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Development of an algebraic theory of the collective motions in atomic nuclei

Specialization: 01.04.16 – nuclear and particle physics

ABSTRACT
for acquiring the scientific degree
of doctor of the physical and mathematical sciences

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1 Introduction

1.1 Actuality of the problem

The non-relativistic nuclear many-body problem requires the solving of the many-particle Schrodinger equation

$$H\Psi(r_1, \ldots, r_A) = E\Psi(r_1, \ldots, r_A), \tag{1}$$

which is formidable task. Not able to solve the equation (1) because of its complexity, we replace it by the following one

$$H_0\Psi_0 = E\Psi_0, \tag{2}$$

for the restricted Hamiltonian H_0, which we hope to solve. The Hamiltonian H_0 is an appropriately chosen part of H that corresponds to some, e.g. collective, effects of nuclear dynamics using some particular criteria. For instance, as such a criterium can be used the irreducible decomposition of H with respect to some group G. Then, an additional integral of motion (or in quantum-mechanical terms, exact quantum number) appears. The physical picture of the nuclear structure that is described by the wave function Ψ_0 thus provides a dynamical model of the nucleus, generated by the Hamiltonian H_0. Thus, an alternative to the exact solution of the many-particle Schrodinger equation is the algebraic approach to nuclear structure, based on the symmetry.

Symmetry is an important concept in physics. In finite many-body systems, such as atomic nuclei, it appears as time reversal, parity, and rotational invariance, but also in the form of dynamical symmetries [1, 2]. Dynamical symmetry is arguably the most fundamental concept in physics. In nuclear physics, dynamical symmetry was already used implicitly in the early works of Heisenberg [3] and Wigner [4] and more explicitly exploited by Elliott [5] in his $SU(3)$ shell model of nuclear rotations and in the seniority scheme for coupling nucleons in pairs [6, 7, 8]. However, the most useful and extensive application of the concept of dynamical symmetry was within the framework of the popular Interacting Boson Model (IBM) [9]. It turns out that a wide range of dynamical models are solvable by virtue of underlying dynamical symmetries. Indeed, almost all models of nuclear structure have an algebraic structure. In the fully algebraic approach, in which the interaction and physical operators can be expressed solely in terms of the invariant operators of a single group-subgroup chain, according to which the basis states are classified, one obtains analytical solutions for the eigenvalues and eigenfunctions. Such limiting cases are referred to as dynamical symmetry limits or simply *dynamical symmetries*.

It is worth noting that the construction of the first successful models of nuclear structure was due to the observation of patterns in nuclear data and the emergence of simplicity in their interpretation. Patterns are signatures of underlying symmetries and their recognition can be of enormous help in understanding the properties of atomic nuclei. Several symmetries of the nucleus as a system of nonrelativistic interacting nucleons that give rise to the total angular momentum, parity, third projection of the isospin quantum number, and permutational symmetry of nuclear states are in common use in nuclear structure physics. These are the exact integrals of motion which the nuclear wave function must satisfy. This make it possible to construct models of nuclear structure that preserve the exact integrals of motion. Such models are called kinematically correct models [10].

Besides in its simplest, phenomenological form, the algebraic approach exists also in a microscopic version. A model is called fully microscopic (or simply microscopic) if the antisymmetrization is completely involved in the construction of the many-fermion Hilbert space of nucleus. The Lie algebra of observables of the model is then expressible in terms of many-particle coordinates (position, momentum and spin). Moreover, in the microscopic models, the collective effects are derived from all the single-particle degrees of freedom. As will see further, this can be done in a very elegant way, using the group theory, by restricting the model many-body Hamiltonian to the Hilbert state space with a definite $O(A-1)$ symmetry, or which is the same, by projecting its $O(A-1)$-scalar part [11, 12]. In particular, the restricted collective spaces $\mathbb{H}^{(\Lambda\omega)}$ that are considered throughout the dissertation are characterized by a definite $O(A-1)$ (or equivalent to it symplectic bandhead) symmetry (ω), which appears as an additional integral of motion. We will demonstrate that for the phenomenological collective models their Hamiltonians act on the non-physical Hilbert spaces $\mathbb{H}^{(\Lambda\omega)}$ with $(\omega) = (0)$, obtaining in this way a very simplified collective dynamics. For the microscopic collective models, the collective irreducible spaces in which the model Hamiltonians act are characterized by a definite $O(A-1)$ symmetry $(\omega) \neq (0)$, thus providing the state spaces to be Pauli allowed subspaces of the many-particle nuclear Hilbert space.

The many roles [13, 14, 15] that the symmetry plays at each stage in the process of understanding the nuclear phenomena in terms of interacting protons and neutrons are considered in more details. In the present work, we use the following operational sequence in understanding the nuclear structure [15]:

1) Observe the phenomena by recognizing patterns in the experimental data;

2) Construct phenomenological models to describe the patterns;

3) Define the limitations and the domain of validity (refine the models);

4) Understand a given phenomenological model in microscopic terms by embedding it in the microscopic nuclear theory (shell model) which can also be refined;

5) Use the symmetry of obtained microscopic model to define an appropriate shell-model coupling scheme and relevant model spaces, as well as to identify the kinds of shell-model configurations needed to describe the collective excitations in nuclei.

From the above sequence, it follows that one can give a collective model a microscopic interpretation by embedding it in the shell model; i.e., by expressing it as a submodel of the shell model. The embedding problem becomes straightforward once it is recognized that both the shell model and the collective models are algebraic models with dynamical groups. Thus, a collective model becomes a submodel of the shell model if its dynamical group is expressed as a subgroup of a dynamical group for the shell model. An example of a complete algebraic model that is a submodel of the shell model is provided by the Elliott's $SU(3)$ model [5].

The present work explores the properties of atomic nuclei that are indicators of the emergence of simple dynamics associated with the symmetries that are available to a nucleus as a many-body system. In particular, it focuses on the dynamical symmetries associated with the nuclear collective motions. Symmetries that provide good quantum numbers, or even approximately good quantum numbers, are obviously useful. However, symmetry can be used to decompose the nuclear Hilbert state space into a sequence of invariant subspaces with respect to a given dynamical algebra, ordered such that their contributions to the observed states of nuclei are of decreasing importance. A standard example of this is given by the symmetry of the three-dimensional spherical harmonic oscillator shell model which proves to be particularly relevant for many states of near closed-shell and singly-closed shell nuclei. An alternative, based on the dynamical symmetry of the six-dimensional harmonic oscillator which proves to be much more appropriate for the rotational states of heavy deformed nuclei, is presented in this work on both phenomenological and microscopic shell-model levels.

1.2 The aim and tasks

The **ultimate aim** of the present dissertation is the development of a microscopic theory of nuclear collective motions aimed at understanding the nuclear collective dynamics in terms of interacting protons and neutrons. In realizing this aim we use the following powerful **algebraic strategy** of constructing such a microscopic theory. It starts with a phenomenological model in terms of a Lie algebra of observables, which is able to capture many of the collective properties of atomic nuclei. Further we seek a microscopic, many-particle realization of this algebra in terms of the position and momentum coordinates of the particles of the system. It then remains to identify the relevant shell-model irreducible representations of this algebra (known as a spectrum generating algebra or simply dynamical algebra) to obtain a microscopic version of the model.

Following the above strategy in Part I we first develop and apply the symplectic and orthosymplectic extensions of the phenomenological Interacting Vector Boson Model (IVBM) [16] for the description of various nuclear phenomena in heavy strongly deformed nuclei. In IVBM, it is assumed that the low-lying collective dynamics can be described by means of two types of vector bosons (phonons, elementary excitations). Various applications of the IVBM in this part can be considered as an application of the symplectic-based effective theory of nuclear collective motions in the two-component nuclear systems by using effective interactions and effective transition operators in appropriately chosen truncated collective spaces. Further, in Part II, we propose and develop a fully microscopic Proton-Neutron Symplectic Model (PNSM) of collective excitations that aims the microscopic shell-model description of collective properties of heavy nuclei, consisting of protons and neutrons.

In pursuing our strategy, the following tasks are formulated in achieving the aim of present dissertation:

• Clarifying the role of the symplectic $Sp(12, R)$ dynamical group in the IVBM by applying the new dynamical symmetries, which arise as a result of the symplectic extension of the model.

• Study of the first few positive- and negative-parity collective rotational bands, for which new experimental data are obtained up to very high angular momenta, in the heavy even-even nuclei from the rare-earth and actinide mass regions, including some of the fine structure energy level effects.

• Study of the possible shapes in the IVBM. Construction of the phase diagram of IVBM.

• Examining the possibility of obtaining triaxial shapes in the phase structure of the IVBM.

• Study of the triaxiality in atomic nuclei within the framework of the IVBM within its irreducible symplectic collective space.

• Introduction and application of the orthosymplectic extension of the IVBM, which incorporate the fermion degrees of freedom, for studying the collective properties of heavy odd-mass and odd-odd nuclei. Simultaneous description of the low-energy collective spectra in the even-even, odd-A and doubly odd nuclei within the supersymmetric (orthosymplectic) extension.

• Study of the chiral rotation in doubly odd nuclei within the orthosymplectic extension of the IVBM.

• Developing of an algebraic microscopic theory of nuclear collective motions in the two-component many-particle proton-neutron nuclear systems.

• Examining the type and number of collective and intrinsic degrees of freedom in the many-particle two-component proton-neutron nuclear systems.

• Examining the possible collective flows in the PNSM and revealing of its dynamical content. Construction of the simplest kinematically correct nuclear wave functions.

• Understanding the collective dynamics within the proton-neutron symplectic model in microscopic and macroscopic terms.

• Constructing of the shell-model representations of the PNSM in the many-particle Hilbert space.

• Study of the macroscopic, hydrodynamical limits of the PNSM.

• Development of the computational techniques of PNSM, including the explicit calculation of the required isoscalar factors and matrix elements of physically interesting operators.

• Application of the newly proposed PNSM for obtaining the microscopic shell-model structure of the low-lying positive-parity states in the strongly deformed heavy even-even nuclei.

• Obtaining of the microscopic shell-model structure of the low-lying negative-parity states in strongly deformed heavy even-even nuclei within the framework of the PNSM.

• Study of the low-lying collective $E1$ dipole strengths in the extended PNSM.

• Study of adiabatic decoupling of the rotational dynamics from other degrees of freedom. Examining the role of emerging quasi-dynamical symmetries.

1.3 Scientific novelty

In the dissertation new results are obtained for the first time, the main of which are:

• The symplectic extension of the IVBM is developed. The new dynamical symmetries which appear as a result of this extension are studied and applied for description of different nuclear phenomena in heavy even-even nuclei.

• The phase structure of the IVBM is obtained for the first time. The possibility of obtaining triaxial shapes is considered.

• The supersymmetric (orthosymplectic) extension of the IVBM is developed, which allow the collective properties of heavy odd-mass and doubly odd nuclei to be described.

• The orthosymplectic extension is applied for the first time for the description of chiral doublet bands in doubly odd nuclei.

• A fully microscopic PNSM of nuclear collective motions is formulated for the first time by considering the possible collective flows and symplectic geometry of the two-component nuclear system.

• The macroscopic limits of the PNSM, which appear for large dimensional representations, are further obtained. As a result, two simplified models of nuclear collective motion, expressed in simple geometrical terms, arise as macroscopic limiting cases of the PNSM.

• Computational techniques for the practical application of the PNSM at $U(6)$ level are developed, including the explicit calculation of the required isoscalar factors and matrix elements of physically interesting operators.

• The PNSM is applied for the first time in obtaining the microscopic shell-model structure of the positive-parity states in well deformed heavy even-even nuclei, namely ^{166}Er, ^{154}Sm and ^{238}U.

• Revealing of the dynamical and quasi-dynamical symmetries of the underlying proton-neutron collective dynamics in the microscopic structure of positive-parity states in well deformed heavy even-even nuclei.

• The first application of the symplectic-based shell-model approach to the structure of negative-parity states in heavy nuclei, in particular to ^{154}Sm and ^{238}U.

• The central extension of the proton-neutron symplectic model with the semi-direct structure $WSp(12, R) \equiv [HW(6)]Sp(12, R)$ is proposed, which allows to include explicitly in the theory various many-particle correlations which lie outside the enveloping algebra of $Sp(12, R)$.

• Study of the low-lying electric dipole strengths in heavy even-even nuclei in the extended PNSM.

1.4 Scientific and practical significance of the obtained results

By applying different dynamical symmetry limits of the IVBM in Part I, we are able to clarify the role of $Sp(12, R)$ as a dynamical group of the possible collective excitations in the two-component nuclear systems. The latter, in turn, allows us to be more precise in the constructing of relevant model Hamiltonians, adequate for the description of a more complete proton-neutron collective dynamics.

The obtained results in Part II are an important step towards the development of a practical, computationally tractable microscopic theory of nuclear collective motion in the heavy nuclei. From the conceptional point of view, the proposed symplectic-based approach provides a general shell-model framework for studying microscopically the observed collective excitations in the two-component many-particle nuclear systems and represents a further step towards the development of a more general and comprehensive microscopic theory of collective motion in atomic nuclei consisting of protons and neutrons.

The practical significance of the obtained results consists of the fact that the algebraic microscopic theory of proton-neutron collective excitations, which is formulated in the present dissertation, opens the path for studying the microscopic structure of strongly deformed heavy nuclei, for which the conventional shell-shell model techniques are very formidable even for the modern computing facilities. It allows to identify effectively the kinds of shell-model configurations needed to describe the rotational states of strongly deformed heavy nuclei and provides a practical way to involve different many-particle correlations (e.g., dipole, quadrupole, octupole, etc.) of interest in the theory.

1.5 Method

In the present work, we use the elegant part of mathematics known as a representation theory of Lie algebras and Lie groups or simply a group theory. The group theory is the precise way of expressing mathematically different symmetry patterns, but for the present purposes it can just be thought as a multi-dimensional extension of the familiar three-dimensional angular momentum techniques.

The group representation approach allows the construction of a Hamiltonian of a system which is, or nearly so, invariant under a certain group of symmetry transformations. It then allows one to construct basis of states realizing the symmetry and to calculate explicitly the matrix elements of different physically interesting operators, themselves classified by the symmetry. This further allows many properties of atomic nuclei to be investigated using algebraic models, in which one obtains bands of collective states which span irreducible representations of the corresponding dynamical group.

1.6 Main results of the dissertation

The main results of the present dissertation which are raised to the defence are:

• The symplectic extension of the IVBM is developed. The new dynamical symmetries which appear as a result of this extension are studied and applied for description of different nuclear phenomena in heavy even-even nuclei.

• The phase structure of the IVBM is obtained for the first time. The possibility of obtaining triaxial shapes is considered.

• The supersymmetric (orthosymplectic) extension of the IVBM is developed, which allow the collective properties of heavy odd-mass and doubly odd nuclei to be described.

• The orthosymplectic extension is applied for the first time for the description of chiral doublet bands in doubly odd nuclei.

• A fully microscopic PNSM of nuclear collective motions is formulated for the first time by considering the possible collective flows and symplectic geometry of the two-component nuclear system.

• The macroscopic limits of the PNSM, which appear for large dimensional representations, are further obtained. As a result, two simplified models of nuclear collective motion, expressed in simple geometrical terms, arise as macroscopic limiting cases of the PNSM.

• Computational techniques for the practical application of the PNSM at $U(6)$ level are developed, including the explicit calculation of the required isoscalar factors and matrix elements of physically interesting operators.

• The PNSM is applied for the first time in obtaining the microscopic shell-model structure of the positive-parity states in well deformed heavy even-even nuclei, namely ^{166}Er, ^{154}Sm and ^{238}U.

• The first application of the symplectic-based shell-model approach to the structure of negative-parity states in heavy nuclei, in particular to ^{154}Sm and ^{238}U.

• Study of the dynamical and quasi-dynamical symmetries of the underlying proton-neutron collective dynamics in the microscopic structure of positive- and negative-parity states in strongly deformed heavy even-even nuclei.

• The central extension of the proton-neutron symplectic model with the semi-direct structure $WSp(12,R) \equiv [HW(6)]Sp(12,R)$ is proposed, which allows to include explicitly in the theory various many-particle correlations which lie outside the enveloping algebra of $Sp(12,R)$.

• Study of the low-lying collective $E1$ dipole strengths in strongly deformed heavy even-even nuclei within the extended PNSM shell-model framework.

1.7 Approbation

The results obtained in the present work were reported on many international conferences (see the list below) and were also discussed at different seminars, given at the Laboratory of Theoretical Physics (Dubna, Russia) and the Institute of Nuclear Research and Nuclear Energy (Sofia, Bulgaria). Concrete titles, places and time of some of these conference papers can be found below in the list of references [B1-B11].

1.8 Publications

The main results of dissertation are published in the referred journal articles [A1-A19] and in the full text conference proceedings [B1-B11] (see the List of publications on which the dissertation is based on). These papers are published as follows: Phys. Rev. C - **11**, Eur. Phys. J. A - **3**, J. Phys. G: Nucl. Part. Phys. - **1**, Nucl. Phys. A - **1**, Int. J. Mod. Phys. E - **3**, J. Phys.: Conf. Ser. - **3**, EPJ Web of Conferences - **2**, AIP Conf. Proc. - **1**, conference book's proceedings - **5**.

1.9 List of scientific publications on which the dissertation is based

Publications in refereed journals

[A1] H. Ganev, V. P. Garistov, and A. I. Georgieva, *Description of the ground and octupole bands in the symplectic extension of the interacting vector boson model*, Phys. Rev. **C 69**, 014305 (2004).

[A2] H. G. Ganev and A. I. Georgieva, *Transition probabilities in the U(6) limit of the Symplectic Interacting Vector Boson Model*, Phys. Rev. **C 76**, 054322 (2007).

[A3] H. G. Ganev, *Collective states of the odd-mass nuclei within the framework of the interacting vector boson model*, J. Phys. G: Nucl. Part. Phys. **35**, 125101 (2008).

[A4] H. G. Ganev, A. I. Georgieva, S. Brant, and A. Ventura, *New description of the doublet bands in doubly odd nuclei*, Phys. Rev. **C 79**, 044322 (2009).

[A5] H. G. Ganev and S. Brant, *Structure of the doublet bands in doubly odd nuclei: The case of* ^{128}Cs, Phys. Rev. **C 82**, 034328 (2010).

[A6] H. G. Ganev, *Phase Structure of the Interacting Vector Boson Model*, Phys. Rev. **C 83**, 034307 (2011).

[A7] H. G. Ganev, *Triaxial shapes in the interacting vector boson model*, Phys. Rev. **C 84**, 054318 (2011).

[A8] H. G. Ganev, *Transition probabilities in the U(3,3) limit of the symplectic IVBM*, Phys. Rev. **C 86**, 054311 (2012).

[A9] H. G. Ganev, *Axial asymmetry in the IVBM*, Eur. Phys. J. **A 49**, 55 (2013).

[A10] H. G. Ganev, *Simultaneous description of low-lying positive and negative parity bands in heavy even-even nuclei*, Phys. Rev. **C 89**, 054311 (2014).

[A11] H. G. Ganev, *Collective degrees of freedom of the two-component nuclear system*, Eur. Phys. J. **A 50**, 183 (2014).

[A12] H. G. Ganev, *U(6)-phonon model of nuclear collective motion*, Int. J. Mod. Phys. **E 24**, 1550039 (2015).
[A13] H. G. Ganev, *Shell-model representations of the proton-neutron symplectic model*, Eur. Phys. J. **A 51**, 84 (2015).
[A14] H. G. Ganev, *Some* $U(n_1+n_2) \supset U(n_1) \otimes U(n_2)$ *isoscalar factors*, Int. J. Mod. Phys. **E 26**, 1750057 (2017).
[A15] H. G. Ganev, *Matrix elements of the proton-neutro symplectic model*, Int. J. Mod. Phys. **E 27**, 1850021 (2018).
[A16] H. G. Ganev, *Structure of the low-lying positive-parity states in* ^{154}Sm, Phys. Rev. **C 98**, 034314 (2018).
[A17] H. G. Ganev, *U(6) quasi-dynamical symmetry in* ^{238}U, Nucl. Phys. **A 987**,112 (2019).
[A18] H. G. Ganev, *Microscopic structure of the low-lying negative-parity states in* ^{154}Sm, Phys. Rev. **C 99**, 054305 (2019).
[A19] H. G. Ganev, $E1$ *transitions in the extended proton-neutron symplectic model*, Phys. Rev. **C 99**, 054304 (2019).

Full-texts in conference proceedings

[B1] H. G. Ganev, A. I. Georgieva, S. Brant, and A. Ventura, *Structure of the doublet bands in doubly odd nuclei with mass around 130*, Proceedings of the XXVIII International Workshop on Nuclear Theory (June 22-27, 2009, Rila Mountains, Bulgaria), ed. S. Dimitrova, Printed by BM Trade Ltd., Sofia, Bulgaria 2010, pp. 177.
[B2] H. G. Ganev and A. I. Georgieva, *Simultaneous Description of Even-Even, Odd-Mass and Odd-Odd Nuclear Spectra*, AIP Conf. Proc. **1203**, 17 (2010).
[B3] H. G. Ganev, *Phase Structure of the Interacting Vector Boson Model*, Proceedings of the XXIX International Workshop on Nuclear Theory (June 20-26, 2010, Rila Mountains, Bulgaria), ed. A. Georgieva and N. Minkov, (Published by Heron Press, Sofia, 2010), pp. 119.
[B4] H. G. Ganev, *Triaxiality in the IVBM*, Proceedings of the XXXI International Workshop on Nuclear Theory (June 24-30, 2012, Rila Mountains, Bulgaria), ed. A. Georgieva and N. Minkov, (Published by Heron Press, Sofia, 2012), pp. 204.
[B5] H. G. Ganev, *On the structure of triaxial nuclei*, Proceedings of the 4-th International Conference on Current Problems in Nuclear Physics and Atomic Energy (Institute for Nuclear Research, Kyiv, 2013), pp. 390.
[B6] H. G. Ganev, *Nuclear shapes in the Interacting Vector Boson Model*, Proceedings of the XXXII International Workshop on Nuclear Theory (June 23-29, 2013, Rila Mountains, Bulgaria), ed. A. Georgieva and N. Minkov, (Published by Heron Press, Sofia, 2013), pp. 141-150.
[B7] H. G. Ganev, *Negative parity states in the IVBM*, J. Phys. Conf. Ser. **533**, 012015 (2014).
[B8] H. G. Ganev, *Contraction limits of the proton-neutron symplectic model*, EPJ Web of Conferences **107**, 03012 (2016).
[B9] H. G. Ganev, *The proton-neutron symplectic model of nuclear collective motions*, J. Phys. Conf. Ser. **724**, 012016 (2016).
[B10] H. G. Ganev, *Structure of the low-lying positive parity states in the proton-neutron symplectic model*, J. Phys. Conf. Ser. **1023**, 012013 (2018).
[B11] H. G. Ganev, *U(6) dynamical and quasi-dynamical symmetry in strongly deformed heavy nuclei*, EPJ Web of Conferences **194**, 05002 (2018).

1.10 Structure of the dissertation

The dissertation consists of an Introduction, 12 Chapters, and a Conclusion with a list of the main results obtained by the author, references and a list of the publications on which the dissertation is based. The volume consists of two parts and is presented on 282 pages, contains 96 figures, 21 tables and a list of cited references consisting of 414 items.

The Part I represents the phenomenological approach to nuclear structure within the framework of the phenomenological algebraic Interacting Vector Boson Model [16] in its symplectic and orthosymplectic extensions. It contains 7 Chapters in which the new extensions of the IVBM are developed and exploited in a full account. Symplectic and orthosymplectic dynamical symmetries allow the change of the number of excitation quanta or phonons building the collective states providing for larger representation spaces and richer subalgebraic structures to incorporate more complex nuclear spectra.

The Part II represents the microscopic approach to nuclear collective motion. In this part it is shown how the phenomenological IVBM can be generalized in a way to be compatible with the proton-neutron composite structure of the nucleus. Along this line, by considering the possible collective flows and the symplectic geometry of the two-component nuclear systems, a fully microscopic Proton-Neutron Symplectic Model is formulated as a generalization of both the IVBM and the one-component $Sp(6,R)$ symplectic model. The latter is often referred to as a microscopic collective model of nucleus. In Part II it is proved that the IVBM is a very particular case of the PNSM and corresponds to the two-fluid irrotational-flow collective model of Bohr-Mottelson type, which contains only two irreducible $Sp(12,R)$ subspaces − the even (scalar) and the odd (one-particle) nuclear Hilbert spaces that correspond to the case of even and odd-mass nuclei, respectively.

Part I: The phenomenological approach

2 The Interacting Vector Boson Model

In Chapter 2, the IVBM is presented: its building blocks, the general rotationally invariant Hamiltonian and the role of $Sp(12,R)$ as a dynamical group of the model. The algebraic structure of the IVBM is given by a lattice of $Sp(12,R)$ subgroups and the consideration of four dynamical symmetry limits, when the Hamiltonian can be written as a linear combination of the Casimir operators of a single reduction chain only.

3 The unitary dynamical symmetry limit

In Chapter 3, the unitary dynamical symmetry (DS), defined by the reduction

$$\begin{array}{ccccccc} Sp(12,R) \supset & U(6) \supset & SU(3) & \otimes & U(2) & \supset SO(3) \otimes & U(1), \\ & [N]_6 & (\lambda,\mu) & \Longleftrightarrow & (N,T) & K \quad L & T_0 \end{array} \tag{3}$$

is considered aiming the description of the first positive- and negative-parity bands up to high angular momenta for many strongly deformed even-even nuclei from the rare-earth and actinide mass regions [A1,A10,B7]. For this purpose, the following Hamiltonian is used [A1,A10,B7]:

$$H = aN + bN^2 + \alpha_3 T^2 + \beta_3 L^2 + \alpha_1 T_0^2, \tag{4}$$

which is diagonal in the basis labeled by the quantum numbers of the subgroups of group-subgroup chain (3). As an example, in Fig. 1 we compare our theoretical predictions [A10,B7] for the energies of the first excited positive and negative parity bands observed in ^{226}Ra and ^{230}Th with experiment [18] and the results of some other collective models incorporating octupole or/and dipole degrees of freedom.

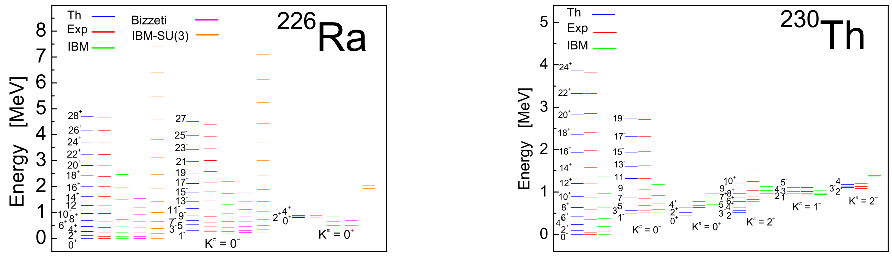

Figure 1: Comparison of the theoretical energies for the low-lying positive and negative parity bands in ^{226}Ra and ^{230}Th with experiment and some other collective models incorporating octupole or/and dipole degrees of freedom.

For the practical application of this DS the symplectic basis in the $Sp(12, R)$ irreducible space is constructed and the role of symplectic generators as transition operators between different basis states is clarified [A2]. Further, the tensorial properties of the $Sp(12, R)$ generators are considered with respect to the the unitary DS chain, which allow the matrix elements of $Sp(12, R)$ operators and any function of them to be calculated [A2] in a purely algebraic way by exploiting the generalized Wigner-Eckart theorem. With the help of the obtained matrix elements, the reduced $B(E2)$ and $B(E1)$ transition probabilities between the collective states of the ground and first $K^{\pi} = 0^{-}_{1}$ negative-parity band are compared with experiment, as given for example in Fig. 2 for ^{226}Ra.

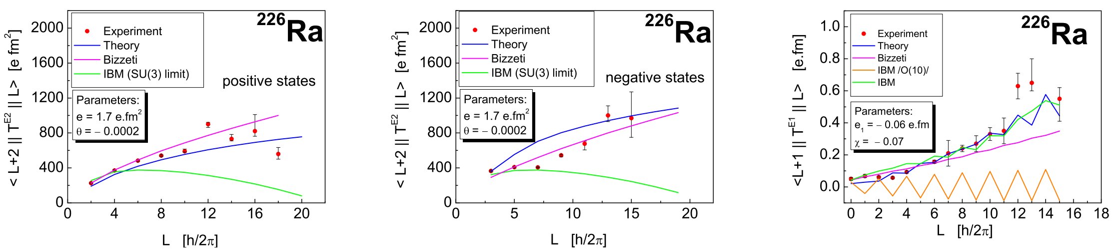

Figure 2: Comparison of theoretical and experimental values for the matrix elements of the intraband $E2$ transitions in the ground state band and $K^{\pi} = 0^{-}$ band, as well as the interband $E1$ transitions between the states of the GSB and $K^{\pi} = 0^{-}$ band, in ^{226}Ra. For comparison, the theoretical predictions of some other collective models incorporating octupole or/and dipole degrees of freedom are also shown.

In the practical calculations, the algebraic notion of the "yrast" states is introduced as states with a given L, built up of minimal number of vector excitations N – the eigenvalue of the total number of bosons. The correspondence between the observed collective states and the symplectic basis states, based on this notion, leads to the appearance of a vibrational term in the eigenvalues of the Hamiltonian, which affects the "yrast" energies. This term plays the role of an interaction between the different bands under consideration, and in particular is responsible for the correct reproduction of the odd-even staggering of the lowest positive- and negative-parity band energies, as shown in Fig. 3 for ^{226}Ra and ^{230}Th. Another examples of the staggering patterns are given in [A1,A10].

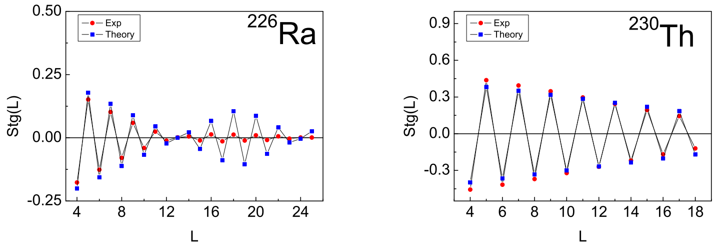

Figure 3: Theoretical and experimental staggering function $Stg(L) = [6\Delta E(L) - 4\Delta E(L-1) - 4\Delta E(L+1) + \Delta E(L+2) + \Delta E(L-2)]/16$ *in* ^{226}Ra *and* ^{230}Th.

4 Geometrical structure of the IVBM

In Chapter 4, the geometrical structure [A6,B3,B6] of the IVBM which corresponds to a specific ground state configuration is obtained by means of the IVBM coherent states [B3,B6]:

$$\mid N; r_1, r_2, \theta \,\rangle = \frac{1}{\sqrt{N!}} (B^\dagger)^N \mid 0 \,\rangle \tag{5}$$

with

$$B^\dagger = \frac{1}{\sqrt{r_1^2 + r_2^2}} \left[r_1 p_z^\dagger + r_2 (n_x^\dagger sin\theta + n_z^\dagger cos\theta) \right], \tag{6}$$

where $\mid 0 \,\rangle$ is the boson vacuum. The geometric properties of the ground states of nuclei within the framework of the IVBM can then be studied by considering the energy functional

$$E(N; r_1, r_2, \theta) = \frac{\langle N; r_1, r_2, \theta | H | N; r_1, r_2, \theta \rangle}{\langle N; r_1, r_2, \theta | N; r_1, r_2, \theta \rangle}. \tag{7}$$

By minimizing $E(N; r_1, r_2, \theta)$ (7) with respect to r_1, r_2, and θ , $\partial E/\partial r_1 = \partial E/\partial r_2 = \partial E/\partial\theta = 0$, one obtains the equilibrium "shape" corresponding to any boson Hamiltonian, H. It is convenient to introduce a new dynamical variable $\rho = r_2/r_1$ which together with the parameter θ determine the corresponding "shape".

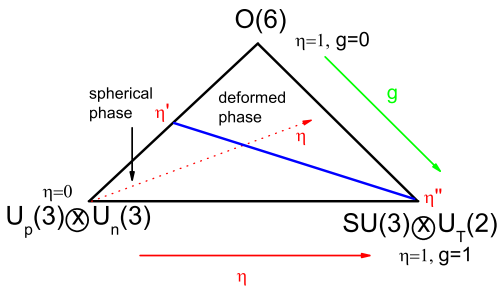

Figure 4: Phase diagram of IVBM. The corners of the triangle correspond to dynamical symmetries.

The basis nuclear shapes – namely, the spherical, axially deformed prolate and γ-unstable – are obtained in the $U_p(3) \otimes U_n(3)$, $SU(3) \otimes U_T(2)$ and $O(6)$ DS limits of the IVBM, correspondingly. The phase diagram of the generalized IVBM Hamiltonian

$$H = (1-\eta) N_n + \frac{\eta}{N-1} \left[-g\widetilde{Q} \cdot \widetilde{Q} + (1-g) P^\dagger P \right]. \tag{8}$$

is constructed and studied [A6,B3]. The three terms in (8) correspond to the three dynamical symmetries: $U_p(3) \otimes U_n(3)$, $SU(3) \otimes U_T(2)$ and $O(6)$. We have two control parameters: η and g; hence the resulting phase diagram is two dimensional one. The phase shape diagram corresponding to the IVBM Hamiltonian (8) can be depicted as a triangle as shown in Fig. 4 with each corner denoting a dynamical symmetry. For $\eta = 0$ one obtains the $U_p(3) \otimes U_n(3)$ or the *vibrational* limit; for $\eta = 1$ one encounters the two limiting cases of *deformed* shapes discussed above: $g = 0$ ($O(6) - \gamma$-unstable rotor) and $g = 1$ ($SU(3)-$axial rotor).

In some cases, the quantum phase transitions can take place between different ground state configurations or "shapes" of the system, occurring at zero temperature as a function of the corresponding control parameter. The order of the phase transitions may be determined with the standard approach (see, for instance, Ref. [9]). The shape-phase transitions that take place in the IVBM are studied in detail in the end of Chapter 4.

5 Triaxial shapes in the IVBM

In Chapter 5, it is shown how the triaxial shapes can be obtained in the IVBM by different perturbations of the $SU^*(3)$ phase structure [A7,B4]. In particular, the two types of perturbation terms on the $SU^*(3)$ energy surface, namely the inclusion of a Majorana interaction and an $O(6)$ term, are considered. For example, the scaled energy surface of the $SU^*(3)$ DS Hamiltonian, to which a Majorana term is added, i.e.

$$H_I = k\frac{1}{N-1}C_2[SU^*(3)] + a\frac{1}{N-1}M_3, \tag{9}$$

is given in Fig. 5 in the form of a three-dimensional plot and a contour plot, respectively. This corresponds to a stable triaxial minimum which becomes deeper and deeper with the increasing of the absolute value of the parameter a. It is further shown that a more accurate angular momentum projection procedure [B6] also changes the topology of the pure $SU(3)$ and $SU^*(3)$ energy surfaces, leading to oblate and maximum triaxial shapes, respectively.

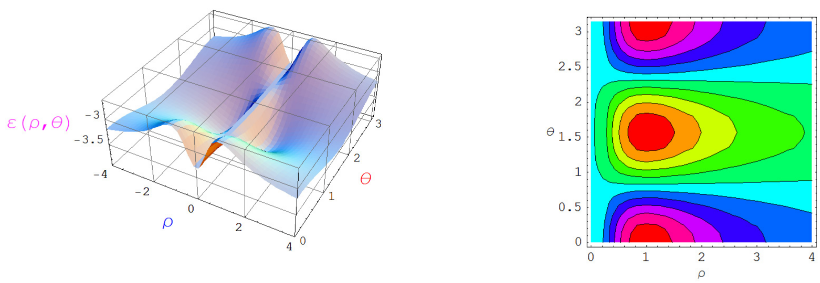

Figure 5: The scaled energy surface $\varepsilon(\rho, \theta)$ in the $SU^*(3)$ limit when a Majorana term is added. The values of the model parameters used are $k = -1$, $a = -3$. For the contour plot only the region $\rho > 0$ is depicted.

6 The $U(3,3)$ dynamical symmetry limit

In Chapter 6, the $U(3,3)$ dynamical symmetry [A8,A9,B5] of the IVBM which is defined by the reduction

$$Sp(12,R) \supset U(3,3) \supset U_p(3) \otimes \overline{U_n(3)} \supset U^*(3) \supset SO(3)$$

is introduced for studying the axial asymmetry in heavy even-even nuclei. This DS is appropriate for nuclei in which the one type of particles is particle-like and the other is hole-like, as for example, in the Os-Pt region. The symplectic basis is constructed along this DS chain [A8]. The effect of a Majorana interaction on the energy of the non-perturbed $U(3,3)$ DS Hamiltonian is examined [A7,B4,B5]. The inclusion of a Majorana term to the model Hamiltonian allows the range from a γ-rigid to γ-unstable structures of the γ-band to be covered by the considered perturbation of the $U(3,3)$ DS Hamiltonian. Thus, the following Hamiltonian

$$H = H_{U(3,3)} + aM_3, \tag{10}$$

where

$$H_{U(3,3)} = a_1 M^2 + b(N_n^2 - N_p^2) + a_3 C_2[SU^*(3)] + b_3 C_2[SO(3)], \tag{11}$$

is used to study the axially-asymmetric properties of heavy even-even nuclei. The Hamiltonian (10) is applied for the calculation of the excitation energies of the ground and γ bands in ^{192}Os, ^{190}Os, and ^{112}Ru [A9,B4,B5], which are assumed in the literature to possess triaxial shapes. For example, the theoretical predictions [A9,B4,B5] for ^{192}Os and ^{190}Os, compared with experiment, are presented in Fig. 6.

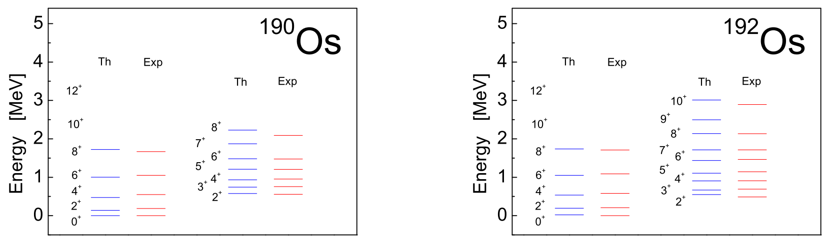

Figure 6: Excitation energies for GSB and γ band in ^{190}Os and ^{192}Os.

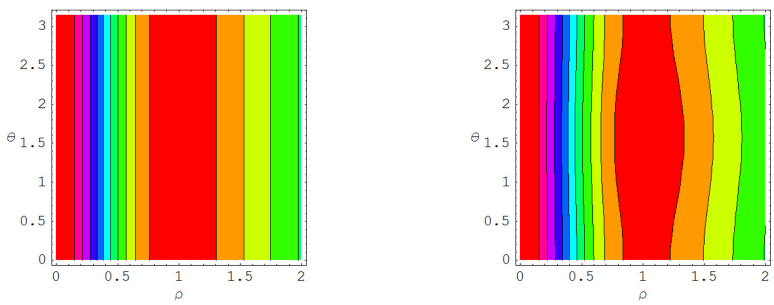

Figure 7: A contour plot of the scaled energy surfaces $\varepsilon(\rho,\theta)$ corresponding to the Hamiltonian (10) for ^{190}Os and ^{192}Os isotopes, respectively. Only the region $\rho > 0$ is depicted.

There is a long-standing debate about the nature of the spectra of Os isotopes. Some authors consider these nuclei as being $O(6)$-like with γ-unstable energy surfaces with a prolate minimum [19], while other as asymmetric rotor [20, 21], which assumes rigidity in the γ degrees of freedom. The Os isotopes considered here have been treated in terms of the IBM in the transition region from the rotor to the γ-unstable limit [22]. In Ref.[23], these isotopes are considered as a textbook example of this transition. In Ref.[24] it was shown that the empirical deviations from the $O(6)$ limit of the IBM, in the Os-Pt region, can be interpreted by introducing explicitly triaxial degrees of freedom, suggesting a more complex and possibly intermediate situation between γ-rigid and γ-unstable properties. Indeed, as it can be seen from the presented examples, the experimentally observed level spacings in the γ band are more regular. In terms of the potentials, this means that the true potentials are γ-dependent. The ground state energy surfaces for the two axially asymmetric nuclei ^{190}Os and ^{192}Os, given in Fig. 7, show a nearly γ-flat potential with very shallow triaxial minimum for the ground state in ^{192}Os, while for ^{190}Os a typical for the $O(6)$ limit θ-unstable (or in IBM terms a γ-unstable) potential is observed. In other words, the potential obtained in the present approach for ^{192}Os is indeed a slightly γ-dependent, representing the case of mixing of γ-flat and γ-rigid structures. Geometrical picture of these two nuclei, obtained within the perturbed $U(3,3)$ DS, is also supported by the odd-even energy staggering patterns that are shown in Fig. 8.

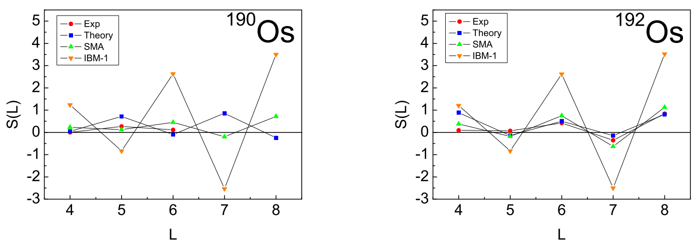

Figure 8: Calculated and experimental staggering of the γ band in ^{190}Os and ^{192}Os isotopes.

Further, the tensor properties of the symplectic generators with respect to the $U(3,3)$ DS are considered, which allow to calculate the matrix elements [A8] of the basic $Sp(12,R)$ operators. With the help of the latter, the $B(E2)$ and $B(M1)$ transition probabilities [A8,A9] between the states of the ground and γ bands are calculated and compared with experiment for ^{190}Os and ^{192}Os.

7 The orthosymplectic extension of the IVBM

In order to incorporate the intrinsic spin degrees of freedom into the symplectic IVBM, we extend the dynamical algebra of $Sp(12,R)$ to the orthosymplectic algebra of $OSp(2\Omega/12,R)$ [A3]. For this purpose we introduce a particle (quasiparticle) with spin j and consider a simple core plus particle picture. Thus, in addition to the boson collective degrees of freedom (described by dynamical symmetry group $Sp(12,R)$) we introduce creation and annihilation operators $a_m^\dagger$ and a_m ($m = -j,\ldots,j$), which satisfy the anticommutation relations

$$\{a_m, a_{m'}^\dagger\} = \delta_{mm'}, \quad \{a_m^\dagger, a_{m'}^\dagger\} = \{a_m, a_{m'}\} = 0. \tag{12}$$

All bilinear combinations of a_m^+ and $a_{m'}$ generate the fermion pair Lie algebra of $SO^F(2\Omega)$ [27]. Further, the fermion, Bose-Fermi and supersymmetry dynamical symmetries are shortly considered, which still lead to exact analytical solutions for the odd-A and doubly odd nuclei. The orthosymplectic (supersymmetric) extension of the IVBM is thus defined through the chain

[A3]:

$$
\begin{array}{ccccc}
OSp(2\Omega/12,R) & \supset & SO^F(2\Omega) & \otimes & Sp^B(12,R) \\
 & & & & \Downarrow \\
 & & \Downarrow & \otimes & U^B(6) \\
 & & & & N \\
 & & & & \Downarrow \\
 & & SU^F(2) & \otimes & SU^B(3) \otimes U_T^B(2) \\
 & & I & & (\lambda,\mu) \Longleftrightarrow (N,T) \\
 & & & \searrow & \Downarrow \\
 & & & \otimes & SO^B(3) \otimes U(1) \\
 & & & & L \qquad\qquad T_0 \\
 & & & \Downarrow & \\
 & & Spin^{BF}(3) & \supset & Spin^{BF}(2). \\
 & & J & & M_J
\end{array}
\tag{13}
$$

We use the following Hamiltonian of the combined boson-fermion system

$$
H = aN + bN^2 + \alpha_3 T^2 + \beta_3' L^2 + \alpha_1 T_0^2 + \eta I^2 + \gamma' J^2 + \zeta M_J^2, \tag{14}
$$

which is diagonal in the basis that is classified according to Eq.(13). By considering the simplest particle-core coupled-type physical picture, e.g., the states of the odd-A nuclei are obtained as a result of the coupling of a particle with intrinsic spin taking a single j-value to a boson core whose excitation states belong to an $Sp(12,R)$ irrep. By using an algebraic notion of yrast states (i.e. the states with given J, which minimize the energy with respect to the number of bosons N) the experimentally observed collective states of odd mass nuclei are mapped onto the $SU(3)$ stretched states of the symplectic basis, which allows the energy levels of the ground and first excited bands to be reproduced very well up to very high angular momenta. As an illustration, the excitation energies of the ground and first few excited bands in ^{157}Gd and ^{163}Dy are shown in Fig. 9.

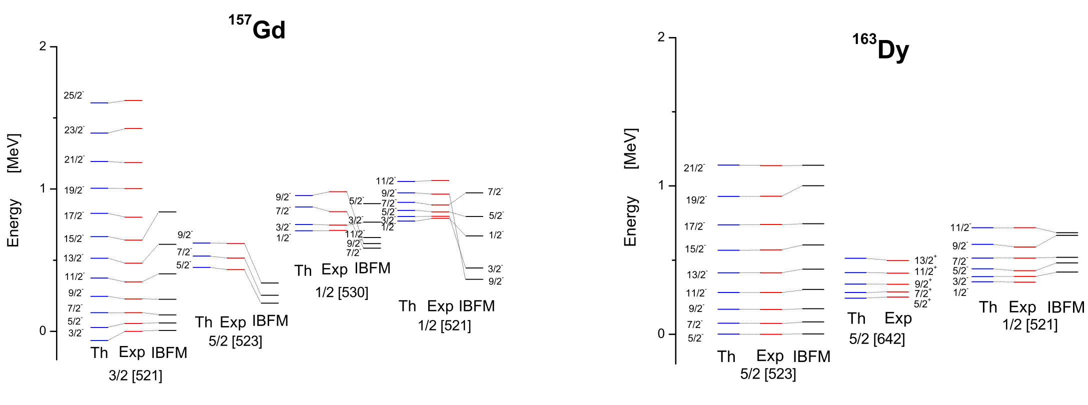

Figure 9: Comparison of the theoretical and experimental energies for the ground and first excited bands in ^{157}Gd and ^{163}Dy, respectively.

The important role of the symplectic structure of the model for the proper reproduction of the intraband $B(E2)$ behavior for odd-A nuclei is revealed [A3]. The theoretical predictions [A3] for the two nuclei ^{157}Gd and ^{163}Dy are compared with the experimental data in Fig. 10. The

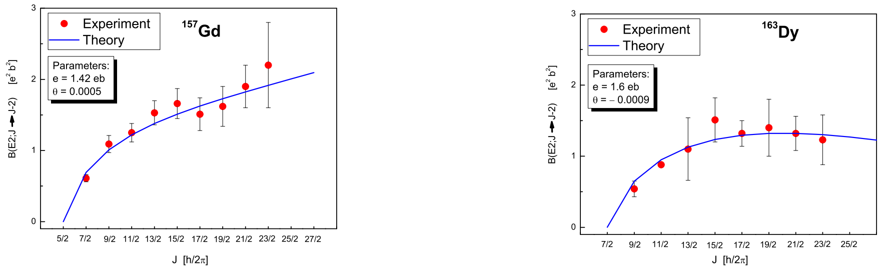

Figure 10: Comparison of the theoretical and experimental values for the ground band $B(E2)$ transition probabilities in the ^{157}Gd and ^{163}Dy, respectively.

orthosymplectic extension of the IVBM is further exploited for the simultaneous description of the collective states in even-even, odd-mass and doubly odd nuclei for two sets of neighboring nuclei with various collective properties [B2].

8 Chiral doublet bands

In Chapter 8, the chiral rotation in some doubly odd nuclei from the $A \sim 130$ region is studied in the supersymmetric extension of the IVBM [A4,A5,B1], which still leads to exactly solvable limit that yields a simple and straightforward application to real nuclear systems. In the calculations, a consistent procedure that includes the analysis of the even-even and odd-even neighbors is employed, which leads to a purely collective interpretation of the chiral doublet bands. It is shown that the good agreement between theoretical and experimental band structures is a result of the mixing of the basic rotational and vibrational collective modes which is traced back to the level of the even-even cores. This allows for the correct reproduction of the high-spin states of the collective bands and the correct placement of the different bandheads. As an example, the excitation energies of the yrast and side bands in ^{132}La and ^{134}Pr are shown in Fig. 11.

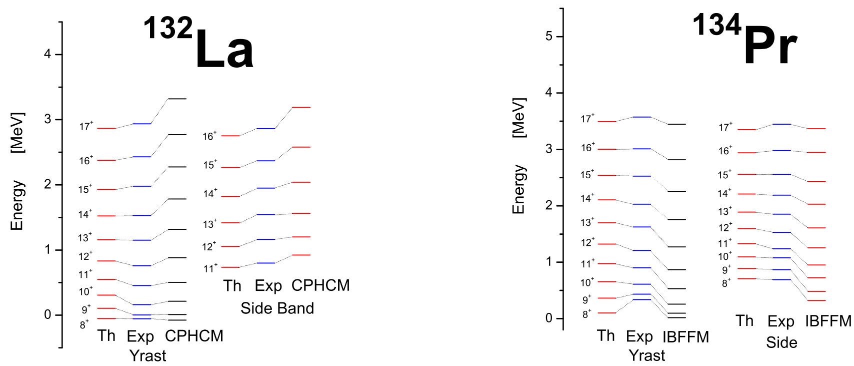

Figure 11: Comparison of the theoretical and experimental energies for the yrast and side bands in ^{132}La and ^{134}Pr, respectively. The theoretical predictions of CPHCM/IBFFM are shown as well for comparison.

The important role of the symplectic terms entering in the corresponding transition operators is revealed for the correct reproduction of the behavior of both $B(E2)$ and $B(M1)$ strengths, which are crucial for establishing the nature of the twin bands. The theoretical predictions for the intraband $B(E2)$ and $B(M1)$ values between the collective states of the yrast band in ^{134}Pr nucleus are compared with the experimental data [28] in Fig. 12. Additionally, for the case of ^{128}Cs, it is demonstrated that the observed odd-even staggering of both $B(E2)$ and $B(M1)$ values could be reproduced by the introduction of an appropriate interaction term of quadrupole type

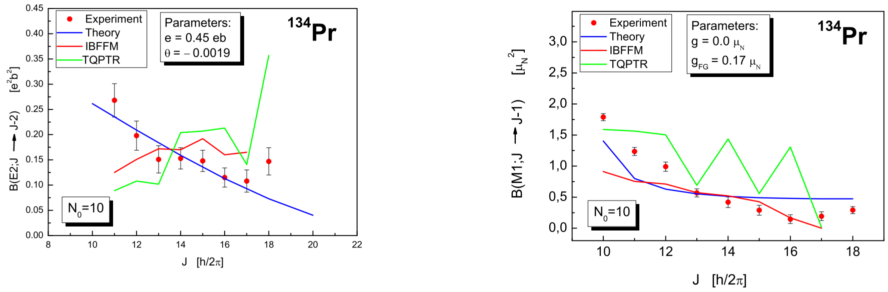

Figure 12: Comparison of the theoretical and experimental values for the $B(E2)$ and $B(M1)$ transition probabilities between the states of yrast band in ^{134}Pr. The theoretical predictions of the IBFFM and TQPTR are shown as well.

[A5], which produces such a staggering effect in the transition strengths.

Part II: The microscopic shell-model approach

9 The Proton-Neutron Symplectic Model

In Chapter 9, a fully microscopic proton-neutron symplectic model [A11,B9] of collective motions is formulated by considering the symplectic geometry and possible collective flows in the two-component many-particle nuclear systems. To sketch this, we start with the translationally-invariant relative Jacobi vectors in the two-component many-particle nuclear system. The position $x_{is}(\alpha)$ and momentum $p_{is}(\alpha)$ operators of these $m = A - 1$ Jacobi vectors, together with the identity operator, close the Heisenberg-Weyl Lie algebra $hw(6m) = \{x_{is}(\alpha), p_{jt}(\alpha), I\}$ with $s,t = 1,2....,m$, $i,j = 1,2,3$, and $\alpha, \beta = p,n$. The many-particle nuclear configuration space is $\mathbb{R}^{6(A-1)}$ and can be decomposed into a product of collective and intrinsic submanifolds which could be represented as an orbit of the direct product group $GL(6,R) \otimes O(A-1)$. Then the many-particle nuclear phase space could be represented as an orbit of the direct product group $Sp(12,R) \otimes O(A-1)$ which is a subgroup of the full dynamical group of the whole system $Sp(12(A-1),R)$, formed by all Hermitian bilinear combinations of position and momentum operators of the Jacobi quasiparticles. The nuclear Hilbert space of spatial wave functions of the corresponding quantum system then decomposes into a direct sum of irreducible $Sp(12,R) \otimes O(A-1)$ subspaces. It follows that the group $Sp(12,R)$ is the group of pure collective excitations in the proton-neutron system and the collective observables of the PNSM can be represented by the following $O(A-1)$ invariant one-body operators [A11]:

$$Q_{ij}(\alpha,\beta) = \sum_{s=1}^{m} x_{is}(\alpha)x_{js}(\beta), \tag{15}$$

$$S_{ij}(\alpha,\beta) = \sum_{s=1}^{m} \Big(x_{is}(\alpha)p_{js}(\beta) + p_{is}(\alpha)x_{js}(\beta) \Big), \tag{16}$$

$$L_{ij}(\alpha,\beta) = \sum_{s=1}^{m} \Big(x_{is}(\alpha)p_{js}(\beta) - x_{js}(\beta)p_{is}(\alpha) \Big), \tag{17}$$

$$T_{ij}(\alpha,\beta) = \sum_{s=1}^{m} p_{is}(\alpha)p_{js}(\beta). \tag{18}$$

The possible collective flows are obtained by considering different motion groups that can be formed by certain subsets of the symplectic generators (15)−(18). In particular, the operators (17) that constitute the motion group $SO(6)$ represent the infinitesimal generators of rigid rotations in the 6-dimensional space. From another side, the shear operators (16) are the infinitesimal generators of irrotational-flow rotations and vibrations. The operators $S_{ij}(\alpha,\beta)$ together with the angular momenta $L_{ij}(\alpha,\beta)$ close under commutation and span the Lie algebra $gl(6,R) \equiv \{S_{ij}(\alpha,\beta), L_{ij}(\alpha,\beta)\}$ of the $GL(6,R)$ motion group. The latter allows for the separate treatment of the collective dynamics of proton and neutron subsystems, as well as the combined proton-neutron collective excitations. By enlarging the set of $GL(6,R)$ momentum operators with other symplectic generators one obtains some other dynamical subalgebras of $Sp(12,R)$. Among the subalgebras of the latter are, for example, the general collective motion in six dimensions $GCM(6)$ and the coupled two-rigid rotor model $ROT_p(3) \otimes ROT_n(3) \supset ROT(3)$ Lie algebras. The $CCM(6)$ algebra introduces the $SO(6)$ intrinsic vortex degrees of freedom which coupled to the giant resonances allows for the continuous range of rotational dynamics from rigid to irrotational flow. It is also shown [A11] that the $GCM(6)$ and $Sp(12,R)$ models appear as hydrodynamic irrotational-flow collective models of the two-component nuclear system that include 21 collective irrotational-flow degrees of freedom, augmented by a $SO(6)$ and $U(6)$ intrinsic structure, respectively, associated with the vortex degrees of freedom.

The full dynamical group $Sp(12(A-1),R)$ of the whole many-particle system allows the separation of the nuclear variables into kinematical (internal) and dynamical (collective) ones, which in turn allows to determine the number and type of collective degrees of freedom in the two-component many-particle nuclear systems purely by a group-theoretical consideration of the relevant coordinate transformation of the microscopic many-particle configuration space. The simplest kinematically-correct nuclear wave functions (i.e., microscopically translationally-invariant, which preserve the observed integrals of motion) are constructed [A11,B9] in terms of collective and their complementary intrinsic coordinates and are represented correspondingly as a product of collective and intrinsic components.

Further, the representation theory of the PNSM in the many-particle shell-model Hilbert space is considered [A13]. This can be naturally obtained by introducing the standard creation and annihilation operators of harmonic oscillator quanta

$$b^{\dagger}_{i\alpha,s} = \sqrt{\frac{m_\alpha \omega}{2\hbar}}\Big(x_{is}(\alpha) - \frac{i}{m_\alpha \omega} p_{is}(\alpha)\Big), \qquad b_{i\alpha,s} = \sqrt{\frac{m_\alpha \omega}{2\hbar}}\Big(x_{is}(\alpha) + \frac{i}{m_\alpha \omega} p_{is}(\alpha)\Big). \tag{19}$$

Then the symplectic generators take an alternative form in terms of all bilinear combinations of the harmonic oscillator raising and lowering operators that are $O(m)$ invariant [A13]:

$$F_{ij}(\alpha,\beta) = \sum_{s=1}^{m} b^{\dagger}_{i\alpha,s} b^{\dagger}_{j\beta,s}, \quad G_{ij}(\alpha,\beta) = \sum_{s=1}^{m} b_{i\alpha,s} b_{j\beta,s}, \tag{20}$$

$$A_{ij}(\alpha,\beta) = \frac{1}{2}\sum_{s=1}^{m} (b^{\dagger}_{i\alpha,s} b_{j\beta,s} + b_{j\beta,s} b^{\dagger}_{i\alpha,s}). \tag{21}$$

The relation of $Sp(12,R)$ irreducible representations of the PNSM to the shell-model classification of the basis states is considered by extending of the state space to the direct product space of $SU_p(3) \otimes SU_n(3)$ irreps, generalizing in this way the Elliott's $SU(3)$ model [5] for the case of two-component system. Thus, the following reduction chain [A13]:

$$\begin{array}{ccccccccccccc} Sp(12,R) & \supset & U(6) & \supset & SU_p(3) \otimes SU_n(3) & \supset & SU(3) & \supset & SO(3) & \supset & SO(2) \\ \sigma & & n\rho \;\; E \;\; \gamma & & (\lambda_p,\mu_p) \;\; (\lambda_n,\mu_n) \;\; \varrho & & (\lambda,\mu) \;\; K & & L & & M \end{array} \tag{22}$$

is used to completely specify the basis of an $Sp(12, R)$ irrep. The PNSM then appears as a natural multi-major-shell extension of the generalized proton-neutron $SU(3)$ scheme which takes into account the core collective excitations of monopole and quadrupole, as well as dipole type associated with the giant resonance vibrational degrees of freedom. It is shown that each $Sp(12, R)$ irreducible representation is determined by a symplectic bandhead or an intrinsic $U(6)$ space which can be fixed by the underlying proton-neutron shell-model structure, so the theory becomes completely compatible with the Pauli principle. It is demonstrated that this intrinsic $U(6)$ structure is of vital importance for the appearance of the low-lying collective bands without involving a mixing of different symplectic irreps [A13,B9]. Thus, within the PNSM shell-model framework, the full many-body Hilbert space is organized vertically into different symplectic slices or cones. Each symplectic slice represents an irreducible collective space of the microscopic collective model and is a small fraction of the full nuclear state space. Further, such organization of the model space allows to build up the required quadrupole collectivity without the use of an effective charge. Then, the full many-body Hilbert space decompose into a direct sum of different symplectic irreducible representations. The structure of the $Sp(12, R)$ irreducible collective subspaces is that of the coupled product of 21-dimensional vibrator, corresponding to the giant resonance degrees of freedom and an intrinsic $U(6)$ structure that is related to the valence proton-neutron degrees of freedom, respectively. This is schematically shown in Fig. 13. The $U(6)$ intrinsic structure, in turn, contains many $SU(3)$ multiplets that are appropriate for the description of different rotational bands. Thus, it is demonstrated that the full range of low-lying collective states could, in principle, be described by a microscopically based intrinsic $U(6)$ structure that is renormalized due to the coupling to the giant resonance vibrations.

Summarizing, the chain (22) defines a shell-model coupling scheme for detail microscopic calculations of nuclear properties and provides a natural mechanisms for identifying the relevant shell-model subspaces.

10 Many-particle limits of the PNSM

In Chapter 10 are considered the many-particle (also referred to as macroscopic or hydrodynamic) limits [A12,B8] of the PNSM which show how a given microscopic discrete system starts to behave like a continuous fluid and reveal further its physical content. The macroscopic limits, which take place at large dimentional representations, are obtained by purely algebraic way using the formal expansion-contraction group-theoretical procedure [29, 30]. The algebraic approach thus allows to interpret a given microscopic algebra of collective observables at macroscopic level in simple geometrical terms. Consequently, it is shown that as a result of the contraction, two new simplified macroscopic models of nuclear collective motion appear. The first one is the $U(6)$-phonon model

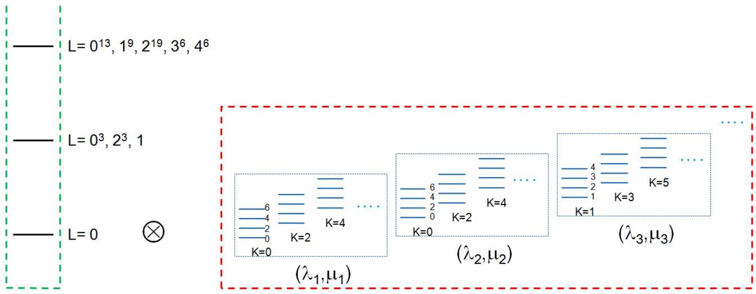

Figure 13: The structure of $Sp(12,R)$ irreducible collective spaces for the PNSM is that of a coupled product of a 21-dimensional vibrator, related to the core collective excitations associated with the giant resonance degrees of freedom, and a microscopically based intrinsic $U(6)$ structure, related to the valence shell proton-neutron degrees of freedom that contains many $SU(3)$ multiplets appropriate for the description of different low-lying collective bands.

[A12,B8] with the semi-direct product structure $[HW(21)]U(6)$, which is shown to be actually an alternative formulation of the original proton-neutron symplectic model in the familiar IBM-terms. It is obtained in the limit $2n << \sigma_0$, in which the symplectic raising and lowering generators behave like boson operators. The full correspondence of the all $Sp(12,R)$ algebra generators and that of $[HW(21)]U(6)$ is given by [A12,B8]:

$$\begin{aligned}
&F^0(p,p) \leftrightarrow \sqrt{2\sigma_0}s^\dagger_p, \quad F^2_M(p,p) \leftrightarrow \sqrt{2\sigma_0}d^\dagger_{M,p},\\
&F^0(n,n) \leftrightarrow \sqrt{2\sigma_0}s^\dagger_n, \quad F^2_M(n,n) \leftrightarrow \sqrt{2\sigma_0}d^\dagger_{M,n},\\
&F^0(p,n) \leftrightarrow \sqrt{2\sigma_0}s^\dagger_\delta, \quad F^2_M(p,n) \leftrightarrow \sqrt{2\sigma_0}d^\dagger_{M,\delta},\\
&F^1_M(p,n) \leftrightarrow \sqrt{2\sigma_0}p^\dagger_M, \quad A^L_M(\alpha,\beta) \leftrightarrow A^L_M(\alpha,\beta),
\end{aligned} \tag{23}$$

and their hermitian conjugate counterparts. $\sigma_0 = (\sigma_1 + \ldots + \sigma_6) + \frac{6}{2}(A-1)$ is the eigenvalue of the harmonic oscillator Hamiltonian for the lowest weight state with energy $\hbar\omega$ and n determines the energy of $2n\hbar\omega$ excited space. Note that beyond the s and d bosons (phonons) which represent the giant monopole and quadrupole vibrational quanta, respectively, the dipole giant degrees of freedom appear that are represented by the components of the p-boson which arise from the contraction of the symplectic proton-neutron raising/lowering operators. The set of operators $\{s^\dagger_\tau, d^\dagger_{M,\tau}, p^\dagger_M, s_\tau, d_{M,\tau}, p_M, I\}$ $(\tau = p, n, \delta)$ generates the vibrational $hw(21)$ phonon algebra. The later consists of the IBM-3 building blocks [9] plus the extra degrees of freedom represented by the components of the dipole p-boson.

At large $SU(3)$ quantum numbers, the intrinsic substructure $SU_p(3) \otimes SU_n(3) \supset SU(3)$ of $U(6)$, associated with the proton-neutron valence shell degrees of freedom, further reduces to $ROT_p(3) \otimes ROT_n(3) \supset ROT(3)$, i.e. to that of two coupled rigid rotors (two-rotor model). Thus, the second model which appears in double contraction limit of the PNSM is the two-rotor model with the $ROT_p(3) \otimes ROT_n(3) \supset ROT(3)$ algebraic structure [B8]. The latter, in contrast to the original two-rotor model [31], is shown to be not restricted to the case of two coupled axial rotors. In this way, the second contraction limit of the PNSM is shown to provide the phenomenological two-rotor model [31] with a simple microscopic foundation. In this way, in double contraction limit, the $sp(12,R)$ algebra reduces to the coupled two-rotor model algebra $rot_p(3) \oplus rot_n(3) \supset rot(3)$

and a phonon algebra $hw(21)$ of the giant resonance vibrational degrees of freedom. The full range of low-lying collective states could then be described as two-rotor model states, renormalized by their coupling to the giant resonance vibrations.

11 The PNSM matrix elements

In Chapter 11, the computational technique [A14,A15] required for practical application of the PNSM at $U(6)$ level is developed. The aim is to obtain the PNSM matrix elements, which are obtained by purely algebraic way by using a generalized Wigner-Eckart theorem with respect to the symmetry-adapted basis of the PNSM. The application of this theorem depends upon the knowledge of the corresponding isoscalar factors (IFs) which were not available. Thus, as a first step it is shown what kinds of IFs appear in the diagonalization of the model Hamiltonian of the two-component many-body systems [A14]. Some of relevant isoscalar factors, needed for the calculation of PNSM matrix elements, are further obtained using a building-up procedure [A14]. With the help of obtained IFs, the matrix elements [A15] of the $Sp(12,R)$ generators of the PNSM are next obtained in a $U(6)$-coupled basis in the space of fully symmetric representations. This allows further the matrix elements of any physical operator of interest, such as the relevant transition operators or the collective potential, to be calculated. As an illustration, the matrix elements of the basic irreducible tensor terms which appear in the $U(6)$ decomposition of the long-range full major-shell mixing proton-neutron quadrupole-quadrupole interaction $Q_p \cdot Q_n$ are presented [A15].

12 Structure of the low-lying positive-parity states

Using the obtained matrix elements for the collective potential, in Chapter 12 the PNSM is firstly applied [A16,A17,B10,B11] to the simultaneous description the low-lying states of the lowest ground, β and γ bands in three strongly deformed heavy nuclei, namely ^{166}Er [B10], ^{154}Sm [A16] and ^{238}U [A17]. For this purpose, the algebraic model Hamiltonian

$$H_I = N\hbar\omega - \frac{1}{2}\chi\big[Q_p \cdot Q_n - (Q_p \cdot Q_n)_{TE}\big] - \xi C_2[SU(3)] + aL^2, \tag{24}$$

is diagonalized in a $SU_p(3) \otimes SU_n(3)$ symmetry-adapted basis for ^{166}Er, and a $U(6)$-coupled basis, respectively, for ^{154}Sm and ^{238}U which is restricted to the state space spanned by the fully symmetric $U(6)$ irreps. As an illustration, the excitation energies of the three positive-parity bands under considerations in ^{154}Sm are shown in Fig. 14. For ^{238}U, since the full major-shell-mixing $Q_p \cdot Q_n$ interaction favors the horizontal mixing of different $SU(3)$ multiplets over the vertical one, a more general Hamiltonian is used to determine the microscopic structure of the low-lying collective states, in which $Q_p \cdot Q_n$ is replaced by its (in-shell) $U(6)$-restricted part $\widetilde{Q}_p \cdot \widetilde{Q}_n$, and a rather general vertical mixing term that lies in the enveloping algebra of $Sp(12,R)$ is introduced

$$\begin{aligned} H_{II} = & N\hbar\omega - \frac{1}{2}\chi\widetilde{Q}_p \cdot \widetilde{Q}_n - \xi C_2[SU(3)] + aL^2 \\ & - k\sum_{\alpha\neq\beta}\Big(A^2(\alpha,\alpha)\cdot G^2(\beta,\beta) + G^2(\alpha,\alpha)\cdot G^2(\beta,\beta) + h.c.\Big). \end{aligned} \tag{25}$$

Correspondingly, the intraband $B(E2)$ transition probabilities between the states of the ground band in ^{154}Sm and ^{238}U are given in Fig. 15.

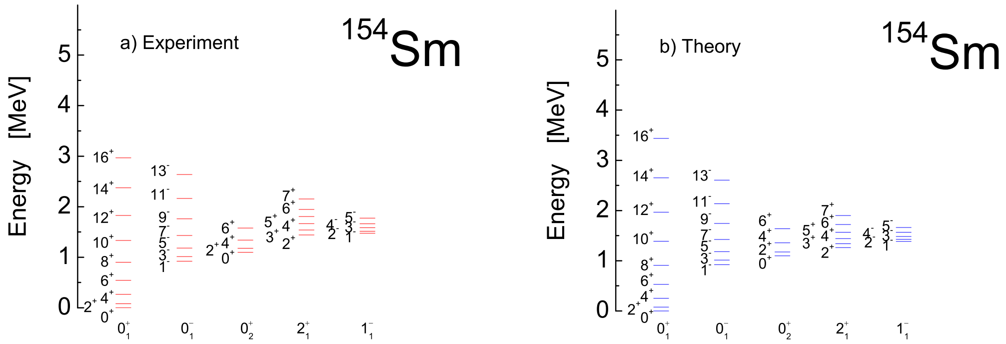

Figure 14: Comparison of experimental energy levels a) with the theory b) for the low-lying positive-parity ground, β and γ bands and negative-parity $K^{\pi} = 0_1^-$ and $K^{\pi} = 1_1^-$ bands in ^{154}Sm.

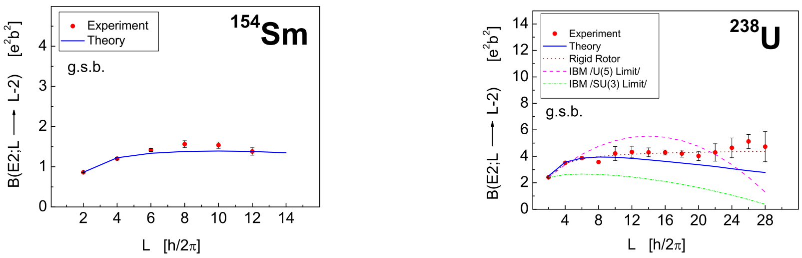

Figure 15: Calculated and experimental intraband $B(E2)$ values between the states of the ground band in ^{154}Sm and ^{238}U, respectively. No effective charge is used. For comparison, the theoretical prediction of some other collective models are also shown in the case of ^{238}U.

It is shown that a good description of the energy levels of these bands for the all three nuclei, as well as the intraband $B(E2)$ transition strengths between the states of the ground band (and of γ band for ^{166}Er) is obtained without the use of an effective charge [A16,A17,B10,B11]. As an illustration, the intraband ground band $B(E2)$ values in ^{154}Sm and ^{238}U are compared with experiment in Fig. 15. For ^{166}Er, the results for the microscopic structure show the presence of a good $SU(3)$ dynamical symmetry [B10]. For the other two nuclei ^{154}Sm and ^{238}U, we show the relevant $SU(3)$ (for ^{154}Sm) and $U(6)$ (for ^{238}U) decompositions of their wave functions in Figs. 16 and 17, respectively. The calculations show that when the collective quadrupole dynamics is covered already by the symplectic bandhead structure, as in the case of ^{154}Sm, the results show the presence of a very good $U(6)$ dynamical symmetry [A16,B11]. In the case of ^{238}U, when we have an observed enhancement of the intraband $B(E2)$ transition strengths, then the results show small admixtures from the higher major shells and a highly coherent mixing of different irreps which is manifested by the presence of a good $U(6)$ quasi-dynamical symmetry [A17,B11] in the microscopic structure of the collective states under consideration. The (parameter-free) results for $B(E2)$ collectivity, obtained for the three nuclei ^{166}Er, ^{154}Sm and ^{238}U, are shown to be very close to those of the phenomenological one-parameter rigid rotor model. The close agreement between the rigid rotor and the PNSM is a strong implication that they effectively describe the same rotational dynamics, albeit in the sense of quasi-dynamical symmetry. But, the most important point is that the PNSM calculations allow to identify the kinds of shell-model configurations needed to describe the rotational states of strongly deformed nuclei.

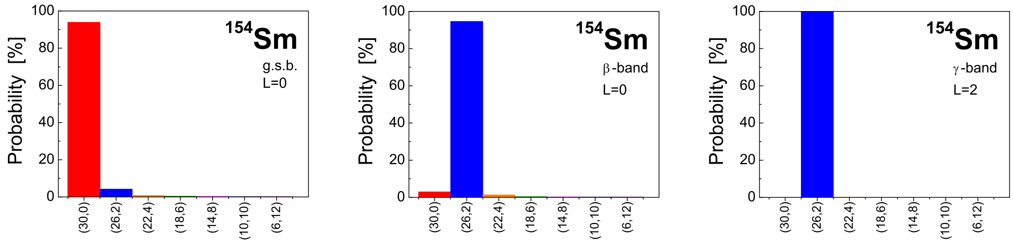

Figure 16: Calculated SU(3) probability distributions for the wave functions for the 0^+ states of the ground and β bands, and for the 2^+ state of the γ band in ^{154}Sm.

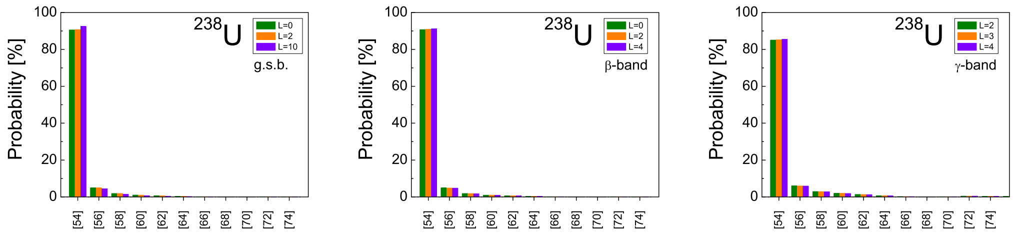

Figure 17: Calculated $U(6)$ probability distributions for the wave functions of the ground, β, and γ bands in ^{238}U for three different angular momentum values.

13 Structure of the low-lying negative-parity states

In Chapter 13, the new microscopic theory is further applied for the first time for obtaining the microscopic structure of the low-lying negative parity states [A18,A19] of the $K^\pi = 0^-_1$ and $K^\pi = 1^-_1$ bands in ^{154}Sm and ^{238}U without the introduction of additional degrees of freedom, inherent to other approaches to odd-parity nuclear states. For this purpose, the following slightly modified Hamiltonians

$$H_I = N\hbar\omega - \frac{1}{2}\chi\big[Q_p \cdot Q_n - (Q_p \cdot Q_n)_{TE}\big] - (\xi + \xi_{sym})C_2[SU(3)] + aL^2 + \epsilon(N_{b.h.} - N_0), \qquad (26)$$

and

$$\begin{aligned} H_{II} =& N\hbar\omega - \frac{1}{2}\chi\widetilde{Q}_p \cdot \widetilde{Q}_n - (\xi + \xi_{sym})C_2[SU(3)] + aL^2 \\ & - k\sum_{\alpha\neq\beta}\Big(A^2(\alpha,\alpha) \cdot G^2(\beta,\beta) + G^2(\alpha,\alpha) \cdot G^2(\beta,\beta) + h.c.\Big) + \epsilon(N_{b.h.} - N_0), \end{aligned} \qquad (27)$$

are diagonalized respectively for ^{154}Sm and ^{238}U in a $U(6)$-coupled basis, restricted to state space spanned by the fully symmetric $U(6)$ irreps of the lowest odd irreducible representation of $Sp(12, R)$. It is shown that a good description of the energy levels of the two bands under consideration, as well as the reproduction of some energy splitting quantities which are usually introduced in the literature as a measure of the octupole correlations, is obtained for these two nuclei. The excitation energies of the negative-parity states of the $K^\pi = 0^-_1$ and $K^\pi = 1^-_1$ bands, together with the positive-parity states, were shown in Fig. 14 for ^{154}Sm.

It is further shown that practically there are no admixtures from the higher shells in the microscopic structure of low-lying collective states with negative-parity in ^{154}Sm and that this points to the presence of a very good $U(6)$ dynamical symmetry [A18]. Additionally, it is shown that the structure of the collective states under consideration for this nucleus shows also the

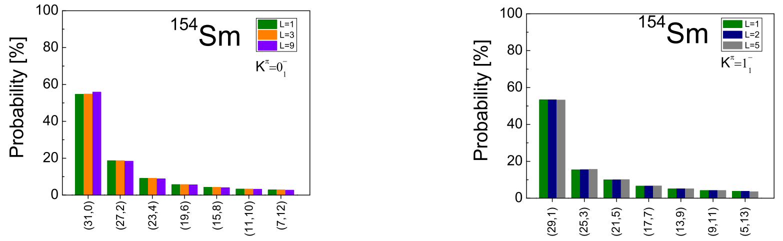

Figure 18: Calculated SU(3) probability distributions for the wave functions of the $K^\pi = 0_1^-$ and $K^\pi = 1_1^-$ bands in ^{154}Sm for three different angular momentum values.

presence of a good $SU(3)$ quasi-dynamical symmetry [A18], as is evident, e.g., from Fig. 18. For ^{238}U, likewise the positive-parity states, the microscopic structure of the low-lying negative-parity states [A19] show small admixtures from the higher major shells and a highly coherent mixing of different irreps (a good $U(6)$ quasi-dynamical symmetry).

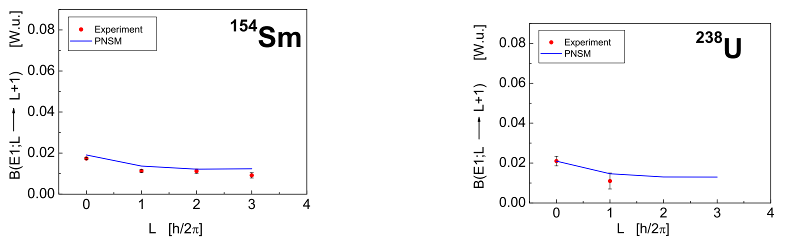

Figure 19: Comparison of the calculated $B(E1)$ values in Weisskopf units between the states of the ground band and $K^\pi = 0_1^-$ band in ^{154}Sm and ^{238}U, respectively, with experiment. No effective charge is used.

Further, the low-energy $E1$ transitions are tackled within the framework of PNSM. In order to study the electric dipole strengths, the explicit matrix elements of dipole operator are obtained [A19]. To achieve this, we make the central extension [A19] of the proton-neutron symplectic model that has the semi-direct structure wsp(12,R) $\equiv$ [hw(6)]sp(12,R), which in contrast to the sp(12,R) algebra among its generators contains the electric dipole operator, which allows to calculate the reduced $E1$ transition strengths. It is demonstrated that this extension introduces $1\hbar\omega$ $1p$-$1h$ (one proton or one neutron Jacobi particle raised by one shell) excitations [A19] to the PNSM $2\hbar\omega$ like particle $1p$-$1h$ (one proton or neutron Jacobi particle raised by two major shells) and proton-neutron $2\hbar\omega$ $2p$-$2h$ (one proton and one neutron Jacobi particles raised by one shell) core excitations. In this way, all kinds of np-nh shell-model configurations with any even or odd number of harmonic-oscillator quanta are incorporated in the theory. It is demonstrated also that any collective operator of physical interest that can be brought in the form of an arbitrary function of either even or odd power in the many-particle position and momentum coordinates will lie in the enveloping wsp(12,R) algebra. Next, the theoretical values of the low-energy $B(E1)$ transition strengths between the states of the ground band and $K^\pi = 0_1^-$ band in ^{154}Sm and ^{238}U are compared with experiment in Fig. 19.

14 Summary and conclusions

In the present dissertation, we have developed an algebraic theory of the collective motions in atomic nuclei starting from the phenomenological Interacting Vector Boson Model in which the collective excitations are considered as built up from two vector bosons or elementary excitation quanta (phonons). Development of the IVBM concerned its mathematical structure (new dynamical symmetry limits, matrix elements, different extensions) as well as its application to diverse collective phenomena in the even-even, odd-mass and odd-odd heavy nuclei, including the simultaneous description of observed collective states in families of neighboring odd and even A nuclei with varying collective properties. A simple description and a clear collective interpretation of the obtained results were obtained within the IVBM using mainly the solutions of its exact dynamical symmetry limits. Further, using the algebraic strategy proposed in the Introduction we demonstrated how the phenomenological theory, presented in particular by the IVBM, can be given a microscopic foundation. In this way by using the elegant group-theoretical methods and the approach initiated by Tomonoga [32] in 1950s, followed by Cusson, Weaver, Bidenharn [33, 34, 35, 36, 37], Ui [38], Rowe and Rosensteel [13, 39, 40, 41] during the 1960s and 1970s in obtaining a microscopic version of the Bohr-Mottelson collective model which is expressed in terms of all single particle coordinates and compatible with the Pauli principe, we extended their results to the case of two-component proton-neutron nuclear systems. Thus, by considering the possible collective flows and the symplectic geometry of the proton-neutron system we have formulated a fully microscopic proton-neutron symplectic model of nuclear collective motions with $Sp(12, R)$ spectrum generating algebra. The latter extends the microscopic collective model known as (one-component) $Sp(6, R)$ symplectic model to the case of two-component many-particle nuclear systems by embedding $Sp(6, R) \subset Sp(12, R)$. This allows the separate treatment of the proton and neutron excitations, as well as the more complete proton-neutron combined dynamics. Simultaneously, the PNSM extends also the IVBM by augmenting the latter with an intrinsic structure related to the vortex dynamics. The addition of intrinsic vortex degrees of freedom is very important to the microscopic theory of nuclear collective motion, because by their coupling to the irrotational-flow collective dynamics one is able to obtain the observed low-lying states of atomic nuclei. Moreover, the vortex degrees of freedom related to the intrinsic motion of all protons and neutrons allow to ensure the proper permutational symmetry of nuclear wave functions and realize them as vectors in the many-particle Hilbert subspaces of the microscopic shell model. In other words, by exploiting the algebraic strategy we extend the phenomenological IVBM in such a way (supplying it with a microscopic intrinsic $U(6)$ structure) that a new model, fulfilling the microscopic translationally-invariant requirements, emerges as a submodel of the nuclear shell model by embedding its dynamical algebra in the infinite-dimensional spectrum generating algebra of one-body operators of the shell model.

Bibliography

[1] D. J. Rowe and J. L. Wood, *Fundamentals of Nuclear Models: Foundational Models*, (World Scientific Publisher Press, Singapore, 2010).

[2] *Dynamical Groups and Spectrum generating Algebras*, Volumes 1 and 2, edited by A. Bohm, Y. Ne'eman, and A. O. Barut (World Scientific Publishing Co. Pte. Ltd., Singapore, 1988).

[3] W. Heisenberg, Z. Phys. **77**, 1 (1932).

[4] E. Wigner, Phys. Rev. **51**, 106 (1937).

[5] J. P. Elliott, Proc. R. Soc. London, Ser. A **245**, 128 (1958); **245**, 562 (1958).

[6] B. H. Flowers, Proc. R. Soc. **A 212**, 248 (1952).

[7] A. K. Kerman, Ann. Phys. **12**, 300 (1961).

[8] K. Helmers, Nucl. Phys. **23**, 594 (1961).

[9] F. Iachello and A. Arima, *The Interacting Boson Model* (Cambridge University Press, Cambridge, 1987).

[10] V. Vanagas, Fiz. Elem. Chastits At. Yadra. **7**, 309 (1976) [Sov. J. Part. Nucl. **7**, 118 (1976)].

[11] V. Vanagas, Fiz. Elem. Chastits At. Yadra. **11**, 454 (1980) [Sov. J. Part. Nucl. **11**, 172 (1980)].

[12] V. V. Vanagas, *Algebraic foundations of microscopic nuclear theory* (Nauka, Moscow, 1988) (in Russian).

[13] D. J. Rowe, Rep. Prog. Phys. **48**, 1419 (1985).

[14] D. J. Rowe, Prog. Part. Nucl. Phys. **37**, 265 (1996).

[15] Rowe, AIP Conf. Proc. **1541**, 104 (2013); See also, arXiv:1304.6115 [nucl-th].

[16] A. Georgieva, P. Raychev, and R. Roussev, J. Phys. **G8**, 1377 (1982).

[17] M. Moshinsky and C. Quesne, J. Math. Phys. **11**, 1631 (1970).

[18] National Nuclear Data Center (NNDC), http://www.nndc.bnl.gov/

[19] K. Nomura *et al.*, Phys. Rev. **C 83**, 054303 (2011).

[20] A. S. Davydov and G. F. Filippov, Nucl. Phys. **8**, 237 (1958).

[21] N. Redon *et al.*, Phys. Lett. **B181**, 223 (1986).

[22] R. F. Casten and J. A. Cizewski, Nucl. Phys. **A 309**, 477 (1978).

[23] R. F. Casten, *Nuclear Structure from a Simple Perspective* (Oxford University, Oxford, 1990).

[24] R.F. Casten, P. Von Brentano, K. Heyde, P. Van Isacker, J. Jolie, Nucl. Phys. **A 439**, 289 (1985).

[25] A. A. Raduta and P. Buganu, Phys. Rev. **C 83**, 034313 (2011).

[26] G. Thiamova, Eur. Phys. J. **A 45**, 81 (2010).

[27] B. G. Wybourne, *Classical Groups for Physicists*, (Wiley, New York, 1974).

[28] D. Tonev et al., Phys. Rev. **C 76**, 044313 (2007).

[29] E. Inönu and E. P. Wigner, *Proc. Nat. Acad.* **39** (1953) 510.

[30] R. Gilmore, *Lie Groups, Lie Algebras and Some of Their Applications* (Wiley, New York, 1974).

[31] N. Lo Iudice and F. Palumbo, Phys. Rev. Lett. **41**, 1532 (1978).

[32] S. Tomonaga, Prog. Theor. Phys. **13**, 467 (1955).

[33] L. Weaver, L. C. Biedenharn, Phys. Lett. **B 32**, 326 (1970).

[34] L. Weaver and L. C. Biedenharn, Nucl. Phys. **A185**, 1 (1972).

[35] L. Weaver, R. Y. Cusson and L. C. Biedenharn, Ann. Phys. (N.Y.) **77**, 250 (1973).

[36] O. L. Weaver, R. Y. Cussion and L. C. Biedenharn, Ann. Phys. (N.Y.) **102**, 493 (1976).

[37] B. Buck, L. C. Biedenharn, and R. Y. Cusson, Nucl. Phys. **A 317**, 205 (1979).

[38] H. Ui, Prog. Theor. Phys. **44**, 153 (1970).

[39] G. Rosensteel and D. J. Rowe, Ann. Phys. **96**, 1 (1976).

[40] D. J. Rowe and G. Rosensteel, Phys. Rev. Lett. **38**, 10 (1977).

[41] D. J. Rowe and G. Rosensteel, Ann. Phys. **126**, 198 (1980).