Numerical solution of the isoscaling problem in nuclear collisions

T. I. Mikhailova

Laboratory of Information Technologies, Joint Institute for Nuclear Research, 6, Joliot-Curie str., Dubna, Moscow region, Russia, 141980

Abstract

Nuclear collisions at low and intermediate energies are of interest because they can give the knowledge of the equation of state of nuclear matter, they are also a tool to produce new isotopes. According to statistical hypothesis the curves representing the ratios of isotope yields for two reactions with the same target, but slightly different beam ions should have linear character and be parallel to each other (isoscaling effect). If this hypothesis is correct then the value of symmetry energy nuclear matter can be determined by the coefficient of the slope of these curves. In this report the Fortran program that allows to extract the value of the mutual slope from the set of approximately parallel lines taking into account the statistical errors of the data is presented. It also makes it possible to verify the reliability of the statistical hypothesis for the data of interest. The isoscaling effect in the reactions ^{48}Ca on ^{9}Be and ^{40}Ca on ^{9}Be at energy 140 MeV per nucleon is analyzed. It is shown that the isotopes around atomic mass number $A = 20$ are the good candidate for determination of the value of symmetry energy.

Keywords

heavy-ion collisions, grand-canonical approach, statistical decay, least-square method, Fortran code, conference proceedings

1. Introduction

heavy-ion reactions are interesting to study because they help to understand the global features of nuclear matter and to produce new isotopes which don't exist in nature. When to heavy ion collides they can form a compound nucleus or break into pieces (fragments) depending on the energy of the reactions and thr distance between the centres of colliding ions (impact parameter). Both processes involve many nucleons and thus can be described in terms of grand canonical approach. Usually the nucleus produced in the collision is excited (have considerable intrinsic energy)and evaporate lighter particles. If the excited system is equilibrated it will decay statistically, which means that yields of isotopes with Z neutrons and N neutrons will de defined as [1]

$$Y(N,Z) = S(N,Z)\,\exp\left(\frac{E_b(N,Z)}{T}\right)\exp\left(\frac{N\mu_n}{T} + \frac{Z\mu_p}{T}\right), \tag{1}$$

here $S(N,Z)$ is the contribution from the secondary decay, E_b – ground state binding energy of the fragment, T is a temperature of the excited nucleus and $\mu_n(p)$ – chemical potentials of the system. If we compare two systems with similar excitation energies and not very different masses, but different N/Z ratio we can obtain a simple formula for there ratio

$$R_{ab}(N,Z) = Y_a(N,Z)/Y_b(N,Z) = C\exp\left(\frac{N\Delta\mu_n}{T} + \frac{Z\Delta\mu_p}{T}\right), \tag{2}$$

40 here $\Delta\mu_n = (\mu_n)_a - (\mu_n)_b$. If we plot $R_{ab}(N,Z)$ dependences as a function of Z or N in semi-logarithmic scale we will obtain a set of parallel lines. This is called the isoscaling effect.

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tmikh@jinr.ru (T. I. Mikhailova)
0000-0001-7141-7610 (T. I. Mikhailova)

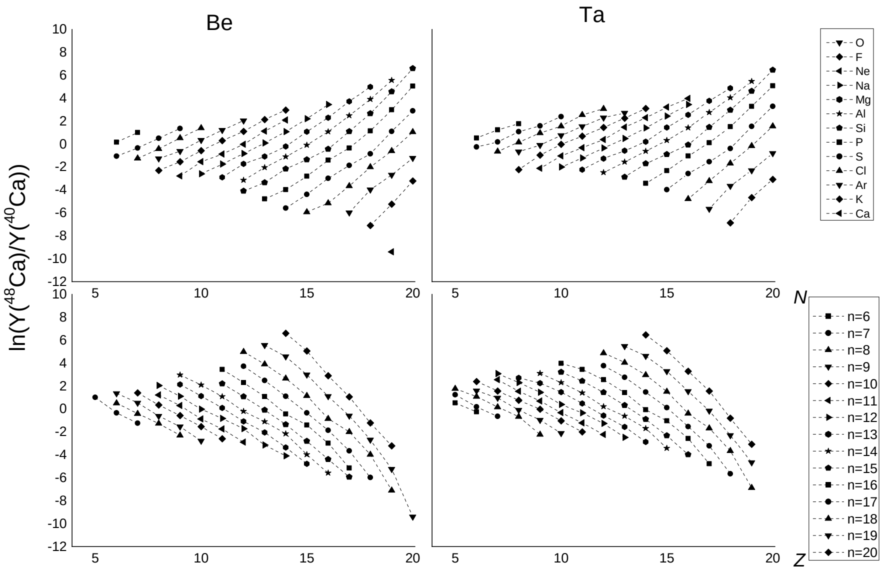

Figure 1: Isoscaling systematics for the reactions ^{48}Ca on ^{9}Be and ^{40}Ca on ^{9}Be at 140 MeV and ^{48}Ca on ^{181}Ta and ^{40}Ca on ^{181}Ta at 140 MeV as a function of Z and N, data from [2]

To check the validity of this assumption one needs a tool to find the coefficients of this parallel lines and mean square root errors. In this report the algorithm based on the least square method with weights which take into account the experimental errors of measurements is proposed.This method is applied to the ratio of yields of isotopes produced in two similar reactions: ^{48}Ca on ^{9}Be and ^{40}Ca on ^{9}Be at 140 MeV per nucleon [2]. The Fortran program implementing this method shows good results.

2. Studying the problem of parallel lines

In Fig. 1 the isoscaling ratio for the reactions ^{48}Ca on ^{9}Be and ^{40}Ca on ^{9}Be at 140 MeV per nucleon are shown. One can see that it is now clear if the hyposesis is valid or not. Or it is valid only for light isotopes and not very good for heavy ones. To answer this question one needs a tool which will help to find the parameters of these 'parallel' lines and to check the deviations. The problem is stated as following: there are $j = 1, \ldots, K$ set of data (K different chemical elements detected in the experiment). For each element the ratio is calculated for N_j different isotopes. The block of data includes: n_j – neutron number, R_{ab} – the value of ratio and δR_{ab} – experimental error of measurements. The task is to verify the hyposesis that the dependence of $(R_{ab})_j$(y– value) on n_j (x – value) is linear

$$y_{jn} = a\, x_{jn} + b_j,$$

here a – common coefficient determining the slope of the lines, b_j, $j = 1, \ldots, K$ – the coefficients that determine the position of the line. We assume that the weights determining each point has the value $p_j = \delta R_{ab}^{-1}$.

In order to find the coefficients a, b_j the least square method is applied. Let $\Phi(a, b_1, b_2, \ldots, b_k)$be the function determining the coefficients a, b_j. this function depends on $K + 1$ parameters:

$$\sum_{n=1}^{N_1} p_{1n}\,(a\,x_{1n} + b_1 - y_{1n})^2 + \sum_{n=1}^{N_2} p_{2n}\,(a\,x_{2n} + b_2 - y_{2n})^2 + \cdots + \sum_{n=1}^{N_K} p_{Kn}\,(a\,x_{Kn} + b_K - y_{Kn})^2 = \tag{3}$$

$$\sum_{j=1}^{K}\sum_{n=1}^{N_j} p_{j1n}\,(a\,x_{jn}+b_j-y_{jn})^2=\Phi(a,b_1,b_2,\ldots,b_k)$$

Differentiating the function $\Phi(a,b_1,b_2,\ldots,b_k)$ on the parameter a, we obtain the following equation:

$$\frac{\delta\Phi}{\delta a}=\sum_{j=1}^{K}\sum_{n=1}^{N_j} 2\,p_{jn}\,(a\,x_{jn}+b_j-y_{jn})x_{jn}=0$$

Or in more expanded view:

$$a\sum_{j=1}^{K}\sum_{n=1}^{N_j} p_{jn}\,x_{jn}^2+\sum_{j=1}^{K} b_j\sum_{n=1}^{N_j} p_{jn}\,x_{jn}-\sum_{j=1}^{K}\sum_{n=1}^{N_j} p_{jn}\,y_{jn}x_{jn}=0$$

Differentiating the function $\Phi(a,b_1,b_2,\ldots,b_k)$ on p[arameter b_j, with $j=1,\ldots,K$, give us the equation:

$$\frac{\delta\Phi}{\delta b_j}=2\sum_{n=1}^{N_j} p_{jn}\,(a\,x_{jn}+b_j-y_{jn})=0$$

or in expanded view:

$$a\sum_{n=1}^{N_j} p_{jn}\,x_{jn}+\sum_{n=1}^{N_j} p_{jn}\,b_j-\sum_{n=1}^{N_j} p_{jn}\,y_{jn}=0$$

In this way the problem of finding the parameters a and b_j lead to the task of solving the system of linear equations

$$\begin{bmatrix} \sum\limits_{j=1}^{K}\sum\limits_{n=1}^{N_j} p_{jn}\,x_{jn}^2 & \sum\limits_{n=1}^{N_1} p_{1n}\,x_{1n} & \sum\limits_{n=1}^{N_2} p_{2n}\,x_{2n} & \cdots & \sum\limits_{n=1}^{N_K} p_{Kn}\,x_{Kn} \\ \sum\limits_{n=1}^{N_1} p_{1n}\,x_{1n} & \sum\limits_{n=1}^{N_j} p_{1n} & 0 & \cdots & 0 \\ \sum\limits_{n=1}^{N_2} p_{2n}\,x_{2n} & 0 & \sum\limits_{n=1}^{N_j} p_{2n} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sum\limits_{n=1}^{N_K} p_{Kn}\,x_{Kn} & 0 & 0 & \cdots & \sum\limits_{n=1}^{N_j} p_{Kn} \end{bmatrix} \cdot \begin{bmatrix} a \\ b_1 \\ b_2 \\ \vdots \\ b_K \end{bmatrix} = \begin{bmatrix} \sum\limits_{j=1}^{K}\sum\limits_{n=1}^{N_j} p_{jn}x_{jn}y_{jn} \\ \sum\limits_{n=1}^{N_1} p_{1n}\,y_{1n} \\ \sum\limits_{n=1}^{N_2} p_{2n}\,y_{2n} \\ \vdots \\ \sum\limits_{n=1}^{N_K} p_{Kn}\,y_{Kn} \end{bmatrix}$$

In this work it is solved by Gauss method [3]. The program is written in Fortran language.In Fig. 2 the results of calculations are shown with red dashed lines. The square-roots deviations of the calculated parallel lines from ther experimental results are: for element $\sigma=8.E-3$ for boron (B), 0.16 for carbon (C), 7.E-2 for nitrogen (N), 0.2 for oxygen (O), 0.24 for fluorine (F), 9.E-2 for neon (Ne), 7.E-2 for sodium (Na), 5.3E-002 for magnesium (Mg), 6.E-2 for aluminum, 0.28 for silicon (Si), 0.12 for phosphorus (P), 8.E-2 for sulfur (S),0.19 for chlorine (Cl),

One can see that for light nuclei the agreement with the isoscaling hypothesis is better than for heavier ones.

3. Conclusions

In this report the Fortran code that make it possible to calculate isoscaling coefficients is presented. It shows that isoscaling model is valid for production light isotopes. This program can also be used to understand the another characteristics of nuclear reactions, for instance Q_{gg} problem (the dependence of yields of isotopes produce in deep-inelasctic collisions on thermal energy of the reaction).

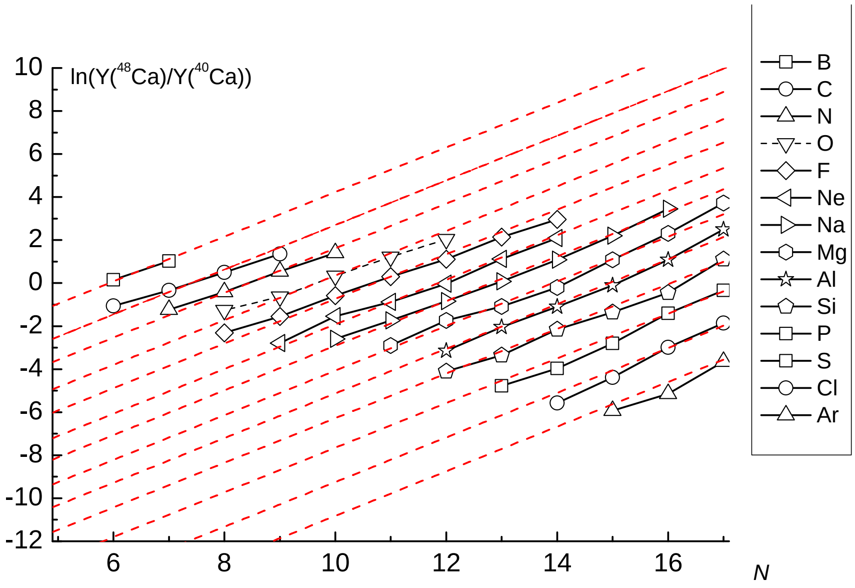

Figure 2: Isoscaling systematics for the reactions ^{48}Ca on ^{9}Be and ^{40}Ca on ^{9}Be at 140 MeV as a function of N, data from [2]

Conflicts of Interest: The authors declare no conflict of interest.

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