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ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

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## ДЕПОНИРОВАННАЯ ПУБЛИКАЦИЯ

JOINT INSTITUTE FOR NUCIE.IR RESEARCH Laboratory of Nuclear P.ooblems

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V. I. Komarov, S.Tesch

CALCULATION OF ABSORPTION FACTORS IN NUCLEAR REACTIONS WITH MEDIUM-ENERGY PROTONS


En, mitctena
Dubna, July $19^{\prime \prime} 8$


#### Abstract

Nuclear transparencies for reactions at intermediate energies in straight-line approximation are eatimated. Some peculiarities as nuclear matter distributions and total cross sections for several nuclear processes are discussed.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.


1. Introduction

During passage of projectiles and secondary nuclear reaction products through nuclear matter absorption losses arise as the result of the interaction of these particles with the nucleons of the nucleus. If one intend to compare the experimental data with other processes or theoretical predictions, the absorption effects must be considered. These absorption losses in dependence on the nuclear processes under study and the chosen nuclear matter distributions can be of considerabl amount.

The calculation method described in this paper has been used to estimate absorption factors for nuclear reaction cross sections of medium-energy protons on light nuclei. In processes of quasielastic scattering on clusters ( $p, N X$ ) and fragmentation processes with pion production $(\mathrm{p}, \pi \mathrm{X})$, where $\mathrm{X}=2,3^{3},{ }^{3}, 4^{4} \mathrm{He}$, absorption factors were needed to compare the data with shell model predictions of effestive cluster numbers /1/. Studying the energy dependence of quasielastic deuteron knock-out we had to consider the absorption effects to get information on the reaction mechanism in comparison to elastic pd backward scattering /2/. Recently it was shown experimentally that a considerable part of the inclusive proton spectrum in backward direction is correlated with energetic protons in forward direction $/ 3 /$. To estimate the anount of this part we had also to calculate the corresponding absorption effect on the forwardemitted proton.

Before us Kalinkin and Shmonin /4/ estimated absorption losses to understand the experimental yield of deuterons in the
kinematical regions of quasielastic scattering and pion production. As the method described in ref. / 4/is of standart type, it can be applied also to other processes. For instance, the same method was used to extract information on the quark structure of hadrons by calculating the quark absorption probability in nuclear matter /5/. Also in electron scattering processes, e.g. of type (e, $e^{\prime} d$ ) it was shown that wave distortions of the outgoing particles must be included to extract information on effective cluster numbers and short-range correlations in nuclei (see, e.g., /6,7/). For the process 6 $\mathrm{Li}_{i}$ (e, $e^{\prime} p$ ) we compared our estimations of the nuclear transparency with optical model calculations (see, ref. /6/). With the corresponding proton-nucleon crcss sections for energies of 100 MeV and higher we found complete agreement with the mean value of $1 p$ and $1 s$ proton absorption.
2. Description of the calculation method

General remarks. The calculation method is based on the fact that at some hundreds of $M \in V$ for the primary protons as well as for the emitted particles (condition of large momentum transfer) the straight-line approximation can be used. The procedure desoribed in this paper :.s completely analogeous to the one applied in ref. /4/. Th:s method needs only simple conceptions of the target nucleus .. geometrical characteristics and density distributions of nuclea: matter. A problem of course is the density distribution suitable for clusters in nuclei, which are in general unknown. In the salculations it turned out the absorption values to depend ratier sensitive on these density
distributions. Beyond it into the absorption estimations enter the total cross sections for the interaction of primary and emitted particles with the rucleons.

Calculation method. For the nuclear process $p+A \rightarrow X+\ldots$ (Knockout of particle $X$ by protons $p$ from a nucleus with mass number A) the absorption factor is given by the following relation

$$
\begin{equation*}
\gamma=\frac{\int_{-\infty}^{+\infty} d^{2} b d z \rho_{x}(b, z) \Gamma_{p}^{7}(b, z) \Gamma_{x}(b, z)}{\int_{-\infty}^{+\infty} d^{2} b d z \rho_{x}(b, z)} \tag{1}
\end{equation*}
$$

if the absorption of the projectile and outgoing particle is taken into account. Using the multiple scattering theory one gets for the particles $p$ and $X$ the expressions

$$
\begin{align*}
& \Gamma_{p}(b, z)=\exp \left(-\sigma_{p N} T_{-}(b, z)\right)  \tag{2a}\\
& \Gamma_{x}(b, z)=\exp \left(-\sigma_{x N} T_{+}(b, z)\right) \tag{2b}
\end{align*}
$$

where the profile functions $T_{-}(b, z)$ ) and $T_{+}(b, z)$ ) are expressed in terms of the nuclear density $\rho(b, z)$

$$
\begin{align*}
& T_{-}(b, z)=A \int_{-\infty}^{z} d z \rho(b, z)  \tag{3a}\\
& T_{+}(b, z)=A \int_{z}^{+\infty} d z \rho(b, z) \tag{3b}
\end{align*}
$$

Relation (aa) gives the probability that the projectile p reaches the point $Z$ with impact parameter $b$ without any interaction, while expression (lb) is the probability for particle $X$ leaving the nucleus from point ( $b, z$ ) without interaction.

Density distributions. For: nuclear matter distributions according to relations (3a), (3b) one can use well known formula with parameters taken form eA scattering experiments (see, e.g., /8/). For light nuc.lei it is customary to use

$$
\begin{equation*}
\rho(b, z)=c\left(1+\eta \frac{b^{2}+z^{i-}}{a_{0}^{2}}\right) \exp \left(-\frac{b^{2}+z^{2}}{a_{0}^{2}}\right) \tag{4}
\end{equation*}
$$

with $y=\frac{z-2}{3}$, $z$ being the charge number of the target nucleus. To get the correct normalization

$$
\begin{align*}
& 4 \pi \int_{0}^{\infty} \rho(r) r^{2} d r=1,  \tag{5}\\
& \text { Lion constant } c \text { is } \quad C=\frac{2}{Z a_{0}^{3} \pi^{3 / 2}}
\end{align*}
$$

the normalization constant $c$ is $C=\frac{2}{7 a_{0}^{3} \pi^{3 / 2}}$
For very light nuclei we used also formula (4) or a pure Gaussian distribution with the parameter $\boldsymbol{a}_{0}$ calculated under the condition the mean square radii of these distribulions to be the same as for the exact distribution. The astinations due to expression (1) with these approximated distributions agreed within an accurasy of about 5\%.

On the other hand, in dependence on the reaction process under study the corresponding density distributions $\rho_{x}(b, z)$ in relation (1) must be chosen suitably. For instance, in the independent - particle model, the probability of finding $n$ nucleons inside the volume element near point $r$ is propertional to $\rho^{n}(\sim)$, that is decreasesidlyar to the nuclear surface. Therefore, effective cluster numbers extracted from experiment can be essentially overestimated, if one uses values from calculations with $\rho_{x}=\rho^{n}(r)$ (see, eeg. /4/). To choose the nearly correct density distributions one can utilize the commonly accepted assumption that they are of more or less peripheral character. An idea of the dependence
of $\gamma$ estimations on different nuclear matter distributions $\rho_{X}(r)$ may give ref. $/ 1 /$ and the tables in the appendix, where we used $\rho_{x}=\rho(b, z)$ of type (4) and $\rho_{x}=\rho^{4}(b, z)$ with $n=2$ and $1 / 2$. To get the correct normalization (5),

$$
C=\frac{4 \sqrt{2}}{\left.a_{0}^{3} \pi^{3 / 2}\left(z+\frac{15}{72}, z^{2}-4 z+4\right)\right)}
$$

for the $n=2$ case is found. jor $n=1 / 2$ integral (5) must be solved numerically. As an example, one gets $C=$ $0.006755 \mathrm{fm}^{-3}$ for ${ }^{12} \mathrm{C}$ with $a_{0}=1.64 \mathrm{fm}$.

Cross sections. To calculate the absorption factor $\gamma$ the corresponding cross sections for the projectile and final particles are needed (see expression (Da), (2b)). For the total cross section of the primary proton one uses $/ 9 /$

$$
\sigma_{P N}=\frac{1}{2}\left(\sigma_{p p} \text { tot }+\sigma_{p n} \text { tot }\right)
$$

The cross section $\sigma_{X N}$ must be taken in accordance with the particle $X$ measured, the experimental setup, angular acceptance of detectors, etc. For the analysis of deuteron knock-out we used /10/.

$$
\sigma_{d N}=\sigma_{p p}+\sigma_{p h}-\frac{\left(1-\rho_{p} \rho_{n}\right) \sigma_{p p} \sigma_{p n}}{4 \pi\left(D+A_{p}+A_{n}\right)}
$$

with the corresponding parameters $\rho, A, D$ for medium-energy nucleons. It should be noted that the screening term of the deuteron total cross section tares only a small effect on the absorption. We found an enlargement of $\gamma$ of about $5 \%$ without this correction term.

For heavier fragments $X$ in part one can find experimental: data /11/. Experimentally unknown cross sections must be calc-
ulated with the help of the nucleon-nucleon cross sections $\sigma_{N N}$ at the corresponding reduced kinetic energies. Comparisons with experimental data show the.t for fragments $3_{H, ~}{ }^{3} \mathrm{He}$ and ${ }^{4}$ He these cross section values nust be reduced by about $15 \%$ and $20 \%$, respectively.

For the knock-out of heavj particles from light nuclei it should be taken into account theat in this case the mass number of the final nucleus is considerably reduced leading to a corresponding reduction of the absorption effect. Therefore the value $A$ in relation (3b) should be replaced by $A-N_{X}$, $N_{X}$ being the mass number of the particle $X$. As an example we note this procedure leading tio a $\gamma$ reduction of about $10 \%$ for the ${ }^{4} \mathrm{He}$ Knock-out from ('arbon at 1 GeV .

In relation (1) instead o!. $\quad \Gamma_{x}(b, z)$ a product of several probabilities can appear, if one considers more than one particle in the final state, for instance, in processes with two or three outgoing protons as studied in $/ 3,12 /$.

Numerical method. If one uses in the case of light nuclei density distributions of type (4), than the integrals of relation (1) essentially can be solved with Hermite-Tshebyshey polynomials /13/. This metiod in the computer code presented in this work has been employed. For the denominator of expression (1) one gets in tiais manuer

$$
\int_{-\infty}^{+\infty} d^{2} b d z \rho(b, z)=\sum_{n=1}^{N} \sum_{k=1}^{K} f\left(b_{n}, z_{k}\right) b_{n} A_{n} A_{k}
$$

The values $A_{n}, A_{k}$ for the crossing points $b_{n}, Z_{k}$ are tabulated /13/. A remainder of the numerator is integrated by using the Simpson formula. It nas been proved for the $\gamma$ estimation within an accuracy of $3 \%$ to be sufficient to
use $N=5$ and $K=8$. For Fermi-type nuclear matter distributions of heavy nuclei $\gamma$ is calcu:ated by using standart methods of two-fold integration.

## Appendix

As an example the absorption factor $\gamma$ for the process $p+{ }^{12} \mathrm{C} \rightarrow 2 \mathrm{p}+\ldots$ at 640 MeV is estimated. In expression (1) the density distribution $\rho_{x}(b, z)$ of form (4) is used. The cross sections for the forward and backward emitted protons are chosen in accordance with the experimental conditions of ref. 13/. With all three cross sections $\sigma^{2}=0$ one gets of course $\gamma=1$. The common factor of the integrals due to relation (1) is omitted. This factor is $2 \pi a_{0}{ }^{3} C$, yielding the denominator value 1 within the accuracy of the numerical calculation of the integral.

In table 1 the absorption factors $\gamma\left(\rho^{n}\right)$ with $n=1 / 2,1,2$ for process $p+{ }^{12} C \rightarrow 2 p+\ldots$ are listed. Table 2 contains absorption coefficients $\gamma(\sqrt{\rho})$ and $\gamma(\rho)$ for several knock-out reactions, at which only the absorption effect due to the heavy fragment is taken into account.


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Absorption factors for process $p_{0}+{ }^{12} C \rightarrow p_{1}+p_{2}+\ldots$ with cross sections $\sigma_{0}, \sigma_{1}, \sigma_{2}$ for primary and outgoing protons eccording to the experimental conditions

| $\sigma_{0}$ | $\sigma_{1}$ | $\sigma_{2}$ | nbsorption factors |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $(\mathrm{mb})$ | $(\mathrm{mb})$ | $(\mathrm{mb})$ | $\gamma(\sqrt{\rho})$ | $\gamma(\rho)$ | $\gamma\left(\rho^{2}\right)$ |
| 38 | - | - | 0.151 | 0.51 | 0.44 |
| 38 | 20 | - | 0.47 | 0.33 | 0.23 |
| 38 | 20 | 15 | 0.42 | 0.27 | 0.18 |
| 38 | - | 15 | 0.34 | 0.42 | 0.35 |
| - | 20 | - | 0.74 | 0.66 | 0.61 |

Absorption factors for seversl knock-out processes with protons on ${ }^{12} C$

| Process | Momentum <br> ( $\mathrm{GeV} / \mathrm{c}$ ) | Absorption $\gamma(\sqrt{\rho})$ | $\gamma(\rho)$ |
| :---: | :---: | :---: | :---: |
| ( $\mathrm{p}, \mathrm{Nd}$ ) | 1.6 | 0.37 | 0.22 |
| ( $\mathrm{p}, \mathrm{N}^{3} \mathrm{He}$ ) | 1.8 | 0.34 | 0.20 |
| ( $\mathrm{p}, \pi^{3} \mathrm{He}$ ) | 1.5 | 0.31 | 0.17 |
| (p, $\mathrm{N}^{4} \mathrm{He}$ ) | 1.9 | 0.29 | 0.15 |
| ( $\mathrm{p}, \pi^{4} \mathrm{He}$ ) | 1.6 | 0.25 | 0.11 |

PROGRAM LIABP
C－－ー－ーIITERACTION OF PRUTONS WITH GARBON－I 2
C ———EXAMPLE：ICIDENCEENEKGY b4．MEV
C－－－－CROES SECTICNS TAKEN IN ACCORDANCE KITH TH E EXPEKIMENTAL CONDITION C－O－－USE PROTON UITH EWERGY 28 ，MEV IN FORWARD DLRECTION
C－－－－－UNE PRDTON UITH FEERGY＂TOUMEV IN BACKWARD GIRECIION
C－m－meninary ant finial protúns IN ThE ABSORPTIO N CALCULATION INCLUDEU
UJMENSION YU，（5，8）．FUZ（5），FU3（5i8）
COMPOM／NULL／E（5），（6（8）
COMMOM／POL）／AB（S），AZ（8）．
COMMgN／VAR CONST，A，C，AG，SP1，SP2，SHB
CCNSQ＝．02672088
GONSW＝0．754851
REAL 8 ，（B（I）$L=1,5$ ）
$\triangle$ PCRIAT（5F：8）
REAE 5 ，（i（I）$I=1,8$ ）
5 FCRIAT（8F，7）
REAL 5：1（AB（I），I＝1，5）
S1 FORTAT（5F ．9）
RLAD 5 ，（42（I），$=1,0$ ）
5 FCRIGT（RF 9）
PRITT2
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FORMAT（／／3 X，IZHCGEFFIGIENTS／／）

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（5 PRIITO，I，B：I），AE（i）
मRIIT2 2
 1
U0 $7 \quad I=18$

6 PCRNAT（H 5XIASX2！日5．9）
KEAR $1, A$ C $A$
FORIAT（3F，5）
GUNSTE？（（：＊A＊＊3＊SQRT（3．14～59？＊＊3））
$A D=F S C:$, ，
HKIUT25
25 FCRIATC／／，X，＇NASS NUMBER＇， $1, X$, CHARGEI，

＊＇IN＊EGRAL IITHOUT ABSOKPTION＇）
PRIAT 3 ，A C，A AD
PRIMT 99
りO $1 . \quad I=15$
KHO＝CONST＊（．＊（C－6．）＊ $\mathrm{C}(\mathrm{I}) * * 2 / 3)$
＊＊EXP（－B（I）＊＊$)$
KAD＝B（I）＊
 $X^{*}(-B(1) * * 2)$
RHOOERAD＊＊＊$C O N S Q$（ $\left(1, \mp(C-2) * B,(I) * * 2^{\prime} 3.\right) *$ ＊ 6 ）＊EXH（－2＊B（I）＊＊ 2 ）
RHOU＝RAP＊＊ $2 *$ CONSW＊SQRTS：．（C－2）＊B（I）＊＊2／S
．d＊EXP（－．5＊B（I）＊＊2）
PKINT 1 ，B（I），RAD，RHO，RHOL，RHOQ，KHOW
GON：Y HUE
99 FORMAT（／94X，4HBCI）， $\mathbf{T X}^{\mathrm{X}, \text { OHRADIUS，} 3 X, 3 H R H O, 3 X}$

```
    SS WITH KHOmLINEAR/SQUARE/SQUA\ERUOT/\
101 FORMAT(10X,2F10.3,4E10,3/1)
    3 FORMAT (/1;X,3FEO.2,F2%.0)
2 & FORMAT(//2OX,'INCIDENCE ENERG/=1,FG,2,SHME
                    V,9OX, TOTAL JROSS SECII
        *UN=',F6,2,2HMB)
    2 FORMAT(2F10.5)
26 FORMAT(/IIGX, IENERGIES ANO CRISS SEGTIUNS
                                    FOR FORWARD AVD BACKWARU
        * PrOTONS'/)
3.{ FORMAT(1GX,19HFORWARD PROTON &ITH,5X,FS,GI
                                    3HMEV,5X,F8,3,2HMB,2OX,
        * L%HACKWARD PROTON WITH,5X,F6.2,3HMEV,3X,F
                        8,3,2HMG)
    READ 2, ENUL,SPY
    READ 2 ,ENB,SPG
    PRINT 208 ,ENUL,SPY
    REAO 2, ENZ,SP2
    PRINT2.
    PRINT SCQ ,ENZ,SPZ,ENB,SPB
    SP1=SPq/10.
    SP2=SP2/10.
    SPB=SPE/10.
    CALL SIMP(FU3)
    CALL FU12(FUQ,FU2)
    GAM=
    vo 11 L= 1,5
    vo l: J=1,8
    1.GAM=GAM*FU1(I;J)*FU2(I)*FU3($,J)*AB(I)/FS&
                                    (0,)
    PRINT 15 GAM
    95 PORMAT(//ZX,'ABSORPTION GAMMM=',F9O,0)
    PRINT3OO
3: FORMAT(//3OX,'ABSORPTION IN DEPENDENCE UN
                                    IMPACT PARAMETER'/?
    PRINT 3O2
3.2 FORMAT(/32X,4HB(1),15X,6HRADIUS,1SX,9HUGAM
                                    MA/DB/)
    00 61 I=1,5
    RAD=B(I)*AS
    BEAB=0.
    v0 6 J=1,8
    6. BEAB=BEAB+FU1(I,J)/B(I)*FUZ(I)*FU3(I,Nな*Ex
                        P(-B(I)**2 )/AD
    61 PRINT O2 ,B(I),RAD,BEAB
    62 FORMAT(1HO,2OX,3F2,1%)
    END
    SUBRCUTINE SINP(FU3)
    DIMENSION S(5,8), FUS(5,8)
    COMMON/NULL/B(5),Z(8)
    COMMON/VAR/CONST,A,C:AG,SPQ,SPZ,SPB
    G=2.
    H=0.2
    \MAX=3
    vo 106 l=1,5
    vo 1, J=1.8
    S(I,J)=(1.*(C -2,)*(B(I)**2+Z(\)**2)/s,
        )*EXP(-< (N)**2)
    DO 11 JJ=1,JMAX
    LZ#Z(J)*H*JJ
    G=6,-G
```



```
                    2)/3,)*ExP(-2Z**2 )*(G
    S(I,N)=S(I,N)*H/3.
1. FU3(I,J)=EXP(- (SP2=SPB-SPq)*CONSI*AU*A*
```

    **exf(-R(I)** \(\because)\) )
    CONTIWUE
    RETURN
    ERD
    SHBROUTINE :U?Z(rU4.fUZ)
    UIMENSION Fノ, 5,0\() \quad F U_{2}(5)\)
    COMION/MULL/3(5), (4) (8)
    COMION/POLY/AB(5),AZ(8)
    COMTOH/VAR'CJNST,M, C:A*,SP1,SP?, SH甘
    DO \(1 \quad \mathrm{I} \equiv, \mathrm{S}\)
    F2 = .
    \(\Delta 0-J=1.8\)
    \(H=+(C \quad-2) *.(B(I) * * 2 * Z(J) * * 2) / s\)
    Fい: (I, J) =F *\&Z(J) * b (I)
    $2 F 2=F 2+(\ldots+(C \quad-6) *.(B(I) * * 2 / 3 . *(J) * * 2 / 3$
. ) $) * A(J) * E X P(-B(\downarrow) * * 2)$
TFLZ(I) $=E X P\left(-C O N S T * A_{V} * A * F 2 *(S P 1+S P B)\right)$
RETURN
Eln D
HNCTIOH FSQ(ALFHA)
C(ANDN/NULL/B(5), L(8)
CGMTOIN/POLY/AB(5),AZ(8)
CCANON: YAR:CONST, M, C, A , SP1,SP2, SPB
$F S Q=A L D F: A$
山C Im.,
VC $1 \quad J=1,8$
$F S Q=F S Q+B(I) *(9 .+(C \quad-2) *.(B(I) * * 2 *(J) *$
$\star() / 3) \quad * A B.(I) * A Z(J)$
EHO
ФAAA: : BAMACHHO :33 KAPT (VOTP.)
* $\ddot{E} E$
*END FILE

