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RECOVERING THE CONVERGENCE OF A STRUCTURE FUNCTION RECONSTRUCTION FROM THE QCD MOMENTS IN THE NEXT-TO-LEADING ORDER

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Abstract

It is found that the convergence of a structure function reconstruction from the QCD moments is destroyed provided the latter are calculated with the help of the usual next-to-leading order anomalous dimensions $\gamma(N)$. The convergence is recovered when $\gamma(N)$ at even/odd N, responsible for the Q^2 evolution of crossing even/odd structure functions, are analytically continued to odd/even N. Simple analytical formulae for the corresponding corrections to $\gamma(N)$ in the singlet sector are obtained based on the known nonsinglet result.

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It is well known that the methods of "approximate" reconstruction of a structure function [1−3] provide a simple and a fast solution of the complicated system of the QCD integro-differential evolution equations with any required accuracy. These methods are based on a structure function expansion in a series of orthogonal polynomials. Especially simple solutions are obtained in terms of the usual moments

$$f(N) = \int_0^1 x^{N-2} f(x)dx. \quad (1)$$

Thus, e.g., QCD predicts the N-th moment of a nonsinglet structure function $F_k^{NS}(x,Q^2)$ in the form (see, e.g., reviews [4,5]):

$$F_k^{NS}(N,Q^2) = C_k^{NS}(N,Q^2)\cdot A^{NS}(N,Q_0^2)\cdot[\alpha(Q^2)/\alpha(Q_0^2)]^{\gamma^{NS(0)}(N)/2\beta_0}\cdot$$
$$\cdot\{1+{}^1/_{4\pi}[\alpha(Q^2)-\alpha(Q_0^2)]\cdot[\gamma^{NS(1)}(N)-\gamma^{NS(0)}(N)\beta_1/\beta_0]/2\beta_0\}, \quad (2)$$

where $A^{NS}(N,Q_0^2)$ (to be determined from experiment) is the moment of a certain linear combination of the parton densities at a reference value Q_0^2 of the 4-momentum transfer squared, $C_k^{NS}(N,Q^2)$ is the Wilson coefficient function (proportional to the moment of the corresponding hard cross section), $\beta_0 = 11 - {}^2/_3 f$, $\beta_1 = 102 - {}^{38}/_3 f$ (f is the number of active flavours) and

$$\gamma^{NS}(N,Q^2) = (\alpha/4\pi)\,\gamma^{NS(0)}(N) + (\alpha/4\pi)^2\,\gamma^{NS(1)}(N) + \cdots \quad (3)$$

is the nonsinglet anomalous dimension. Both $C_k(N,Q^2)$ and $\gamma(N,Q^2)$ are known up to the next-to-leading order in the QCD coupling constant α. In particular, the quantities $\gamma^{(1)}(N)$ have been calculated in [6] for nonsinglet and also for singlet sector and represented in a simple analytical form in [7]. The coefficients of the series of orthogonal polynomials used in refs. [1,2] are just certain linear combinations of the QCD moments given in eq. (2). It is known [2,8−10] that a fast convergence of the reconstruction series can be achieved in the case of Jacobi polynomial expansion with an appropriate weight function. We have found, however, that the convergence is destroyed at "large" N ($N \approx 11$ is critical in the case of Jacobi polynomials) if, as usual, the QCD moments were calculated with the help of the next-to-leading order anomalous dimensions of refs. [6,7]. The convergence is recovered provided that the values of the latter at odd or even N are corrected based on the requirement of an analytic behaviour (see below).

It is well known [11] (see also [5,12]) that the anomalous dimensions, obtained with the operator product expansion formalism, in the next-to-leading order predict only even/odd moments of crossing even/odd structure functions. To find out the evolution of the moments at any N, an analytic continuation of $\gamma^{(1)}(N)$ should be performed for even and odd N separately. Thus the quantity $\gamma^{(1)}(N)$ can be written as [11]

$$\gamma^{(1)}(N) = \gamma_\alpha^{(1)}(N) + (-1)^N \gamma_\beta^{(1)}(N), \tag{4}$$

where $\gamma_\alpha^{(1)}(N)$ and $\gamma_\beta^{(1)}(N)$ can be analytically continued, e.g., to the imaginary N values. Their Mellin transforms correspond, in the nonsinglet case, to the splitting functions [12,13] $P_{qq}(x)$ and $P_{q\bar{q}}(x)$. The Q^2 evolution of crossing even/odd structure functions is governed by the anomalous dimensions [11,12]

$$\gamma\pm(N) = \gamma_\alpha(N) \pm \gamma_\beta(N) \tag{5}$$

$(\gamma_\beta^{(0)}(N) = 0)$. In particular, the evolution of a nonsinglet or singlet structure function $F_2(x,Q^2)$ measured in deep inelastic charged lepton-nucleon scattering is described by the anomalous dimensions

$$\gamma_+(N) = \begin{cases} \gamma(N), & N \text{ even}, \\ \gamma(N) + 2\gamma_\beta(N), & N \text{ odd}. \end{cases} \tag{6}$$

Similarly, $\gamma_-^{NS}(N)$ (relevant, e.g., for $F_2^{\nu p} - F_2^{\bar{\nu} p}$) equals $\gamma^{NS}(N)$ and $\gamma^{NS}(N) - 2\gamma_\beta^{NS}(N)$ for odd and even N, respectively. Since $\gamma_\beta^{(1)}(N)$ is quite small and vanishes very fast with increasing N ($\gamma_\beta^{NS(1)} \sim 1/N^6$) [11], it is often neglected in the literature, including all the papers dealing with the structure function reconstruction from the QCD moments, except for ref. [9]. However, as stated above, such a neglect has a fatal consequence for the convergence of the reconstruction series. The reason is very simple – the contributions of the moments are weighted by factorially large coefficients. This, together with a lot of cancellations, make the final result sensitive to slight changes in the moments.

The simple analytical expression for $P_{q\bar{q}}^{(1)}(N) = -\frac{1}{8}\gamma_\beta^{(1)NS}(N)$ obtained in ref. [12], rather than the "numerical" result of ref. [11], can be used to solve the problem in the nonsinglet case. Similar expressions for $\gamma_\beta^{(1)}(N)$ can be found also in the singlet sector. Thus, using eqs. (5.32) - (5.39) of ref.

[12] and noting that the contributions to $\gamma_\beta^{(1)}(N)$ arise not only from the terms in $\gamma^{(1)}(N)$ containing the factor $(-1)^N$ explicitely, but also from the ones including alternating series $S_n'(^1/_2N)$ and $\tilde{S}(N)$, it may be seen that the combination

$$a_N\cdot(-1)^N + b_N\cdot S_2'(^1/_2N) + c_N\cdot[S_3'(^1/_2N) - 8\tilde{S}(N)]$$

in $\gamma^{(1)}(N)$ leads to

$$\gamma_\beta^{(1)}(N) = a_N + b_N\cdot(-1)^N\cdot B(N) + c_N\cdot(-1)^N\cdot C(N),$$
$$B(N) = S_2'(^1/_2N) - 2S_2(N) + \zeta(2),\ \ C(N) = S_3'(^1/_2N) - 8\tilde{S}(N) - 4S_3(N) - 2\zeta(3), \qquad (7)$$

where $\zeta(z)$ is the Riemann zeta function, $\zeta(2) = \pi^2/6$, $\zeta(3) = 1.20205\ 69031\ 59594$. The resulting expressions for $\gamma_\beta^{(1)}$ are given below:

$$\gamma_\beta^{FF(1)}(N) = \gamma_\beta^{NS(1)}(N) = -16(C_F^2 - {}^1/_2C_FC_A)\{(2N^2+2N+1)/N_1^3 + (-1)^N[(2S_1(N) - 1/N_1)B(N) + {}^1/_2C(N)]\},$$
$$\gamma_\beta^{FA(1)}(N) = -8C_AT_R[(3N^4+15N^3+29N^2+50N+44)N^2/N_2^3 - (-1)^N B(N)2(N^2+N+2)/N_2],$$
$$\gamma_\beta^{AF(1)}(N) = -8C_FC_A[(N^3+N^2+4N+2)/N_1^3 - (-1)^N B(N)(N^2+N+2)/N_3], \qquad (8)$$

where $N_1 = N\cdot(N+1)$, $N_2 = N_1\cdot(N+2)$, $N_3 = N_1\cdot(N-1)$. We do not present here an obvious formula for the "correction" to the gluon-gluon anomalous dimension of refs. [6,7] as the latter slightly differs from the generally accepted result of refs. [14,15] in the term proportional to C_A^2. This result has been deduced from a plausible supersymmetric relation between the four elements of the singlet anomalous dimension matrix [15] and agrees with the gluon-gluon splitting function calculated in ref. [13]. Since the explicit terms $\sim(-1)^N$ in the singlet anomalous dimensions $\tilde\gamma(N)$ of ref. [14] have been absorbed with $\eta = (-1)^N = 1$ (without loss of generality), eq. (6) should be modified by the replacements $\gamma \to \tilde\gamma$ and $\gamma_\beta \to \tilde\gamma_\beta$, where $\tilde\gamma_\beta$ includes only the terms arising from the alternating series:

$$\tilde\gamma_\beta^{AA(1)}(N) = C_A^2(-1)^N\{[32(N^2+N+1)/N_4 - 16S_1(N)]B(N) - 4C(N)\}, \qquad (9)$$

$N_4 = N_3\cdot(N+2)$.

ИНФОРМАЦИОННАЯ КАРТА

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РЕФЕРАТ

Обнаружено нарушение сходимости реконструкции структурной функции из КХД моментов в случае, когда последние вычисляются в следующем порядке теории с помощью обычных аномальных размерностей $\gamma(N)$. Сходимость восстанавливается, если $\gamma(N)$ при четных/нечетных N, ответственные за Q^2 эволюцию кроссинг четных/нечетных структурных функций, аналитически продолжить к нечетным/четным N. На основании известного несинглетного результата получены простые аналитические формулы для соответствующих поправок к $\gamma(N)$ в синглетном секторе.

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