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A B S T R A C T

Various effects connected with the capture of μ- meson by light nuclei with spin ½ not accompanied by proton or neutron emission are investigated. Calculations show that the form factor gives a significant correction to these effects.

I. INTRODUCTION

Recently there has appeared a general interest in the influence of the form factor on processes involving the weak interactions/1/. The influence of the form factor on the process of μ - capture by light nuclei without emitting neutrons or protons has been examined by several authors/2/. Strictly speaking, these formulas are suitable for initial state nucleus with spin zero only, since the rate of capture and the state of polarization of the final state nucleus sensitively depends upon the hyperfine structure of μ -mesic atoms. This has been pointed out by Bernstein et al., and Shmushkevich/3/. Thus, it will be interesting to study the influence of the form factor on the μ -capture without emitting a neutron, with the effect of the hyperfine structure taken into account.

II. THE ASSUMPTIONS

The Hamiltonian of weak interaction with form factor can be written as follows:

$$H_W = \frac{G}{\sqrt{2}} \bar{\psi}_p (V_\alpha + A_\alpha) \psi_n \; \bar{\psi}_\nu \gamma_\alpha (1+\gamma_5) \psi_\mu$$

$$V_\alpha = \gamma_\alpha + \frac{\mu'}{2m_p} \sigma_{\alpha\beta} K_\beta \qquad (1)$$

$$A_\alpha = \lambda \gamma_\alpha \gamma_5 + \frac{i f'}{m_\mu} \gamma_5 K_\alpha$$

where $K = p_p - p_n$., $\sigma_{\alpha\beta} = \frac{1}{2i}(\gamma_\alpha \gamma_\beta - \gamma_\beta \gamma_\alpha)$, λ represents the ratio of the coupling constant of Gamow-Teller and the coupling constant of Fermi and is equal to 1.2 in the case of β -decay, term of μ' is the 'weak magnetism', μ' is the sum of anomalous gypomagnetic ratio, and $\mu' = \mu_p + \mu_n - 1$ 3.7 /4/, terms of f' is the induced pseudoscalar, the order of magnitude of f' is estimated as 8 λ in the case of the μ -meson captured by the proton/5/.

When the μ -meson is captured in nucleus, the matrix element can be expressed as

$$M.E = \frac{G}{\sqrt{2}} \int d\tau \langle \nu | \gamma_\alpha (1+\gamma_5) | \mu \rangle \langle f | \sum_{i=1}^{A} \tau_i^{(-)} (V_\alpha + A_\alpha) | i \rangle \, \delta(\vec{r} - \vec{r}_i) \qquad (2)$$

in which ν, μ, f, i specify the wave functions of the neutrino, the μ -meson, the final and the initial states of the nucleus respectively. $\tau_i^{(-)}$ is the operator changing a proton into neutron and $d\tau$ is the nuclear volume element. Before the capture, the μ -meson is nearly at rest, thus the non-relativistic wave function of μ -meson can be used. The momentum of the protons in the initial state is also neglected. This will give to an error no more than a few percent as pointed by Primakoff and Joffe[2]. After such non-relativistic approximation having been made the matrix element can be expressed as

$$M.E = \frac{G}{2}\Big[\langle v_\nu | 1-\vec{\sigma}\cdot\vec{n} | v_\mu\rangle \langle f | \sum_{i=1}^{A} \tau_i^{(-)} (V - P\vec{\sigma}\cdot\vec{n}) e^{-i\vec{q}\cdot\vec{r}_i} \varphi_\mu | i\rangle$$

$$- \langle v_\nu | (1-\vec{\sigma}\cdot\vec{n})\vec{\sigma} | v_\mu\rangle \langle f | \sum_{i=1}^{A} \tau_i^{(-)} A\,\vec{\sigma} e^{-i\vec{q}\cdot\vec{r}_i} \varphi_\mu | i\rangle \qquad (3)$$

where $\vec{q}$ represents the momentum of the neutrino, $\vec{n}$ is the unit vector in a direction of momentum of the neutrino, $\varphi_\mu = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_\mu}\right)^{3/2} e^{-\frac{Zr}{a_\mu}}$ is the Coulomb wave function of the μ -meson, $a_\mu = \frac{\hbar^2}{m_\mu e^2}$ is the Bohr radius of the μ -mesic hydrogen atom, v_ν and v_μ are the spin wave functions of the neutrino and the μ -meson.

$$V = 1 + \frac{\beta}{2}, \qquad A = \lambda + \frac{\mu\beta}{2}, \qquad P = \frac{\beta}{2}(f+\mu) \qquad (4)$$

in which $\beta = \frac{q}{m_p}$, $\mu = \mu' + 1$, $f = f' - \lambda$

Here we only calculate the process of μ -capture by a nucleus with spin ½ and after capture it transits to a state of spin ½ or 3/2, without emitting neutron. The μ - meson is depolarized during the slowing down and during the jumping down from the outer orbits into the inner orbits. It loses a further part of its polarization and transmits it to the nucleus while a definite hyperfine structure state is formed. Since it is well known that the hyperfine splitting of the ground state level in μ -mesic hydrogen is much larger than the $\hbar/\tau$, where τ is the μ -meson half-life, the hyperfine structure states are incoherent. The corresponding density matrices of the initial states are respectively:

$$\rho_+ = \frac{1}{4}\left(1 + K\vec{j}(\vec{\sigma}_p + \vec{\sigma}_\mu) + \frac{1}{3}\vec{\sigma}_p\cdot\vec{\sigma}_\mu\right) \qquad (5)$$

$$\rho_- = \frac{1}{4}\left(1 - \vec{\sigma}_p\cdot\vec{\sigma}_\mu\right) \qquad \text{(singlet state)}$$

as shown by Shmushkevich[6]. It can be seen from formula (5) that in the triplet state the proton and the μ -meson polarization vectors are identical and equal to $K\vec{j}$, which means that ½ of the polarization of the μ -meson is transmitted to the nucleus.

Shmushkevitch's theory seems to be supported by the recent experiment carried out by Ignatenko et al. They measured the polarization of the μ -meson captured in the K - orbit of phosphorous nucleus with spin ½, which turns out to be 8.5%[7]. This value is just one half of the polarization of μ -meson captured by a nucleus with spin zero which is about 17%. It seems in this case, one half of the polarization transfers ot the nucleus.

In order to carry out the calculation, it is convenient to assume there is a population of the two hyperfine structure states. Let b be the percentage of the singlet state, then the density matrix before the capture is

$$\begin{aligned} \rho &= (1-b)\rho_+ + b\rho_- \\ &= 1 + \vec{\xi}_p \cdot \vec{\sigma}_p + \vec{\xi}_\mu \cdot \vec{\sigma}_\mu + \epsilon\, \vec{\sigma}_p \cdot \vec{\sigma}_\mu \end{aligned} \tag{6}$$

where $\epsilon = \frac{1}{3}(1-4b)$, $\vec{\xi}_p = \vec{\xi}_\mu = (1-b)K\vec{j}$

The value of $\vec{\xi}_\mu$ is easily obtained by measuring assymetry coefficient of μ -meson decay before the capture. The value of ϵ is easily proved to be 1/4, with the assumption that the μ -meson jumps to the ground state and emits γ rays. But if there is a certain mechanism which further produces the transition between the two hyperfine structure states, then the value of b is no more than 1/4. However, the value of b can still be estimated from the measurement of the assymetry parameter of μ -meson decay, if we assume the μ -meson has the same polarization as predicated by the theory before these mechanisms operate, i.e. 8.5%, and then compare the theoretical value with the experimental value, since the transition from the triplet to the singlet state causes an additional depolarization.

III. THE RESULTS OF CALCULATIONS

After the capture, the nucleus transits to a state of spin ½ and 3/2, thus we consider these two processes separately. Following the standard procedures of calculation, we obtain the formula of the transition probability, the angular distribution, the polarization of the nucleus in the direction of μ -meson and in the direction of recoil nucleus. The angular correlation between the $\nu-\gamma$ and $\mu-\beta$ is also calculated. We give here the shorter formulae for the special case: $\vec{\xi}_p = \vec{\xi}_\mu = \vec{\xi}$, a = 1/4, ϵ = 0, while the general formulae are given in the appendix. Thus, the total transition probability is

$$\begin{aligned} W &= \frac{G^2 Z^3}{2\pi^2 a_\mu^3} \cdot N_0\, q^2 \left(1 - \frac{q}{A m_p}\right) \\ N_0 &= (1+\beta)\,|M_F|^2 + \left(\lambda^2 + \frac{\lambda\beta}{2}(2\mu - f)\right)|M_{G.T.}|^2 \\ |M_F|^2 &= \frac{1}{2J_i+1}\left|\langle f|\sum_{i=1}^{A} \tau_i^{(-)} e^{i\vec{q}\cdot\vec{r}_i}\varphi_\mu|i\rangle\right|^2 \\ |M_{G.T.}|^2 &= \frac{1}{2J_i+1}\left|\langle f|\sum_{i=1}^{A} \tau_i^{(-)}\vec{\sigma}\, e^{i\vec{q}\cdot\vec{r}_i}\varphi_\mu|i\rangle\right|^2 \end{aligned} \tag{7}$$

It is shown by Joffe that the matrix element of M_F and $M_{G.T.}$ can be expressed in terms of the matrix element in β - decay as follows:

$$M_F = M_F^{\beta}\left(1 - \frac{1}{6} q^2_{\mu}\langle r^2\rangle_e\right)$$
$$M_{G.T.} = M_{G.T.}^{\beta}\left(1 - \frac{1}{6} q^2\langle r^2\rangle_A\right) \qquad (8)$$

$\langle r^2\rangle_e$ and $\langle r^2\rangle_A$ are the square of the radius of charge corresponding to the vector transition and axial vector transition respectively. The numerical values of M_F^{β} and $M_{G.T.}^{\beta}$ can be obtained from the experimental values in β -decay, $\langle r^2\rangle_e$ can be calculated with the help of experimental data of inelastic scattering of electrons by nuclei. Since the matrix element $\psi_f^* \vec{\sigma} \psi_i$ has the same form as the magnetic dipole transition, then $\langle r^2\rangle_A$ can be put equal to the square of the radius in the magnetic radiative transition of a nucleus belonging to the same isotopic multiplet. However, in these formulae the contribution of the quadrupole moment is neglected, which has been estimated by Joffe to be no more than a few percent/2/.

The angular distribution of the neutrino with respect to the direction of μ -meson is

$$W = 1 - a\,\vec{\xi}\cdot\vec{n}$$

$$a = \frac{1 + \beta - \frac{2}{\sqrt{3}}\left(\lambda - \frac{\beta}{2}(f - \lambda)\right) Re\,\rho + \frac{1}{3}(\lambda^2 - \lambda\beta f)\rho^2}{1 + \beta + \left(\lambda^2 + \frac{\lambda\beta}{3}(2\mu - f)\right)\rho^2}, \quad J' = \frac{1}{2}$$
$$\qquad (9)$$
$$a = -\frac{2\lambda^2 + \lambda\beta(3\mu + f)}{3\lambda^2 + \lambda\beta(2\mu - f)}, \quad J' = \frac{3}{2}, \quad \rho = \frac{M_{G.T.}}{M_F}$$

J' means the total angular momentum of the final state of the nucleus. It can be seem from formula (9), that the coefficient of angular distribution of the pure Gamow-Tellor transition is independent of the matrix element, and the form factor gives great correction to the assymetry coefficient. This means that we can detect the form factor by this experiment. For the transition of nucleus from spin ½ to ½, the coefficient of angular distribution depends onthe ratio of the matrix element, which can be taken from the experimental values of the β -decay.

The nucleus polarization in the direction of μ -meson is

$$W(m') = 1 + B\,\frac{m'}{j'}\,\vec{\xi}\cdot\vec{n}_j$$

$$B = \frac{1 + \beta - \frac{2}{\sqrt{3}}\left(\lambda + \frac{\beta}{6}(3\lambda + 2\mu - f)\right) Re\,\rho + \frac{1}{3}\left(\lambda^2 + \frac{\lambda\beta}{3}(2\mu - f)\right)\rho^2}{1 + \beta + \left(\lambda^2 + \frac{\lambda\beta}{3}(2\mu - f)\right)\rho^2}, \quad J' = \frac{1}{2} \qquad (10)$$

$$\beta = 2 , \quad J' = \frac{3}{2}$$

$\vec{n}_j$ is the direction of polarization of nucleus for pure Gamow-Tellow transition. Although the coefficient B is independent of the matrix element, but it does not depend upon any coupling constants also. This leads to the conclusion that we can not obtain information about the form factor by measuring the polarization in the direction of μ -meson. But, strictly speaking, this conclusion is only correct for terms of lower order of β . If terms of β^2 are taken into account, they will give a correction up to 5-10%. The correction is given in the Appendix (formula /21/).

The formula for the polarization of the nucleus along the direction of the neutrino is

$$W(m') = 1 + C\frac{m'}{J'}\vec{n}\cdot\vec{n}_j + D\left(m'^2 - \frac{5}{4}\right)\left(\frac{1}{3} - (\vec{n}\cdot\vec{n}_j)^2\right)\delta_{J',\frac{3}{2}}$$

$$C = \begin{cases} \dfrac{2}{3}\,\dfrac{(\lambda^2 + \lambda\mu\beta)\rho^2 + \sqrt{3}\left(\lambda + \frac{\beta}{2}(\lambda - f)\right)R_e\rho}{1 + \beta + \left(\lambda^2 + \frac{\lambda\beta}{3}(2\mu - f)\right)\rho^2} , & J' = \frac{1}{2} \\[2ex] \dfrac{\lambda^2 + \lambda\mu\beta}{\lambda^2 + \frac{\lambda\beta}{3}(2\mu - f)} , & J' = \frac{3}{2} \end{cases}$$

$$D = -\frac{\frac{1}{2}\beta\lambda(\mu + f)}{\lambda^2 + \frac{\lambda\beta}{3}(2\mu - f)} , \quad J' = \frac{3}{2} \tag{11}$$

The angular correlation between the $\mu - \gamma$ is not of great interest here, since it is always isotropic and independent of the form factor being present or not. But the angular correlation between the $\nu - \gamma$ is very sensitive to the form factor. It has also the advantage of being independent of the matrix element. Here we list a number of special cases while the general formulae are put in the Appendix. Let J_i , J', J_f represent the initial state, the intermediate state, and the final state, respectively, then the formulae for transition $J_i \rightarrow J' \rightarrow J_f$ are

$$\begin{aligned} & 1 - \frac{3D}{3+D}\cos^2\theta && (0 \rightarrow 1 \rightarrow 0) \\ & 1 + \frac{3D}{6-D}\cos^2\theta && (0 \rightarrow 1 \rightarrow 1) \\ & 1 - \frac{3D}{30+D}\cos^2\theta && (0 \rightarrow 1 \rightarrow 1) \\ & 1 - \frac{3D}{6+2D}\cos^2\theta && \left(\tfrac{1}{2} \rightarrow \tfrac{3}{2} \rightarrow \tfrac{1}{2}\right) \end{aligned} \tag{12}$$

$$1 + \frac{6D}{15-D}\cos^2\Theta \qquad \left(\tfrac{1}{2} \to \tfrac{3}{2} \to \tfrac{3}{2}\right)$$

$$1 + \frac{3D}{30-5D}\cos^2\Theta \qquad \left(\tfrac{1}{2} \to \tfrac{3}{2} \to \tfrac{5}{2}\right)$$

Θ is the angle between the neutrino and the γ - ray emited by the excited nucleus. It can be seen, that the form factor gives great contribution to the angular correlation. For example, in the transition $0 \to 1 \to 0$, the correlation is isotopic if the form factor is absent, but if form factor is present, it will give an anisotropy up to 50%.

The angular correlation between the μ -meson and the β -ray is

$$\begin{aligned} & 1 + E\,\vec{\xi}\cdot\vec{v}_e \\ & E = 1 - \frac{\beta^2(f+\mu)^2}{12\lambda^2} \qquad (0 \to 1 \to 0) \\ & E = 2 - \frac{\beta^2(f+\mu)}{12\lambda^2} \qquad \left(\tfrac{1}{2} \to \tfrac{3}{2} \to \tfrac{1}{2}\right) \end{aligned} \tag{13}$$

$\vec{v}_e$ is the velocity of the electron. In general, this correlation is difficult to detect, except in certain case when the lifetime of the nucleus after capture is so short, for example, the transition ${}_6C^{12} \rightleftarrows {}_5B^{12}$, ${}_6C^{13} \rightleftarrows {}_5B^{13}$ that may be possible to maintain the polarization of nucleus after capture in a proper magnetic field. It is difficult to detect the form factor in this way, but the transition $\frac{1}{2} \to \frac{3}{2} \to \frac{1}{2}$ can be used to detect whether the nucleus is polarized by μ -meson, and also to detect the spirality of μ -meson as good as the transition from $0 \to 1 \to 0$, since different spirality of μ -meson leads to different angular correlation in opposite sign.

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APPENDIX

Here we put down the general formulae which we have obtained up to terms of order of β^2. Of course, the terms given here are not complete enough because we have neglected the momentum of the proton in the initial state. However, in certain circumstances, owing to the great coefficient of anomalous magnetic moment and pseudoscalar, it becomes as great as terms of order of β. The formulae about the angular distribution of electron and γ -ray for oriented nuclei, are taken from V.B. Berestetsky et al /8/, and Falkoff and Ling /9/.

A. The transition from ½ ⟶ ½.

1/ The total transition probability

$$W = \frac{G^2 Z^3}{2\pi^2 a_\mu^3} \gamma^2 \left(1 - \frac{\gamma}{A m_p}\right) N_0$$

$$N_0 = \left(1+\frac{\beta}{2}\right)^2 |M_F|^2 + \left(\lambda^2 + \frac{\lambda\beta}{3}(2\mu - f) + \frac{\beta^2}{12}(2\mu^2+f^2)\right) |M_{G.T.}|^2$$
$$- 2\epsilon \left[\lambda^2 + \frac{\lambda\beta}{3}(2\mu-f) - \frac{\beta^2}{12}\mu(2f-\mu)\right] |M_{G.T.}|^2$$
$$- 2\sqrt{3}\,\epsilon \left[\lambda + \frac{\beta}{6}(3\lambda+2\mu-f) + \frac{\beta^2}{12}(2\mu-f)\right] Re\, M_F M_{G.T.} \qquad (14)$$

2/ The angular distribution of recoil nucleus

$$W(\theta) = 1 - \vec{\xi}_p \cdot \vec{n} \frac{N_1}{N_0} - \vec{\xi}_\mu \cdot \vec{n} \frac{N_2}{N_0}$$

$$N_1 = \frac{2}{3}\Big[\left(\lambda^2 + \lambda\mu\beta + \frac{\mu^2\beta^2}{4}\right) |M_{G.T.}|^2$$
$$- \sqrt{3}\left(\lambda + \frac{\beta}{2}(f-\lambda) - \frac{\beta^2}{4} f^2\right) Re\, M_F M_{G.T.}\Big] \qquad (15)$$
$$N_2 = \left(1+\frac{\beta}{2}\right)^2 |M_F|^2 - \frac{1}{3}\left(\lambda^2 + \lambda\beta(2\mu+f) + \frac{\beta^2}{a}(2\mu^2 - f^2)\right) |M_{G.T.}|^2$$

3/ The polarization of neucleus in the direction of polarization of μ -meson

$$W(m') = 1 + \vec{\xi}_p \cdot \vec{n}_j \, 2m' \frac{N_5}{N_0} + \vec{\xi}_\mu \cdot \vec{n}_j \, 2m' \frac{N_6}{N_0}$$

$$N_5 = \left(1+\frac{\beta}{2}\right)^2 |M_F|^2 - \frac{1}{3}\left(\lambda^2 + \frac{\lambda\beta}{3}(2\mu-f) + \frac{\beta^2}{12}(2\mu^2+f^2)\right) |M_{G.T.}|^2$$

$$N_6 = -\frac{2}{\sqrt{3}}\left[\lambda + \frac{\beta}{6}(3\lambda+2\mu-f) + \frac{\beta^2}{12}(2\mu-f)\right] Re\, M_F M_{G.T}$$
$$+ \frac{2}{3}\left[\lambda^2 + \frac{\lambda\beta}{3}(2\mu-f) - \frac{\beta^2}{12}\mu(2f-\mu)\right] |M_{G.T.}|^2 \qquad (16)$$

m' is the magnetic quantum number of final state nucleus.

4/ The polarizalion of nucleus in the direction of neutrino

$$W(m') = 1 + \vec{n}\cdot\vec{n_j}\ 2m' \frac{N_4}{N_0}$$

$$N_4 = \frac{2}{3}\Big[\big(\lambda^2+\lambda\mu\beta+\frac{\mu^2\beta^2}{4}\big)|M_{G.T.}|^2+\sqrt{3}\big(\lambda+\frac{\beta}{2}(\lambda-f)-\frac{\beta^2}{4}f\big)\,Re\,M_F M_{G.T.}$$

$$-\epsilon\Big[(1+\frac{\beta}{2})^2|M_F|^2+\frac{1}{3}\big(7\lambda^2+\beta\lambda(4\mu-3f)+\frac{\beta^2}{3}(2\mu^2-4\mu f+f^2)\big)|M_{G.T.}|^2 \qquad (17)$$

$$+\frac{4}{\sqrt{3}}\big(\lambda+\frac{\beta}{2}(\lambda+\mu)+\frac{\beta^2}{4}\mu\big)\,Re\,M_F M_{G.T.}\Big]$$

B. The transition from ½ ⟶ 3/2.

1/ The total transition probability

$$W = \frac{G^2 Z^3}{2\pi^2 a_\mu^3} N_0'\, q^2\big(1-\frac{q}{Am_p}\big)$$

$$N_0' = \Big[(1+\epsilon)\big(\lambda^2+\frac{\lambda\beta}{3}(2\mu-f)-\frac{\beta^2}{12}\mu(2f-\mu)+\frac{\beta^2}{12}(\mu+f)^2\big]|M_{G.T.}|^2 \qquad (18)$$

2/ The angular distribution of recoil nucleus

$$W = 1+\frac{1}{3}\frac{N_1'}{N_0'}\vec{\xi_p}\cdot\vec{n}+\frac{1}{3}\frac{N_2'}{N_0'}\vec{\xi_\mu}\cdot\vec{n}$$

$$N_1' = \big(\lambda^2+\lambda\mu\beta+\frac{\mu^2\beta^2}{4}\big)|M_{G.T.}|^2$$

$$N_2' = \big(\lambda^2+\lambda\beta(2\mu+f)+\frac{\beta^2}{4}(2\mu^2-f^2)\big)|M_{G.T.}|^2 \qquad (19)$$

3/ The polarization of nucleus along the direction of polarization of μ -meson

$$W(m') = 1+\frac{2m'}{3}\frac{N_3'}{N_0'}\vec{\xi_p}\cdot\vec{n_j}+\frac{2m'}{3}\vec{\xi_\mu}\cdot\vec{n_j}\frac{N_4'}{N_0'}$$

$$N_3' = \Big[\lambda^2+\frac{\lambda\beta}{3}(2\mu-f)+\frac{\beta^2}{12}(2\mu^2+f^2)\Big]|M_{G.T.}|^2$$

$$N_4' = \Big[\lambda^2+\frac{\lambda\beta}{3}(2\mu-f)-\frac{\beta^2}{12}\mu(2f-\mu)\Big]|M_{G.T.}|^2 \qquad (20)$$

if $\vec{\xi_p} = \vec{\xi_\mu}$, then

$$W = 1 + \left(2 - \frac{\beta^2(f+\mu)^2}{12(1+\epsilon)\lambda^2}\right) \frac{2m'}{3}\, \vec{\xi}\cdot\vec{n}_j \tag{21}$$

4/ The polarization of nucleus along the direction of emitting neutrino

$$W(m') = 1 + \frac{2m'}{3}\frac{N_5'}{N_0'}\,\vec{n}\cdot\vec{n}_j + \frac{N_6'}{N_0'}\left(m'^2 - \frac{5}{4}\right)\left(\frac{1}{3} - (\vec{n}\cdot\vec{n}_j)^2\right)$$

$$N_5' = (1+\epsilon)\left(\lambda + \frac{\mu\beta}{2}\right)^2 |M_{G.T.}|^2 \tag{22}$$

$$N_6' = -\left[(1+\epsilon)\left(\frac{\beta\lambda}{2}(\mu+f) + \frac{\beta^2}{4}(\mu+f\mu)\right) - \frac{\beta^2}{8}(\mu+f)^2\right] |M_{G.T.}|^2$$

C. The angular correlation for neutrino and γ -rays.

$$W(\theta) = \sum_{M_f, m'} W(m')\, P_{M_f, m'}(\theta)$$

$$P_{M_f,m'}(\theta) = \left| C^{J_f, M_f; L, M}_{J', m'} \right|^2 |E_L|^2 F_L^M(\theta) + \left| C^{J_f, M_f; L, M}_{J', m'} \right|^2 |M_{L-1}|^2 F_{L-1}^M(\theta)$$

$$+\; C^{J_f, M_f; L, M}_{J', m'}\, C^{J_f, M_f; L-1, M}_{J', m'} \left(E_L M_{L-1}^* + c.c.\right) F^M_{L,L-1}(\theta)$$

$$F_L^M(\theta) = \frac{4\pi}{L(L+1)}\left[2M^2 |Y_L^M|^2 + (L+M)(L-M+1)|Y_L^{M-1}|^2\right.$$

$$\left. +\; (L+M+1)(L-M)|Y_L^{M+1}|^2\right]$$

$$F^M_{L,L-1}(\theta) = -4\pi\left(\frac{2L+1}{2L-1}\cdot\frac{L^2-M^2}{L^2(L^2-1)}\right)^{\frac{1}{2}} \times \tag{23}$$

$$\times \left[2M|Y^M_{L-1}|^2 + (L-M-1)|Y_L^{M+1}|^2 - (L+M-1)|Y^{M-1}_{L-1}|^2\right]$$

D. The angular distribution of β -ray along direction of polarization of μ -meson.

$$W(\theta) = \sum_{m'} W(m')\left(1 - \bar{\beta}_{m'}\, v\cos\theta\right)$$

$$\bar{\beta}_{m'} = \frac{\Delta_{j'm'} + \dfrac{2m'\,\delta_{j_f,j'}}{\sqrt{j'(j'+1)}}}{1 + |\rho'|^2\,\delta_{j_f,j'}}\,, \quad \Delta_{j'm'} \begin{cases} \dfrac{m'}{j'} & j_f = j'-1 \\ \dfrac{m'}{j'(j'+1)} & j_f = j' \\ -\dfrac{m'}{j'+1} & j_f = j'+1 \end{cases} \tag{24}$$

$$\rho' = \frac{M_F^{\beta}}{M_{G.T.}^{\beta}}$$

j_f is the spin of final state nucleus after β -decay.

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