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3 Colored Quarks

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3.1 INTRODUCTION

A concept of colored quarks as fundamental fermions possessing a specific quantum number, color, and like leptons, being elementary constituents of matter, forms the foundation of current theoretical ideas about the world of elementary particles and atomic nuclei.

When in 1964 Gell-Mann [1] and Zweig [2] suggested the existence of quarks as hypothetical particles that make up mesons and baryons (i.e. every strongly interacting particles observed) quarks were imagined as purely mathematical objects. Using them, the then discovered unitary *SU* (3) symmetry of strong interactions [3] could be described in the most simple and elegant way. These particles, which possess fractional electric charges and are not observable in free state, eventually gained an appropriate physical interpretation.

First of all, constructing hadrons from quarks having spin 1/2 lead to a contradiction with the Pauli principle for systems of particles with half integer spin.

The problem of statistics was not, however, the only obstacle standing in front of the theory. The following questions remain unanswered: why in nature do only those systems which correspond to three quarks or quark-antiquark pairs exist? Why are there no indications of the existence of other multiquark systems?

Of special interest is the question as to whether quarks could exist in the free state (the problem of quark confinement or non-emission).

In 1965 an analysis of these issues led Bogolyubov, Struminsky, and Tavkhelidze [4], as well as Nambu and Han [5], Freund [6], and Miyamoto [7] to the fundamental idea that the quarks possess a new and hitherto unknown quantum number afterwards called *color* [8].

For more than 15 years this idea lies at the base of elementary particle physics. Once hadron spectroscopy problems could be solved, the colored quarks hypothesis led on to the development of the quantum chromodynamics (QCD) of strong interactions, and enlivened numerous versions of the "grand unification" theory (GUT).

The present paper considers the main topics for the development of the theory of colored quarks and exposes a number of the important achievements for the physics of elementary particles, atomic

nuclei, and high energies [9-11] which were obtained from this theory.

The paper begins with a discussion on the dynamic treatment of hadrons as composite quark systems and the construction of form factors and amplitudes of various processes, in which hadrons take part (Sec. 3.2).

A dynamic quark model [4, 12] proposed in 1964 at JINR (Dubna) offered a systematic description of both static and observed parameters of elementary particles (magnetic moments, axial-vector constants of weak transitions, etc.), and hadron form factors [13]. These studies gave an impetus to the development of the current quark models of elementary particles, among which the quark bag model [14, 15] and quark parton model are the most popular.

An important step for the developing the dynamic hadron theory was made by Nambu, who first introduced vector fields, i.e. the carriers of the color interaction, which became the prototype of the QCD gluon field [16]. QCD, whose rapid progress we have observed in the last few years [17], originated then as a result of a unification of the hypothesis of colored quarks and the color SU^c (3) symmetry with the principle of local gauge invariance by Yang and Mills [18].

It is important to emphasize in this respect that the Greenberg hypothesis of the parafermi statistics of quarks [19-20], as shown in Sec. 3.3, does not admit the gauge SU^c (3) symmetry, since this symmetry forms the basis of QCD and the Greenberg hypothesis is thus a physically unacceptable alternative to the hypothesis of colored fermi quarks [21].

It is obviously not possible in a paper of this size to elucidate all the achievements of QCD, the development of which was a substantial advance for the theory of strong interactions.

Secs. 3.4 and 3.5 show how QCD and the ideas of composite hadrons can describe a wide range of phenomena from approximate scale invariance to automodel behavior in a consistent theoretical way and can substantiate the quark counting method for high momenta transfer processes.

The scale invariance discovered when inclusive and deep inelastic processes were investigated is one of the most universal regularities in high energy physics [22].

The automodel hypothesis formulated in [23] results in unified approach to the study of the scale properties of high-energy, strong, weak, and electromagnetic interactions based on similarity principles and dimensional analysis. The compatibility of the automodel hypothesis asymptotic behavior with the fundamentals of quantum field theory has been rigorously corroborated in [24], where a one-to-one relation has been found between the automodel's amplitude and cross-sectional asymptotes for deep inelastic processes and the behavior of a local currents products near the light cone.

In 1973 the automodel hypothesis and the ideas about hadrons' quark structure led to the formulation of the quark counting rules. These specify the pattern of the asymptotic behavior of amplitudes and cross sections of the various exclusive processes depending on a degree of "complexity" of the partaking hadrons [25, 26].

The quark counting formulas describe the numerous experimental data on the elementary particle scattering surprisingly well and enable straightforward information about the number of elementary hadron constituents to be taken from the experiments.

In the last few years the idea of colored quarks and fundamental QCD forces has started penetrating into the theory of nuclear reactions.

It is to be noted that the most immediate manifestation of the quark nuclear structure is an experimentally observed law of the exponential falling of the electromagnetic form factor of deuteron at high momenta transfers, which agrees well with the quark counting formula and exhibits the presence of a hard 6-quark deuteron structure [27, 28].

Sec. 3.6 treats the problems of allowing for the quark degrees of freedom in describing pure nuclear phenomena, especially those taking place at high energies and momenta transfers. A possibility is, in particular, indicated of exciting the "hidden" color in nuclear matter and of a number of other consequences [29].

The final Sec. 3.7 is dedicated to discussing the unified gauge theories of strong and electromagnetic interactions having a spontaneously broken color symmetry and integral charge quarks [30, 31].

3.2 COLORED QUARKS AND HADRON DYNAMICS

3.2.1 A Hypothesis of Colored Quarks

According to the colored quarks hypothesis formulated for the first time in [4-7], quarks obey Fermi-Dirac statistics, each type of quark appearing in three unitarily equivalent states:

$$q = (q_1, q_2, q_3),$$

which differ in the values of a new quantum number called afterwards the *color*. Since only three quarks were known when the new quantum number was introduced, i.e. u, d, and s, the colored quark model became known at the three-triplet model.

The wave function of an observed baryonic family, which can be approximated for spin unitary symmetry by a fully symmetric 56 component tensor Φ_{abc}, was assumed to be fully symmetric with respect to the color variables of the three constituent quarks:

$$\psi_{ABC}(x_1, x_2, x_3) = \frac{1}{\sqrt{6}} \varepsilon_{\alpha\beta\gamma}\Phi_{abc}(x_1, x_2, x_3), \tag{3.1}$$

where $A = (\alpha, a)$, $B = (\beta, b)$, and $C = (\gamma, c)$.

From this assumption it can be concluded that the observed mesons and baryons are neutral relative to the new quantum number and conform to singlet states, if expressed in terms of unitary $SU^c(3)$ symmetry corresponding to the quantum number. So, for example, the known mesons and baryons are built up from quarks and antiquarks in the following fashion:

$$\bar{q}^{\alpha}(1)\, q_{\alpha}(2) \quad \text{mesons},$$
$$\varepsilon^{\alpha\beta\gamma} q_{\alpha}(1)\, q_{\beta}(2)\, q_{\gamma}(3) \quad \text{baryons}, \tag{3.2}$$

where the color indices α, β, γ each have only three possible values.

Thus, starting from the colored quark hypothesis, the Pauli principle requirements can be met both for spin quarks as well as hadron spectroscopy.

Colored quarks can have both fractional and integral charges. In the latter case the quarks could be created in high energy particle collisions without violating the basic law of electric charge conservation and would be, generally speaking, unstable particles decaying into known hadrons and leptons [30].

Below we shall discuss the consequences which result from the hypothesis that charge on the quarks is integral (see Sec. 3.7).

Let us stress that the introduction of a new quantum number led to the quarks being seen as normal physical objects accessible to direct or indirect observation.

3.2.2 Dynamic Quark Models

The introduction of colored fermi quarks as fundamental physical particles has opened a way to the dynamic description of elementary particles.

It is the absence of quarks in the free state that has been difficult to explain. Yet explaining this phenomenon, known as quark confinement or non-emission, is one of the crucial problems now facing elementary particle physics. Although the confinement problem will obviously be solved after experiment, a number of attempts have been made to explain the "permanent detention" of quarks inside hadrons in a logically consistent way. In particular, the quark "bag" model has been suggested.

The quark bag model originated in the work done at Dubna (the "Dubna quark bag") [4, 13] and at MIT [14].

A dynamic quark model, whose development started in 1964 at Dubna, was based on the assumption that quarks were very heavy objects bound in hadrons by immense forces. On the one hand, these forces ensure a large mass defect of the quarks in the hadrons, and on the other prevent them from being emitted [1].

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[1] The description in terms of heavy, composite [11] quarks is, in a sense, complementary to the description in terms of light or "current" quarks

Quark confinement is not unconditional and in principle quarks can be freed if the hadrons acquire a sufficiently high energy.

The dynamic composite model offers a systematic description both of the static observable properties of elementary particles (μ, g_A/g_V, etc.) and the hadron form factors. Note that among the others it gave the first satisfactory explanation of the enhancement of the magnetic moment of a heavy quark bound inside a hadron. This effect could be simply described by the Dirac equation for a quark bound by a scalar field described by a rectangular potential well with $U(r) = -U_0\theta(r_0 - r)$, and in the presence of an external magnetic field **H** by

$$[E + i\boldsymbol{\alpha}(\nabla + ie\mathbf{A})]\psi = \beta M^*\psi; \quad r \leqslant r_0, \tag{3.3}$$

where $M^* = M - U_0$ and $\mathbf{H} = \text{curl } \mathbf{A}$.

Solving equation (3.3) in the limit of an infinitely heavy mass M of the free quark at a fixed value of the effective mass M^*, which hereafter we will assume is zero, we can obtain the following expression for magnetic moment of the bound quark:

$$\mu = \frac{e}{2E}\cdot\frac{4Er_0 - 3}{6(Er_0 - 1)} \simeq 0.83\frac{e}{2E}; \quad \left(E \simeq \frac{2.04}{r_0}\right). \tag{3.4}$$

We emphasize that the finiteness of the magnetic moment of an infinitely heavy, bound quark can be inferred from an assumption about the scalar nature of the binding potential but does not obtain in the vector case, for example.

This result makes it possible to obtain a good qualitative estimate of the absolute values of the nucleon's magnetic moment, taking one third of the nucleon's mass as the energy of the bound quark and using $SU(6)$ symmetry, i.e.

$$\mu_p \simeq 3 \text{ n.m}, \quad \mu_n \simeq -2 \text{ n.m}$$

and a number of other relations.

Comparing these results with the experimental values reveals the importance of studying the relativistic corrections for the matrix elements of the electromagnetic and weak currents of composite particles. The nature of these corrections can be demonstrated in the most simple and elegant fashion in terms of a model of quasi-independent quarks.

In this model the quarks that make up a hadron move independently within some self-consistent scalar potential $U(r)$, their binding to which compensates for their mass[2]. If the weak and electromagnetic interactions are introduced in a minimal way [32],

$$i\partial_\mu \to i\partial_\mu + \begin{cases} eA_\mu & \text{electromagnetic interaction,} \\ \dfrac{G}{\sqrt{2}}\tau^{\pm}\gamma_5 l_\mu^{\pm} & \text{weak interaction,} \end{cases} \tag{3.5}$$

where A_μ is the electromagnetic potential, $l_\mu^{\pm}$ charged weak lepton currents, and G the Fermi constant for weak interactions, we will obtain the following for the ratio of the axial to the vector constants of the weak interaction, g_A/g_V, and for the magnetic moment of proton, say,

$$g_A/g_V = -\frac{5}{3}\langle\uparrow|\sigma_z|\uparrow\rangle, \tag{3.6a}$$

$$\mu_p = \frac{e}{2E_q}\langle\uparrow|\sigma_z + L_z|\uparrow\rangle. \tag{3.6b}$$

Here $\sigma_z/2$, L_z, and E_q are respectively the spin and orbital moments and the energy of an individual, bound quark in a nucleon whose projection of total angular momentum is

$$\langle\uparrow|J_z|\uparrow\rangle = \left\langle\uparrow\left|\frac{1}{2}\sigma_z + L_z\right|\uparrow\right\rangle = \frac{1}{2}. \tag{3.7}$$

Whence it is not difficult to find out that

$$g_A/g_V = -\frac{5}{3}(1 - 2\delta), \tag{3.8a}$$

$$\mu_p = \frac{e}{2E_q}(1 - \delta), \tag{3.8b}$$

where the parameter δ,

$$\delta = \langle\uparrow|L_z|\uparrow\rangle = -i\int d^3r\, \psi^*[\mathbf{r}\times\nabla]_z\psi, \tag{3.9}$$

specifies the value of the relativistic corrections. For an ultrarelativistic case, when $\langle\mathbf{q}^2\rangle/E_q^2 \sim 1$, we have $\delta \sim 1/6$ which yields a correction of the order of 30% for the g_A/g_V ratio. This example shows up the degree to which the effect of relativistic corrections could be essential for predictions of non-relativistic quark models. So-called configuration mixing is another indication of the role of relativistic effects. This is when, in the most general vector representation of a baryon (composed of three quarks), P- and D-waves add to the S-wave contribution [34]. The spin unitary part of the wave function will then not be fully symmetric, being a superposition of the contributions corresponding to 20-, 56-, and 70-tuple representations of the $SU(6)$ group.

The dynamic composite models led to a qualitative explanation and quantitative description of an entire set of transmutational

which is a more adequate notion when analyzing a hadron's point structure. Light quark confinement is associated with the non-Abelian nature of the fundamental QCD interaction.

[2] A. Salam has figuratively called this effect an "Archimedian bath" [33].

particle and resonance processes. We specially mention the quark model of electromagnetic and weak meson decays [35, 36] which has been developed using the dynamic approach.

This model offers an explanation of the weak leptonic decays of the pseudoscalar π- and K-mesons and the electromagnetic decays of the vector meson resonances (ρ^0, ω^0, and ϕ^0) into electron-positron pairs as the annihilation of quarks and antiquarks bound inside these mesons. The widths of corresponding decays are governed by the values of the wave functions of bound quark-antiquark pairs in matching coordinates [35], i.e.

$$\Gamma(\pi\to\mu\nu)=\frac{G^2\cos^2\theta}{2\pi^2}\,m_\mu^2\left(1-\frac{m_\mu^2}{m_\pi^2}\right)|\psi_\pi(0)|^2, \tag{3.10}$$

$$\Gamma(V^0\to e^+e^-)=\frac{16\pi\alpha^2}{3m_V^2}\,g_V^2|\psi_V(0)|^2;\;(V^0=\rho^0,\,\omega^0,\,\phi^0,\,\ldots), \tag{3.11}$$

where

$$g_\rho=1/\sqrt{2};\quad g_\omega=1/3\sqrt{2};\quad g_\phi=-1/3;$$

G is the weak Fermi constant, and θ the Cabbibo angle[3].

An analysis of the data on the widths on the meson resonances based on these formulas led to the known conclusion about a dependence of distance scale (effective sizes) on the quantum numbers of a bound system (the Weisskopf-Van Royen paradox), e.g. [36],

$$\frac{|\psi_K(0)|^2}{|\psi_\pi(0)|^2}\simeq\frac{m_K}{m_\pi},\ \text{etc.} \tag{3.12}$$

In the case of the decay $\pi^0\to 2\gamma$ the annihilation model indicates that the width of this decay is proportional to the number of different quark colors.

The constituent annihilation model and formulas (3.10-3.11), allowing for the first QCD corrections to it form a basis for the contemporary theoretical analysis of the various decay modes of the members of a new family of heavy particles, namely the J/ψ_{g^-}, and γ-mesons.

3.2.3 Hadron Currents and Form Factors

The above examples illustrate the importance of allowing for relativistic corrections when a hadron's local currents and form factors are being constructed. A consistent solution of this problem can only be given using the relativistically covariant description of

[3] An allowance for the renormalization of the magnetic moment and axial vector, weak constant of the bound quarks is made by introducing additional factors $(1-2\delta)$ and $(1-\delta)$, into formulas (3.10) and (3.11), respectively.

the system of bound particles within the scope of quantum field theory.

One of the first attempts to construct the local currents of composite particles was performed for [13]. In this work a relativistically covariant equation for mesons and baryons was postulated and this marked the beginning of relativistic, composite quark models.

The equation proposed, the choice of which was dictated by a requirement there should be a generalized $SU(6)$ invariance and that the large mass of the bound quarks was compensated for, enabled many relationships to be obtained for the vertices and form factors of weak and electromagnetic transitions with hadron participation.

It has been proposed in [37, 38] that the three dimensional, dynamic (quasipotential) equations in quantum field theory [39] be used to construct the local currents of the composite particles.

With this approach the matrix current's elements for the bound particle systems will be defined by expressions of the type

$$\langle P',\beta|J_\mu(0)|P,\alpha\rangle$$

$$=\int\psi^*(\mathbf{q}_i')\,\widetilde{\Gamma}_\mu(P',\,\mathbf{q}_i'|P,\,\mathbf{q}_j)\,\psi(\mathbf{q}_j)\prod_{k=1}^{n}d\mathbf{q}_k'\,d\mathbf{q}_k, \tag{3.13}$$

where $\psi(\mathbf{q}_i)$ is the simultaneous wave function of a bound state in the C.M.S.[4]

The construction of the vertex integral operator $\widetilde{\Gamma}(P',\,\mathbf{q}_i'\,|\,P,\,\mathbf{q}_j)$ and the quasipotential equation for the wave function $\psi(\mathbf{q}_i)$ is a basic problem for the simultaneous approach to the description of a system of interacting particles [40-42].

For the case of two spin particles (the quarkonium) the quasipotential wave function equation reduces to the form

$$(\gamma_0E-\widetilde{M})\,\psi=0, \tag{3.14a}$$

$$\gamma_0\psi\equiv\gamma_0^{(1)}\psi=\gamma_0^{(2)}\psi, \tag{3.14b}$$

where E is the total energy of the system, and $\widetilde{M}$ the mass operator which is a function (generally nonlocal) in momentum $\mathbf{q}$, as well as in $i\nabla_q$, $\sigma_{1,2}$, and E [41].

For non-interacting particles the mass operator is merely

$$\widetilde{M}=2W=2\sqrt{m^2+\mathbf{q}^2}. \tag{3.15}$$

[4] Below in Sec. 3.5 when the asymptotic behavior of hadron form factors are studied for the high momenta transfer range, the three dimensional dynamic equations in the variables of the "light front" (i.e. on the zero plane) are used.

Given an interaction, the operator $\widetilde{M}$ reads

$$\widetilde{M}\psi = 2\sqrt{m^2+\mathbf{q}^2}\,\psi(\mathbf{q}) + \int V(E;\,\mathbf{q},\,\mathbf{q}')\,\psi(\mathbf{q}')\,d\mathbf{q}'. \tag{3.16}$$

The quasipotential $V(E;\,\mathbf{q},\,\mathbf{q}')$ which enters this expression is specified by the relation

$$\overline{G}^{-1} = \overline{G}_0^{-1} - \frac{1}{2\pi i}V, \tag{3.17}$$

where $\overline{G}^{-1}$ and $\overline{G}_0^{-1}$ are operators for the inverse bitemporal Green functions of two interacting and two free particles, respectively. In the C.M.S. and in the Foldy-Watthausen representation they are

$$\overline{G}(E;\,\mathbf{q},\,\mathbf{q}') = \Lambda T_{\mathbf{q}}^{+}\left\{\int_{-\infty}^{+\infty} dq_0 \int_{-\infty}^{+\infty} dq_0' G_{\mathbf{p}=0}(E;\,q,\,q')\right\} T_{\mathbf{q}'}^{+}\Lambda. \tag{3.18}$$

Here

$$T_{\mathbf{q}} = \frac{(m+W-\boldsymbol{\gamma}_1\cdot\mathbf{q})\,(m+W+\boldsymbol{\gamma}_2\cdot\mathbf{q})}{2W\,(m+W)}\,; \quad W=\sqrt{m^2+\mathbf{q}^2} \tag{3.19}$$

is the operator for the unitary Foldy-Watthausen transformation for a two particle system, and

$$\Lambda = \frac{1+\gamma_0(1)\,\gamma_0(2)}{2} \tag{3.20}$$

is the projection operator of the 16-component bispinors onto a subspace which can have an inverse operator (3.17).

The work [37] demonstrated how the dynamic moments of local vector and axial currents for the bound states of two spin particles can be found from the three-dimensional quasi-potential equations.

Suppose a quark interaction is introduced with the weak homogeneous and time dependent external vector (V_μ^i) and axial-vector (A_μ^i) fields thus:

$$\gamma_\mu\partial_\mu \to \hat{D} = \gamma_\mu\partial_\mu + i\lambda^\alpha V_\mu^\alpha\gamma_\mu + i\lambda^\alpha A_\mu^\alpha\gamma_5\gamma_\mu \tag{3.21}$$

(here λ^α are the generators of the flavor group).

As a result, the mass operator becomes a function of the external field and can be expanded into the powers of the field, viz.

$$\widetilde{M}_{V,A} = \widetilde{M} + \delta\widetilde{M}_{V,A} + \ldots, \tag{3.22}$$

where the first non-vanishing correction is controlled by a variation of the two-particle Green function,

$$\delta\widetilde{M}_{V,A} = 2\pi i\overline{G}^{-1}\cdot\delta\overline{G}_{V,A}\overline{G}^{-1} \tag{3.23}$$

and so $\delta G_{V,A}$ can be found by standard perturbational methods.

The vector and axial charges of the particle system,

$$Q^\alpha = \left\langle \text{bound}\,\middle|\,\int J_0^\alpha(x)\,dx\,\middle|\,\text{bound}\right\rangle,$$
$$\mathbf{Q}^\alpha = \left\langle \text{bound}\,\middle|\,\int J_{5i}^\alpha(x)\,dx\,\middle|\,\text{bound}\right\rangle\ (i=1,\,2,\,3), \tag{3.24}$$

can be defined in terms of the total energy variation with the external fields present, i.e.

$$\delta E = V_0^\alpha Q^\alpha + \mathbf{A}^\alpha\mathbf{Q}_5^\alpha + \ldots. \tag{3.25}$$

Whence (in the lowest order of perturbation theory for the vertex operator) it follows:

$$Q^\alpha = \int \psi^*(\mathbf{q})\,\{\lambda_1^\alpha + \lambda_2^\alpha\}\,\psi(\mathbf{q})\,d\mathbf{q}, \tag{3.26a}$$

$$\mathbf{Q}_5^\alpha = \int \psi^*(\mathbf{q})\,\{\lambda_1^\alpha\boldsymbol{\Delta}_1 + \lambda_2^\alpha\boldsymbol{\Delta}_2\}\,\psi(\mathbf{q})\,d\mathbf{q}, \tag{3.26b}$$

where

$$\boldsymbol{\Delta} = \frac{m}{W}\left[\boldsymbol{\sigma} + \mathbf{q}\frac{(\boldsymbol{\sigma}\cdot\mathbf{q})}{m\,(m+W)}\right],\quad W=\sqrt{m^2+\mathbf{q}^2}. \tag{3.27}$$

These are exactly the same charges for which the *SU* (6) algebra was first postulated. In a nonrelativistic approximation, when $\mathbf{q}^2 \ll m^2$, $\boldsymbol{\Delta} \simeq \boldsymbol{\sigma}$, *SU* (6) algebra is trivially satisfied for quantities (3.26a, b) given that the two particle states are completely described by the wave functions ψ ($\mathbf{q}$).

However, as was shown above, relativistic corrections may be significant. For example, if the operator $\boldsymbol{\Delta}$ is averaged over a spherically symmetric state, we find that

$$\langle\boldsymbol{\Delta}\rangle = \boldsymbol{\sigma}\left(1-\frac{\langle\mathbf{q}^2\rangle}{3m^2}\right) + O(1/m^4). \tag{3.28}$$

This agrees with the model of quasi-free quarks, where the renormalization of the axial constant is controlled by a factor in [13], viz.

$$(1-2\delta) = 1-\frac{\langle\mathbf{q}^2\rangle}{3E_q^2} + O(1/E_q^4). \tag{3.29}$$

In other words, the corrections of the order of $1/m^2$ are needed by relativistic kinematics and are independent of the nature of the particle's interaction.

In the paper by Gell-Mann and Dashen [43] the problem of deriving relativistic quark dynamics from the current algebra in a system with an infinite momentum (the "$P_z = \infty$" system) was posed.

The above analysis allows us to state that the results for the charges on the vector and axial vector currents obtained in these papers can be reproduced without reference to the commutation relations of current algebra at infinite momentum [44].

In fact, the diagonal matrix current elements for the $P_z = \infty$ states are related to the current's diagonal matrix elements at rest by the Lorentz transformation:

$$\langle J_0\rangle_{P_z\to\infty} \to \frac{1}{\sqrt{1-\beta^2}} [\langle J_0\rangle_{\mathbf{P}=0}] + \beta \langle J_z\rangle_{\mathbf{P}=0}]_{\beta\to 1}. \tag{3.30}$$

As a result, using (3.26), we find for the limiting values of the vector and axial vector charge matrix elements that

$$\lim_{P_z\to\infty} \frac{1}{P_0} \left\langle \int J_0^\alpha d\mathbf{x} \right\rangle = \int \psi^*(\mathbf{q}) \{\lambda_1^\alpha + \lambda_2^\alpha\} \psi(\mathbf{q})\, d\mathbf{q}, \tag{3.31a}$$

$$\lim_{P_z\to\infty} \frac{1}{P_0} \left\langle \int J_{50}^\alpha d\mathbf{x} \right\rangle = \int \psi^*(\mathbf{q}) \{\lambda_1^\alpha \Sigma_1 + \lambda_2^\alpha \Sigma_2\} \psi(\mathbf{q})\, d\mathbf{q}, \tag{3.31b}$$

where

$$\Sigma_1 = \frac{m}{W+q_z} \left[\sigma_z(1) + \frac{\sigma(1)\cdot \mathbf{q}}{m} \left(1 + \frac{q_z}{m+W}\right)\right], \tag{3.32}$$

$$\Sigma_2 = \frac{m}{W-q_z} \left[\sigma_z(2) - \frac{\sigma(2)\cdot \mathbf{q}}{m} \left(1 - \frac{q_z}{m+W}\right)\right].$$

Note that relativistic corrections of the order of $1/m^2$ to axial constant of the two spin particles, which is defined by (3.32), have the same form as before, namely,

$$\langle \Sigma_i \rangle = \sigma_z(i) \left(1 - \frac{\langle \mathbf{q}^2\rangle}{6m^2}\right) + O(1/m^4). \tag{3.33}$$

It is not difficult to show that

$$[\Sigma_1]^2 = [\Sigma_2]^2 = 1, \tag{3.34}$$

whence the validity of the $SU(3) \times SU(3)$ algebra follows for the charges of vector and axial octet currents of two free quarks. Thus, the three-dimensional dynamic equation can serve as an effective means to check the current algebra of composite particles at infinite momenta.

The results above pertained to the charges, i.e. to the matrix current elements at zero momentum transfer. In [44] it was generalized to slowly varying external fields. This allows the higher dynamic moments of the particle system's currents, such as magnetic and electric dipole moments, to be found.

In particular, evaluating a variation of two spin particles' energy, given homogeneous electric and magnetic fields,

$$\delta E = \mathbf{M}\cdot\mathbf{H} + \mathbf{D}\cdot\mathbf{E}, \tag{3.35}$$

we find the following expressions for the magnetic and electric dipole moments of the system (in the lowest approximation of perturbation theory for the vertex operators):

$$\mathbf{M} = \int \psi^*(\mathbf{q}) \{e_1\mathbf{m}_1 + e_2\mathbf{m}_2\} \psi(\mathbf{q})\, d\mathbf{q}, \tag{3.36a}$$

$$\mathbf{D} = \int \psi^*(\mathbf{q}) \{e_1\mathbf{d}_1 + e_2\mathbf{d}_2\} \psi(\mathbf{q})\, d\mathbf{q}, \tag{3.36b}$$

where

$$\mathbf{m}_i = \frac{1}{2W}(\mathbf{L} + \boldsymbol{\mu}_i), \quad \mathbf{L} = -i\left[\mathbf{q} \times \frac{\partial}{\partial \mathbf{q}}\right], \tag{3.37}$$

$$\boldsymbol{\mu}_i = \frac{m+W}{2W}\left[\boldsymbol{\sigma}_i + \mathbf{q}\frac{(\boldsymbol{\sigma}_i\cdot\mathbf{q})}{(m+W)^2}\right], \tag{3.38}$$

$$\mathbf{d}_i = \gamma_0 \left\{\frac{i}{2}\frac{\partial}{\partial\mathbf{q}} + i\,\frac{[\mathbf{q}\times\sigma(i)]}{W(m+W)}\right\}.$$

Relativistic corrections to the magnetic moment of a bound quark that were found by averaging over a spherically symmetric state, i.e.

$$\langle \mathbf{m}_i \rangle = \frac{1}{2m}\boldsymbol{\sigma}_i\left(1 - \frac{\langle \mathbf{q}^2\rangle}{6m^2} + O(1/m^4)\right) \tag{3.39}$$

agree with results from the quasi-free quark model. In this model renormalization of the magnetic moment of the bound quark is governed by the factor

$$(1-\delta) = \left(1 - \frac{\langle \mathbf{q}^2\rangle}{6E_q^2}\right) + O(1/E_q^4) \tag{3.40}$$

Relativistically covariant quasi-potential equations have been used in [45] to construct form factors for composite particles at arbitrary momentum transfer values.

3.2.4 QCD as the Gauge Theory of Colored Quarks and Gluons

One of the most important implications of introducing color into elementary particle physics is the development of QCD, i.e. the gauge theory of colored quarks and gluons treated as an up-to-date basis of strong interaction theory.

QCD has emerged as a result of bringing the local gauge invariance idea of Yang and Mills [18] into the color $SU^c(3)$ symmetry [17]. An important role in developing QCD was played by the concept of vector particles introduced in 1965 by Nambu. These are the carriers of interaction between the colored quarks and were the prototypes of the QCD vector fields of gluons [16].

Hopes for a theoretical explanation of quark confinement or non-emission[5] are pinned on the non-Abelian nature of a QCD gauge invariance group.

A most important feature of QCD is the asymptotic freedom [46] associated with the discovered weakening of short-range interactions between quarks, i.e. with the increasing momentum transfer:

$$\alpha_s(q^2) \sim \frac{12\pi}{(11N_c - 2N_f)} \frac{1}{\ln q^2/\Lambda^2},$$

where N_c and N_f are the numbers of colors and flavors, respectively, and Λ the fundamental scale.

The QCD's asymptotic freedom corresponds to the idea that the quasi-free quarks, which originate in the dynamic, composite hadron models where light quarks whose mass is "erased" by the interaction, are effectively confined only by the walls of the potential well and remain practically free inside the confinement region (the "Dubna quark bag").

Unifying the heuristic quark bag picture and QCD has led to the development of a number of the modern approaches to hadron dynamics [47, 48].

Theorists at MIT [15] have introduced the assumption that there is some constant density of bag volume energy which is specified by universal parameter $B \simeq \Lambda^4$. This assumption guarantees the stability of a quark bag which has zero complete color with respect to the quark-gluon fields filling the bag.

In a number of works there have been some attempts to substantiate the quark bag model on the basis of a nontrivial topological structure of the QCD vacuum state which is related to instantons (i.e. the effects of tunneling between classically degenerated states which differ by the value of their so-called topological gauge potential number and which conform to zeroth QCD field strengths [47]).

In particular, the application of the finite energy sum rules [49] to analyze the annihilation of e^+e^- pairs into hadrons and to describe the dynamics of quark-antiquark systems (the quarkonium) has led to the following values for the average vacuum fields of light quarks and for a physical vacuum's energy density [50]:

$$\begin{aligned} \langle \bar{u}u \rangle &= \langle \bar{d}d \rangle \simeq -(0.25\ \mathrm{GeV})^3, \\ \varepsilon &= -\frac{9}{32\pi} \langle \alpha_s G^a_{\mu\nu} G^a_{\mu\nu} \rangle \simeq -(0.25\ \mathrm{GeV})^4 \end{aligned} \tag{3.41}$$

5 Note that the "elusiveness" of a rigorous proof of color confinement in the non-Abelian theories, in spite of considerable efforts and isolated partial advances in this direction, could indicate that color confinement is, by way of a mathematical analogy, the "Fermat" theorem of contemporary particle theory. In other words, quark confinement or non-emission cannot, generally speaking, be inferred from the first principles of QCD.

(here $G^a_{\mu\nu}$ are the color gluon's field strengths, $a = 1, 2, \ldots, 8$). The negative sign of ε leads to a positive volume density of the bag energy. However, since the quantity B used in describing the hadrons in the quark bag model is almost an order of magnitude less than the energy density of the physical vacuum $|\varepsilon|$ signifies, apparently, that the physical vacuum inside the hadrons is "destroyed" only partially [48].

Note that recently, in connection with studying a possibility of a spontaneous breakdown of color symmetry, the theory is entered by the colored scalar particles of small mass equally with quarks and gluons. A principal corollary of similar theories is the existence of a new hadron family that include the scalar color fields [51-53].

It will be shown below (see Sec. 3.7) that given a condensate of the scalar fields

$$\langle \alpha_s^{1/6} \varphi^+ \varphi \rangle \sim (1\ \mathrm{GeV})^2, \tag{3.42}$$

here $b = \frac{1}{4}\left(11 - \frac{2}{3}N_f - \frac{1}{6}N_s\right)$, N_s being the number of fundamental scalar triplets and allowing for large perturbative corrections in the QCD with scalar fields, the masses of the new hadrons can amount to several tens of a GeV [53]. Observing states such as these would be equivalent to discovering a new flavor, namely, scalar quarks.

3.3 PARASTATISTICS AND COLOR

3.3.1 Parafermi Statistics for Quarks

The first attempt to solve the problem of quark statistics was initiated in 1964 by Greenberg [19] and has been used assuming that quarks are parafermions of rank 3. In this framework, baryons can be explained and described by completely symmetric spin-unitary wave functions. In particular, the baryons built up of three quarks with the identical quantum numbers have been explained, e.g.

$$\Delta^{++}\ (J_z = 3/2) \text{ or } \Omega^-\ (J_z = 3/2).$$

Earlier works (see, e.g. [20]) have emphasized that the use of parafermi statistics for quarks and the introduction of a new quark quantum number (color) together with an appropriate color $SU^c(3)$-symmetry are not equivalent approaches in elementary particle theory and lead, generally speaking, to different physical consequences. Even so, in the last few years a number of works have appeared in which they have been erroneously equated [54].

It is essential to understand this problem properly not only for quark theory, but also for the future development of elementary particle theory. In this section we want to clarify the problem and, using just a brief analysis of the most essential features of both approaches, we shall show that the use of parastatistics is not equivalent to the introduction of quark color and the appropriate gauge $SU^c(3)$-symmetry, which form the basis of QCD.

Remember first of all that parafermi statistics are governed by the following set of the trilinear permutation relations

$$[\psi(x), [\bar{\psi}(y), \psi(z)]_-]_- = -2iS(x-y)\psi(z), \quad (3.43)$$

where $S(x-y)$ is the ordinary permutation function of the free fermi fields. As has been shown by Green [55], who was the first to introduce parastatistics, a parafermi field of rank P allows the covariant representation

$$\psi(x) = \sum_{i=1}^{P} \psi_i(x) \quad (3.44)$$

to be made, where the fields $\psi_i(x)$, commonly called the Green parafield components, obey bilinear permutation relations of an anomalous type, viz.

$$[\psi_i(x), \bar{\psi}_j(y)]_\pm = -i\delta_{ij}S(x-y). \quad (3.45)$$

Here the sign "+" (the anticommutator) corresponds to the same components $(i=j)$ and, the sign "—" (the commutator) to different ones $(i \neq j)$.

Using a parafermi field represented in terms of Green components, it is not difficult to show that in the interesting case of $P = 3$ we have, in contrast to ordinary fermi fields:

$$\psi^2(x)\,|\,0\rangle \neq 0, \quad \psi^3(x)\,|\,0\rangle \neq 0. \quad (3.46)$$

The last condition solves the problem of baryon spectroscopy that has required it to be possible to place up to the three identical particles into the same quantum state. Note also that the symmetric character of the completely spin-unitary (the $SU(6)$-part) wave function of observed baryons is because the only composite operator which possesses a unit baryon number in this approach is the operator described in [54, 56] and follows from the ordinary anticommutational Fermi-Dirac relations, namely,

$$[[\psi^a(x), \psi^b(x)]_+, \psi^c(x)]_+, \quad (3.47)$$

where a, b, and c are the spin-unitary field indices.

However, in addition to the ordinary mesons and baryons, for which

$$M \sim [\psi, \bar{\psi}^-]_-, \quad B \sim [[\psi, \psi]_+, \psi]_+, \quad (3.48)$$

a consequence of the normal permutation relations for composite operators leads to systems with the quantum numbers corresponding to a diquark and quark-meson, namely,

$$D \sim [\psi, \psi]_-, \quad F \sim [[\psi, \psi]_+, \bar{\psi}]_+. \quad (3.49)$$

These have no analogues in the spectrum of the hadron states observed.

It should be emphasized that for the parafield to be presented as a sum of Green components does not signify, *per se*, that there are any internal degrees of freedom, i.e. it is only a convenient mathematical means just like the higher spins in the theory of angular momentum can be constructed as a sum of several 1/2 spins. Besides, the anomalous character of the permutation relations of Green components does not allow any physical sense to be directly attached to them.

There is, however, one transformation, i.e. the so-called Klein transformation, that is nonlinear and nonlocal but allows the permutation relations to be reduced to a normal, canonical form. The transformation is of the form [57]

$$\psi_i \to \theta^{-1}\psi_i\theta \equiv \dot{\Psi}_i, \quad \begin{aligned} \Psi_1 &= \psi_i K, \\ \Psi_2 &= i\psi_2 K, \\ \Psi_3 &= \psi_3. \end{aligned} \quad (3.50)$$

The operator K is described by the relations

$$\begin{aligned} \psi_1 K &= K\psi_1; \\ \psi_2 K &= -K\psi_2; \quad KK^+ = K^2 = 1, \\ \psi_3 K &= -K\psi_3; \end{aligned} \quad (3.51)$$

and can be selected in the following form:

$$K = \exp i\pi(N_2 + N_3); \quad N_i = \int d^3x\, \psi_i^+\psi_i. \quad (3.52)$$

It can be easily checked that the Ψ_i fields, obtained from the initial Green parafermi field components as a result of the Klein transformation, follow the normal permutation relations:

$$\begin{aligned} [\Psi_i(x), \quad \Psi_j(y)]_+ &= -i\delta_{ij}S(x-y), \\ [\Psi_i(x), \quad \Psi_j(y)]_+ &= 0. \end{aligned} \quad (3.53)$$

Let us stress that the Klein transformation changes the form of the permutation relations and is, therefore, either noncanonical or nonunitary. If the Klein transformation does not change the character of the theory of free fields, then the possibility of applying this transformation to interacting fields generally puts extremely strong restrictions upon the theory, i.e. the so-called superselection rules [58].

In the case we are interested in, i.e. parafermi-statistics of rank 3, as Govorkov showed for the first time in 1966 [59], the relevant superselection rules can be formulated as a requirement of theory invariance given in terms of the normal fermi-fields $\Psi_i = \theta^{-1}\psi_i\theta$ with respect to the transformations of the SO (3)-rotation group of the three-dimensional matter space. In other words, the space of the admissible physical states of the parafields is mapped by the Klein transformation onto the set of those elements Ψ_i of the Fock normal fermi fields space, which are invariant relative to SO (3)-transformations of these fields.

Thus, the observed quantities from the initial theory of parafields of rank 3 will be specified by the vacuum averages of the operators that are invariant relative to SO (3)-transformations of the Ψ_i-fields.

3.3.2 Parastatistics and Gauge Symmetry

If locality or microcausality are required, extremely severe restrictions are imposed on the possible forms for parafield interaction Lagrangians [56]. In particular, attempt to construct a parafermion gauge interaction requires parabosons of the same rank to be introduced [60].

Indeed, out of the two parafermion vector currents,

$$j_\mu(x) = \frac{1}{2}[\bar{\psi}, \gamma_\mu\psi]_-, \tag{3.54a}$$

$$j'_\mu(x) = \frac{1}{2}[\bar{\psi}, \gamma_\mu\psi]_+, \tag{3.54b}$$

only the first is a local operator and can be associated with the electromagnetic current. The second current (3.54b) is generally nonlocal (except for the case when $P = 2$) and satisfies paraboson-type trilinear permutation relations on a spacelike surface.

Let us introduce a paraboson vector field $B_\mu(x)$, which obeys the following permutation relations:

$$[B_\mu(x), [B_\nu(y), B_\lambda(z)]_+]_- = ig_{\mu\nu}D(x-y)B_\lambda(z) + ig_{\mu\lambda}D(x-z)B_\nu(y), \tag{3.55}$$

where $D(x-y)$ is the scalar field permutation function. As was the case of the parafermi fields, the field $B_\mu(x)$ can be given as a sum of Green components:

$$B_\mu(x) = \sum_{i=1}^{P'} B^i_\mu(x) \tag{3.56}$$

with the fields $B^i_\mu(x)$ obeying anomalous permutation relations like

$$[B^i_\mu(x), B^j_\nu(y)]_- = -ig_{\mu\nu}D(x-y) \quad (i=j),$$
$$[B^i_\mu(x), B^j_\nu(y)]_+ = 0 \quad (i \neq j). \tag{3.57}$$

Generalizing the Klein transformation $(P = P')$,

$$K = \exp i\pi (N_2 + N_3),$$
$$N_i = \int d^3x\, \psi^\dagger_i\psi_i + \int d^3x\, B^{-i}_\mu \overleftrightarrow{\partial_0} B^i_\mu, \tag{3.58}$$

leads to the normal form of the permutation relations of the Green components both for the parafermi- and parabose-fields.

Following the work in [60], we can see that the most general form of the Lagrangian of a parafermion interaction with vector fields that is compatible with the locality requirement is the following:

$$L = -\frac{1}{4}F^2_{\mu\nu} - \frac{1}{4}G^2_{\mu\nu} - \frac{1}{2}\left[\bar{\Psi}, \not{D}\Psi\right]_- + e j_\mu A_\mu + g'\left[j'_\mu, B_\mu\right]_+, \tag{3.59}$$

where

$$\not{D} = i\gamma_\mu\partial_\mu - m,$$
$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + ig\frac{1}{2}\left[B_\mu, B_\nu\right]_-, \tag{3.60}$$

and A_μ and F_μ being the vector potential and strength tensor of the electromagnetic field, respectively.

We stress that the requirement $P = P' = 3$ emerges automatically as a condition of self-consistancy when constructing a local Lagrangian which contains only Yukawa trilinear couplings of the parafermions with the vector fields.

Passing over to Green field components and using the Klein θ-transformation,

$$\Psi_i = \theta^{-1}\psi_i\theta, \quad B^i_\mu = \theta^{-1}B^i_\mu\theta,$$

from (3.54), and when $g = g'$, we get a Lagrangian of the gauge theory with the SO (3)-symmetry group, thus:

$$L = -\frac{1}{4}F^2_{\mu\nu} - \frac{1}{4}G^2_{\mu\nu} - \bar{\Psi}\not{D}\Psi + eA_\mu\left[\bar{\Psi}, \gamma_\mu\Psi\right]_- + g\left(\mathcal{B}_\mu, \left[\bar{\Psi} \times \gamma_\mu\Psi\right]_+\right), \tag{3.61}$$

where

$$G_{\mu\nu} = \partial_\mu\mathcal{B}_\nu - \partial_\nu\mathcal{B}_\mu + g\left[\mathcal{B}_\mu \times \mathcal{B}_\nu\right].$$

Note that both fermions (Ψ) and bosons ($\mathbf{B}_\mu$) are transformed here via a three-dimensional (vector) representation of the SO (3)-group.

We shall finish by proving the limited equivalence of the gauge theory with local *SO* (3)-symmetry and the theory of parafields of rank 3 which was given in [60]. As opposed to QCD, this theory has only three gluons and diquarks are absent in the particle spectrum, as are fermions with quark-meson quantum numbers and other exotic hadrons. Besides, the gauge theory with *SO* (3)-symmetry only posses-ses asymptotic freedom if the number of flavors does not exceed two which contradicts experiment.

Thus, the hypothesis of parafermi statistics for quarks is not equivalent to the introduction of color, or a colored *SO* (3)-symmetry, since it leads to results which are unacceptable on physical grounds.

3.4 HADRON QUARK STRUCTURE AND AUTOMODEL BEHAVIOR AT HIGH ENERGIES

3.4.1 Scale Laws in Particle Physics

The study of interaction processes at high energies and momentum transfers is of primary importance for understanding dynamics of strong interactions and elementary particle structure. The scale properties of these processes have been intensively studied over the last decade, both theoretically and experimentally. All the interaction types, strong, electromagnetic, and weak, exhibit automodel, asymptotic behavior or approximate scale invariance in some form.

In this section we will show how the scale properties can be described starting from the most general, model independent laws of physical similarity, using dimensional analysis, and considering elementary particle quark structure.

Experiments with the high energy particles produced in modern accelerators are now a chief source of information about the elementary particles structure and properties of the fundamental hadron constituents.

As the energy of the interacting particles is increased, which corresponds to ever shorter distances, large numbers of secondaries are produced, since new inelastic channels are opened. The diversity of the processes of interparticle transmutations, the complexity of experimental observation and description of various end products of reactions at sufficiently high collision energies make many of the traditional methods of investigation untenable.

An entirely new approach to studying inelastic high energy particles interactions was proposed by A. A. Logunov in 1967 and has come to take the place of traditional methods. The basis of this method called the inclusive method is that only secondaries of a given sort are observed in the final state of reaction. This allows a model independent description to be made of high energy multiparticle processes based on the fundamentals of quantum field theory [22, 61].

A theory of inclusive reactions has been developed in the works of Logunov and his disciples[6] and has led to the establishment of a number of rigorous asymptotic relations and limitations on the high energy interaction cross sections, and demonstrated the fruitfullness of the approach in analyzing such important regularities as scale invariance.

As soon as the proton accelerator at the High Energy Physics Institute (HEPI) in Serpukhov, which was then the world's largest, was commissioned in 1969, a remarkable feature of scale invariance in the inclusive hadron reactions was discovered when the secondary spectra were studied.

The discovery of scale properties of inclusive processes and their theoretical investigation by R. P. Feynman [62], C. N. Yang [63] and others has deepened our ideas about strong interactions and given a new impetus to the development of the inclusive approach.

The investigations of deep inelastic processes in inclusive electron-nucleon scattering that were carried out in SLAC (Stanford) led to the discovery in 1968 of scale reaction properties (Bjorken scaling). These indicate the existence of a "hard", pointlike structure of a nucleon (the quark-parton structure).

Several years earlier, in 1964, the pointlike behavior of the total cross sections of lepton-hadron interactions was suggested by M. A. Markov [64] from purely theoretical considerations of the dominant role of the channels opening anew, compared to the suppression factor owing to hadron form factors. Experiments on the other big accelerators including those at CERN (Geneva) and Fermilab (Batavia, USA) have supported the pointlike behavior of the deep inelastic scattering of neutrinos and antineutrinos with nucleons. In other words, the effective nucleon size seems to have disappeared in these interactions.

3.4.2 Automodel Principle

In 1969 it was suggested [23] that the experimentally discovered scale properties shown by the electron-nucleon interactions might be assumed to be common for all deep inelastic lepton-hadron processes and could be derived in a model independent fashion using dimensional analysis and physical similarity laws.

An automodel principle was formulated in these works that was universal for describing the scale properties of the widely differing processes of deep inelastic interactions between elementary particles. Essentially, the automodel principle was an assumption that the form factors, and other measurable quantities of deep inelastic processes, were independent at the asymptotic limit of high energies and momentum transfers, of any dimensional parameters (such as

6 See the second paper in the present collection.

particle masses, strong interaction radius, etc.), which can fix the scale of measuring lengths or momenta. Thus, deep inelastic form factors appear to be homogeneous functions of relativistically invariant kinematic variables, whose homogeneity is specified by dimensional analysis[7].

Consider, for example, a deep inelastic process interaction where leptons transfer a momentum q to hadrons having momenta p_i. At the so-called Bjorken limit of $\nu_i \sim s_{ij} \sim |q^2| \gg p_i^2 = m_i^2$ (m_i is mass of the ith hadron) and for fixed values of the dimensionless ratios of the large kinematic invariants ν_i/q^2, s_{ij}/q^2, where $\nu_i = qp_i$, $s_{ij} = p_ip_j$ $(i \neq j)$, an observed parameter, $F(q, p_i)$, of the process under study behaves with a momenta scale transformation

$$q_\mu \to \lambda q_\mu, \quad p_{i\mu} \to \lambda p_{i\mu} \tag{3.62}$$

that corresponds to the automodel principle like a homogeneous function of order $2k$, viz.:

$$F(q, p_i) \to F(\lambda q, \lambda p_i) = \lambda^{2k} F(q, p_i), \tag{3.63}$$

where $2k$ is the physical dimension of the quantity $F(q, p_i)$, i.e.

$$F(q, p_i) = m^{2k}. \tag{3.64}$$

It is not difficult to see that the most general version of the form factor that satisfies these requirements is

$$F(q, p_i) = (q^2)^k f(\nu_i/q_i, s_{ij}/q^2), \tag{3.65}$$

where the function f depends only on the dimensionless ratios of the large kinematic variables that are constant in the Bjorken limit.

For electron-nucleon deep inelastic scattering where the differential cross section is specified by the formula

$$\frac{d^3\sigma}{dk^3} = \frac{4\pi\alpha^2}{q^4}\left[\cos^2\frac{\theta}{2} W_2(q^2, \nu) + \sin^2\frac{\theta}{2} W_1(q^2, \nu)\right], \tag{3.66}$$

(θ is the electron scattering angle in the laboratory frame) the automodel principle leads to scale invariant behavior of the form factors W_1 and W_2. This was found for the first time by Bjorken [65], i.e.

$$\nu W_2(q^2, \nu) = F_2(q^2/\nu), \quad W_1(q^2, \nu) = F_1(q^2/\nu), \tag{3.67}$$

since

$$[W_1(q^2, \nu)] = m^0, \quad [W_2(q^2, \nu)] = m^{-2}.$$

Applying the automodel principle to other lepton-hadron processes has led to a set of important results. In particular, a scale law

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[7] Automodel behavior in high energy physics is closely analogous to the similarity or automodel property in gas- and hydrodynamics [68] (the term automodel has been borrowed therefrom).

was found which for the first time describes the spectrum of the muon pairs produced in the high energy proton-proton collision $p + p \to \mu^+ + \mu^- +$ hadrons, namely [67],

$$\frac{d\sigma}{dM} \sim \frac{1}{M^3}\Psi\left(\frac{M}{E}\right), \tag{3.68}$$

where M is the effective mass of the muon pair and E the initial energy of the colliding particles. Experimental studies of this process initiated in 1970 by L. Leberman's group in Brookhaven supported this scale law demonstrating the universality of automodel asymptotic behavior for a wide class of deep inelastic lepton-hadron interactions [68].

Note that in the case of pure, high energy, hadronic collisions the automodel principle leads to so-called Feynman scaling for the inclusive cross sections of the production of secondaries having the limited transverse momenta (with respect to the collision axis), viz.

$$\left(E\cdot\frac{d^3\sigma}{dp^3}\right)_{a+b\to c+\ldots} = f(p_t, p_z/E). \tag{3.69}$$

That is to say, the collisions in this case result in the inclusive spectra being dependent only on the ratios of the longitudinal momentum components of the isolated secondaries and on the energy p_z/E of primaries. This scale law is derived by analogy with a "flat" explosion in hydrodynamics and uses generalized dimensional analysis of the independent units of measurement of the lengths and momenta along and perpendicular to the collision axis.

Thus, the experimentally observed scale properties of elementary particle interactions can be described in a unified manner based on the automodel principle, which starts from physical similarity laws and dimensional analysis.

At the same time it should be asked to what extent the automodel asymptotic behavior is compatible with the fundamentals and requirements of quantum field theory, such as the conditions of locality, microcausality, and spectrality.

This problem has been fully studied by N. N. Bogolyubov, V. S. Vladimirov, and A. N. Tavkhelidze, who have found sufficient and, in certain cases, necessary conditions for the existence of the automodel asymptote in quantum field theory. One result of this approach has been the establishment of an exact correlation between the automodel asymptote of observables, i.e. amplitudes and cross sections and interaction properties, at extremely short distances [24].

Although providing a theoretical basis from which to understand the general, model independent features of scale regularities, a similar axiomatic approach cannot, quite naturally, pretend to give a concrete form for the functions of the dimensionless ratios of the kinematic variables. These functions characterize the automodel asymptote and their form is defined by the dynamics of interaction. The additional information required to specify the form of these functions can be found by considering the composite quark nature of hadrons. In particular, the form factors of the deep inelastic lepton-hadron processes in the Bjorken-Feynman quark-parton model are expressed in terms of the distribution functions of the elementary point constituents of hadrons or the partons, i.e. quarks, antiquarks, and gluons.

The quantum mechanical correactions to this model will be discussed below (see Sec. 3.5).

3.4.3 Quark Counting Rules

Some especially interesting and important consequencies have resulted from the idea that hadrons have a composite nature. These consequences are associated with the deep inelastic or exclusive interactions of hadrons, in particular, when binary reactions of the large angle high-energy, hadron scattering are considered. In this kinematic region all the energy and momentum transfers are high and, consequently, we deal with interactions which are concentrated within a region of mainly short distances and time intervals and where the "hard" pointlike quark structure of hadrons must be exhibited in an explicit manner.

In 1973, a general formula was established [25], on the basis of the automodel principle and composite hadron nature. This controls the character of the energy dependence of the differential cross section for an arbitrary binary reaction for large angle high energy $(E = \sqrt{s})$ scattering and the form factors asymptote at high momentum $(Q = \sqrt{-t})$ transfers, viz.

$$\frac{d\sigma}{dt}(ab \to cd) \sim s^{-(n_a+n_b+n_c+n_d-2)}, \qquad F_a(t) \sim t^{-(n_a-1)}, \tag{3.70}$$

where $n_{i=a,b,c,d}$ are the numbers of elementary constituents of the hadrons participating in the reaction.

This formula is known as a quark counting formula and establishes a direct correlation between the rate of exponential diminishing of the differential cross section for an exclusive binary reaction of large-angle scattering and the energy and degree of complexity of the participating particles, i.e. the number of their elementary constituents.

The discovery of the quark counting formulas has afforded many opportunities to investigate the quark structure of hadrons and light atomic nuclei experimentally [25, 26].

Following [25], we shall dwell briefly on the derivation of formulas (3.70) which were based on dimensional analysis techniques

("dimensional quark counting"). An advantage of this approach to the derivation of quark counting formulas is its universality and independence of special details of a composite structure model of a hadron.

Consider a general binary reaction, $a + b \to c + d$. Assume that at the high energy and momentum transfer limit, particle a behaves like a composite system containing n_a point constituents, quarks, say. The vector of state of such a system can be written thus:

$$| a\rangle = \hat{N}_a \, | n_a, \text{ quarks}\rangle, \tag{3.71}$$

where the symbol $\hat{N}_a$ denotes the operation of multiplying the vector of the free quarks' state by a suitable system wave function and integrating (summing) over the quark variables.

The binary reaction differential cross section v can be given in the form

$$\frac{d\sigma}{dt}(ab \to cd) = \mathrm{Tr}\left(\prod_{i=a,b,c,d} \hat{\rho}_i \frac{d\hat{\sigma}}{dt}\right), \tag{3.72}$$

where

$$\hat{\rho}_i = \hat{N}_i \times \hat{N}_i^{+}, \tag{3.73a}$$

$$\frac{d\hat{\sigma}}{dt} = \frac{1}{s^2} \, | \langle n_a, n_b \, | \, T \, | \, n_c, n_d\rangle |^2. \tag{3.73b}$$

The dimension of a single particle state that is normalized in a elativistically invariant way is known to be

$$[\, | \text{single-particle}\rangle] = m^{-1},$$

whence the dimensions of the operator factors $\hat{\rho}_i$ and of $d\hat{\sigma}/dt$ which describe corresponding multiquark process follow, i.e.

$$[\hat{\rho}_i] = m^{2(n_a-1)}, \tag{3.74a}$$

$$\left[\frac{d\hat{\sigma}}{dt}\right] = m^{-2(n_a+n_b+n_c+n_d-2)}. \tag{3.74b}$$

Assuming, in accordance with the automodel principle, that short-range quark interactions are scale invariant, i.e. independent of dimensional dynamic parameters, we arrive at the conclusion of the exponential fall-off of quantity (3.73b) as energy and momentum transfer decrease, as does the differential cross section of the exclusive reaction, i. e.

$$\frac{d\sigma}{dt}(ab \to cd) \to \left(\frac{1}{s}\right)^{n_a+n_b+n_c+n_d-2} f(t/s). \tag{3.75}$$

The function $f(t/s)$ depends only on a ratio of the large kinematic variables, or equivalently on the scattering angle, and is a dimensional quantity with the natural scale being the effective particle size. Thus, the exponential asymptotic law (3.75) indicates a factorization of the effects of short- and long ranges.

The exponential fall off law (3.70) for the hadron form factor can be found by treating a special case of the exclusive reaction, i.e. the scattering of a structureless lepton from a hadron composed on n_a quarks.

Quark counting rules can be generalized for more complicated exclusive reactions as well.

Note that applying the aforementioned considerations to an analysis of inelastic production processes of particles with high transverse momenta, P_t, in high energy hadron collisions leads to point-like asymptote in the inclusive cross section, i.e. $E\frac{d^3\sigma}{dP^3} \sim P_t^{-4}$ [69-71]. This contradicts available experimental data and has forced a number of authors (see, for example [26]) to search for reasons behind the possible suppression of elementary quark-quark short-range interactions, and also to assume that in these processes, composite systems, i.e. mesons, baryons, etc., behave as elementary constituents.

In particular, if the production of high P_t particles in the inclusive reaction $ab \to c + \ldots$ is conditioned by hard scattering of a quark from a composite system of n_c quarks, then the cross section must have the asymptote $P_t^{-4n_c}$ to match the quark counting rules, i.e. P_t^{-8} (c = pion), P_t^{-12} (c = nucleon), etc.

Further study of these processes, however, shows a deviation from the canonical behavior (P_t^{-4}) of inclusive cross section at high P_t to be correlated with scale invariance (Bjorken scaling) violation in deep inelastic lepton-hadron scattering. The nature of this phenomenon is now the object of intense theoretical studies within the framework of QCD and the composite picture of hadrons.

3.5 QUARK COUNTING RULES AND QCD

3.5.1 Exponential Asymptotic Behavior of Exclusive Processes

In Sec. 3.4 we have shown that the notion of a composite quark structure of hadrons together with the fundamentals of local quantum field theory provides a basis for understanding the major dynamic regularities of high energy particle interactions.

A special position among them is taken by the so-called power laws of particle physics which include the quark counting rules. These establish a direct correlation between the rate of the exponential

fall off of the inclusive reaction cross section with energy and particle complexity, i.e. the number of elementary constituents, namely,

$$\frac{d\sigma}{dt}(ab \to cd) \sim s^{-(n_a+n_b+n_c+n_d-2)}, \quad (3.76)$$

$$F_a(t) \sim t^{-(n_a-1)}. \quad (3.77)$$

A quark counting formula describes, surprisingly well, numerous experimental data on high energy particle scattering and allows immediate information about the number of hadron elementary constituents to be inferred from experiments.

It is of interest to note that experimental results on electron-deuteron scattering at high energies and momentum transfers indicate an applicability of the quark counting fundamentals to the nuclear interactions as well. An analysis of the data of relativistic nuclear physics corroborates this conclusion (see the discussion in Sec. 3.6).

We stress that the exponential asymptotic behavior of exclusive cross sections as predicted by quark counting rules differs qualitatively from Regge mode behavior which results in exponentially low probabilities of high momentum transfer interactions.

The success of quark counting formulas has made it imperative to substantiate it within QCD. Below we shall discuss various approaches to the solution of the problem as well as some new results produced in this direction.

3.5.2 QCD Corrections to Quark Counting Formulas

A number of current works have been dedicated to the corroboration of quark counting rules within QCD. Most of them rest on some summing technique of perturbative QCD diagrams and are applicable, generally speaking, only to short distances (high momentum transfers) [73-76].

The key role in such an approach is played by a statement about factorizing all infrared-divergent contributions (corresponding to long distances) in the form of some structure factors of a hadron wave function type which cannot be found within perturbation theory. The remaining finite factors, which dictate the asymptote of the exclusive amplitudes in the high energy and momentum transfer limit, are given by a renormalized series in a perturbative QCD.

Although a number of difficult questions arise when deriving the factorization and remain inadequately answered, the outcome of this approach to the study of the asymptotics of exclusive processes is worthy of attention. In particular, it is unlikely that it can be rigorously proved that the assumed expansion into a perturbational series and retention of only the diagram's main asymptote in the sum of this series will not destroy the pole structure of bound states of the appropriate Green functions [72].

In Sec. 3.5.3 we shall discuss another approach to the problem of corroborating the quark counting rules which rests on a dynamic treatment of the composite systems in the three-dimensional formulation of quantum field theory on the zero plane.

We now briefly enumerate some of the results of studying exclusive processes in perturbative QCD.

(1) *Meson Form Factors*

The asymptote of the meson electromagnetic form factors have the following form (neglecting higher twist contributions):

$$F(Q^2) = 12\pi C_F \frac{\alpha_s(Q^2)}{Q^2} \times \Big| \sum_{n=0,\,2,\,\ldots} a_n \left(\ln \frac{Q^2}{\Lambda^2}\right)^{-\gamma_n} \Big|^2 (1 + O(\alpha_s,\, m/Q)). \tag{3.78}$$

The logarithmic corrections to the exponential asymptote of the form factor are dictated by the QCD's effective charge behavior, viz.

$$\alpha_s(Q^2) = \frac{g_s^2}{4\pi} \sim \frac{4\pi}{\beta} \left(\ln \frac{Q^2}{\Lambda^2}\right)^{-1}, \quad \beta = 11 - \frac{2}{3} N_f \tag{3.79}$$

and by the value of the anomalous dimensions

$$\gamma_n = \frac{C_F}{\beta} \left\{ 1 + 4 \sum_{k=2}^{n+1} \frac{1}{k} - \frac{2\delta_{h_q,\, -h_{\bar{q}}}}{(n+1)(n+2)} \right\} \tag{3.80}$$

($C_F = \frac{n_c^2 - 1}{2n_c}$), which depend on the way the spins of the quark and antiquark meson constituents are added. If h_q and $h_{\bar{q}}$ are the corresponding helicities we have

$$\delta_{h_q,\, -h_{\bar{q}}} = \begin{cases} 0 \text{ for parallel spins } (\rho_{h=\pm 1}), \\ 1 \text{ for antiparallel spins } (\pi,\, \rho_{h=0}). \end{cases} \tag{3.81}$$

The a_n coefficients in formula (3.78) are defined in terms of the meson's system wave function and cannot, in general, be found using perturbative QCD techniques.

Retaining only the first term in formula (3.78) in the asymptotically high momentum transfer limit, we shall obtain

$$F(Q^2) \to 16\pi f^2 \frac{\alpha_s(Q^2)}{Q^2} \left(\ln \frac{Q^2}{\Lambda^2}\right)^{-\frac{2C_F}{\beta}|h_q + h_{\bar{q}}|}, \tag{3.82}$$

where the normalization factor f^2 is controlled by the physically measurable transition constants and it takes the following values for π- and ρ-mesons:

$$f^2 = \begin{cases} f_\pi^2 \simeq (93 \text{ MeV})^2 & \pi \to \mu\tilde{\nu}, \\ f_\rho^2 \simeq (152 \text{ MeV})^2, & \rho \to e^+e^-. \end{cases} \tag{3.83}$$

According to theoretical estimates, the asymptotic formula (3.82) can be applied far beyond the momentum trasfers attainable in modern accelerators.

We also present the leading asymptote of a pion's transitional form factor that is associated with the $e\pi \to e\gamma$ process. This asymptote has a purely exponential form, viz.

$$F_{\pi\gamma}(Q^2) \to 2f_\pi/Q^2. \tag{3.84}$$

(2) *Baryon Form Factors*

The asymptotic baryon magnetic form factor has the form

$$G_M(Q^2) \to -\langle e_{-\parallel}\rangle \left[C \frac{\alpha_s(Q^2)}{Q^2} \right]^2 \times \left(\ln \frac{Q^2}{\Lambda^2}\right)^{-\frac{2C_F}{\beta}|h_{q_1} + h_{q_2} + h_{q_3}|}, \tag{3.85}$$

where $\langle e_{-\parallel}\rangle$ is the average electric charge of constituent quarks which have helicities h_q that are opposite to the baryonic ones.

Note that at the limit of $SU(6)$-symmetry we get

$$\langle e_{-\parallel}\rangle = \frac{1}{6}(\mu - 3Q) = \begin{cases} 0 & \text{(proton)}, \\ -1/3 & \text{(neutron)}, \end{cases} \tag{3.86}$$

where μ is the magnetic moment and Q the electric baryon charge.

The nucleon electric form factor is exponentially suppressed in contrast to the magnetic one; i.e.

$$G_E/G_M \to \text{const} \cdot \frac{m^2}{Q^2}. \tag{3.87}$$

This is because the amplitudes are suppressed as the helicities change (here $\Delta h = \pm 1$). Baryon form factors which have the helicity $|h| > 1$ are suppressed in an analogous fashion.

(3) *Large Angle Scattering*

The differential cross section of the large angle high energy scattering has the following asymptote:

$$\frac{d\sigma}{dt}(ab \to cd) \to \left(\frac{\alpha_s(Q^2)}{Q^2}\right)^{n-2} \times \left(\ln \frac{Q^2}{\Lambda^2}\right)^{-2\sum_i \gamma_i} f_{ab\to cd}(\theta), \tag{3.88}$$

where

$$n = n_a + n_b + n_c + n_d,$$
$$\gamma_i = |h_i| C_F/\beta, \quad i = a,\, b,\, c,\, d.$$

As in every other exclusive process, the amplitudes dominate which satisfy the conservation law of hadron helicities, viz.

$$h_a + h_b = h_c + h_d. \tag{3.89}$$

This law is the result of the vector nature of a QCD interaction.

There is virtually no information about the form of the angular dependence of a binary reaction differential cross sections that is controlled by the functions $f_{ab\to cd}(\theta)$, nor is there any information about the values of the absolute cross section.

The unsolved problem of the so-called "pinch" singularities has to be noted here. Their contributions to large angle scattering amplitudes are characterized by the rate of the exponential decrease of the cross sections, which is smaller than was predicted by quark counting rules (the Landshoff paradox). More investigations must be made before a statement about the suppression of the pinch singularity contribution by Sudakov form factors and the factorization of exclusive binary reactions can be made.

3.5.3 The QCD Description of Composite Systems

We now discuss another approach to the problem of being able to give a consistent account of the quantum mechanical effects when describing high momentum transfer, hadron interactions.

The basis of these methods are the dynamic quasi-potential equations for composite particles given in the "light front" variables which were first introduced by Dirac. The convenience of light front variables is first that when high energy and momentum transfer interactions are studied, particle masses play no significant role and can be taken to vanish[8]. This makes it unacceptable to describe the process in terms of C.M.S. variables. Besides, a theory of composite systems in terms of light front variables most closely approximates the description of interacting hadrons in Feynman's parton model, which imparts the required clearness to rigorous results.

In order to describe the simplest composite system, i.e. a meson which has a 4-momentum P and a set α of quark and antiquark quantum numbers, we choose a gauge invariant two-point function of the Bethe-Salpeter amplitude type [77], i.e.

$$\chi_P(x_1, x_2 \mid C) = \mathrm{Tr}\,\langle 0 \mid T\left(\psi(x_1)\,\overline{\psi}(x_2)\exp\int_{x_1}^{x_2} ig\hat{A}_\mu\,dx_\mu\right) \mid P, \alpha\rangle, \tag{3.90}$$

[8] Obviously, by neglecting exponential corrections of the $O(m/Q)$ type, where Q is the large transferred momentum. This corresponds to the higher twist contributions to the asymptote of the quantum mechanical perturbative theory diagrams.

where the operation T involves both the chronological ordering of the fermion's quark fields and the ordering of the variables of the non-Abelian vector field $\hat{A}_\mu = \frac{1}{2}\hat{\lambda}^a A^a_\mu$ along an arbitrary integration contour C which joins the points x_1 and x_2. In formula (3.90) a trace is taken in the color variables so that all the color of the meson system that is described by the vector of state $\mid P, \alpha\rangle$ vanishes.

In the case of a baryon system, the gauge invariant amplitude generally depends on three contours C_i ($i = 1, 2, 3$) which joins the points x_i ($i = 1, 2, 3$) to some arbitrarily chosen point z, namely,

$$\chi_B = \langle 0 \mid T(\mathscr{E}^{abc}(x_1, x_2, x_3 \mid C_i)\,\psi_a(x_1)\,\psi_b(x_2)\,\psi_c(x_3)) \mid B\rangle, \tag{3.91}$$

$$\mathscr{E}^{abc}(x_1, x_2, x_3 \mid C_i) = \varepsilon^{a'b'c'}\left[e^{ig\int_z^{x_1} dx\cdot\hat{A}}\right]^a_{a'}\cdot\left[e^{ig\int_z^{x_2} dx\cdot\hat{A}}\right]^b_{b'}\left[e^{ig\int_z^{x_3} dx\cdot\hat{A}}\right]^c_{c'}, \tag{3.92}$$

with an assumed ordering along each contour.

Note that as a consequence of formulas (3.91) and (3.92) all color of the three quark fields vanishes only at the limit of matching coordinates x_i (or in any other state which is symmetric in coordinates, this corresponding to a requirement of standard $SU(6)$-symmetry). Yet in general the summed quark color is compensated by gluon field color.

The dependence of (3.90) and (3.91) on the arbitrary contours is obviously an inevitable result of requiring gauge invariance for the QCD description of the composite systems. To justify such an approach we note that gauge invariance must be primarily understood as a requirement of observability. The quantities like (3.90) and (3.91) pertain undoubtedly to the observable, i.e. measurable quantities, since they control the probabilities of such processes as, for example, $\pi \to \mu\bar{\nu}$, $\psi \to e^+e^-$, etc. In addition, they are in the expressions for the exclusive amplitudes of hadron interactions. As will be shown below, we can thus exploit gauge invariant wave functions to construct composite systems' form factors and in doing so factorize the infrared singularities in perturbative QCD in a natural way.

Mandelstam, for example, has used a gauge invariant description for electromagnetic systems [78]. Note, however, that in quantum electrodynamics physical quantities remain invariant if the gauging of either the fermion fields ($\psi \to e^{i\alpha Q}\psi$) or electromagnetic potentials ($A_\mu \to A_\mu + \partial_\mu\beta$) alone are changed by virtue of the current conservation of the charged fermion fields, i.e. $\partial_\mu j_\mu = 0$. This means that the formally gauge invariant Green functions can be used for calculations with perturbative QCD.

Since the gluons carry the color charge, only the sum current of fermions and bosons, i.e. $\partial_\mu (j_\mu + ig [A_\nu, F_{\mu\nu}]) = 0$, is known to be conserved in QCD. As a result, those quantities which vary in common gauge transformations of the fermion and color vector fields generally appear to be infrared divergent, i.e. they simply do not exist. The exceptions are those quantities which, by definition, allow only short-range contributions. Thus, the QCD wave function of the bound system whose dynamics is tangibly controlled by long range interactions must be defined in a gauge invariant manner.

Let us discuss briefly how an amplitude like (3.90) depends on the shape of the contour C. By parametrizing the point z_μ on the contour C joining the points $x_{1\mu}$ and $x_{2\mu}$ as

$$z_\mu = z_\mu (s); \quad 0 = s = 1$$

(with $z_\mu (s = 0) = x_{1\mu}$, $z_\mu (s = 1) = x_{2\mu}$) we find that the amplitude variation (3.90) with respect to the contour shape is:

$$\frac{\delta\chi}{\delta C_\mu (z)} = \Gamma_\mu (x_1, x_2, z/C)$$

$$= \mathrm{Tr} \langle 0 \mid T \left\{ \psi (x_1) \bar{\psi} (x_2) \, ig \, \dot{z}_\nu (s) F_{\mu\nu} (z) \, e^{ig \int_{x_1}^{x_2} dx \cdot \hat{A}} \right\}$$

$$\times \mid P, \alpha \rangle, \quad \dot{z}_\mu (z) = dz_\mu (s)/ds. \quad (3.93)$$

It is not difficult to see that the first variation of quantity (3.90) with respect to the contour's shape is proportional to the amplitude of the bound quark, antiquark, and gluon system which has the quantum numbers of the initial meson system.

Using the QCD equations of motion for the quark-gluon fields we get

$$\gamma_\mu (\partial_\mu + ig\hat{A}_\mu) \psi = 0,$$

$$\partial_\nu \hat{F}_{\mu\nu} + ig [\hat{A}_\nu, \hat{F}_{\mu\nu}] = j_\mu, \quad (3.94)$$

$$\mathrm{Tr} \, \lambda^a j_\mu = g \left(\bar{\psi} \gamma_\mu \frac{1}{2} \lambda^a \psi \right),$$

and we find that the relation

$$\frac{\partial}{\partial z_\mu} \Gamma_\mu (x_1, x_2, z \mid C) = ig\dot{z}_\mu (s)$$

$$\times \mathrm{Tr} \langle 0 \mid T \left\{ \psi (x_1) \bar{\psi} (x_2) \, j_\mu (z) \, e^{ig \int_{x_1}^{x_2} dx \, \hat{A}} \right\} \mid P, \alpha \rangle,$$

links the divergence of Γ_μ with the four quark amplitude.

All the above show that any dependence gauge-invariant ampltudes have on the contours' shape should not be significant when

considering high momentum transfer processes, where the quark counting rules and perturbative computations reveal that any contributions configurations with more than the minimum number of elementary constituents make are suppressed exponentially (the "higher twist" contributions).

The exact equations of motion for the QCD gauge invariant amplitudes of composite systems obviously should define the dependence not only on the end points (the coordinates of the constituent quarks), but also on the shape of the connecting contours. So, for example, a generalization of the Dirac operator on the contour is

$$\hat{D}_x = \gamma_\mu \left[\partial/\partial x_\mu - \int_0^1 ds \, \xi_{\mu\nu} (z) \, \delta/\delta C_\nu (z) \right] \quad (3.96)$$

which is dependent on the following displacement field:

$$\xi_{\mu\nu} (z) = \lim_{\Delta x \to 0} \frac{\Delta z_\mu (s)}{\Delta x_\nu}, \quad (3.97)$$

which is described on the contour C (see Fig. 3.1).

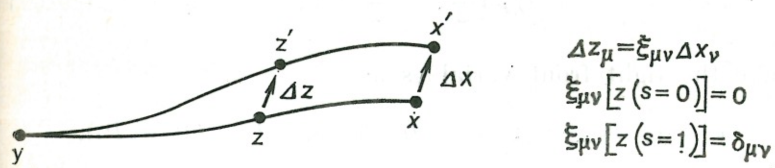


Fig. 3.1. The displacement field on the contour $\xi_{\mu\nu} (z (s))$, which determines the variation of the shape of contour C when the end point is displaced.

The problem of deriving and investigating a whole system of equations and relations which specify a gauge invariant amplitudes lies beyond the limits of the present paper [79].

We shall, however, stress that a dynamic QCD description of a composite system can be developed on the basis of the three-dimensional equations for a wave function related to a gauge invariant amplitude like (3.90), (3.91) for a fixed contour shape.

In particular, when the system's dynamics is described in terms of the light front variables, i.e. on the zero plane,

$$x_\mu = (x_1 - x_2)_\mu, \quad x^+ = x_0 + x_z = 0, \quad (3.98)$$

it is convenient to choose the contour C as lying on hyperplane (3.98). The most convenient gauging here for the vector field is the axial one, viz.

$$A^+ = A_0 + A_z. \quad (3.99)$$

Indeed, the gauge invariant amplitude is readily seen to be explicitly dependent only on the transverse dynamic degrees of freedom of the vector field, i.e.

$$\exp\left[ig\int\limits_C dz\cdot\hat{A}\right]=\exp\left[-ig\int\limits_C d\mathbf{z}_\perp\cdot\mathbf{A}_\perp\right], \tag{3.100}$$

where C lies on the zero plane $(x_1-x_2)^+=0$ and $A^+=0$.

When the interactions have a high momentum, Q, we are interested in the role played by the small impact parameters such that $(x_1-x_2)_\perp\sim 1/Q$. Whence it follows that the exponential factor can be dropped from the definition of the gauge-invariant amplitudes like those of (3.90) and (3.91), and the relation will remain accurate to exponential m/Q corrections.

Below we shall briefly show the fundamentals of the theory behind QCD's three-dimensional dynamic equations for composite systems.

If, in the case of meson systems, we use the transformation to the momentum representation, viz.

$$\varkappa_P(q)=\int dx\, e^{ixq}\chi_P\left(\frac{x}{2},\ -\frac{x}{2}\Big|\, C\right), \tag{3.101}$$

and define the light front variables as

$$x^\pm=\frac{1}{2}(x_0\pm x_z);\quad q^\pm=(q_0\pm q_z);\quad P^\pm=(P_0\pm P_z), \tag{3.102}$$

then we can introduce a quantity

$$\overline{\chi_P}(q^+,\ \mathbf{q}_\perp)=\int\limits_{-\infty}^{+\infty}dq^-\chi_P(q), \tag{3.103}$$

which is dependent on the gauge invariant amplitude (3.90) only in the zero plane, i.e. at $x^+=(x_1-x_2)^+/2=0$.

The meson system wave function will then be defined by the relation

$$\Psi_P(x,\ \mathbf{q}_\perp)=\mathrm{Tr}\,(\hat{\pi}\overline{\chi_P}(q^+,\ \mathbf{q}_\perp), \tag{3.104}$$

where the quantity $x=\frac{1}{2}+q^+/P^+$ varies between $0\leqslant x\leqslant 1$ by virtue of the so-called "projection" properties of χ_P (3.103).

The presence and form of the operator $\hat{\pi}$ in formula (3.104) is controlled by the procedure chosen to project the multipoint Green functions of the Dirac quark fields onto the zero-plane.

A detailed discussion of related problems can be found in [72, 77].

As has been shown in [77], the operator $\hat{\pi}$ can be chosen, in accordance with the total spin of the quark-antiquark system, in the following form:

$$\hat{\pi}=\begin{cases}\gamma_5\gamma^+ & (h_q+h_{\bar q}=0),\\ \gamma_\perp\gamma^+ & (h_q+h_{\bar q}=\pm 1).\end{cases} \tag{3.105}$$

In this way the function (3.104) satisfies the quasi-potential equation thus:

$$\left[M^2-\frac{\left(\mathbf{q}_\perp+\left(x-\frac{1}{2}\right)\mathbf{P}_\perp\right)^2+m^2}{x(1-x)}\right]\psi_P(x,\ \mathbf{q}_\perp)$$
$$=\int\limits_0^1 dx'\int d^2q'_\perp V(x,\ \mathbf{q}_\perp;\ x',\ \mathbf{q}'_\perp)\,\psi_P(x',\ \mathbf{q}'_\perp), \tag{3.106}$$

where $P^2=P^+P^--\mathbf{P}_\perp^2=M^2$, and the quasi-potential is defined in the standard way using an inversion, i.e. finding the operator which resolves the appropriate four-point Green function when it is projected onto the zero plane of initial and final states.

In the approach to describe composite systems developed here using the gauge invariant amplitudes (3.90), the following expression is natural for the four-point Green functions:

$$G(x_1,\ x_2\mid x_1',\ x_2')=\langle 0\mid T\{0(x_1,\ x_2\mid C)\,O(x_1',\ x_2'\mid C'\}\mid 0\rangle, \tag{3.107}$$

where

$$O(x_1,\ x_2\mid C=\mathrm{Tr}\left(\psi(x_1)\,\overline{\psi}(x_2)\,e^{ig\int_{x_1}^{x_2}dx\cdot\hat{A}}\right) \tag{3.108}$$

and is the gauge invariant operator which is bilinear in terms of the quark fields and dependent on the arbitrary contour C which joins the points x_1 and x_2. The limiting value of the Green function (5.32) at $g_s=0$ (in well chosen gauging of the vector fields) will be denoted by $s(x_1,\ x_2;\ x_1',\ x_2')$.

According to the general method of the Logunov-Tavkhelidze approach, the relation of the quasi-potential V to the Fourier-images of the Green functions projected onto the zero-plane can in quantum field theory be given in the following symbolic form:

$$(2\pi i)^{-1}V=\overline{G}^{-1}-\overline{S}^{-1}=\overline{S}^{-1}\,(\overline{SKG})\,\overline{G}^{-1}, \tag{3.109}$$

$$G=S+SKG. \tag{3.110}$$

The projection operation here assumes the axial gauging of the vector fields and the contours C and C' which lie on the zero plane (see formula (3.10)) as well as the convolution with the operator π prescribed by formula (3.105), namely,

$$\bar{G}(x, \mathbf{q}_\perp; x', \mathbf{q}'_\perp) = \int_{-\infty}^{+\infty} dq^- dq'^- \operatorname{Tr}[\hat{\pi} G_P(q, q') \hat{\pi}]. \quad (3.111)$$

The resolvent of the integral operator (3.111) is defined by the following relation:

$$\int_0^1 dx' \int d\mathbf{q}'_\perp \bar{G}(x, \mathbf{q}_\perp; x', \mathbf{q}'_\perp) \bar{G}^{-1}(x', \mathbf{q}'_\perp; x'', \mathbf{q}''_\perp) = \delta(x - x'') \delta(\mathbf{q}_\perp - \mathbf{q}''_\perp). \quad (3.112)$$

In [74, 77] expression for the contribution of the single gluon exchange diagram to the quark-antiquark interaction quasi-potential has been found by standard perturbational means, viz.

$$V = \bar{S}^{-1} \cdot (\overline{SK_1 S}) \cdot \bar{S}^{-1} + O(\alpha_s^2),$$
$$K_1 = \frac{\pi \alpha_s}{K^2 + i0} C_F \gamma_\mu^{(1)} \cdot \gamma_\mu^{(2)} \left[g_{\mu\nu} - \frac{n_\mu k_\nu + n_\nu k_\mu}{(nk)} \right] \quad (3.113)$$

in the axial gauging, i.e. at $n_\mu^2 = 0$, $\mathbf{n} \cdot \mathbf{k} = k^+ = k_0 + k_z$.

Depending on quark and antiquark helicity values, the single-gluon exchange quasi-potential will be of the form

$$\left\{ \begin{matrix} h_q + h_{\bar{q}} = 0 \\ h_q + h_{\bar{q}} = \pm 1 \end{matrix} \right\} : V(x, \mathbf{q}_\perp; x', \mathbf{q}'_\perp)$$
$$= \frac{8\pi G \alpha_s}{x(1-x) y(1-y)} \left[(y(1-y) \mathbf{q}_\perp^2 + x(1-x) \mathbf{q}'^2_\perp) \right.$$
$$\times \left(\left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} + \frac{\varepsilon(x-y)}{(x-y)} \right) + \mathbf{q}_\perp \cdot \mathbf{q}'_\perp \left. \left\{ \begin{matrix} xy + (1-x)(1-y) \\ 0 \end{matrix} \right\} \right]$$
$$\times \left[\frac{\theta(y-x)}{(x-y)\left(\frac{\mathbf{q}_\perp^2}{x} + \frac{\mathbf{q}'^2_\perp}{1-y} \right) - (\mathbf{q}_\perp - \mathbf{q}'_\perp)^2 - i0} \right.$$
$$\left. + \frac{\theta(x-y)}{(y-x)\left(\frac{\mathbf{q}_\perp^2}{1-x} + \frac{\mathbf{q}'^2_\perp}{y} \right) - (\mathbf{q}_\perp - \mathbf{q}'_\perp)^2 + i0} \right]. \quad (3.114)$$

Note that using axial gauging leads to the occurrence of singular expressions like $\varepsilon(x - y)/(x - y)$ in equation (3.114), which should be regularized. As was shown in [74, 77] the Ward identities enable the regularization of the quasi-potential singularities to be related to corresponding Green function renormalizations whilst allowing for their QCD multiloop corrections.

A study of the asymptote of the quasi-potential equations for the high transverse momenta range led Brodsky and Lepage to formulate the so-called evolution equation:

$$\frac{\partial}{\partial \tau} \phi(x, \tau) = \frac{C_F}{\beta} \int_0^1 dy\, V(x, y) \phi(y, \tau) \quad (3.115)$$

for the function

$$\phi(x, \tau) \equiv \int_0^{Q^2} d\mathbf{q}_\perp^2 \psi(x, \mathbf{q}_\perp^2),$$

where

$$\tau = \frac{1}{4\pi} \beta \int_{Q_0^2}^{Q^2} dk_\perp^2 \alpha_s(k_\perp^2)/k_\perp^2 \simeq \ln\left[\frac{\alpha_s(Q_0)^2}{\alpha_s(Q^2)} \right].$$

The integral kernel of equation (3.115) is determined by the asymptote of the single gluon exchange quasi-potential:

$$V(x, \mathbf{q}_\perp; x', \mathbf{q}'_\perp) \to \frac{4\pi\alpha_s}{x(1-x)} (q_\perp^2) \cdot V(x, y) \quad (3.116)$$
$$q_\perp \to \infty,$$
$$q'_\perp \text{ is fixed,}$$

and it has the following form:

$$V(x, y) = 2 \frac{1-x}{1-y} \left[\left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} + \frac{1}{x-y} \right] \theta(x-y)$$
$$+ 2 \frac{x}{y} \left[\left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} - \frac{1}{x-y} \right] \theta(y-x), \quad (3.117)$$

where the standard Gel'fond-Shilov regularization of the singularity is assumed; for example

$$\left(\frac{\theta(x-y)}{x-y}, \varphi(y) \right) = \int_0^x dy \frac{\varphi(y) - \varphi(x)}{x-y}. \quad (3.118)$$

Note that the eigenfunctions of the integral operator (3.117) are Gegenbauer polynomials, i.e. $\psi_n = G_n^{3/2}(2x - 1)$, viz.

$$\int_0^1 dy\, V(x, y) \psi_n(y) = (1 + \lambda n) \psi_n(x), \qquad \lambda_n = -\beta \gamma_n / C_F, \quad (3.119)$$

and the eigenvalues are defined by the dimensions of the nonsinglet quark distributions γ_n which are determined by a QCD single loop approximation (see formula (3.80)).

If we expand the solution of the evolution equations in the eigenfunctions of the operator $V(x, y)$, we find that

$$\phi(x, \tau) = x(1-x) \sum_{n=0,\,2,\,\dots} c_n \psi_n(x) \left[\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right]^{\gamma_n}. \qquad (3.120)$$

This result has been used in [73-75] to study the logarithmic corrections to the exponential asymptote of the meson form factors predicted by the quark counting rules.

3.5.4 The Hadron Form Factors at High Momentum Transfers

Using the pion electromagnetic form factors as examples, we can show how the QCD of composite systems enables the amplitude asymptote of exclusive hadron interactions to be investigated in the high momentum transfer range.

In the dynamic approach of describing composite systems, pion form factors are defined by the following general expression:

$$\langle \pi(P') | J_\mu(0) | \pi(P) \rangle = (P' + P)_\mu F_\pi(t)$$

$$= \int_0^1 dx\, dx' \int d^2 q_\perp\, d^2 q'_\perp \overline{\psi}_{P'}(x', \mathbf{q}'_\perp)$$

$$\times \mathcal{T}_\mu(P', x', \mathbf{q}'_\perp | P, x, \mathbf{q}_\perp)\, \psi_P(x, \mathbf{q}_\perp). \qquad (3.121)$$

The vertex operator $\mathcal{T}_\mu = \overline{G}^{-1} \overline{R}_\mu \overline{G}^{-1}$ is defined in terms of the following five-point Green function:

$$R_\mu = \langle 0 | T \{ 0(x'_1 x'_2 | C')\, J_\mu(z)\, O(x_1, x_2 | C) \} | 0 \rangle, \qquad (3.122)$$

where $O(x_1, x_2, | C)$ is the gauge invariant bilinear operator of (3.108) and $J_\mu = \overline{\psi}(z)\, e\gamma_\mu \psi(z)$ the electromagnetic quark current. The projection of the chosen contour C_i shape onto the zero-plane $R \to \overline{R}$ and the corresponding gauge conditions are discussed in [72].

Since the vertex operator $\mathcal{T}_\mu$ (as opposed to the five-point Green function R_μ) has no poles in the bound states whose existence is controlled by long-range interaction, perturbative QCD techniques can be used to find $\mathcal{T}_\mu$. This is one of the advantages of the approach being developed here.

Something else important is that the factorization of infrared singularities (the "factorization" theorem) allows the hadron form factor to be expressed naturally as an integral over the wave functions which accumulate long range contributions with relevant nonperturbational effects. The integral kernel, $\mathcal{T}_\mu$, is defined by a perturbative QCD renormalized series in the constant $\alpha_s(Q^2)$.

The relation of the factorization theorem becomes more transparent once the expression for the pion factor is transformed to

$$F_\pi \propto \int \overline{\psi}_0 [\overline{S}^{-1} (\overline{G \Gamma G})\, \overline{S}^{-1}] \psi_0. \qquad (3.123)$$

Here the relations used result from the dynamic equation:

$$\overline{G}^{-1} \cdot \psi = \overline{S}^{-1} \cdot \psi_0,$$

$$\psi = \psi_0 + \overline{S} \cdot V \psi, \quad \overline{G}^{-1} = \overline{S}^{-1} - V, \qquad (3.124)$$

where ψ_0 is the part of the composite system wave function which corresponds to the contribution of only the "soft", i.e. long range component responsible for the quark confinement. The "hard" (short-range) QCD interactions are taken into account by the potential V.

Let us write out the first few terms of the perturbational series and appropriate Feynman diagrams for the vertex operator viz.

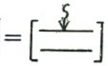
$$\tilde{\Gamma} \equiv \overline{S}^{-1} \cdot (\overline{G \Gamma G}) \cdot \overline{S}^{-1}.$$

Using the expansions of four-point and reduced five-point Green functions into perturbational series, we get

$$G = S + SK_1S + SK_2S + SK_1SK_1S + \dots,$$

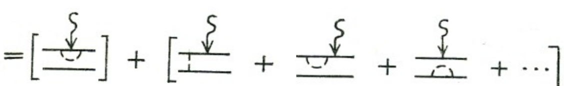
$$\Gamma = \Gamma_0 + \Gamma_1 + \Gamma_2 + \dots, \qquad (3.125)$$

and find that,

$$\tilde{\Gamma}_0 = \overline{S}^{-1} (\overline{S \Gamma_0 S})\, \overline{S}^{-1} = [\Gamma_0] =$$  , (3.126a)

$$\tilde{\Gamma}_1 = \overline{S}^{-1} (\overline{S \Gamma_1 S})\, \overline{S}^{-1} + \overline{S}^{-1} (\overline{S K_1 S \Gamma_0})\, \overline{S}^{-1} + \overline{S}^{-1} (\overline{S \Gamma_0 S K_1 S})\, \overline{S}^{-1}$$

$$= [\Gamma_1] + [K_1 S \Gamma_0 + \Gamma_0 S K_1]$$

$$=$$  (3.126b)

where symmetric diagrams are dropped from the last brackets. The brackets in formulas (3.126) signify a transformation to a mass's surface which assumes, for example, a substitution in "light front" variables like

$$P_i^+ = x_i \cdot P^+, \quad P_i^- = \frac{1}{P^+} \frac{m^2 + \mathbf{p}_{\perp i}^2}{x_i} \tag{3.127}$$

for the momentum components of ith parton in a system having a total momentum P.

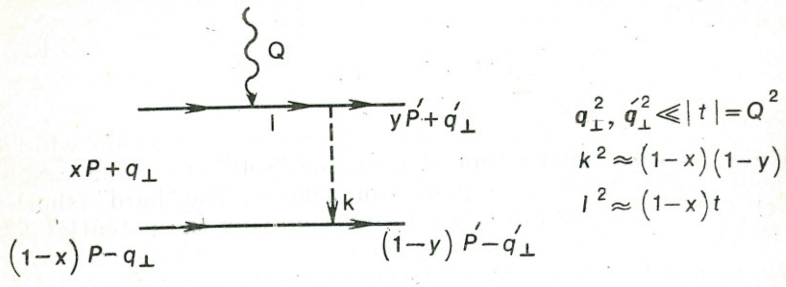


Fig. 3.2. One of the four diagrams of single gluon exchange dominating in the asymptotic limit of high transfer momenta Q.

The main contribution to the asymptote of the pion form factor is given by the diagram of the quark-antiquark single gluon exchange (Fig. 3.2).

Computations yield

$$F_\pi(t) = \frac{4\pi\alpha_s(Q^2)}{Q^2} C_F \left| \int_0^1 \frac{dx}{x(1-x)} \int_0^{Q^2} d^2q_\perp \psi(x, \mathbf{q}_\perp) \right|^2 . \tag{3.128}$$

Assuming further that in the region under consideration, once the wave function $\psi(x, \mathbf{q}_\perp)$ has been integrated over the relative transverse momenta, according to the quark counting rules for the parton distribution, it behaves as follows:

$$\int d^2q_\perp \psi(x, \mathbf{q}_\perp) \simeq C[x(1-x)]^{2n_\pi - 3}, \quad x \sim 0.1. \tag{3.129}$$

If $n_\pi = 2$ and we use the condition of normalization to a known constant of decay $\pi \to \mu\tilde{\nu}$, i.e.

$$2\sqrt{n_c} \int_0^1 dx \int d^2q_\perp \psi(x, \mathbf{q}_\perp) = f_\pi, \tag{3.130}$$

we find the leading asymptote of the pion form factor:

$$F_\pi(t) \to \frac{16\pi\alpha_s(Q^2)}{Q^2} f_\pi^2. \tag{3.131}$$

This result coincides with that obtained on the basis of the perturbational techniques (cf. (3.82)). Substituting the evolution equations solution for a partially integrated pion wave function (3.120) into equation (3.128), we find the aforementioned asymptotic expansion of the pion form factor (3.78).

Note that the pion form factor value predicted by formulas (3.82) and (3.131) is several times less than that given by experiment at the momentum transfer values achieved today. This may suggest, first, that the asymptotically high momenta range lies substantially higher than the value customarily assumed, and, secondly that it is necessary to study both the exact form of the composite system wave function and the contributions of the perturbational and non-perturbational corrections to the form factor asymptote.

3.5.5 Angular Dependence of Binary Reactions

The angular dependence of binary reactions contains the important information about the nature of the quark processes that take place in short-range hadron interactions with high momentum transfer.

The study of the angular dependence of the binary reactions in a number of theoretical models has led to the establishment of the so-called generalized quark counting rules [80]. These control the main contributions to the asymptotes of large-angle scattering differential cross sections, in terms of the topologies of the various quark diagrams corresponding to the process:

$$T(ab \to cd) \sim \sum_i T_i, \tag{3.132}$$

where i is the quark diagram topology.

Each of the contributions in (3.132) is a homogeneous function (up to the logarithmic corrections) of the large kinematic variables:

$$T_i \sim s^{-\alpha} t^{-\beta} u^{-\gamma}, \tag{3.133}$$

with the exponents, which correspond to the topology of the quark diagram, obeying the condition

$$2(\alpha + \beta + \gamma) = \sum_{j=a,b,c,d} (n_j - 1), \tag{3.134}$$

which ensures the appropriate exponential decrease in the differential cross section with increasing energy.

The exponents α, β and γ are individually controlled both by the quark diagram's topology and by the nature of the kinematic singularities of the appropriate helicity amplitudes.

If, as is shown in [80], the kinematic singularities in each reaction channel are isolated in the form

$$T_i(\lambda,\ \lambda') = \left(\cos\frac{\theta}{2}\right)^{|\lambda+\lambda'|}\left(\sin\frac{\theta}{2}\right)^{|\lambda-\lambda'|} a_i \qquad (3.135)$$

(here θ is the scattering angle in the scattering channel, say, $a + b \to c + d$, and $\lambda = \lambda_a - \lambda_b$, $\lambda' = \lambda_c - \lambda_d$, where the λ_i are the hadron helicities) and the generalized quark counting rules are used for the reduced amplitudes a_i, a formula for the angular dependence can be obtained for a wide class of binary reactions.

By way of example, the angular dependence formula for the pion-proton elastic scattering can be given in symbolic form as:

$$\frac{d\sigma}{dt}(\pi p \to \pi p) = \frac{1}{s^2}\left|T_{++}\right|^2 = \frac{1}{s^2}\left|\cos\frac{\Theta_s}{2}\underset{A/st^2}{\supset\subset} + \cos\frac{\Theta_u}{2}\underset{B/ut^2}{\asymp}\right|^2, \qquad (3.136)$$

where, by assuming asymptotic γ_5-invariance, a T_{++}-amplitude contribution alone is presented, which conserves the proton helicity. Thus only the quark diagrams are taken into account and they are topologically flat.

Using the expressions for half-angle cosine scattering in the s- and u-channels,

$$\cos\frac{\theta_s}{2} = \sqrt{-\frac{u}{s}}, \quad \cos\frac{\theta_u}{2} = \sqrt{-\frac{s}{u}},$$

we can reduce the angular dependence formula (3.136) to the form $z = \cos\theta_s$)

$$\frac{d\sigma}{dt}(\pi p \to \pi p) = \frac{1}{s^8}\frac{(1+z)}{(1-z)^4}\left|A + \frac{4B}{(1+z)^2}\right|^2. \qquad (3.137)$$

We can find the angular dependences for the other binary reactions in a similar way.

The development of QCD computational techniques for exclusive processes means that the problem of corroborating and improving the generalized quark counting rules, which are at present only heuristic, can be approached.

Below we will briefly show, without much substantiation, the results of the angular dependence studies in QCD for the simplest binary reaction, i.e. the elastic quark-pion scattering [81].

The scattering amplitude, assuming γ_5-invariance, has the form

$$T(q\pi \to q\pi) = \left(\bar{u}_{q'}\frac{\hat{P}+\hat{P}'}{2}u_q\right)B(s,\ t), \qquad (3.138)$$

with

$$T_{++} = \sqrt{-us}\,B(s,\ t), \quad T_{+-} = 0.$$

Treating the pion as a composite system yields

$$B(s,\ t) = \int_0^1 dx\,dx' \int d^2\mathbf{q}_\perp\, d^2\mathbf{q}'_\perp \bar{\psi}(x',\ \mathbf{q}'_\perp)\,\mathscr{B}(x', \mathbf{q}'; x, \mathbf{q}_\perp)\,\psi(x,\ \mathbf{q}_\perp), \qquad (3.139)$$

where the integral operator $\mathscr{B}$ is defined in terms of the six-point Green function R_6 projected onto the zero plane, i.e. $\mathscr{B} = \overline{G}^{-1}\cdot\overline{R}_6\cdot\overline{G}^{-1}$, with $\psi(x,\ \mathbf{q}_\perp)$ being the pion wave function.

As in the case of the vertex operator $\mathscr{T}_\mu$, which specifies the pion form factor, $\mathscr{B}$ does not contain pole singularities corresponding to the bound states and, as a consequence, can be found using perturbation theory.

The principal asymptotic contributions to the amplitude of the quarkpion scattering will generally be governed by 16 perturbative graphs of order α_s^2. In the limit of many colors, N_c, the contribution of the topologically nonflat quark diagrams appears to be suppressed by the ratio $1/N_c$ compared to the flat ones. In this way we get

$$\mathscr{B} \simeq \frac{(4\pi\alpha_s)^2}{x(1-x)\ y(1-y)}\left\{C_F^2\left[\frac{(1-x)(1-y)}{s^2} + \frac{(x+y)}{st}\right] - \frac{1}{2}C_F C_V\left[\frac{1}{st} - \frac{(1-x-y)}{s^2}\right] \pm (s \leftrightarrow u)\right\} \qquad (3.140)$$

($\pm$ stand for odd and even amplitude charges).

Integrating (3.140) with the wave functions (3.129), we obtain the following angular dependence for the large-angle quark-pion scattering:

$$\frac{d\sigma}{dt}(q\pi \to q\pi) \simeq \left(\frac{C_F f_\pi \alpha_s}{s}\right)^4\left(1 - \sec^2\frac{\theta}{2}\right)^2, \qquad (3.141)$$

which fully agrees with the generalized counting quark rules.

3.5.6 A Quark Counting of Anomalous Dimensions of Inclusive Processes

As indicated in the previous section, the assumption about the dominating role of the elementary quark-quark processes and the exact scale invariance of the hadron structure functions leads to point-like asymptote for inclusive cross section at high transverse momenta, P_t.

It was noted, however, that the deviation from the canonical asymptotic law, P_t^{-4}, that has been observed experimentally reflects a scaling violation in deep inelastic lepton-hadron processes.

Studies of the QCD corrections to the point-like, exponential asymptotes of deep inelastic and inclusive reactions at high transverse momenta have shown that the form of the logarithmic factors which define the deviation from the scaling in these processes is controlled by the hadron's quark structure. The value of the exponent of the large $\log Q^2$ (Q is the large transferred momentum), the so-called anomalous dimensions of the inelastic form factors, turns out to be controlled by the number of quarks that constitute the hadrons that take part in the reactions.

A general formula was established in [82] which defines the leading logarithmic corrections to the canonical point-like asymptote of an arbitrary deep inelastic or inclusive reaction between interacting particles at high momentum transfers in terms of the active and passive quarks which relate to the reacting hadrons.

This formula, called the quark counting rule of anomalous dimensions, expresses an element of the differential cross section of an arbitrary inclusive reaction in the following form:

$$d\sigma = d\sigma_0 \frac{(1-x)^{S-1}}{\Gamma(S)} \left[(\log Q^2)^{\gamma(S)+r\ln(1-x)}\right]^H. \qquad (3.142)$$

Here $d\sigma_0$ is the appropriate element of the elementary point-like interaction of quarks and/or gluons, and r some group factor equal to 16/25 when $N_c = 3$ and $N_f = 4$. The value of

$$S = \sum_{i(\text{hadrons})} 2(n_i - 1) \qquad (3.143)$$

is the double number of the passive quarks (the spectators) related to the participating hadrons and H the full number of active quarks taking part in an elementary hard scattering event, which coincides with the full number of hadrons taking part in the reaction.

The quantity

$$\gamma(n) = -\frac{r}{4}\left[1 + 4\sum_{k=2}^{n} \frac{1}{k} - \frac{2}{n(n+1)}\right] \qquad (3.144)$$

is the standard expression (in the single-loop perturbative approximation) for the anomalous dimension of the so-called non-singlet part of the quark-to-hadron fragmentation function.

Consider, by way of an illustration of the general formula (3.142), the deep inelastic electron-nucleon scattering (Fig. 3.3), for which we shall have

$$E\frac{d^3\sigma}{dk^3} \propto \left(\frac{\alpha}{Q^2}\right)^2 \frac{(1-x)^{A-1}}{\Gamma(A)} \left[(\log Q^2)^{\gamma(A)+r\ln(1-x)}\right], \qquad (3.145)$$

where $Q^2 \sim k_T^2 \gg m^2$, $A \equiv 2(n_A - 1)$, and $\alpha \simeq 1/137$.

For the inclusive hadron jet production with the high transverse momentum $P_t^2 = Q^2$ (see Fig. 3.4) formula (3.142) yields

$$E\frac{d^3\sigma}{dP^3} \propto \left(\frac{\alpha_s(P_t^2)}{P_t^2}\right)^2 \frac{(1-x)^{A+B-1}}{\Gamma(A+B)} \times \left[(\log P_t^2)^{\gamma(A+B)+r\ln(1-x)}\right]^2, \qquad (3.146)$$

where $A = 2(n_A - 1)$, and $B = 2(n_B - 1)$.

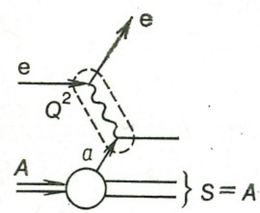


Fig. 3.3. Deep inelastic scattering $e + A \to e + \ldots$, $S = A = 2(n_A - 1)$, $H = 1$.

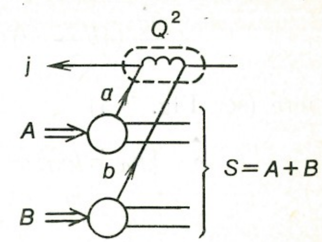


Fig. 3.4. Inclusive formation of the jet $A + B \to j + \ldots$, $S = A + B = 2(n_A + n_B - 2)$, $H = 2$.

An inclusive hadron production with high transverse momentum (Fig. 3.5), as opposed to (3.146), will have the following cross section:

$$E\frac{d^3\sigma}{dP^3} \propto \left(\frac{\alpha_s(P_t^2)}{P_t^2}\right)^2 \frac{(1-x)^{A+B+C-1}}{\Gamma(A+B+C)} \times \left[(\log P_t^2)^{\gamma(A+B+C)+r\ln(1-x)}\right]^3. \qquad (3.147)$$

This set of the applications of the quark counting rule of anomalous dimensions (3.142) can, if desirable, be continued.

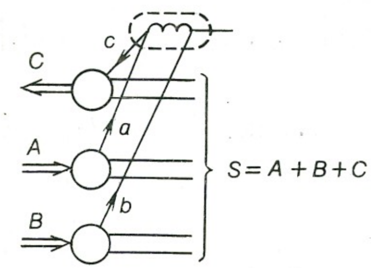


Fig. 3.5. Inclusive hadron production $A + B \to C + \ldots$, $S = A + B + C = 2(n_A + n_B + n_C - 3)$, $H = 3$.

A simple and physically transparent derivation of formula (3.142) can be made starting from the model of the hard scattering of hadron constituents [82, 83].

Indeed, the hard scattering model, for example, in the cases of the inclusive production of a high P_t jet in a hadron collision leads to the following cross section formula:

$$E\,\frac{d^3\sigma}{dP^3}(A+B\to \text{jet}+\ldots)$$
$$=\frac{1}{\pi}\sum_{a,\,b,\,c}\iint dx_a\,dx_b F_{a/A}(x_a,\ Q^2)\,F_{b/B}(x_b,\ Q^2)$$
$$\times\frac{d\hat\sigma}{d\hat t}(ab\to cd)\cdot\hat s\delta(\hat s+\hat t+\hat u), \tag{3.148}$$

where (see Fig. 3.4)

$$\hat s=(p_a+p_b)^2\sim x_a x_b s;\ \ \hat t=(p_a-p_c)^2\sim x_a t;$$
$$\hat u=(p_b-p_c)^2\sim x_b u,$$

and $\frac{d\hat\sigma}{d\hat t}(ab\to cd)$ is the relevant hard scattering cross section defined by the sum of the Born diagrams of perturbative QCD.

The hadron structure function in the first QCD logarithmic approximation (at $x\sim 1$) has the following form:

$$F_{a/A}(x,\ Q^2)\propto\frac{(1-x)^{\bar A-1}}{\Gamma(\bar A)}\exp(c\xi),$$
$$\bar A=A+r\xi=2(n_A-1)+r\xi, \tag{3.149}$$

where

$$\xi=\ln\left[\ln\frac{Q^2}{\Lambda^2}\Big/\ln\frac{Q_0^2}{\Lambda^2}\right],\quad r=16/25,$$
$$c=r(\ln 2-1/2)\simeq 0.12.$$

Substituting (3.149) into (3.148) and changing the integration variables, we obtain

$$E\,\frac{d^3\sigma}{dp^3}(A+B\to\text{jet}+\ldots)\propto\left(\frac{\alpha_s(P_t^2)}{P_t^2}\right)J(x_1,\ x_2,\ p_t^2), \tag{1.150}$$

where

$$J\equiv\frac{(1-x_1)^{\bar A-1}\cdot(1-x_2)^{\bar B-1}}{\Gamma(\bar A)\,\Gamma(\bar B)}$$
$$\times\int_0^1\int_0^1\frac{u^{\bar A-1}v^{\bar B-1}e^{2c\xi}}{[1-u(1-x_1)]^2\,[1-v(1-x_2)]^2}\,du\,dv$$
$$\times(1-x_1x_2)\,\delta[uv(1-x_1-x_2)-(1-u-v)]. \tag{3.151}$$

Here we used the following notations:

$$x_1=-t/(u+s)=x_R\frac{\sin^2\theta/2}{1-x_R\cos^2\theta/2},$$
$$x_2=-u/(t+s)=x_R\frac{\cos^2\theta/2}{1-x_R\sin^2\theta/2}, \tag{3.152}$$
$$x_R=-(u+t)/s=1-M^2/s.$$

where M^2 is the square of the missing mass, and θ the scattering angle of the a and b constituents in their center of mass system.

As x_R approaches unity, the integral (3.151) can be computed explicitly as

$$J=\frac{(1-x_R)^{A+B-1}}{\Gamma(A+B)}\,\frac{\left[(\log Q^2)^{\gamma(A+B)+r\ln(1-x_R)}\right]^2}{\left(\cos^2\frac{\theta}{2}\right)^{\bar A}\cdot\left(\sin^2\frac{\theta}{2}\right)^{\bar B}}, \tag{3.153}$$

where

$$\gamma(n)=-r\psi(n+1)+c\simeq-\frac{r}{4}\left[1-\frac{2}{n(n+1)}+4\sum_{k=2}^{n}\frac{1}{k}\right],$$
$$c=r\left(\ln 2-\frac{1}{2}\right)\simeq 0.12$$

and $\psi(n)$ is the digamma function.

It is interesting to note that the dependence of initial hadron momenta on the fraction x_R at $x_R\sim 1$ and the high momenta transfer Q, which follows from formula (3.153), exhibits an effective "collectivization" of the constituents pertaining to different hadrons. Indeed, the whole x_R-dependence enters via the effective structure function of the unified dihadron system $D=(A+B)$ which unifies the A and B constituents, i.e.

$$F_{(a+b)/(A+B)}(x_R,\ Q^2)\propto\frac{(1-x_R)^{s-1}}{\Gamma(\bar s)}; \tag{3.154}$$
$$\bar S=\bar A+\bar B=2(n_A-1)+2(n_B-1)+2r\xi.$$

The a and b components are isolated from here and carry away the x_R fraction of the total momentum of the dihadron D.

The contribution of the two-loop corrections and the double logarithmic factors (the quark form factors), which modify formula (3.142) of the hard scattering model near the phase space boundary, was investigated in [83].

3.6 QUARKS IN NUCLEI

3.6.1 Quark Degrees of Freedom of Nuclei

The role of the quark degrees of freedom in describing nuclear phenomena, especially those at high energies and momenta transfer, is a topical item of contemporary nuclear physics.

The starting point of the majority of the works in this direction is an attempt to answer the following question: does the simple fact that the nucleons, which make up known atomic nuclei, are composed of quarks allow us to understand better regularities of purely nuclear phenomena such as the high excitation behavior of nuclear matter, short-range nuclear structure, etc.?

In other words, can atomic nuclei be understood simply as multiquark systems? Under what conditions could quark degrees of freedom be exhibited explicitly?

These questions introduce us to a new and promising branch of research which could lead to radical changes in our notions of the world of atomic nuclei. Undoubtedly, the notion of colored quarks and gluons as the fundamental constituents of matter could shed new light on the nuclear properties of matter and on the nature of nuclear forces.

We shall now briefly discuss only some aspects of the quarks-in-nuclei problem referring to details to the original and review papers.

3.6.2 Nuclear Form Factors at High Momentum Transfer

The most straightforward indication of the quark structure of nuclei stems from the experimental data on evolution of deuteron's electromagnetic form factor at high momentum transfer. This is in a good agreement with the quark counting predictions [27, 28], i.e.

$$F_A(t) \sim t^{-(3A-1)}. \tag{3.155}$$

Here $3A$ is the total number of the valence quarks inside a nucleus composed of A nucleons. Formula (3.155) predicts the following exponential fall in the nuclear form factors, viz.

$$\begin{aligned} &F_{\mathrm{D}}(t) \sim t^{-5}, \\ &F_{^3\mathrm{He}}(t) \sim F_{\mathrm{H}^3}(t) \sim t^{-8}, \\ &F_{^4\mathrm{He}}(t) \sim t^{-11}, \ \text{etc.} \end{aligned} \tag{3.156}$$

Available experimental data enable us to trace a "leveling off" trend of the deuteron form factor $(q^2)^5 F_{\mathrm{D}}(q^2)$ which is normalized in an appropriate way.

The data under discussion have been obtained at SLAC (Stanford, USA) and they concern the effective form factor, $F_{\mathrm{D}}(t) = [A(t)]^{1/2}$, which is controlled by an elastic electron-deuteron scattering cross section [27]:

$$\frac{d\sigma}{dt} = \left(\frac{d\sigma}{dt}\right)_{\mathrm{Mott}} \left[A(q^2) + B(q^2)\tan^2\frac{\theta}{2}\right], \tag{3.157}$$

where ($\eta = q^2/4M_{\mathrm{D}}^2$)

$$A(q^2) = G_0^2(q^2) + \frac{2}{3}\eta G_1^2(q^2) + \frac{8}{9}\eta^2 G_2^2(q^2) \tag{3.158}$$

is the combination of squares of the electric dipole and magnetic quadrupole form factors, respectively.

As the SLAC data show, the deuteron electromagnetic form factor in the $0.8 \leqslant q^2 \leqslant 6.0$ (GeV) transferred momenta range behaves almost exponentially close to $F_{\mathrm{D}}(q^2) = (q^2)^{-5.0\pm0.5}$. This is in remarkable agreement with the quark counting rules and accords with an expected six-quark deuteron structure.[9] Note that for the nuclei, ^{3}He and ^{4}He, the momentum transfers are too sufficiently high as yet to come to any definite conclusions about their quark structures.

3.6.3 Deuteron as a Six-Quark System

The results above show that deuteron's short-range behavior, i.e. at high momentum transfers, is more adequately described in terms of quarks than nucleons.

Does it follow therefrom that deuteron should be considered as a six-quark system, bound together by color QCD forces like, say, a quark bag?

To anticipate a little, note that the analysis of this question leads to the concept "hidden color", a notion which plays a significant role in the physical interpretation of exotic multiquark systems (dibaryons, tribaryons, tetrabaryons, etc.) and in the description of the short-range features of nuclear matter.

Suppose we discuss the properties of a six-quark system as having deuteron quantum numbers and described by the quark bag model. All six quarks are assumed to belong to a ground state with the energy $E_q(j^P = 1/2^+) = 2.04/R$ ($m_q \simeq 0$) in a static, spherically symmetric cavity of radius R.

The structure of multiquark system's wave function is defined by the following basic principles:

(1) **the Pauli principle,**

i.e. a complete antisymmetrization of all quark variables including spin, isospin, and color;

(2) **the zero color request,**

which by virtue of an assumed colorlessness of a multiquark system allows only singlet group representations of the colored $SU^c(3)$-symmetry.

For a system of six quarks all of which are at the same energy level, these requirements lead to a wave function which can be written in

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9 An analysis of cumulative particle production processes in nuclear collisions at relativistic energies agrees with these conclusions [84].

a symbolic form as

$$\Psi(6q)=\frac{1}{\sqrt{20}}\Big(1-\sum_{\substack{i=1,2,3\\ j=4,5,6}} P_{ij}\Big)\ \underset{\text{colour}}{\begin{array}{|c|c|}\hline 1&4\\\hline 2&5\\\hline 3&6\\\hline\end{array}}\times\underset{\text{spin-isospin}}{\begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4&5&6\\\hline\end{array}}, \qquad (3.159)$$

where the P_{ij} are the permutation operators of the spin (S), isospin (I), and color quark indices. Moreover, the color and spin-isospin parts of the wave function ($SU^{SI}(4)$-symmetry) are denoted by the appropriate Young schemes.

Thus, the six-quark system described by the static, spherically symmetric quark bag model has a wave function which corresponds to the 50 component representation of the spin-isospin group of the SU^{SI} (4)-symmetry (the quark analogue of the Wigner symmetry group in nuclear spectroscopy).

The reduction

$$SU^{SI}(4)\to SU^{S}(2)\times SU^{I}(2) \qquad (3.160)$$

splits the 50-plet into a sum of $(2J+1,\ 2I+1)$ type terms, i.e.

$$50=(3,\ 1)+(1,\ 3)+(5,\ 3)+(3,\ 5)+(7,\ 1)+(1,\ 7). \qquad (3.161)$$

The first of these has the quantum numbers of deuteron, the second one, of the virtual ${}^1S_0$-state, etc.

In connection with the aforementioned experimental indications of deuteron's hard six-quark structure it is natural to ask, "can the six-quark system which has the wave function corresponding to the first term in expression (3.161) be identified with real deuteron?" To answer the question, we must analyze the baryon composition of the six-quark system under consideration. This concerns the relative weights of the various configurations obtained by dividing the initial six-quark system into two subsystems with three quarks each, i.e.

$$[6q]_{B=2}\to[3q]_{B=1}+[3q]_{B=1}. \qquad (3.162)$$

In this way we come inevitably across the notion of "hidden color", i.e. the presence in the decomposition of the six-quark system (3.162) of components in which individual three-quark subsystems have nonzero color (colored baryons B_c) [86].

Taking, as a concrete example, a six-quark state having deuteron's quantum numbers, we find the following for the relative weights of the various dibaryon configurations:

$$D_{6q}\to PN\ (1/9);\ \Delta\Delta\ (4/45);\ B_cB_c\ (4/5). \qquad (3.163)$$

Such a large value for the admixture of the hidden color configuration (80%) for the six-quark system, as described by the quark bag model, does not enable it to be identified with real deuteron.

3.6.4 Hidden Color in Nuclear Matter

To reconcile the results of the experiments that measure deuteron's form factor with the large weight of the hidden color component in a six-quark bag, it has to be assumed that real deuteron, which is mainly a weakly bound proton-neutron system, has only low, but finite, probability of going over into the six-quark state as described by the quark bag model. Qualitatively, this can be expressed by the simple formula

$$|D\rangle=a|PN\rangle_{\substack{\text{weakly bound}\\ \text{system}}}+b|6q\rangle, \qquad (3.164)$$

where $|a|^2+|b|^2=1$.

The admixture of the six-quark component in deuteron can be determined from the experimental data on deuteron's electromagnetic form factor at high momenta transfer. These yield [85] $\alpha=|b|^2\simeq 7\cdot 10^{-2}$. Using this value as well as the weights of various components in the six-quark bag with deuteron's quantum numbers, we can readily find an estimate of the exotic admixtures in a real deuteron, namely

$$\Delta\Delta\text{-component}\sim 0.6\%,\quad \text{"hidden color"}\sim 5.6\%. \qquad (3.165)$$

This estimate obviously has a qualitative character and does not allow for a number of important details, such as the presence of D-waves, the weak non-orthogonality of two vectors of state in formula (3.164), etc.

In any case we conclude that the ordinary, orthodox theory of nuclear matter, which does not allow for the color of fundamental constituents, quarks and gluons, is incomplete.

The search for the experimentally observed consequences of hidden nuclear color is one of the most important problems of relativistic nuclear physics.

One feasible way of research could be to study deuteron electrodisintegration at high energy and momenta transfer. This produces two baryons or hadron jets which are emitted at large angles and have the quantum numbers of the colored three-quark systems contained in the deuteron. Starting from an assumption about the smallness of the ranges corresponding to the dynamic mechanism of the processes under consideration, we must expect the following relation etween the various reaction channels.

$$d\sigma(\gamma^* D \to PN) : d\sigma(\gamma^* D \to \Delta\Delta) : d\sigma(\gamma^* D \to B_c B_{\bar{c}}) = 5:4:36. \quad (3.166)$$

We refer the reader for details to the original papers and now only show that the differential cross section of the deuteron electro-disintegration in exclusive channels behaves in the following manner in the high momentum transfer (Q) range, if allowance is made for QCD corrections:

$$\frac{d\sigma}{dQ^2} \sim \left(\frac{\alpha_s(Q^2)}{Q^2}\right)^{11} (\log Q^2)^{-\gamma}, \quad (3.167)$$

where

$$\gamma = \frac{2C_F}{\beta}(|h_D| + |h_{B_1}| + |h_{B_2}|)$$

is the anomalous dimension (h_D and h_{B_i} are the helicities of the deuteron and final baryons). The rate of the exponential fall-off in cross section (3.167) as momentum transfer increases agrees with the quark counting rules and consists of the canonical exponent which is equal to two plus $\sum_{i=D, B^1 B_2} (n_i - 1)$.

The search for hidden color in nuclei is difficult both because of the smallness of the relevant effects and because it is not possible directly to observe the hidden color. The discovery of more explicit manifestations of quark color and QCD fields in nuclei would, therefore, be of great importance. We would like to indicate in this connection the interesting possibility that highly excited states of nuclear matter might exist and which could be mainly pure hidden color excitations [29].

The analysis performed on this shows that at excitation energies of the order of $\Delta E \sim 0.5$ GeV, the state widths are controlled by the hidden color "discharge" via a single-gluon exchange and comprise the values $\Gamma_{c\bar{c}} \leqslant 10$ MeV, thus making them accessible for observation.

3.6.5 Quantum Theory of Nuclear Forces

When discussing the role of quark degrees of freedom in the description of purely nuclear interactions, it would be quite natural to ask whether it is possible to understand the principal features of nuclear forces from a knowledge of fundamental QCD interactions. Is it possible, in particular, to determine the nuclear parameters and hadron-hadron interaction constants such as, say, the pion-nucleon coupling constant, in terms of the principal QCD constants?

As we have shown above, the quark model provides a natural explanation of a short-range repulsive core of forces acting between two nucleons. As is indicated in a number of texts, quark exchange between different nucleons is one source of exotic nuclear components and, in addition, it leads to the occurrence of multi-nucleon forces in nuclear matter.

Below we shall give some of the arguments as to how QCD could explain the long-range components of nuclear forces. This is generally done in a phenomenological manner as an exchange of "white" (i.e. colorless) particles, i.e. pions, vector mesons, etc. This is a difficult problem, inasmuch as at long ranges we come across the strong coupling and confinement phenomena.

To simplify the issue, we shall try to answer a simple question, "do two quark bags interact at interbag separation distances exceeding two bag radii?" Obviously this should be understood in a positive way, since it is difficult to imagine a quantum object as having a rigidly fixed boundary. Indeed, the quantum fluctuations of a quark bag's surface over time will result in the formation of the interbag joins (where the color fields are non-vanishing) which would permit quarks to tunnel from one quark bag to another. The tunnelling probability has an exponentially low value at large relative separations, R, between the bag centers, namely, $\sim\exp(-\mu R)$ when $R \gg 2a$ (a is the bag radius), this corresponding to the Yukawa interaction fall-off law.

As is shown in [29], the evolution of a system of two quark bags separated by the distance R ($R > 2a$) is described by the amplitude:

$$\langle R | e^{-iHT} | R \rangle = \int d^3\sigma \int_{\sigma \text{ is fixed}} d\psi \, d\bar{\psi} dA e^{iS[\psi, A]}. \quad (3.168)$$

Here σ is the three-dimensional hypersurface in space-time that confines the domains with non-zero functions of the colored quark-gluon fields. Over large evolution times, T, amplitude (3.168) has an asymptote at $\sim\exp[-iTU(R)]$, where $U(R)$ is the interaction static potential of the bags. The amplitude of a single tunnelling act of a quark pair from different bags through the joint under the potential barrier $\Delta E \gtrsim 1/\tau$ (τ being the fluctuation time or joint thickness) is proportional to the product $T \cdot U(R)$. According to the quasiclassical nature of the tunnelling, this amplitude is exponentially small, namely,

$$U(R) \sim \exp[-S_0(R)], \quad (3.169)$$

where $S_0(R)$ is governed by the action integral taken over the region of joint and minimized over its thickness at a large and fixed R.

As can be shown [29]

$$S_0(R) \to \mu R \text{ as } R \to \infty, \quad (3.170)$$

where μ is defined by the energy spectrum minimum of the quark-antiquark system, i.e. $\mu = m_\pi$. The result is nothing but the long-range part of the Yukawa interaction which can thus be explained by fundamentals QCD.

3.7 BROKEN COLOR SYMMETRY AND INTEGRAL-CHARGED QUARKS

3.7.1 The Problem of Quark Charges

Even the first work on the three-triplet model have shown that colored quarks could possess both the fractional and integral electric and baryon charges.

The assumption of an exact color symmetry and the non-observability of color is compatible only with fractional charges. However, owing to quark confinement, the straightforward detection of the electric charge of individual quarks is a major problem.

Charge is known to play a double role in quantum electrodynamics. On one hand, it is an integral of motion, whose values govern the states of observed particles, and on the other, it is the interaction constant which is renormalized due to vacuum polarization effects. It is perhaps possible to speak of quark charges within the framework of QCD as only effective constants that characterize the electromagnetic quark interaction at sufficiently small separation distances. Therefore, the values of the electric charge on quarks can be established by comparison with the electric charges on leptons at the same separation distances.

In the case of integrally-charged quarks, color symmetry is not as yet being broken (either locally or globally) at least in electromagnetic interactions. Indeed, in models which have integral-charged quarks the electromagnetic current is the sum of the $SU^c(3)$-group singlet and octet terms, i.e.

$$J_\mu^{\text{em}} = J_\mu^{c=0}\,(1) + J_\mu^{c\neq 0}\,(8). \tag{3.171}$$

So, in the three-triplet model, i.e.

$$u = (u_1,\ u_2,\ u_3),\quad d = (d_1,\ d_2,\ d_3),\quad s = (s_1,\ s_2,\ s_3) \tag{3.172}$$

integral quark charges,

$$Q_u = (1,\ 1,\ 0);\ Q_d = Q_s = (0,\ 0,\ -1), \tag{3.173}$$

are chosen according to the formula

$$Q_q = Q_{\text{GMZ}} + Y_c. \tag{3.174}$$

Here Q_{GMZ} is the standard fractional charge in the Gell-Mann-Zweig model, and

$$Y_c = \frac{1}{\sqrt{3}}\lambda_8 = (1/3,\ 1/3,\ -2/3) \tag{3.175}$$

is one of the generators (the 8th component) of the color group.

Generally, we have in integral charged quark model

$$Q_q = Q_{\text{GMZ}} + Q_c;\ Q_c = T^a \cdot C_a, \tag{3.176}$$

where T^a is an array of the eight generators of the $SU^c(3)$-group in the basic representation. It follows from the hermiticity condition of electric charge, i.e. $Q_c^+ = Q^c$, that if an appropriate selection of the representation basis is made, the color part of the charge Q_c can be reduced to the diagonal form viz.

$$Q_c = aT_3 + b\frac{1}{\sqrt{3}}T_8 = \left(a + \frac{b}{3},\ -a + \frac{b}{3},\ -\frac{2b}{3}\right). \tag{3.177}$$

The requirement that the charges be integral leads to $2a$ and $(a + b)$ being integers. Quark charges do not exceed unity at $a = 0$ or $b = 1$ and this corresponding to selection (3.175). The same is valid when $a = b = -1/2$. The last case, however, also converts into (3.175) when the first and third colored quarks are permutated.

It should be stressed that according to the Gell-Mann-Nishijima formula integral-charged quarks must be correlated with integral baryon charges, i.e.

$$Q_q = T_3 + \frac{1}{2}B_q;\quad B_q = (1,\ 1,\ -1). \tag{3.178}$$

The dependence of quark charges on their color state obviously leads to a breakdown of global color symmetry in electromagnetic interactions. This is exhibited, for example, in the mass-splitting of colored quark triplets, etc. However, as has been demonstrated in the works cited, hadron neutrality relative to the color (i.e. matching the observed mesons and baryons with singlet color wave functions), guarantees the disappearance of any manifestation of the aforementioned breakdown of global color symmetry for all observed hadron characteristics (charge, magnetic moment, form factor, etc.).

However, the following two important items must be emphasized. First, it appears to be possible in electromagnetic interactions to excite particle states with a nonzero color, having an energy which is assumed to be fairly high, much higher compared to the mass spectrum of the observed mesons and baryons. Secondly, the integral nature of quark's electric and baryon charges makes it possible for them to transform into leptons and other observable particles. We would eventually draw some conclusion about the instability of quarks which could explain the negative results in the search for them both in the environment and in accelerator experiments. In this way, as Pati and Salam have demonstrated [89], quark instability would not contradict nucleons' high stability or the extreme suppression of the observed effects of baryon charge non-conservation.

In allowing for the QCD quark interaction described by non-Abelian gauge theory, the introduction of integral-charged quarks poses a new problem. The straightforward selection of a quark's electromagnetic current in the form of (3.171) would bring about a breakdown of the local gauge SU^c (3)-symmetry and, as a consequence, an inability to normalize the theory. The problem is removed once spontaneously broken color SU^c (3)-symmetry is considered. This requires new degrees of freedom to be introduced into the theory, e.g., Higgs color scalar fields.

3.7.2 Spontaneously Broken Color Symmetry

The simplest model of strong and electromagnetic interactions with the spontaneously broken color symmetry and integral-charged quarks is the gauge model based on the $SU^c(3) \times U(1)$-group with the scalar triplet φ_a.

The Lagrangian of the model has the following form [31]:

$$L = -\frac{1}{4}(F^0_{\mu\nu})^2 - \frac{1}{4}(F^a_{\mu\nu})^2 + L_\varphi + \bar{q}\left(i\hat{\partial} + \frac{g}{\sqrt{3}}\lambda^a\hat{A}^a + g'Y_q\hat{A}^0 - m_q\right)q + \bar{l}(i\hat{\partial} + g'Y_l\hat{A}^0 - m_l)\,l, \tag{3.179}$$

where

$$F^0_{\mu\nu} = \partial_\mu A^0_\nu - \partial_\nu A^0_\mu; \quad \hat{A}^0 = \gamma_\mu A^0_\mu; \\ F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu; \quad \hat{A}^a = \gamma_\mu A^2_\mu, \tag{3.180}$$

and L_φ is the Lagrangian of the Higgs scalar field:

$$L_\varphi = \left|\left(\partial_\mu - i\frac{g}{\sqrt{3}}\lambda^a A^a_\mu - ig'Y_\varphi A^0_\mu\right)\varphi\right|^2 - h(\varphi^+\varphi)^2 - m^2\varphi^+\varphi. \tag{3.181}$$

If the $SU^c(3) \times U(1)$-symmetry is not broken, the A^0_μ field has to be identified with a photon and so the electromagnetic properties of leptons and hadrons result in the following choice of hypercharges:

$$Y_l = Y_\mu = -1; \quad Y_u = 2/3; \quad Y_d = Y_s = -1/3.$$

When $m^2 < 0$, the symmetry is spontaneously broken, the nonzero vacuum averages of the φ_a fields being chosen in the form

$$\langle\varphi_a\rangle\,(0,\ 0,\ v/\sqrt{2}), \tag{3.182}$$

where v equals $(-m^2/n)^{1/2}$ in the tree approximation.

To match this choice, the new vacuum must be invariant with respect to the $SU'(2) \times U'(1)$-group that is composed of elements like

$$g = \exp i\,\{\lambda^a\omega^a + (\lambda^8 + 2/\sqrt{3})\,\omega\}, \quad a = 1,\ 2,\ 3. \tag{3.183}$$

As a result of a spontaneously broken symmetry, the vector bosons,

$$A^{\pm}_{45} = \frac{1}{\sqrt{2}}(A_4 \pm iA_5), \quad A^{\pm}_{67} = \frac{1}{\sqrt{2}}(A_6 \pm iA_7), \tag{3.184a}$$

$$U = A^8\cos\theta - A^0\sin\theta, \quad \tan\theta = \frac{g'}{g}\left(\frac{3Y_\varphi}{2}\right),$$

and the scalar field,

$$\chi = \frac{1}{\sqrt{2}}(\varphi_3 + \varphi_3^* - \sqrt{2}\,v), \tag{3.184b}$$

gain mass, viz.

$$m^2_{45} = m^2_{67} = g^2v^2/3, \quad m^2_\chi = 2\,|m^2| = 2hv^2 \\ m^2_v = \frac{4v^2}{g}\left(g^2 + g'^2\left(\frac{3Y_\varphi}{2}\right)^2\right). \tag{3.185}$$

The vector field,

$$A = A^0\cos\theta + A^8\sin\theta, \tag{3.186}$$

is the only one massless field to be associated with leptons. For this reason vector field (3.186) must be identified with a photon.

It can be easily derived that the electric charges on quarks and leptons in the case of a spontaneously broken symmetry are equal to

$$Q_e = Q_\mu = e; \quad Q_q = e\left(Y_q + Y_c\left(\frac{3Y_\varphi}{2}\right)\right), \tag{3.187}$$

where $e = g'\cos\theta$.

If we choose $Y_\varphi = 2/3$, then all the particles in the theory will have integral charges.

The theory under consideration differs from standard QED in two ways. First, additional scalar degrees of freedom have been introduced which are necessary to break the color symmetry. Second, as a result of the spontaneous breakdown, the lepton symmetries acquire an additional short-range interaction (the U-boson exchange), and photons become "colored", i.e. they interact in different ways with quarks of different colors.

The Lagrangian mass of the U-boson, m_U, can be chosen as the characteristic scale of the symmetry breakdown. It is known that at momentum transfers when $Q^2 \gg m_U^2$ the initial $SU^c(3) \times U(1)$-symmetry is reestablished and the effective quark charge becomes integral. In the high momentum transfer range the theory differs from QCD only in the occurrence of scalar particles.

The choice of the parameter m_U has been investigated in [31, 53], where the limit $m_U \ll 1$ GeV was established by analyzing the sum rules for the annihilation of e^+e^--pairs into hadrons [49], and by studying the radiative decays of heavy meson resonances and the corrections to the anomalous magnetic moments of muons.

Thus, a model with a spontaneously broken color symmetry and integral-charged quarks does not contradict the available experimental data provided the symmetry breakdown originates at values of momenta beyond the asymptotic freedom range or at distances comparable to the QCD confinement radius.

Lepton electrodynamics simply indicates that the scale of symmetry breakdown cannot be too large. Actually, when $m_U \gtrsim 1$ GeV, a weak coupling mode will act over all distances in strong interactions since the strong interaction constant α_s is "frozen" at the energies lower than a U-boson's mass, i.e. [51]

$$\alpha_s(Q^2) \simeq \alpha_s(m_U^2) \lesssim 1, \qquad Q^2 < m_U^2.$$

Thus, the U-boson must be observed (with a mass close to the Lagrangian mass) as a resonance in the $e^+e^- \to \mu^+\mu^-$-annihilation and having an electronic width

$$\Gamma_{e^+e^-} \simeq \frac{4\alpha^2}{9\alpha_s(m_U^2)} m_U \gtrsim 70 \text{ keV} \tag{3.188}$$

when

$$m_U = 1 \text{ GeV}, \quad \alpha_s(m_U^2) \simeq 0.3.$$

The results of an experimental search of the resonances in the e^+e^--annihilation at the current energies exclude this possibility. In addition, the assumption that m_U values could exceed a few GeV contradicts the achievements of the $SU^c(3)$-symmetry in classifying hadrons.

The occurrence of scalar charged fields in the models considered brings additional contributions to the asymptote of the total cross section of the annihilation of e^+e^--pairs into hadrons, viz.

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \to R_q^{ac} + R_s^{ac}, \tag{3.189}$$

where

$$R_q^{ac} = \sum_q Y_q^2 = 2 \quad \text{(for } u\text{-, } d\text{-, and } s\text{-quarks)}, \tag{3.190a}$$

and

$$R_s^{ac} = \frac{1}{4} \sum_\varphi Y_\varphi^2 = \frac{1}{3} \quad \text{(for a scalar triplet)}. \tag{3.190b}$$

It is important to emphasize that $\sqrt{s} \gg m_U$ is an insufficient condition for formulas (3.189) and (3.190) to be applied to the description of experimental data, i.e. it is necessary to require the corrections to be small including both the perturbational (logarithmic) and non-perturbational (exponential) corrections to the leading asymptote of the annihilation cross section.

The allowance made for two-loop and three-loop corrections in the $\overline{\text{MS}}$ renormalization scheme in the model which has a triplet of scalar fields ($Y_\varphi = 2/3$) [87] yields

$$R_q^{ac} = \sum_q Y_q^2 \left\{1 + \frac{\bar{\alpha}_s}{\pi} + 1.47\left(\frac{\bar{\alpha}_s}{\pi}\right)^2 + \ldots\right\}, \tag{3.191a}$$

$$R_s^{ac} = \frac{1}{3}\left\{1 + \frac{4\bar{\alpha}_s}{\pi} + 36.6\left(\frac{\bar{\alpha}_s}{\pi}\right)^2 + \ldots\right\}. \tag{3.191b}$$

Here the effective quark-gluon coupling constant $\bar{\alpha}_s$ is defined in the following way:

$$\frac{\bar{\alpha}_s}{\pi} = \frac{1}{b \ln s/\Lambda^2_{\overline{MS}}}, \tag{3.192}$$

where

$$b = \frac{1}{4}\left(11 - \frac{2}{3} N_f - \frac{1}{6} N_s\right),$$

and N_f and N_s are the numbers of quark and scalar triplets respectively.

In a rigorous sense, the value of the fundamental scale Λ in models with integral charged quarks differs from the relevant value in standard QCD owing to the additional scalar degrees of freedom. Choosing $\Lambda_{\overline{\text{MS}}} \simeq 100$ to 200 MeV as a reference point, which follows for example from deep inelastic lepton-nucleon scattering, we find that the contribution of the perturbational correction to the annihilation cross section within a scalar sector is significant at least in the energy range $\sqrt{s} \lesssim 10$ GeV.

This shows, perhaps, that there is a production threshold of the physical hadronic states that incorporate color scalar fields their structure and it lies at higher energies. We shall demonstrate below that a theoretical estimate of the mass scale of hadrons (i.e. hadrons having one or more quarks substituted by color scalars) agrees with this conclusion.

3.7.3 Scalar Quarks and New Hadrons

The above analysis of spontaneously broken color symmetry was based on an introduction of fundamental color scalars. One major implication of such models is that a new hadron family made up from quarks and colored scalars strongly bound by QCD forces could exist. The experimental observation of states like this would be commensurable with the discovery of a new flavor, namely, scalar quarks.

As was shown above, all the available experimental data does not preclude the possibility that color symmetry could break down spontaneously, provided the scale of the breakdown in terms of mass is sufficiently small, e.g.

$$\langle \varphi \rangle = v \ll 1 \text{ GeV}.$$

Thus, scalar quarks in models with the spontaneously broken color symmetry should have small mass.

At first sight, low-mass colored scalars, or scalar quarks, should result in new hadrons that have masses of the order of the scale characteristic of low-energy hadron physics ($\sim$1 GeV). This, however, contradicts experiment.

Below we will show that thanks both to the large perturbational corrections in QCD with scalar quarks and to the nonperturbational effects due to a possible scalar condensate, i.e.

$$\langle \alpha_s^{1/b} \varphi^+ \varphi \rangle \simeq -(1 \text{ GeV})^2, \tag{3.193}$$

the mass of a new hadron would be of the order of several tens of a GeV. Thus, the new hadrons could, in principle, be discovered and investigated at accelerators like PETRA, LEP, SPS, and UNK in colliding beams experiments.

A hadron's properties are substantially governed by the structure of the vacuum state. Introducing low-mass scalar quarks into CQD changes the vacuum structure, since in addition to the well-known quark and gluon condensates [50] viz.

$$\langle \bar{q}q \rangle \simeq -(0.25 \text{ GeV})^3, \quad q = u,\, d, \tag{3.194}$$
$$\langle \alpha_s G^a_{\mu\nu} G^a_{\mu\nu} \rangle \simeq +(0.5 \text{ GeV})^4,$$

it emerges that there is a scalar condensate $\langle \varphi^+ \varphi \rangle$.

The value of $\langle \varphi^+ \varphi \rangle$ cannot be determined at the present time in a purely theoretical way. It is, like the quark and gluon field condensates, a phenomenological parameter. We do have, from dimensional considerations, that $\langle \varphi^+ \varphi \rangle = c\Lambda^2$, where Λ is the fundamental scale (the inverse confinement radius) of QCD.

However, QCD coupling is known to be substantially different for different channels and is characterized (depending on the channels' quantum numbers) by different effective scales $\Lambda(\alpha)$.

The computation of the three-loop corrections to the total cross section of the annihilation of e^+e^--pairs into hadrons with fundamental scalar triplets shows that the effective scale in the production channel for the $\varphi^+\varphi^-$-pairs is about an order of magnitude larger than the corresponding scale in the quark-antiquark channel, i.e. $\Lambda(\varphi^+\varphi) \simeq$ $\simeq 9\Lambda(\bar{q}q)$ in the $\overline{\text{MS}}$ renormalization scheme. This plays a significant role in the subsequent discussion[10]. The mass of a bound state of a colored scalar is defined by the average $\langle \varphi^+ \varphi \rangle$ in the same way as the mass of a meson made up from quarks and antiquarks is defined mainly by the quark condensate.

Further we shall consider the bound states of the scalar and normal quarks to have the quantum numbers of the following local currents:

$$\begin{aligned} \rho_s &= \varphi^+ \overleftrightarrow{\nabla}_\mu \varphi, \\ \nabla_\mu &= \partial_\mu - i\frac{g}{2}\lambda^a A^a_\mu - ig'\frac{2}{3}A^0_\mu, \\ \pi_s &= \alpha_s^{1/5}(\varphi^+\varphi), \\ \chi_s &= \alpha_s^{1/2b}(\varphi^+ q), \end{aligned} \tag{3.195}$$

where $b = \frac{1}{4}\left(11 - \frac{2}{3}N_f - \frac{1}{6}N_s\right)$. The quantities $\alpha_s^{1/b}$, $\alpha_s^{1/2b}$ provide the renorm-invariance of the two-point Green functions that correspond to the currents π_s and χ_s in the main logarithmic approximation.

The current and a corresponding particle ρ_s which we call the vector phionium, have quantum numbers $J^{PC} = 1^{--}$. Like the ρ-meson, the ρ_s-particle may be produced in e^+e^- collisions and will manifest itself as a resonance.

The quantum numbers of the scalar phionium π_s are $J^{PC} = 0^{++}$. The bound state of normal and scalar quarks χ_s, has spin 1/2 and can be called the "white" quark. The electric charges of ρ_s and π_s are equal to zero and the charge on the χ_s depends on the flavor of the corresponding quark, for example, $Q_{\varphi^+u} = 0$, $Q_{\varphi^+d} = -1$, etc.

The properties of the bound states of the scalar quarks (3.195) can be studied using QCD's sum rules taking into account the nonperturbational effects.

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10 We should point out in this connection, what was mentioned above (see Sec. 3.3.2) when we discussed the annihilation model of meson decays concerning the dependence of the effective radius of the mesons' quark-antiquark systems on their quantum numbers [35, 36].

Using the well-known techniques of operator expansion and QCD's sum rules [49, 50] we present the basic relations:

$$\frac{1}{M^2}\int_0^\infty ds\, e^{-s/M^2} R_{\rho_s}(s) = \frac{1}{3}\Big\{1+\frac{4}{\pi}\alpha_s(M^2,\ \Lambda_1(\rho_s))$$

$$+\frac{32\pi^2}{M^2}\sigma\,[\alpha_s(M^2,\ \Lambda_2(\rho_s)]^{-1/b}+O(1/M^4)\Big\}, \tag{3.196a}$$

$$\frac{1}{M^2}\int_0^\infty ds\, e^{-s/M^2} R_{\pi_s}(s) = [\alpha_s(M^2,\ \Lambda_1(\pi_s)]^{2/b}$$

$$+\frac{32\pi^2}{M^2}\sigma\,[\alpha_s(M^2,\ \Lambda_2(\pi_s)]^{1/b}+O(1/M^4), \tag{3.196b}$$

$$\frac{1}{M^2}\int_0^\infty ds\, e^{-s/M^2} R_{\chi_s}(s) = [\alpha_s(M^2,\ \Lambda_1(\chi_s)]^{1/b}$$

$$+\left(1+\frac{1}{3\pi}\alpha_s\right)\frac{32\pi^2}{3M^2}\sigma+O(1/M^4), \tag{3.196c}$$

where

$$\sigma \equiv \langle \alpha_s^{1/b}\varphi^+\varphi\rangle, \quad \alpha_s(M^2,\ \Lambda) \equiv \pi/b\ln\frac{M^2}{\Lambda^2}.$$

The quantities $R_{\rho_s}(s)$, $R_{\pi_s}(s)$, and $R_{\chi_s}(s)$ are positive definite spectral densities related to the two-point Green functions which have the corresponding currents ρ_s, π_s, and χ_s, e.g.

$$i\int \langle 0|\,T\,(\rho_s^\mu(x)\,\rho_s^\nu(0))\,|0\rangle\, e^{iqx}\, d^4x$$

$$=(q_\mu q_\nu - q^2 g_{\mu\nu})\,R_{\rho_s}(q^2), \tag{3.197}$$

etc.

The scale parameters Λ_i, which are computed allowing for the perturbational corrections, are equal to

$$\Lambda_1(\rho_s)\cong g\gamma\Lambda_{\overline{MS}},\quad \Lambda_2(\rho_s)\cong 2.7\,\gamma\Lambda_{\overline{MS}},$$
$$\Lambda_2(\pi_s)\cong 3.2\gamma\Lambda_{\overline{MS}},\quad \gamma\cong 1.33 \tag{3.198}$$

(γ being the Euler constant), respectively.

The sum rules for the new hadrons differ qualitatively from the similar quarkonium relations by the contribution of the non-perturbational effects arising here from the terms which are of the order of $1/M^2$, rather than of $1/M^4$. This fact has a simple explanation and is associated with the dimensions of the fields since dim $\langle\varphi^+\varphi\rangle = 2$, i.e. the term with the lowest dimension in the sum rules is proportional to $\langle\varphi^+\varphi\rangle/M^2$ for the scalar field. For the fermion fields, dim $\langle\bar{q}q\rangle =$ $= 3$ and the expansion starts from the $m_q\langle\bar{q}q\rangle/M^4$ and $\langle G_{\mu\nu}^2\rangle/M^4$

term. It is this difference which leads to the significantly larger mass of the new hadrons (the phioniums) as compared with the masses of quarkoniums.

An analysis of the sum rules (3.196) leads to the following estimates for the mass of a phionium:

$$m_{\rho_s'} \simeq (45\pm 5)\,(-\sigma)^{1/2},$$
$$m_{\pi_s}-m_{\chi_s}\simeq\frac{1}{\sqrt{3}}\,m_{\rho_s}, \tag{3.199}$$

where the quantity σ varies within the limits

$$(0.5\ \ \text{GeV})^2 < |\,\sigma\,| < (2\ \text{GeV})^2.$$

Unfortunately, a numerical value for the scalar potential cannot be extracted from the experimental data available, since it does not significantly influence the parameters of the known resonances of quarks. Assuming, for example, that the value for the scalar condensate is related to the phionium scale ($\Lambda_1(\rho_s)\simeq 9\gamma\Lambda_{\overline{MS}}$) in the same way as the quark condensate value is related to the quarkonium scale ($\Lambda_1(\rho)\simeq 1.4\gamma\Lambda_{\overline{MS}}$), we can obtain the following for $\Lambda_{\overline{MS}}\simeq 100$ MeV:

$$\langle\varphi^+\varphi\rangle^{1/2}=\langle\bar{q}q\rangle^{1/3}\cdot\frac{\Lambda_1(\rho_s)}{\Lambda_1(\rho)}\simeq 1.5\ \text{GeV}. \tag{3.200}$$

Thus, the mass characteristic of the new hadrons is significantly larger than that of hadrons constructed from low-mass quarks and can be up to some tens of a GeV.

In a rigorous sense, a small value for the scalar condensate, say, $(-\sigma)^{1/2}\sim 0.5$ GeV, cannot presently be excluded by the theory. In this case the new particles would have been observed in the experiments in the accelerators now in operation. Note that the experimental search for long-lived ($\tau > 10^{-13}$ sec) particles in the e^+e^--annihilation processes [88] yields a limit for the "white" quark mass, namely,

$$m_{\chi s} > 12\ \text{GeV}.$$

In the model under consideration the lightest χ_s-particles must be absolutely stable by virtue of baryonic number conservation. When $SU^c(3)\times U(1)$-theory is embedded into grand unification models, the stability property of the lightest "white" quarks vanishes. In theories with "late" unification (such as $SU(5)$, $SO(10)$), the lifetime τ of χ_s-particles is about 10^{30} years, and in theories with small unification scales (for example, $SU^4(4)$, $SU^2(8)$, etc., [89, 90]) it is about 10^{-8}-10^{+6} sec.

As has been noted above, colored scalar particles with small Lagrangian masses occur naturally in theories which permit weakly broken color symmetry.

A question arises as to how color breakdown influences the properties of the new hadrons, if they do in fact exist.

In this case the vacuum average $\langle \varphi^{*a}\varphi_b \rangle$, in addition to a contribution from the color symmetry's scalar condensate, which has a negative sign (as was shown above), contains a positive addend due to the color breakdown, namely,

$$\langle \varphi^{*a}\varphi_b \rangle = \frac{1}{3}\delta_b^a \langle \varphi^{*\prime}\varphi' \rangle + \frac{1}{2} v^a v_b,$$
$$\varphi_a = \varphi_a' + \frac{1}{\sqrt{2}} v_a, \quad \langle \varphi_a' \rangle = \frac{1}{\sqrt{2}} v_a \neq 0. \qquad (3.201)$$

Obviously, the properties of the new hadrons in theories with spontaneously broken and those with exact color symmetries are practically coincident when $v^2 \ll (\langle \varphi^+\varphi \rangle)$.

Yet the so-called *U*-boson, which occurs in the model and which together with the γ-quantum diagonalizes the mass matrix of the vector particles, is perhaps unobservable, i.e. is not a physical state when there is a weak color breakdown.

The experimental implications of theories with the spontaneously broken color symmetry are discussed in more detail in [53], to which we refer the reader.

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