture)
	-1/2	1	0^-	π	$\vert\pi\rangle = \vert 0\rangle$
	1/2	0	1^-	ω	$\vert\omega\rangle = \varepsilon^{\mu}_{\nu\lambda\sigma} k^\nu b^{+\lambda}_{1/2} b^{+\sigma}_{1/2} \vert 0\rangle$
	3/2	1	2^+	A_2	$\vert A_2\rangle = i\, a_1^{+\mu_1} \varepsilon^{\mu_2}_{\nu\lambda\sigma} k^\nu b^{+\lambda}_{1/2} b^{+\sigma}_{1/2} \vert 0\rangle$
G=+1	0	1	1^-	ρ	$\vert\rho\rangle = b^{+\mu}_{1/2} \vert 0\rangle$
	1	0	2^+	f	$\vert f\rangle = i\, b^{+\mu_1}_{1/2} a_1^{+\mu_2} \vert 0\rangle$
	1	0	0^+	σ'	$\vert\sigma'\rangle = i(b^+_{1/2} \cdot a_1^+ - k \cdot b^+_{1/2}\, k \cdot a_1^+) \vert 0\rangle$

Table 1.

($\varepsilon_{\mu\nu\lambda\tau}$ is the Levi-Civita tensor). Using eqs.(15),(17),(18) we obtain easily the following integral representation for the amplitude (21) (x_2, x_3, x_4 are the integration variables introduced by the propagators)*)

$$
\begin{aligned}
F_{6} = g^4 \iiint_0^1 & \frac{dx_2\, dx_3\, dx_4}{(1-x_2x_3)(1-x_3x_4)}\; x_2^{-\alpha_S^{(12)}}\; x_3^{-\alpha_{\pi,\omega}^{(13)}-1}\; x_4^{-\alpha_S^{(14)}} \\
\times & \left[\frac{1-x_2}{1-x_2x_3}\right]^{-\alpha_S^{(23)}} \left[\frac{1-x_3}{1-x_2x_3}\,\frac{1-x_2x_3x_4}{1-x_3x_4}\right]^{-\alpha_S^{(34)}} \left[\frac{1-x_4}{1-x_3x_4}\right]^{-\alpha_S^{(45)}} \\
\times & \left[\frac{1-x_2x_3}{1-x_2x_3x_4}\right]^{-\alpha_{\pi,\omega}^{(24)}-1} \left[\frac{1-x_3x_4}{1-x_2x_3x_4}\right]^{-\alpha_{\pi,\omega}^{(35)}-1} \left[1-x_2x_3x_4\right]^{-\alpha_S^{(25)}} \\
\times & \left\{ k_4\cdot k_5\; k_2\cdot k_3\; \frac{1}{X_{23}\, X_{45}\, X_{25}} + k_2\cdot k_5\; k_3\cdot k_4\; \frac{x_3}{X_{34}\, X_{25}} \right. \\
& \left. \quad - k_3\cdot k_5\; k_2\cdot k_4\; \frac{x_3}{X_{25}} \right\}
\end{aligned}
\tag{25}
$$

Here we have used the notation $\alpha_{\pi(\omega)}^{(13)} = \frac{1}{2} + \alpha'(k_1+k_2+k_3)^2$ etc. and X_{ij} is an abbreviation for the expression in the bracket raised to the power $-\alpha^{(ij)}$. Note, that the six-point function (21) factorizes on the pion pole ($|\{\ell^a\},|\ell^b\},k\rangle \Rightarrow |\pi(k)\rangle,\ J=0$) into two four pion amplitudes of the type (16). Another way to see this is to expand in eq.(25) the part of the integrand without the factor $x_3^{-\alpha_{\pi,\omega}^{(13)}-1}$ around $x_3=0$, retaining the term $O(x_3^0=1)$ and performing then the x_3-integration. Next, the terms of order $O(x_3)$ of the bracket $\{\dots\}$ yield the factorizing ω-contribution etc. Analogous factorization properties follow in the high

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*) This expression follows also from the slightly modified operator rules and states of the so-called " F_1-picture" by killing the pion "ancestor" state/3/.

energy region where Regge behaviour holds. Let us consider, for example, the Regge limit shown in fig.5. It is defined by the kinematics

$$\left.\begin{array}{l} \bar{s}_{16},\ \bar{s}_{234},\ s_{345},\ s_{34} \to -\infty \\ t_{12},\ s_{23},\ t_{123},\ s_{45},\ t_{56},\ \frac{s_{234}}{s_{34}} = \varkappa, \\ \frac{s_{345}}{s_{34}} = \bar{\varkappa},\quad \frac{s_{16}}{s_{34}} = \varkappa\bar{\varkappa} \end{array}\right\} \text{fix} \tag{26}$$

The contributions to the asymptotic behaviour of the integral representation (25) arise only from the neighbourhood of the point $x_3 = 0$. Expanding the integrand around this point and using the formula

$$\lim_{s\to\infty} \int_0^1 dx\, x^{(-\alpha-1)} e^{-sxA} \sim (-s)^{\alpha} \Gamma(-\alpha)\, A^{\alpha} \tag{27}$$

we easily obtain the following factorizing contributions for the exchange of the π, ω-trajectory ($t_{123} \equiv t$)

$$\begin{aligned} F_{(6)} \underset{|s_{16}|\to\infty}{\sim} & \left[-\alpha(s_{16})\right]^{\alpha_{\pi,\omega}(t)} \Gamma\left(-\alpha_{\pi,\omega}(t)\right) \times \\ & \times \left\{ g^{2} \int d F_{(4)}(x_2) \left[(1-x_2)+\varkappa x_2\right]^{\alpha_{\pi,\omega}(t)} \cdot g^{2} \int d F_{(4)}(x_4) \left[(1-x_4)+\bar{\varkappa} x_4\right]^{\alpha_{\pi,\omega}(t)} \right\} \\ & + \frac{1}{2} \left[-\alpha(s_{16})\right]^{\alpha_{\pi,\omega}(t)} \Gamma\left(1-\alpha_{\pi,\omega}(t)\right) \times \\ & \times \left\{ g^{2} \varepsilon_{\alpha\beta\mu\nu} (k_2)^{\mu} (k_3)^{\nu} \int d B(x_2) \left[(1-x_2)+\varkappa x_2\right]^{\alpha_{\pi,\omega}(t)-1} \cdot \right. \\ & \left. \cdot\, g^{2} \varepsilon_{\alpha\beta\varsigma\tau} (k_4)^{\varsigma} (k_5)^{\tau} \int d B(x_4) \left[(1-x_4)+\bar{\varkappa} x_4\right]^{\alpha_{\pi,\omega}(t)-1} \right\} \\ & + \left(\pi_{A_1} - \text{contributions} \ldots\right) \end{aligned} \tag{28}$$

Here, $dF_4(x)$ and $dB(x)$ are short-hand notations for the integrands of eqs.(7),(8). The factors $[(1-x)+\underline{x}\,x]^{(\alpha_{\pi,\omega}(t)\,(-1))}$ appear here because one of the external particles of the factorized four-point amplitudes is a Reggeon. If we continue the variable $\underline{\alpha}'t=\underline{\alpha}'t_{123}$ to the values $\underline{\alpha}'m_{\pi}^{2}=-\frac{1}{2}$ or $\underline{\alpha}'m_{\omega}^{2}=\frac{1}{2}$, respectively, where $\underline{\alpha}_{\pi,\omega}(m_{\pi}^{2})=0$, $\underline{\alpha}_{\pi,\omega}(m_{\omega}^{2})=1$ these factors will take the value one. In this case the integrals in the first contribution to eq.(28) coincide with the four-pion amplitude (7) whereas the second contribution yields the original Veneziano amplitudes for the process $\underline{\pi}+\underline{\pi}\to\underline{\pi}+\underline{\omega}$ /1/.

Finally, we mention that the integral representation (25) is invariant with respect to a cyclic permutation of the external momenta $k_i\to k_{i+1}$ as can easily be seen by performing a change of integration variables. This important property guarantees crossing symmetry of the total amplitude and was indeed built into the amplitude from the very beginning due to some well-defined transformation properties of the propagator D and the vertex V_{NS} under the projective group/6-13/. Furthermore, the amplitude (25) develops at most $(N-3)=3$ simultaneous poles in Mandelstam variables due to the vanishing of some of the x_i or of some brackets $[\ldots]$ in the integrand.*) These properties generalize the

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*) Such simultaneous poles appear only in Mandelstam channels that are <u>not</u> dual to each other (see. e.g. the configuration of fig.3). A Mandelstam channel of a N-point amplitude corresponds to a partition of the external particles into two groups by drawing a line. Two channels are said to be dual to one another if their associated lines cross each other.

duality concept discussed in Sect.2 for the four-point amplitude to the six (N)-point case.

4. The Neveu-Schwarz-Ramond (N.S.R.) Meson-Fermion Model

To introduce fermions into the dual model Ramond/5/ considered besides the a- and b-operator algebra an additional set of anti-commuting creation and destruction operators, now with integer indices

$$\{ d_m^{\mu}, d_n^{+\nu} \} = - g^{\mu\nu} \delta_{mn} \qquad m,n = 1,2,\ldots,\infty \tag{29}$$

as well as suitable generalizations of Dirac's γ-matrices ($\gamma_5 = i\gamma_1\gamma_2\cdots\gamma_d$, d space-time dimension)

$$\begin{aligned} \Gamma^{\mu}(z) &= \gamma^{\mu} + i\sqrt{2}\,\gamma_5 \sum_{n=1}^{\infty} (d_n^{+\mu} z^n + d_n^{\mu} z^{-n}) \\ \Gamma_5 &= \gamma_5 (-1)^{\sum_{n=1}^{\infty} d_n^+ \cdot d_n} \end{aligned} \tag{30}$$

satisfying the anticommutation relations

$$\begin{aligned} \{ \Gamma^{\mu}(\theta), \Gamma^{\nu}(\theta') \} &= 2 g^{\mu\nu} \delta\left(\tfrac{1}{2\pi}(\theta-\theta')\right) \quad , (z = e^{i\theta}) \\ \{ \Gamma^{\mu}(1), \Gamma_5 \} &= 0 \end{aligned} \tag{31}$$

Furthermore, the spectrum of the fermion sector is determined by a generalized Dirac equation obtained by means of the following correspondence principle

$$(\gamma \cdot p - m_N)|\phi\rangle = 0$$

$$\Rightarrow (\langle \Gamma(z) \cdot P(z) \rangle - m_N)|\phi\rangle = 0 \quad \text{or} \quad \frac{1}{\sqrt{\alpha'}}(F_0 - \sqrt{\alpha'}\, m_N)|\phi\rangle = 0 \tag{32}$$

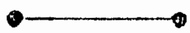
where $\langle \cdots \rangle$ means an averaging procedure

$$\langle A(z) \rangle = \frac{1}{2\pi}\int_0^{2\pi} d\theta\, A(\theta) = \oint_{z=0} \frac{dz}{2\pi i z} A(z) \quad , (z = e^{i\theta}) \tag{33}$$

and $\underline{\underline{P}}^{\mu}(z)$ is a generalized momentum operator of the dual model defined by

$$\underline{\underline{P}}^{\mu}(z) = p^{\mu} + \frac{1}{\sqrt{2\alpha'}} \sum_{n=1}^{\infty} \sqrt{n}\,\left(a_n^{\mu} z^{-n} + a_n^{+\mu} z^{n}\right) \tag{34}$$

The fermion propagator is then given as usually by the inverse of the operator appearing in the generalized Dirac equation (32)

<u>propagator:</u> 

$$D_{(f)} = \frac{1}{\sqrt{\alpha'}\,m_{(N)} - F_{(0)}} = \left[\sqrt{\alpha'}\,m_{(N)} - \sqrt{\alpha'}\,\underline{\gamma}\cdot p + i\underline{\gamma_5} \sum_{n=1}^{\infty} \sqrt{n}\,\left(a_n^{+}\cdot d_n + a_n\cdot d_n^{+}\right)\right]^{-1} \tag{35}$$

or equivalently, $(L_0 = -F_{(0)}^{(2)})$

$$D_{(f)} = \frac{F_{(0)} + \sqrt{\alpha'}\,m_{(N)}}{L_{(0)} + \alpha' m_{(N)}^{2}} = \left(F_0 + \sqrt{\alpha'}\,m_{(N)}\right)\left[\alpha' m_{(N)}^{2} - \alpha' p^{2} - \sum_{n=1}^{\infty} n\left(a_n^{+}\cdot a_n + d_n^{+}\cdot d_n\right)\right]^{-1} \tag{36}$$

The emission (absorption) of a meson from a fermion line is described by the following generalized pseudoscalar meson-fermion coupling (F_2-picture)

<u>meson emission</u>

<u>vertex $V_{(m)}(k)$:</u> 

$$\underline{\underline{V}}_{(m)}(k) = g_R \Gamma_5\, V_0(k) \tag{37}$$

where $V_0(k)$ was defined in eq.(15).

It is now possible to write down the meson-fermion amplitude shown in fig.6 as expectation value of a chain of propagators and vertex operators (solid lines describe fermions)*)

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*) In writing eq.(38) we have taken into account the time direction represented by the direction of the fermion line. In coming particle operators stand now on the right hand side of outgoing particle operators.

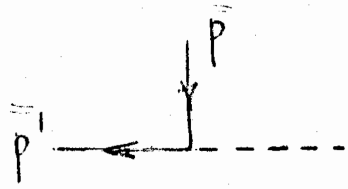
$$T_{N+2} = \bar{u}(\bar{p}')\langle 0,\bar{p}'|V_m(k_N)\frac{1}{\sqrt{\alpha'}m_N - F_0}V_m(k_{N-1})\cdots\frac{1}{\sqrt{\alpha'}m_N - F_0}V_m(k_1)|\bar{p},0\rangle \times u(\bar{p}) \qquad (38)$$

where $\bar{u}(p')$, $u(p)$ are usual Dirac spinors.

Finally, Corrigan and Olive have found an expression for a fermion emission vertex

<u>fermion emission</u>

<u>vertex V_f :</u>



which is given by a rather complicated exponential form, quadratic in the mesonic b-operators and bilinear in the d- and b-operators. The reader interested in more details is referred to their original work[18].

Before discussing explicit examples of meson-fermion amplitudes let us say a few words on the ghost problem of the dual-resonance model. The operator formalism considered above is manifestly Lorentz covariant as it was constructed using the manifestly covariant algebra (10),(29). Then it follows that the timelike components of the oscillator operators generate unphysical negative norm states-"ghosts"

$$\langle 0|a_m^0 a_m^{+0}|0\rangle = \langle 0|b_m^0 b_m^{+0}|0\rangle = \langle 0|d_m^0 d_m^{+0}|0\rangle = -1$$

Now we have learned from the analogous situation in quantum electrodynamics that Lorentz covariance and positivity of the norm can be reconciled by using the Lorentz gauge condition. This condition restricts then the space of allowed physical states. As has been first found by Virasoro[19] for the conventional model and then by other authors also for the N.S.R. model[20] there exists an infinite set of gauge conditions in the dual-resonance model that can be used to eliminate all the ghosts. In the N.S.R.

model such no-ghost theorems hold for space-time dimensions $d \leq d_{crit} = 10$ and unfortunately only for the unphysical masses of the ground state particles given by $\alpha' m_{\pi}^2 = -\frac{1}{2}$, $\alpha' m_N^2 = 0$. Instead of using these theoretically consistent but unphysical mass values it is sometimes convenient in phenomenological applications to take $\alpha' m_{\pi}^2$, $\alpha' m_N^2$ rather as free parameters that will be adjusted to their experimental values (see Sect.6 and below).

Examples

Let us now quote for illustration the meson-fermion amplitudes of fig.7 a-d calculated by means of the above rules. Performing the necessary a,b- and d-operator algebra one easily obtains the following expressions (the ground states of the N.S.R. model are designated as "π" and "N")/10/.

i) $\pi N \to \pi N$

$$F_4(s,t) = g_R^2\, \bar{u}(p')\, \gamma \cdot k_2\, u(p)\, B\left(\tfrac{1}{2} - \alpha_N(s),\, 1 - \alpha_\rho(t)\right)$$

$$F_4(s,u) = g_R^2\, \bar{u}(p')\, \gamma \cdot k_2\, u(p)\, B\left(\tfrac{1}{2} - \alpha_N(s),\, \tfrac{1}{2} - \alpha_N(u)\right)$$

$$\left(\alpha_N(s) = \tfrac{1}{2} + \alpha'(s - m_N^2)\ ,\ \alpha_\rho(t) = \tfrac{1}{2} + \alpha'(t - m_\pi^2)\right) \tag{39}$$

ii) $\pi N \to \rho N$

$$\begin{aligned} F_4^{\lambda}(s,t) = \tfrac{1}{2} g_R\, g\, \bar{u}(p') \Big\{ -k_1 \cdot e(k_2,\lambda) \Big[& B\left(\tfrac{3}{2} - \alpha_N(s),\, -\alpha_{\pi,\omega}(t)\right) \\ & + B\left(\tfrac{1}{2} - \alpha_N(s),\, -\alpha_{\pi,\omega}(t)\right) \Big] \\ + i\, (k_1)^\nu e^\mu(k_2,\lambda)\, \sigma_{\mu\nu}\, & B\left(\tfrac{1}{2} - \alpha_N(s),\, 1 - \alpha_{\pi,\omega}(t)\right) \Big\} \gamma_5\, u(p) \end{aligned} \tag{40}$$

iii) $\pi N \to \omega N$

$$F_{(4)}^{\lambda}(s,t) = g_{R}\cdot g\, i\,(k_1)_{\varsigma}\, \varepsilon^{\mu}_{\;\nu\tau\varsigma}(k_2)^{\nu} e(k_2,\lambda)_{\mu}\, \bar{u}(p')\, \times$$
$$\times \Big\{ -C_1^{\tau\sigma\varsigma} B\left(\tfrac{1}{2}-\alpha_N(s), 2-\alpha_g(t)\right)$$
$$+ C_2^{\tau\sigma\varsigma}\left[-B\left(\tfrac{1}{2}-\alpha_N(s), 1-\alpha_g(t)\right) + \tfrac{1}{2} B\left(\tfrac{1}{2}-\alpha_N(s), 2-\alpha_g(t)\right)\right]\Big\} \tag{41}$$

where

$$C_1^{\tau\sigma\varsigma} = -\frac{1}{2\sqrt{2}}\,\frac{i}{3!}\,\gamma^{5}\gamma^{\alpha}\varepsilon_{\alpha}^{\;\tau\sigma\varsigma}$$
$$C_2^{\tau\sigma\varsigma} = \frac{1}{\sqrt{2}}\left(\gamma^{\sigma} g^{\varsigma\tau} - \gamma^{\tau} g^{\varsigma\sigma}\right)$$

Here, $e_{\mu}(k,\lambda)$ is the polarization vector of the vector mesons ς, ω to helicity λ, $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_{\mu},\gamma_{\nu}]$ and B is the beta-function (8). In the configurations of fig.7 b-d the fermion emission vertex V_f has been used. Finally, the amplitudes with an external ς or ω may be calculated by factorizing corresponding five- and six-point amplitudes in a two or three meson channel, respectively, or by replacing a $|\pi\rangle$ state in the operator definition of the amplitude by the states $|\varsigma\rangle$ or $|\omega\rangle$ of table 1. Note, that the respective trajectories appearing in the meson (t-) or fermion (s-) channels are automatically shifted by integers or half-integers j as it should be. These shifts guarantee that the first poles of the amplitude appear at values $\alpha(m_j^2) = j$ where j is the spin of the lowest resonance lying on the respective trajectory. The leading Regge behaviour is just restored by the kinematical factors appearing in front of the B-functions. Concluding, let us say some words concerning the different coupling schemes of the π, ω and ς mesons of the model to the fermion-antifermion state. By inspection of eq.(40) we see that the pion couples in accord with eq.(37) by a usual pseudoscalar $\bar{u}(p')\gamma_5 u(p)$ coupling. Let us next take the residuum at the ω-pole $\alpha' t = \alpha' m_{\omega}^2 = \frac{1}{2}$. We have

$$\beta_{\pi\omega} = \underset{\alpha_{\pi,\omega}=1}{\mathrm{Res}}\, F_{(4)}(s,t) = \frac{1}{2} g_{\rho}\, g\, \bar{u}(p')\Big[-2\alpha_{N}(s)\, k_1\cdot e(k_2,\lambda) + i\, k_1^{\nu} e^{\mu}(k_2,\lambda)\,\sigma_{\mu\nu}\Big]\gamma_5 u(p) \tag{42}$$

which may easily be rewritten in the form

$$\beta_{\omega} = \frac{i}{2}\Big[g\, \varepsilon^{(\rho)}_{\alpha\tau\sigma}(k_2)^{\alpha} e(k_2,\lambda)^{\tau}(k_1)^{\sigma}\Big]\Big[g_{\rho}\, \varepsilon_{\rho\mu\nu\tau}(p-p')^{\mu}\,\bar{u}(p')\gamma_5 \times \sigma^{\tau\nu} u(p)\Big] \tag{43}$$

We, thus, see that the ω-meson of the N.S.R. model couples only "magnetically" to the $N\bar{N}$ state. This situation has to be contrasted with the ρ meson that couples according to eq.(41) only via the "electrical" coupling $\bar{u}(p')\gamma_{\mu}u(p)$. It is worth mentioning that the N.S.R. model yields also an interesting result for the $N\bar{N}\to N\bar{N}$ amplitude /21/. The reader interested in the art of higher level algebraic manipulations is referred to this work which represents doubtlessly a summit in the operator investigations of the N.S.R. model.

5. Unitarity Corrections to the Regge Trajectories

In the preceding sections we have discussed some properties of the "tree" graphs of the N.S.R. model. These were meromorphic functions in the respective Mandelstam variables with pole singularities on the real axis that reflected the propagation of stable one-particle intermediate states. Due to the unitarity equation (comp. fig.8) the imaginary part of a scattering amplitude should, however, also acquire contributions from many-particle intermediate states leading to normal threshold cuts in the Mandelstam variables. To satisfy unitarity one should, therefore, include higher order loop amplitudes into the dual model. These loop graphs should, in particular, add an imaginary part to the up to now real Regge trajectories so that, due to $\Gamma_{res} = \frac{\mathrm{Im}\,\alpha(s=m^2_{res})}{\alpha' m_{res}}$, the resonances will get a finite width.

For simplicity, we shall restrict us in the following to the discussion of the planar one-loop graph. Factorizing a loop graph on an internal line should give a tree amplitude with two excited lines on their ends (represented by occupation number states) as residuum. These facts suggest to construct the loop graph by "sewing" together the two excited lines of corresponding tree amplitudes after having inserted a propagator between them (comp. fig.9). The summation over the inserted occupation number states is just equivalent to taking the trace of the vertices and propagators appearing in fig.9 (a fter having replaced $\frac{1}{l-\alpha}$ by D)/6/. It thus follows

$$T_4^{(1)} = \int \frac{d^d k}{(2\pi)^d} \, \mathrm{Tr}\left[D\, V(k_1)\, D\, V(k_2)\, D\, V(k_3)\, D\, V(k_4) \right] \tag{44}$$

If one now inserts the integral representation of the meson propagators as well as the vertex operators into eq.(44) one obtains after calculating seperately the a- and b-operator traces as well as the loop momentum integration the final answer/22/ ($\alpha' = \frac{1}{2}$) *)

$$T_4^{(1)} = g^4\, 2^{-4} \frac{1}{(2\pi)^5} \int_0^1 \prod_{i=1}^{4} dx_i \, \frac{1}{\omega^{3/2} \ln^5 \omega} f^{-8}(\omega)\, \varphi_+^{8}(\omega)$$
$$\prod_{1 \le i < j \le 4} \psi(x_{ij}, \omega)^{-k_i \cdot k_j} \sum_P (-1)^P \prod_{j=1}^{2} k_{i_{2j-1}} \cdot k_{i_{2j}}\, \chi^+(x_{i_{2j-1} i_{2j}}, \omega) \tag{45}$$

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*) This expression has been calculated using the operator rules of the F_1 -picture. Moreover, the calculations are done in the "critical" (unphysical) dimension of space-time $d_{crit} = 10$ of the N.S.R. model (only in this dimension the model is ghost free and consistent on the one-loop level).

Here $x_{(i)}$ are the integration variables of the loop propagators $\omega = x_1 x_2 x_3 x_4$, $f(\omega) = \prod_{n=1}^{\infty}(1-\omega^n)$, $\varphi_{(\pm)}(\omega) = \prod_{n=1}^{\infty}(1 \pm \omega^{n-1/2})$, $x_{(ij)} = x_{(i+1)} x_{(i+2)} \dots x_{(j)}$ and the sum in eq.(45) is over all permutations P that group the external particles in pairs, each pairing being counted only once regardless of the ordering in each pair. The functions $\psi(x,\omega)$, $\chi^+(x,\omega)$ appearing in eq.(45) may be expressed by Jacobi's theta functions $\vartheta_{(i)}(\nu|\tau)$ /23/

$$\psi(x,\omega) = -\ln\omega\, \vartheta_1\left(\frac{\ln x}{\ln\omega}\Big| -\frac{2\pi i}{\ln\omega}\right) \Big/ \vartheta_1'\left(0\Big| -\frac{2\pi i}{\ln\omega}\right)$$

$$\chi^+(x,\omega) = \frac{i}{2}\vartheta_2\left(0\Big|\frac{\ln\omega}{2\pi i}\right)\vartheta_4\left(0\Big|\frac{\ln\omega}{2\pi i}\right)\frac{\vartheta_3\left(\frac{\ln x}{2\pi i}\Big|\frac{\ln\omega}{2\pi i}\right)}{\vartheta_1\left(\frac{\ln x}{2\pi i}\Big|\frac{\ln\omega}{2\pi i}\right)} \tag{46}$$

The meson amplitude with an internal fermion loop is obtained quite analogously by inserting the fermion operators in the trace (44)/24/

$$T_4^{(1)} = \frac{1}{2} g_{(R)}^{(4)} 2^{(-4)} \frac{1}{(2\pi)^5}\int_0^1 \prod_{i=1}^4 dx_i \frac{1}{\omega \ln^5\omega} f^{(-8)}(\omega)\varphi_{(0)}^{(8)}(\omega)$$

$$\prod_{1\le i<j\le 4}\psi(x_{(ij)},\omega)^{-k_i\cdot k_j}\sum_P(-1)^P\prod_{j=1}^{2}\sqrt{2}\,k_{(P_{2j-1})}\cdot\sqrt{2}\,k_{(P_{2j})}\,\chi_f^+(x_{(P_{2j-1},P_{2j})},\omega) \times \mathrm{Tr}\{1\} \tag{47}$$

where

$$\varphi_0(\omega) = \prod_{n=1}^{\infty}(1+\omega^n)$$

$$\chi_{(f)}^+(x,\omega) = \frac{i}{2}\vartheta_3\left(0\Big|\frac{\ln\omega}{2\pi i}\right)\vartheta_4\left(0\Big|\frac{\ln\omega}{2\pi i}\right)\frac{\vartheta_2\left(\frac{\ln x}{2\pi i}\Big|\frac{\ln\omega}{2\pi i}\right)}{\vartheta_1\left(\frac{\ln x}{2\pi i}\Big|\frac{\ln\omega}{2\pi i}\right)} \tag{48}$$

These loop amplitudes may be most conveniently studied by performing the following change of variables (Jacobi transformation)

$$\theta_i = \frac{\pi \ln(x_2 x_3 \dots x_i)}{\ln\omega} \qquad 0\le\theta_i\le\pi \quad (2\le i\le 4,\ \theta_1=0)$$
$$\theta_i<\theta_j \text{ if } i<j$$

$$q = \exp(2\pi^2/\ln\omega) \qquad 0\le q\le 1 \tag{49}$$

As we are interested in the explicit form of trajectory corrections due to planar loops we have to investigate the asymptotic behaviour of eqs.(45),(47). Using eq.(47) we obtain, for example,[25]

$$T_4^{(1)}(s,t) \underset{\substack{|s|\to\infty \\ t\ \mathrm{fix}}}{\sim} 8 g_R^2 \left[-\alpha_\rho(s)\right]^{\alpha_\rho(t)} \Big[\ln(-\alpha_\rho(s))\, \Gamma(1-\alpha_\rho(t))\, \Sigma(\alpha_\rho(t)) + \Gamma(1-\alpha_\rho(t))\beta(t) - \Gamma'(1-\alpha_\rho(t))\Sigma(\alpha_\rho(t)) + \Gamma(1-\alpha_\rho(t))\Sigma'(\nu)\big|_{\nu=\alpha_\rho} \Big] \tag{50}$$

where Σ is given by the rather complicated expression

$$\Sigma(\nu) = \frac{1}{4}(\sqrt{2}g_R)^2 (2\pi)^{-10} \int_0^1 dq\, q^{-2} \left(\frac{\varphi(q)}{f(q^2)}\right)^8 \int_0^{\pi} d\bar\theta\, \bar\psi(\theta,q)^t \times$$
$$\times \left[-\frac{\partial^2 \ln\bar\psi}{\partial\theta^2}\right]^{\nu-2} \Big\{ (1-\nu)\left[\bar\chi(\theta,q)\frac{\partial^2\bar\chi}{\partial\theta^2} - \left(\frac{\partial\bar\chi}{\partial\theta}\right)^2\right]$$
$$+ 2(1-\alpha_\rho(t))\bar\chi^2(\theta,q)\left[-\frac{\partial^2}{\partial\theta^2}\ln\bar\psi\right] - \frac{\alpha_\rho^2(t)}{\nu} F^2(q^2)\left[-\frac{\partial^2}{\partial\theta^2}\ln\bar\psi\right]^2\Big\} \tag{51}$$

and $\beta(t)$ is defined by some three-fold integral. Moreover, we have introduced the notations $\bar\chi(\theta,q) = (-\frac{\ln q}{\pi})^{-1}\chi^+_{A}(x,\omega)$, $\bar\psi(\theta,q) = (-\frac{\ln\omega}{\pi})^{-1}\psi(x,\omega)$, $F(q^2) = \lim_{\theta\to 0}(2\sin\theta)\bar\chi(\theta,q)$.
Note, that eq.(50) may be interpreted as the $O(g_R^2)$ term of a total amplitude

$$T_4^{tot}(s,t) = \sum_{m=0}^{\infty} T_{(4)}^{m\text{-loop}}(s,t) \underset{|s|\to\infty}{\sim} 8 g_R^2\, Z(t)\,\beta_{new}(t)\,\Gamma(1-\alpha_\rho^{new}(t))\,(-\alpha(s))^{\alpha_\rho^{new}(t)} \tag{52}$$

expanded in powers of the coupling constant g_R. The corrected expressions for the ρ-trajectory and residue read

$$\alpha_\rho^{new}(t) = \alpha_\rho(t) + \Sigma(\alpha_\rho(t)) + O(g_R^4)$$

$$\beta_{new}(t) = 1 + g_R^2\,\beta(t) + O(g_R^4)$$

moreover,

$$\Sigma(t) = [\,1 - \Sigma'(\nu)\big|_{\nu=\alpha_s} + O(g_R^4)\,]^{-1} \qquad (53)$$

As seen from eq.(51), the correction function $\Sigma(\alpha(t))$ is badly defined due to the singular behaviour of the integrand near $q=0$. One can give a meaning to this integral by performing a suitable regularization and renormalization. As a net result of the renormalization one obtains a finite correction function $\hat{\Sigma}$ as well as renormalized values of the dual coupling constants and of the trajectory slope α' /25,26/.

Concluding, it can be shown that the correction function develops normal threshold cuts in t leading to an imaginary part of $\alpha_s^{new}(t)$ and thus to $\Gamma_{res} \neq 0$. Moreover, the asymptotic behaviour of the meson loop (45) can be obtained from the above results through the formal substitutions $q \to -q$ and $(\sqrt{2}\, g_R)^4 \to -(g)^4$.

In writing eq.(52) we have assumed that the multi-loop graphs contributing to the total meson amplitude (see ref./27/ for the definition of multi-loop amplitudes) possess the factorizing asymptotic behaviour that is necessary to perform the sum over m. Such factorization properties of the asymptotic expressions of multi-loop graphs have been proved for the conventional Veneziano model/10,28/.

6. Phenomenological Applications

We have already mentioned that the N.S.R. model possesses some yet unphysical properties concerning its particle spectrum. Thus, the ghost-free version of the model contains the pion as a tachyon $(\alpha' m_\pi^2 = -\frac{1}{2} < 0)$ lying on a degenerate π, ω - trajectory. Furthermore, the ρ meson and the fermion ground state are mass-less. Finally, unitary loop corrections can be consistently taken into account only for a critical (unphysical) space-time dimension $d_{crit} = 10$. It is the hope of many people that a more realistic dual meson-fermion model without these defects can be constructed that retains some similarity to the original model. A promising step towards this aim has been done recently by Cremmer and Scherk /29/. Let us, however, do also justice to the present N.S.R. model and remember some of its sound and attractive features. First of all, the dual amplitudes of a given process contain only very few parameters and offer a unified description of both the low energy resonance region and of the high energy region where single- or multi-Regge pole exchange is dominant. Moreover, the same amplitudes may describe a large number of different processes related by crossing.

These features are preserved if one replaces in phenomenological applications the unphysical Regge trajectories appearing in the amplitudes of the present model by the experimental ones. Furthermore, instead of treating complicated loop expressions some amount of unitarity can be taken into account by giving an imaginary part to the trajectories tolerating then the appearance of "ancestor" and ghost states. The latter ones are generally believed to be unimportant in the kinematical regions considered. In refs./30,31/ the N.S.R. model has been used as a guide for the construction of more realistic dual meson-fermion amplitudes. The modified amplitudes thus obtained have been applied to a phenomenological study of the production of vector and tensor mesons lying on degenerated trajectories in processes of the type $\pi p \to (\rho^0, f, g^0)n$; $\pi^- p \to \omega n, A_2^- p$ and $K^- p \to \bar{K}^*(890, 1420)N$ /30/ and $K\bar{p} \to \bar{K}^{*0}(890)(\pi \bar{p})$ *) /31/.

Starting point for the quasi-two body amplitudes were formulae of the type of eq.(40),(41) (extended to higher $J=2,3$ resonances). The amplitude of the quasi-three body reaction (comp. fig.10 where one of the three possible graphs with nonexotic quantum number is shown) were calculated by factorizing a six-point meson-fermion amplitude on the spin 1^- state in a two-meson channel and by further modifying this amplitude. Let us say some words on the kind of modifications chosen in these applications. First of all, as a prerequisite for a subsequent shift of the π-trajectory to its physical expression, one had to overcome the unphysical degeneracy of the π, ω - trajectories. The separation of the (π, ω) trajectory can most easily be done by using helicity amplitudes $H_\lambda^{(\pm)}$ with definite parity exchange in the t-channel/32/ (λ is the helicity of the produced vector or tensor meson)

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*) In applying the N.S.R. model to KN reactions mainly the spin and parity content of the model is considered. In distinction to πN reactions there are then no G-parity restrictions.

$$H_{\lambda}^{(\pm)} = H_{\lambda} \mp (-)^{\lambda} H_{-\lambda} \tag{54}$$

As is well known, the amplitudes (54) are dominated to leading order in s by the exchange of trajectories with definite naturality. Hence, the unnatural spin-parity π-trajectory ($P = -(-1)^{J}$) was attributed to H_{λ}^{-}, whereas the natural spin-parity ω-trajectory ($P = +(-1)^{J}$) contributes to H_{λ}^{+}. The N.S.R. model was further modified to include several baryon trajectories ($N_{\alpha}, N_{\gamma}, \Delta$). Fig.11 shows the differential cross sections $d\sigma_{\lambda}^{+}/du$ and $d\sigma_{\lambda}^{-}/du$ for the reactions a) $\pi^{-}p \to \rho^{0} n$, b) $\pi^{-}p \to f n$ and c) $\pi^{-}p \to g^{0} n$ at 17.2 GeV compared with the theoretical predictions for the natural and unnatural spin-parity exchange to helicity λ /30/. In particular, the unnatural spin-parity exchange is described reasonably for all measured helicities of the produced resonances for small t. It is worth mentioning that the dual amplitude predicts a non-zero contribution in the forward direction for the helicity non-flip amplitude in distinction to usual pion exchange models. Finally, the five-point meson-fermion graph with an external vector meson contributes in the single Regge limit (comp. fig.10) to the helicity amplitude as follows

$$H_{\lambda(\bar{s}\bar{u})} \sim e_{\mu}(\lambda)\, \Gamma(1-\alpha_{\pi,\omega}(t)) \left(-\alpha_{16}^{N}\right)^{\alpha_{\pi,\omega}(t)} B^{\mu}_{(\bar{s}\bar{u})} \tag{55}$$

where

$$\begin{aligned} B^{\mu}_{(\bar{s}\bar{u})} = {} & 2\,\bar{u}(k_5)\, k_{4}\!\!\!/\,\gamma\, u(k_6) \Big[\frac{k_1^{\mu}}{(-\alpha_{\pi,\omega}(t))} G\big(-\alpha_{\pi,\omega}(t), \tfrac{1}{2}-\alpha_{45}^{N}, \tfrac{1}{2}-\alpha_{46}^{N}; \beta\big) \\ & - \frac{k_5^{\mu}}{(-\alpha_{16}^{N})} G\big(1-\alpha^{\pi,\omega}(t), \tfrac{3}{2}-\alpha_{45}^{N}, \tfrac{1}{2}-\alpha_{46}^{N}; \beta\big) + \frac{(k_4+k_5)^{\mu}}{(-\alpha_{16}^{N})} G\big(1-\alpha_{\pi,\omega}(t), \tfrac{1}{2}-\alpha_{45}^{N}, \\ & \qquad \tfrac{3}{2}-\alpha_{46}^{N}; \beta\big) \\ & - \frac{\bar{u}(k_5)\gamma_{\nu} u(k_6)}{(-\alpha_{16}^{N})} \Big[g^{\mu\nu}(k_2+k_3)\cdot k_4 - k_4^{\mu}(k_2+k_3)^{\nu} \Big] G\big(1-\alpha_{\pi,\omega}(t), \tfrac{1}{2}-\alpha_{45}^{N}, \tfrac{1}{2}-\alpha_{46}^{N}; \beta\big) \\ & - \frac{\bar{u}(k_5)\gamma_5 \gamma^{\nu} u(k_6)}{(-\alpha_{16}^{N})} \Big[\varepsilon^{\mu}_{\nu\rho\sigma} (k_2+k_3)^{\rho} k_4^{\sigma} \Big] G\big(1-\alpha_{\pi,\omega}(t), \tfrac{1}{2}-\alpha_{45}^{N}, \tfrac{1}{2}-\alpha_{46}^{N}; \beta\big) \end{aligned} \tag{56}$$

Here, the following abbreviations have been used

$$G(a,b,c;z) = B(b,c)\,F(a,b,b+c;\bar{z})$$
$$\underline{\beta} = \frac{\alpha_{16}-\alpha_{235}}{\alpha_{16}}$$
$$\alpha_{16} = \alpha_0 + \alpha'(k_1+k_6)^2 \quad etc. \tag{57}$$

where F is the Gauss hypergeometric function and B the Euler beta function (8).

Modifying this term further in the above mentioned way (and including the remaining other graphs) one obtained predictions for the differential cross sections of unnatural and natural spin-parity exchange as well as for the $\overline{K}^{*0}$(890) density matrix elements ρ_{11}, ρ_{1-1}, $Re\,\rho_{10}$, $Im\,\frac{\rho_{10}-\rho_{01}}{2}$ and $Im\,\rho_{1-1}$ as functions of $t' = |t - t_{min}|$ and the nucleon-pion mass $m_{\pi N}$. As an example, Fig.12 shows the model predictions for ρ_{11}, ρ_{1-1} and $Re\,\rho_{10}$ compared with the data at 10 GeV/c. A better overall agreement with data were found if one additionally included a B-trajectory ($\underline{\pi}B$-model, solid curve, compared to only pion exchange, dashed curve).

The conventional Veneziano model has been applied in the past also to the description of inclusive single particle reactions/33/ reproducing such experimental features as Feynman scaling behaviour and large transverse momentum cutoff . One can easily see that similar results may be obtained also from the N.S.R. model as it preserves the conventional model in its a-operator part. The reader interested in more details of the phenomenological application of dual models is referred to ref./8,9,10/.

<u>Figure Caption</u>

Fig.1a-c: The three noncyclic configurations contributing to the four-pion amplitude.

Fig.2: Graphical representation of duality.

Fig.3: Multiperipheral configuration of a dual six-pion amplitude.

Fig.4: Chew-Frautschi plot of the lowest resonances of the N.S. model.

Fig.5: Single Regge limit of a six-pion amplitude. The wiggled line visualizes the exchanged Reggeon.

Fig.6: (N + 2)-point meson-fermion amplitude (The solid line represents a fermion (N)).

Fig.7a-d: Four-point functions of the processes $\pi N \to \pi N$ (a-b), $\pi N \to \rho N$ (c), $\pi N \to \omega N$ (d)

Fig.8: Graphical representation of the unitarity relation

Fig.9: Sewing procedure for the planar one-loop amplitude

Fig.10: Single Regge limit of a graph contributing to the process $K^- p \to \bar{K}^{*0} (\pi^- p)$. This graph has been obtained by factorizing a six-point function of the N.S.R. model.

Fig.11 (taken from ref.[30]). Differential cross-sections for a) $\pi^- p \to \rho^0 n$, b) $\pi^- p \to f n$ and c) $\pi^- p \to g^0 n$ at 17.2 GeV/c compared with the model predictions for the natural and unnatural spin-parity exchange to helicity λ, $\frac{d\sigma_\lambda^+}{dt}$ and $\frac{d\sigma_\lambda^-}{dt}$, in the Gottfried-Jackson system.

Fig.12 (taken from ref.[31]). The K^{*0}(890) density matrix elements ρ_{11}, ρ_{1-1} and $Re\,\rho_{10}$ as functions of t' and $m_{K\pi}$ (the solid curves are obtained from a model with π-B exchange, the dashed curves correspond to π-exchange alone).

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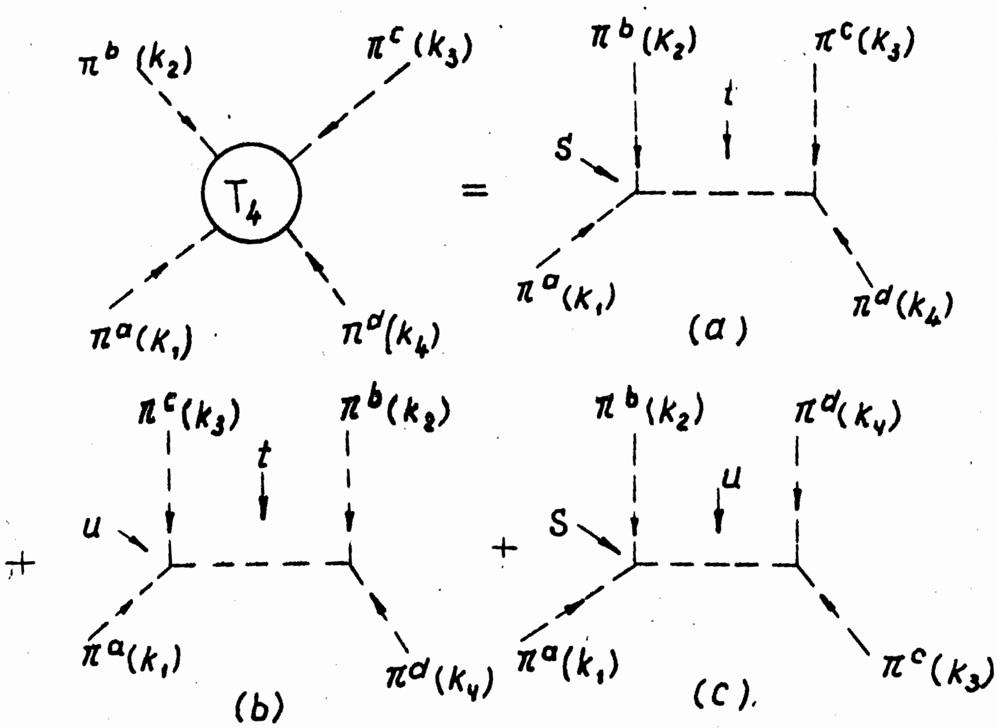
Fig14-c



$\pi^b(k_2)$
$\pi^c(k_3)$
T_4
$\pi^a(k_1)$
$\pi^d(k_4)$
=
$\pi^b(k_2)$
$\pi^c(k_3)$
t
s
$\pi^a(k_1)$
$\pi^d(k_4)$
(a)
+
$\pi^c(k_3)$
$\pi^b(k_2)$
t
u
$\pi^a(k_1)$
$\pi^d(k_4)$
(b)
+
$\pi^b(k_2)$
$\pi^d(k_4)$
u
S
$\pi^a(k_1)$
$\pi^c(k_3)$
(c).

fig. 2

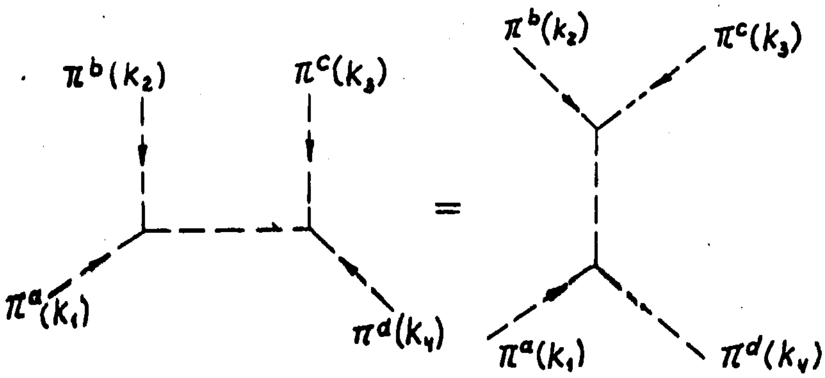


fig. 3

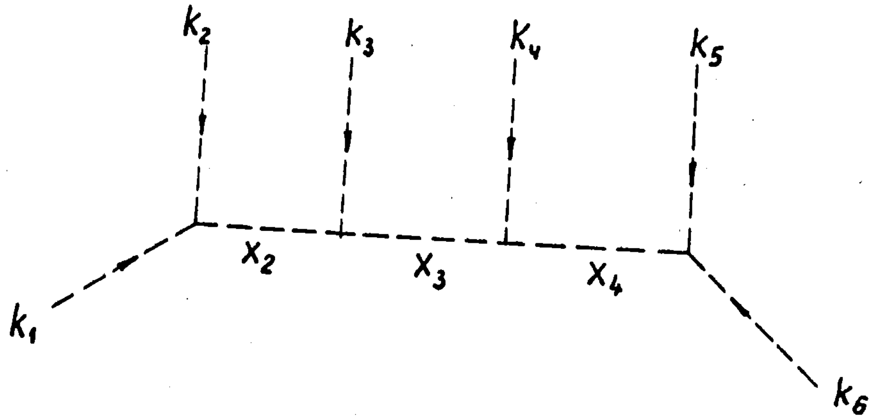


fig. 4

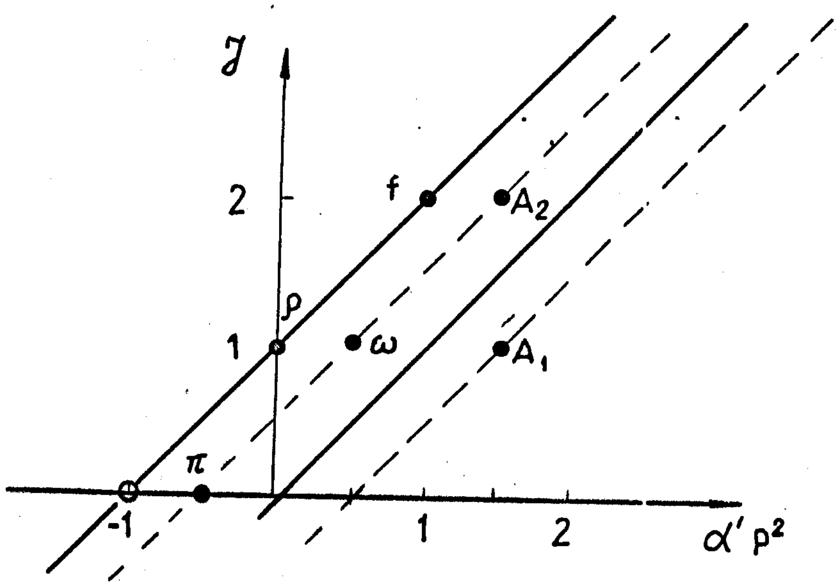


fig. 5

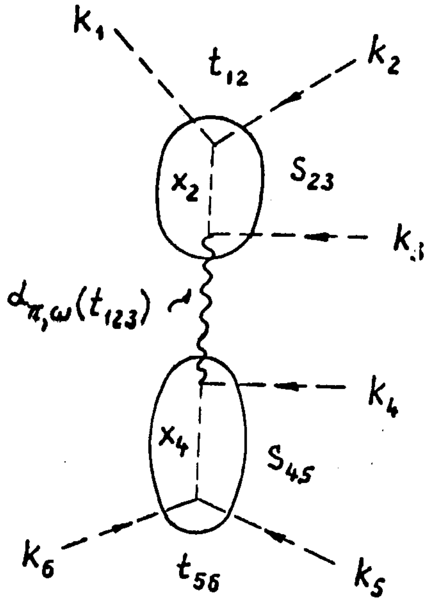


fig. 6

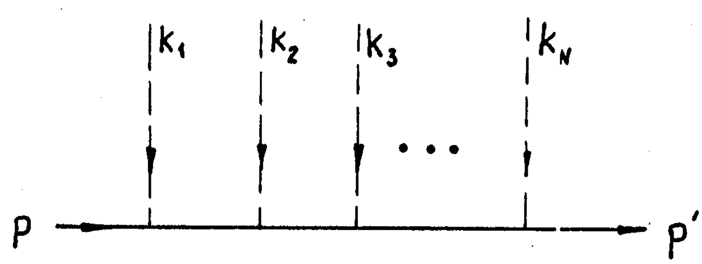


fig 8.7 a-d

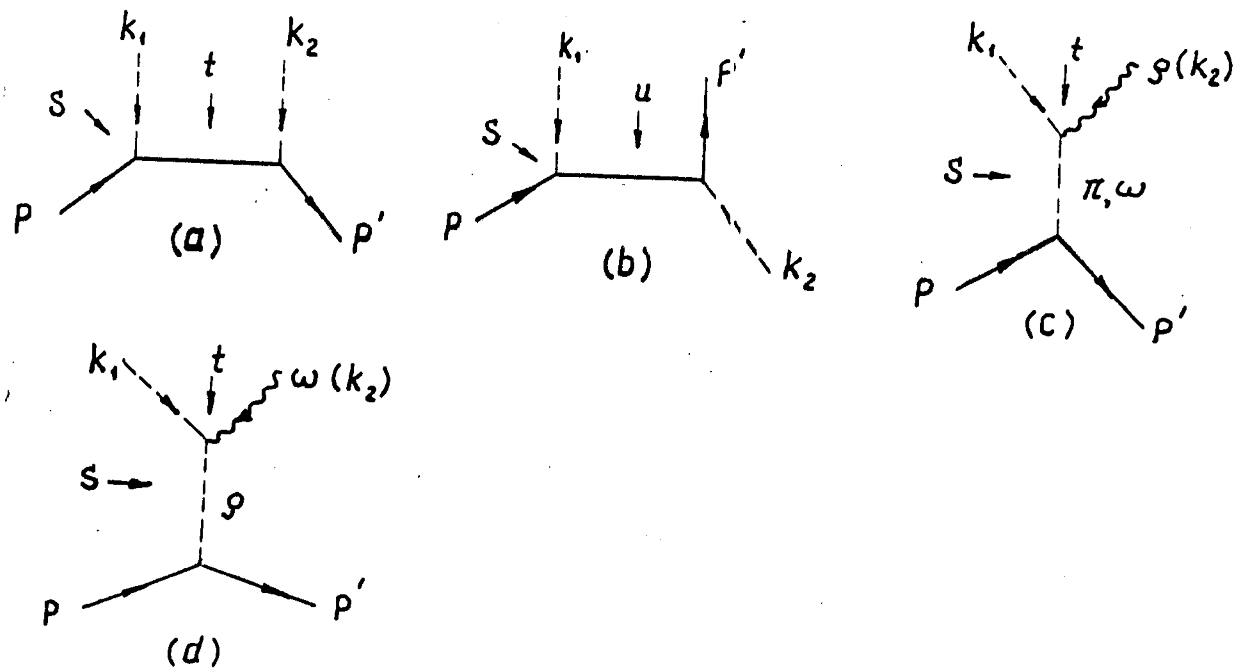


fig 8

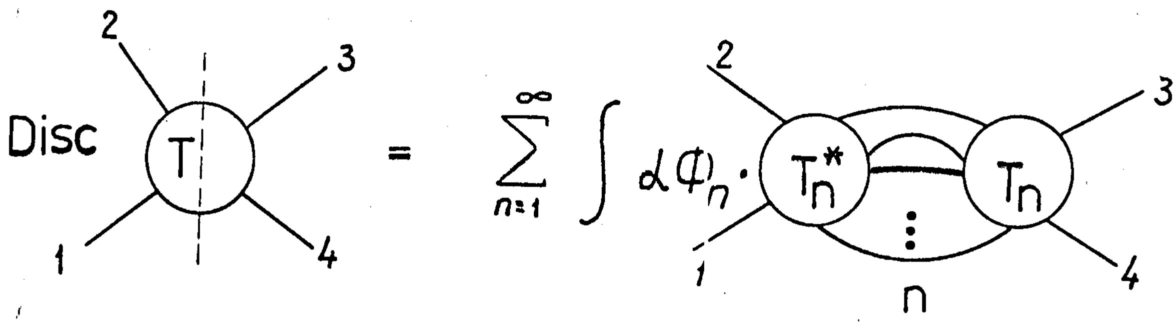


fig. 9

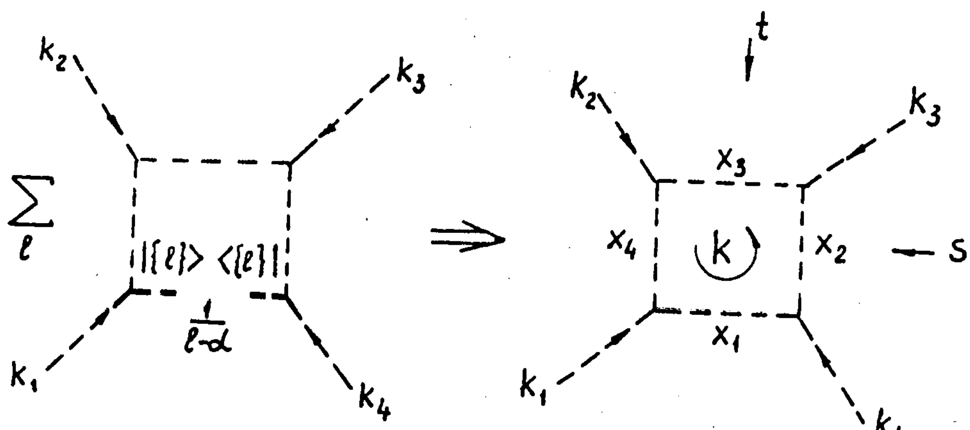
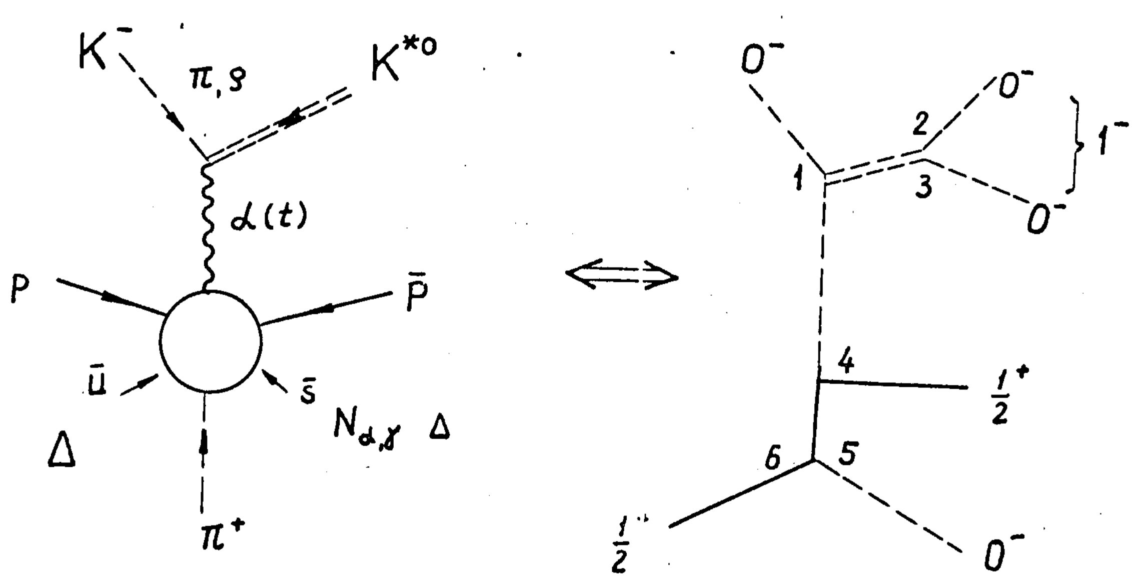
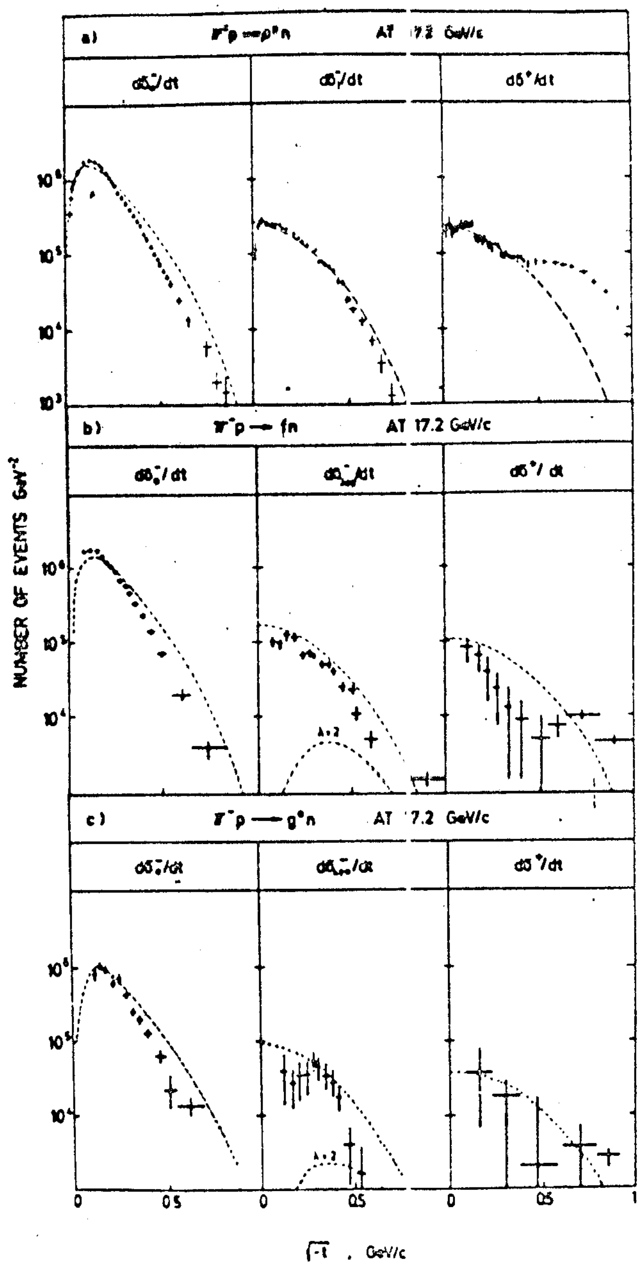

k_2
k_3
$\sum_\ell$
$|\{\ell\}\rangle \; \langle\{\ell\}|$
$\frac{1}{\ell-\alpha}$
k_1
k_4
$\Rightarrow$
t
k_2
k_3
x_3
x_4
k
x_2
s
x_1
k_1
k_4

fig. 10





a) $\pi^- p \to \rho^0 n$ AT 17.2 GeV/c
$d\sigma_0^-/dt$
$d\sigma^-/dt$
$d\sigma^+/dt$
b) $\pi^- p \to f n$ AT 17.2 GeV/c
$d\sigma_0^-/dt$
$d\sigma_{tot}^-/dt$
$d\sigma^+/dt$
$\lambda=2$
c) $\pi^- p \to g^0 n$ AT 17.2 GeV/c
$d\sigma_0^-/dt$
$d\sigma_{tot}^-/dt$
$d\sigma^+/dt$
$\lambda=2$
NUMBER OF EVENTS GeV^{-2}
10^3
10^4
10^5
10^6
0
0.5
1
$\sqrt{-t}$, GeV/c

Fig. 11

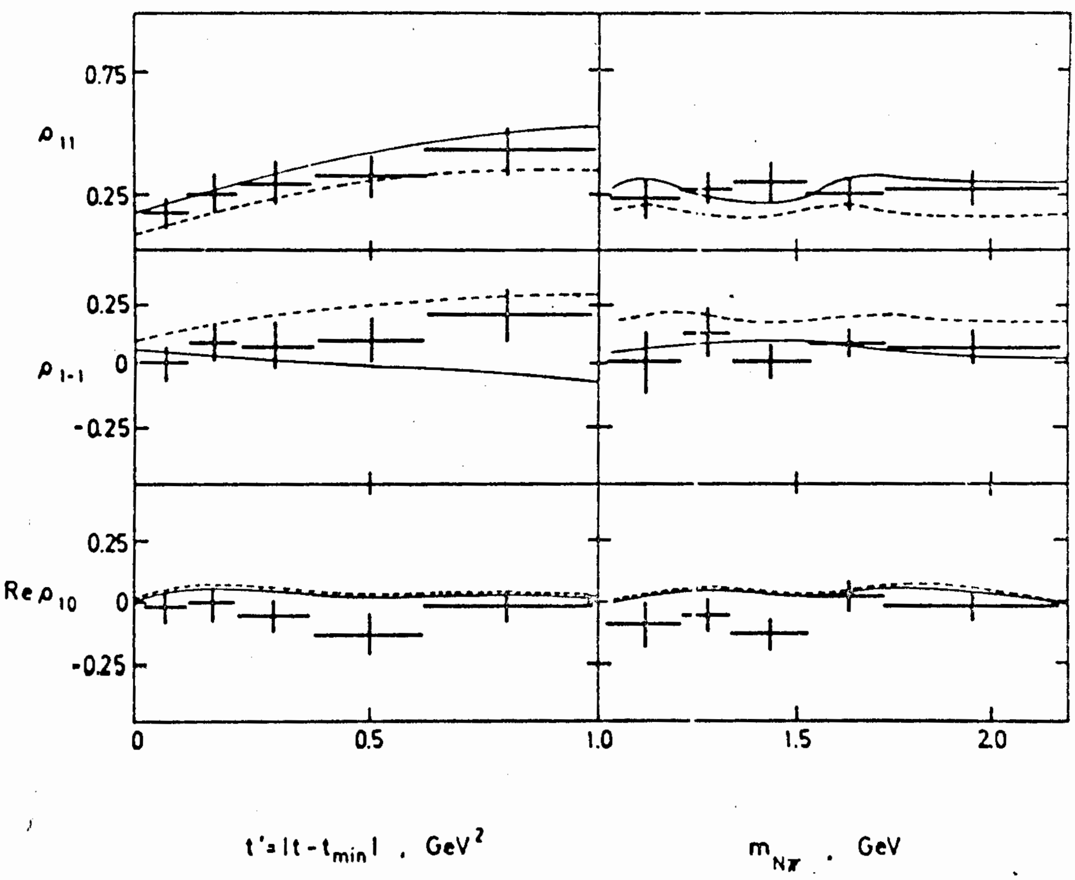

0.75
ρ_{11}
0.25
0.25
ρ_{1-1}
0
-0.25
0.25
Re ρ_{10}
0
-0.25
0
0.5
1.0
1.5
2.0
$t' = |t - t_{min}|$, GeV^2
$m_{N\pi}$, GeV

Fig. 12