# INTRODUCTION TO NEUTRINO[*]

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A brief history of neutrino is presented. Neutrino oscillations are discussed and the most economical mechanism of the generation of Majorana neutrino masses and mixing is considered.

## 1. Introduction

Neutrino physics is an extremely important part of the particle physics. Four Nobel prizes were awarded for discoveries connected with neutrino. In 2015, the Nobel Prize in physics was awarded to T. Kajita and A.B. McDonald "for the discovery of neutrino oscillations, which shows that neutrinos have mass".

In the paper, I will give a short introduction to neutrino. In the first part, I will briefly consider the history of neutrino. Main modern problems of neutrino physics are: neutrino nature (Dirac or Majorana?), neutrino mass spectrum (normal or inverted?), absolute values of neutrino masses, existence of sterile neutrinos *etc.* In the second part of the paper, I will discuss these problems.

## 2. Neutrino history

The history of neutrino started with the famous Pauli letter addressed to participants of the nuclear conference in Tübingen (1930). In this letter, Pauli put forward an idea of existence of *a new, neutral, spin 1/2, light (with a mass not larger than the mass of the electron), penetrating particle* (Pauli called this particle a neutron) which is emitted in the $\beta$-decay of a nucleus together with electron

$$(A, Z) \to (A, Z + 1) + e^- + \text{"}n\text{"}.$$

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Assumption of the existence of the "neutron" was *the only possible explanation of the continuous β-spectrum* in the framework of conservation of energy and momentum in the β-decay.

In 1932, neutron was discovered by Chadwick [1] and neutron–proton structure of nuclei was soon established. Pauli's idea of a new particle was actively discussed in Rome by Fermi and his collaborators. Fermi and Amaldi baptized the Pauli particle with the name neutrino (neutral, small).

In 1934, Fermi proposed the first theory of the β-decay [2]. The main problem for Fermi was to understand how electron–neutrino pair was produced in the decay of nucleus, a bound state of protons and neutrons[1].

Fermi, the author of famous at that time review on Quantum Electrodynamics, understood the β-decay of the neutron

$$n \to p + e^- + \bar\nu \tag{1}$$

by analogy with the electromagnetic process

$$p \to p + \gamma \,. \tag{2}$$

The Hamiltonian of process (2) has the form

$$\mathcal{H}_\mathcal{I}^{\text{EM}}(x) = e \ \bar p(x)\gamma^\alpha p(x) \ A_\alpha(x) \,. \tag{3}$$

In the paper which was called *Tentative Theory of Beta Rays*, Fermi proposed the following Hamiltonian of the β-decay of the neutron

$$\mathcal{H}_\mathcal{I}^{\beta}(x) = G_\text{F} \ \bar p(x)\gamma^\alpha n(x) \ \bar e(x)\gamma_\alpha \nu(x) + \text{h.c.} \,, \tag{4}$$

where $G_\text{F}$ is the Fermi constant.

The Fermi theory was the first (very nontrivial at that time) application of the Quantum Field Theory to the process which is different from electromagnetic processes. Let us also stress that by analogy with QED, Fermi assumed that in the Hamiltonian of the β-decay of the neutron entered the product of vectors.

In spite of similarity, there exists a fundamental difference between Hamiltonians $\mathcal{H}_\mathcal{I}^{\text{EM}}(x)$ and $\mathcal{H}_\mathcal{I}^{\beta}(x)$. In the system $\hbar = c = 1$, which we are using, a fermion field has dimension $M^{3/2}$ and a boson field has dimension $M$. Taking into account that a Hamiltonian has dimension $M^4$, we conclude that the charge $e$ is dimensionless quantity and the Fermi constant has dimension $M^{-2}$. We will discuss the physical reason for this difference later.

The β-decays of nuclei are mainly allowed decays in which electron and neutrino are emitted in the $S$-states. The Fermi theory could explain such allowed decays in which electron and neutrino are produced in singlet states.

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[1] The idea that particles are created in quantum transitions between another particles, which is obvious today, was not so much familiar at that time.

In such decays, the following (Fermi) selection rules are satisfied

$$\Delta J = 0, \qquad \pi_{\mathrm{i}} = \pi_{\mathrm{f}}, \qquad (5)$$

where $J$ is a spin and $\pi$ is parity of a nucleus.

It was observed, however, that allowed $\beta$-decays of nuclei which satisfy Gamov–Teller selection rules [3]

$$\Delta J = \pm 1, 0, \qquad \pi_{\mathrm{i}} = \pi_{\mathrm{f}} \qquad (0 \to 0 \quad \text{transition is forbidden}), \qquad (6)$$

which correspond to emission of electron and neutrino in the triplet state.

This means that in the Hamiltonian of the $\beta$-decay, additional term (terms) must enter and that the analogy with QED can be only partially correct.

The most general Hamiltonian of the $\beta$-decay, in which only fields (but not their derivatives) enter, is the sum of the products of scalar×scalar, vector×vector, tensor×tensor, axial×axial and pseudoscalar×pseudoscalar

$$\mathcal{H}_I^{\beta}(x) = \sum_{i=S,V,T,A,P} G_i\, \bar{p}(x) O^i n(x)\ \bar{e}(x) O_i \nu(x) + \text{h.c.} \qquad (7)$$

Here

$$O^i \to 1\ (S), \qquad \gamma^\alpha\ (V), \qquad \sigma^{\alpha\beta}\ (T), \qquad \gamma^\alpha\gamma_5\ (A),\ \gamma_5\ (P). \qquad (8)$$

In Hamiltonian (7) five (!) interaction constants $G_i$ enter. There was a general belief in the forties and fifties that not all interaction constants were equally important and that there were "dominant" terms in Hamiltonian (7).

During many years, the situation with the Hamiltonian of the $\beta$-decay was uncertain and contradictory: some experiments were in favor of $V$ and $A$ terms, other were in favor of $S$ and $T$ terms. Later, it occurred that some experiments on the study of the $\beta$-decay were wrong.

In 1957, large violation of parity in the $\beta$-decay and other weak processes were discovered. This discovery drastically changed our understanding of the $\beta$-decay, neutrino and weak interaction in general.

The problem started with the so-called $\theta$–$\tau$ puzzle. Namely, it was observed in the fifties that particle(s) with the same mass decayed into $2\pi$ ($\theta$-mode) and $3\pi$ ($\tau$-mode) states with opposite parities. One of the possible solution of the puzzle was an assumption that in the decay of a particle (which today is called $K$-meson) parity is not conserved.

Lee and Yang [4] analyzed existed data and came to the conclusion that there was no proof that parity is conserved in the weak interaction. They proposed different experiments which could allow to check conservation of parity in weak processes.

If the parity is not conserved in the weak interaction, the Hamiltonian is the sum of scalar and pseudoscalar. The most general Hamiltonian of the $\beta$-decay had in this case the form

$$\mathcal{H}_I^\beta(x) = \sum_{i=S,V,T,A,P} \bar{p}(x) O_i n(x) \ \bar{e}(x) O^i \left( G_i - G_i' \gamma_5 \right) \nu(x) + \text{h.c.}, \quad (9)$$

where the constants $G_i$ characterized the scalar part of the Hamiltonian and the constants $G_i'$ characterized the pseudoscalar part.

In the first $\beta$-decay experiment, which was performed by Wu *et al.* [5] soon after the Lee and Yang proposal, large effect of the violation of the parity was discovered. This means that the constants $G_i'$ (or some of them) are of the same order as the constants $G_i$. It was clear that some principles must be found which could decrease the number of interaction constants and simplify the form of the interaction Hamiltonian.

In 1957–58, two such crucial steps were done and correct effective Hamiltonian of the $\beta$-decay and other weak processes was obtained. The first step was **the theory of the two-component neutrino**.

A bit of history. In 1928, Dirac proposed relativistic equation for spin $1/2$ particle. He showed that requirement of the Lorenz invariance can be satisfied if the wave function $\psi(x)$ is a 4-component function. As a result, the Dirac equation has solutions with positive energies as well as solutions with negative energies which (in the framework of QFT) describe antiparticles.

In 1929, Weyl [6] obtained relativistic equations for a spin $1/2$ particle but for a two-component functions. He introduced two-component spinors

$$\psi_{\text{L,R}}(x) = \tfrac{1}{2} \left( 1 \mp \gamma_5 \right) \psi(x). \quad (10)$$

For a particle with a mass $m$ from the Dirac equation, we have two coupled equations

$$i\gamma^\alpha \partial_\alpha \ \psi_{\text{L}}(x) - m \ \psi_{\text{R}}(x) = 0, \qquad i\gamma^\alpha \partial_\alpha \ \psi_{\text{R}}(x) - m \ \psi_{\text{L}}(x) = 0. \quad (11)$$

If $m = 0$, we obtain two decoupled *Weil equations* for the two-component spinors $\psi_{\text{L}}(x)$ and $\psi_{\text{R}}(x)$

$$i\gamma^\alpha \partial_\alpha \ \psi_{\text{L}}(x) = 0, \qquad i\gamma^\alpha \partial_\alpha \ \psi_{\text{R}}(x) = 0. \quad (12)$$

The Weil equations, however, are not invariant under the parity transformation

$$\psi'_{\text{R,L}} \left( x' \right) = \eta \ \gamma^0 \psi_{\text{L,R}}(x), \qquad x' = \left( x^0, -\vec{x} \right), \quad (13)$$

where $\eta$ is an arbitrary phase factor.

In the thirties (and much later), there was a general belief that the conservation of parity is a law of nature. For these reasons, the Weil equations were rejected[2]. After it was discovered that parity is not conserved in the weak interaction, Landau [8], Lee and Yang [9] and Salam [10] proposed *the two-component neutrino theory.* They had different arguments in favor of this theory. Landau required CP invariance, Salam required $\gamma_5$ invariance, Lee and Yang applied the Weil theory to neutrino.

According to the two-component neutrino theory, the neutrino is a massless particle[3] and neutrino field is $\nu_L(x)$ or $\nu_R(x)$. The theory predicted:

1. $G'_i = \pm G_i$. Large violation of the parity in the $\beta$-decay must be observed.

2. The helicity of $\nu(\bar\nu)$ is equal to $-1(+1)$ in the case of the field $\nu_L(x)$ and $+1(-1)$ in the case of the field $\nu_R(x)$.

The first prediction was in agreement with the Wu *et al.* experiment [5]. The crucial confirmation of the two-component neutrino theory was obtained in the classical experiment by Goldhaber *et al.* [11] on the measurement of the neutrino helicity (1958). In this experiment, neutrino helicity was obtained from the measurement of the circular polarization of $\gamma$s produced in the chain of reactions

$$e^- + {}^{152}\mathrm{Eu} \to \nu + {}^{152}\mathrm{Sm}^*, \qquad {}^{152}\mathrm{Sm}^* \to {}^{152}\mathrm{Sm} + \gamma. \qquad (14)$$
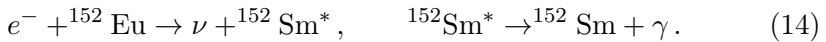
Goldhaber *et al.* [11] concluded "...our result is compatible with 100% negative helicity of neutrino ...". Thus, the two-component theory with neutrino field $\nu_L(x)$ was confirmed.

The second step was universal $V-A$, current $\times$ current theory of the weak interaction by Feynman and Gell-Mann [12], Marshak and Sudarshan [13] (1958). These authors assumed that in the Hamiltonian of the weak interaction enter *left-handed components of all fields.*

Let us consider $\bar{e}_L O^i \nu_L$. We have

$$\bar{e}_L(1, \sigma_{\alpha\beta}, \gamma_5)\nu_L = \bar{e}\frac{1+\gamma_5}{2}(1, \sigma_{\alpha\beta}, \gamma_5)\frac{1-\gamma_5}{2}\nu = 0 \qquad (15)$$

and

$$\bar{e}_L\gamma_\alpha\gamma_5\nu_L = -\bar{e}_L\gamma_\alpha\nu_L. \qquad (16)$$

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[2] "...because the equation for $\psi_L(x)$ ($\psi_R(x)$) is not invariant under space reflection, it is not applicable to the physical reality" [7].

[3] At the time of the discovery of the parity violation, the upper bound of the neutrino mass was about 200 eV (much smaller than the mass of the electron).

Thus, if we assume that left-handed components of all fields enter into the Hamiltonian of the weak interaction, the only possible Hamiltonian of the $\beta$-decay has the form

$$\mathcal{H}_I^\beta = \frac{G_{\rm F}}{\sqrt{2}} 4\, \bar{p}_{\rm L}\gamma^\alpha n_{\rm L}\, \bar{e}_{\rm L}\gamma_\alpha \nu_{\rm L} + \text{h.c.} = \frac{G_{\rm F}}{\sqrt{2}}\, \bar{p}\gamma^\alpha(1-\gamma_5)n\, \bar{e}\gamma_\alpha(1-\gamma_5)\nu + \text{h.c.} \quad (17)$$

Hamiltonian (17) like Fermi Hamiltonian (5) is characterized by only one interaction constant $G_{\rm F}$ [4]. Feynman and Gell-Mann, Marshak and Sudarshan had different arguments in favor of Hamiltonian (17). Let us stress that simplicity could be the argument: the Hamiltonian $\mathcal{H}_I^\beta$ is the simplest possible effective Hamiltonian of the $\beta$-decay which ensure large violation of parity, Fermi and Gamov–Teller transitions, *etc.* It describes all existing data.

Not only $\beta$-decay of nuclei but also $\mu$-decay

$$\mu^\pm \to e^\pm + \nu + \bar{\nu} \qquad (18)$$

and $\mu$-capture

$$\mu^- + (A, Z) \to \nu + (A, Z - 1) \qquad (19)$$

were known in the fifties. Feynman and Gell-Mann, Marshak and Sudarshan proposed *universal theory of the weak interaction* which included also these processes.

The Hamiltonian of $\beta$-decay (17) has a form of the product of the nucleon $\bar{p}_{\rm L}\gamma^\alpha n_{\rm L}$ and lepton $\bar{e}_{\rm L}\gamma_\alpha \nu_{\rm L}$ currents. Feynman and Gell-Mann introduced the notion of *the universal weak charged current*

$$j^\alpha = 2\,(\bar{p}_{\rm L}\gamma^\alpha n_{\rm L} + \bar{\nu}_{\rm L}\gamma^\alpha e_{\rm L} + \bar{\nu}_{\rm L}\gamma^\alpha \mu_{\rm L}) \qquad (20)$$

and assumed that the total Hamiltonian of the weak interaction has the current $\times$ current form

$$\mathcal{H}_I = \frac{G_{\rm F}}{\sqrt{2}}\, j^\alpha\, j_\alpha^+ \qquad (21)$$

with one universal interaction constant $G_{\rm F}$.

In the Feynman and Gell-Mann paper, the origin of the current $\times$ current interaction was briefly discussed: "We have adopted the point of view that the weak interactions arise from the interaction of a current $j_\alpha$ with itself, possibly via an intermediate charged vector meson of high mass".

Thus, they suggested that there existed a heavy, charged vector $W^\pm$ boson and that the Lagrangian of the weak interaction has the form

$$\mathcal{L}_I = -\frac{g}{2\sqrt{2}}\, j_\alpha\, W^\alpha + \text{h.c.}\,, \qquad (22)$$

where $g$ is a dimensionless interaction constant.

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[4] Interesting that title of the Feynman and Gell-Mann paper was *Theory of the Fermi Interaction.*

In the second order of the perturbation theory, interaction (22) generates the current × current effective Hamiltonian (21) which describes low-energy processes with virtual $W$ boson. The Fermi constant $G_F$ is given in this case by the relation

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8\, m_W^2}, \qquad (23)$$

where $m_W$ is the mass of the $W^{\pm}$ boson[5].

We will discuss now two important further steps in the developments of the weak interaction theory. In the Feynman and Gell-Mann, Marshak and Sudarshan papers, only one type of neutrino was considered. However, from the early years of the study of the $\mu$-decay and $\mu$-capture, there was an idea that there existed two types of neutrinos (electron $\nu_e$ which is produced together with $e^+$ and muon $\nu_\mu$ which is produced together with $\mu^+$). In the case of two types of neutrino, the charged current takes the form

$$j^\alpha = 2\,(\bar{p}_L\gamma^\alpha n_L + \bar{\nu}_{eL}\gamma^\alpha e_L + \bar{\nu}_{\mu L}\gamma^\alpha \mu_L)\,. \qquad (24)$$

A neutrino experiment which could check the two-neutrino hypothesis was proposed by Pontecorvo [14] (1959). According to the universal $V{-}A$ theory, the width of the decay $\pi^+ \to e^+ + \nu_e$ is much smaller than the width of the decay $\pi^+ \to \mu^+ + \nu_\mu$

$$\frac{\Gamma(\pi^+ \to e^+ + \nu_e)}{\Gamma(\pi^+ \to \mu^+ + \nu_\mu)} \simeq 1.2 \times 10^{-4}\,. \qquad (25)$$

Thus, if $\nu_\mu$ and $\nu_e$ are different particles, the beam of neutrinos produced in decays of $\pi^+$ is predominantly $\nu_\mu$ beam. In this case, in a neutrino detector $\mu^-$s, produced in the reaction

$$\nu_\mu + N \to \mu^- + X, \qquad (26)$$

will be observed. If $\nu_\mu$ and $\nu_e$ are the same particles, practically equal number of $\mu^-$s and $e^-$s will be observed in the neutrino detector.

The two-neutrino experiment was performed at the Brookhaven [15] (1962). It was the first experiment with neutrino from accelerator. It was proved that $\nu_\mu \neq \nu_e$. For this discovery, in 1988, L. Lederman, M. Schwartz and J. Steinberger were awarded the Nobel Prize ("for the neutrino beam method and the demonstration of the doublet structure of the leptons through the discovery of the muon neutrino").

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[5] As we know today, interaction (22) is a part of the Standard Model electroweak interaction. The modern value for the mass of $W^{\pm}$ boson is $m_W = 80.385 \pm 0.015$ GeV.

The strange particles were included in the $V$–$A$ charged current in 1962 by Cabibbo [16]. From the investigation of semi-leptonic decays of strange particles

$$K^+ \to \mu^+ + \nu_\mu \,, \qquad \Lambda \to n + e^- + \bar{\nu}_e \,, \qquad \Sigma^- \to n + \mu^- + \bar{\nu}_\mu \,, \quad \text{etc.}$$

three phenomenological rules were established:

1. $|\Delta S| = 1$, $\Delta S = S_\text{f} - S_\text{i}$, $S_\text{i}(S_\text{f})$ is strangeness of initial (final) hadron(s).

2. $\Delta Q = \Delta S$, $\Delta Q = Q_\text{f} - Q_\text{i}$.

3. The decays of strange particles are suppressed with respect to decays of nonstrange particles.

For example, from the second rule, it followed that the decay $\Sigma^+ \to n + e^+ + \nu_e$ was forbidden. In fact, it was found that $\Gamma(\Sigma^+ \to ne^+\nu_e)/\Gamma_\text{tot} < 5 \times 10^{-6}$.

In 1964, Gell-Mann and Zweig suggested that strange and nonstrange hadrons are bound states of $u$, $d$ and $s$ quarks. We will build the Cabibbo current from the quark fields. Using Feynman and Gell-Mann, Marshak and Sudarshan prescription (only left-handed fields in the current) from the fields of $u$, $d$ and $s$ quarks, we can build only two currents which change the charge by one

$$j^\alpha(\Delta S = 0) = 2\,\bar{u}_\text{L}\gamma^\alpha d_\text{L} \,, \qquad j^\alpha(\Delta S = 1) = 2\,\bar{u}_\text{L}\gamma^\alpha s_\text{L} \,. \qquad (27)$$

It is obvious that these currents automatically satisfy the rules 1 and 2 [6]. In order to satisfy the rule 3, Cabibbo introduced the parameter, the Cabibbo angle $\theta_\text{C}$. The Cabibbo current had a form

$$j_\alpha^\text{C} = 2\cos\theta_\text{C}\bar{u}_\text{L}\gamma_\alpha d_\text{L} + 2\sin\theta_\text{C}\bar{u}_\text{L}\gamma_\alpha s_\text{L} \,. \qquad (28)$$

Cabibbo demonstrated that the experimental data could be described if we assume that the current has a form of (28). For the parameter $\sin\theta_\text{C}$, it was found the approximate value $0.2$ [7].

Let us notice that the Cabibbo current can be presented in the following short form

$$j_\alpha^\text{C} = 2\bar{u}_\text{L}\gamma_\alpha d_\text{L}^\text{mix} \,, \qquad (29)$$

where

$$d_\text{L}^\text{mix} = \cos\theta_\text{C}d_\text{L} + \sin\theta_\text{C}s_\text{L} \qquad (30)$$

is the Cabibbo "mixture" of the fields of $d$ and $s$ quarks.

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[6] In the sixties, this was a strong argument in favor of suggestion that fundamental weak interaction is interaction of quarks and leptons.

[7] The modern value of the parameter $\sin\theta_\text{C}$ is $\sin\theta_\text{C} = 0.2253 \pm 0.0014$.

Soon, it occurred that there was a problem with the Cabibbo's extension of the charged current. Namely, this current generated a neutral current which changed the strangeness by one. Such a current induced the decay

$$K^+ \to \pi^+ + \nu + \bar{\nu} \tag{31}$$

with a decay rate which was many orders of magnitude larger than the existed experimental upper bound.

The solution of the problem was proposed in 1970 by Glashow, Illiopulos and Maiani [17]. They assumed that existed a fourth "charmed" quark $c$ with the charge $2/3$ and that there was an additional term in the weak charged current

$$j_\alpha^{\mathrm{GIM}} = 2\ \bar{c}_{\mathrm{L}}\gamma_\alpha s_{\mathrm{L}}^{\mathrm{mix}}\,, \tag{32}$$

where

$$s_{\mathrm{L}}^{\mathrm{mix}} = -\sin\theta_{\mathrm{C}}d_{\mathrm{L}} + \cos\theta_{\mathrm{C}}s_{\mathrm{L}} \tag{33}$$

is the mixture of $d_{\mathrm{L}}$ and $s_{\mathrm{L}}$ fields, orthogonal to the Cabibbo mixture (30).

If the $c$ quark, a constituent of hadrons, exists, in this case, a new family of "charmed" particles must exist. This prediction was perfectly confirmed by experiment. In 1974, the $J/\Psi$ particles, bound states of $(c\text{–}\bar{c})$, were discovered. In 1976, $D^+ = (c\bar{d})$, $D^- = (\bar{c}d)$, $D^0 = (c\bar{u})$, $\bar{D}^0 = (\bar{c}u)$ and other charmed particles were discovered.

After these steps in the development of the weak interaction theory, the total charged current of quarks and leptons took the form

$$j_\alpha^{\mathrm{CC}} = 2\ \left(\bar{\nu}_{e\mathrm{L}}\gamma_\alpha e_{\mathrm{L}} + \bar{\nu}_{\mu\mathrm{L}}\gamma_\alpha\mu_{\mathrm{L}} + \bar{u}_{\mathrm{L}}\gamma_\alpha d_{\mathrm{L}}^{\mathrm{mix}} + \bar{c}_{\mathrm{L}}\gamma_\alpha s_{\mathrm{L}}^{\mathrm{mix}}\right)\,, \tag{34}$$

where $d_{\mathrm{L}}^{\mathrm{mix}}$ and $s_{\mathrm{L}}^{\mathrm{mix}}$ are given by relations (30) and (33), respectively.

The current $\times$ current theory with the charged current, given by expression (34), could describe all existed data on the investigation of weak decays and neutrino reactions. Thus, it was confirmed that fields of $d$ and $s$ quarks enter in the charged current in the mixed form.

What about neutrinos? In the seventies, on the basis of an analogy of the weak interaction of quarks and leptons, it was suggested [18–20] that neutrinos had small masses and fields of neutrinos $\nu_{e\mathrm{L}}$ and $\nu_{\mu\mathrm{L}}$ entered into the charged current in a mixed form

$$\nu_{e\mathrm{L}} = \cos\theta\nu_{1\mathrm{L}} + \sin\theta\nu_{2\mathrm{L}}\,, \qquad \nu_{\mu\mathrm{L}} = -\sin\theta\nu_{1\mathrm{L}} + \cos\theta\nu_{2\mathrm{L}}\,. \tag{35}$$

Here, $\nu_1$ and $\nu_2$ are fields of neutrino with masses $m_1$ and $m_2$ and $\theta$ is a neutrino mixing angle. In analogy with quarks, it was assumed that $\nu_{1,2}$ were *Dirac fields possessing conserved lepton number*.

One of the most important consequences of mixing (35) was neutrino oscillations. First ideas of neutrino masses, mixing and oscillations were put forward by Pontecorvo [21, 22] (1957–58) soon after the two-component neutrino theory was proposed and confirmed by the experiment on the measurement of the neutrino helicity. Pontecorvo came to an idea of neutrino oscillations looking for analogy in the lepton world of the famous $K^0 \leftrightarrows \bar{K}^0$ oscillations, proposed by Gell-Mann and Pais and observed in experiments.

$K^0$ and $\bar{K}^0$ are particles with the strangeness $+1$ and $-1$, correspondingly. They are produced in strong interaction processes. The strangeness, however, is not conserved in the weak interaction. Eigenstates of the total Hamiltonian $K_1^0$ and $K_2^0$ particles with definite masses and decay widths, are described by the mixed states[8]

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right), \qquad |K_2^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right). \qquad (36)$$

From these relations, we obviously have

$$|K^0\rangle = \frac{1}{\sqrt{2}}\left(|K_1^0\rangle + |K_2^0\rangle\right), \qquad |\bar{K}^0\rangle = \frac{1}{\sqrt{2}}\left(|K_1^0\rangle - |K_2^0\rangle\right). \qquad (37)$$

When Pontecorvo proposed neutrino oscillations, only left-handed neutrino $\nu_{\mathrm{L}}$ and right-handed antineutrino $\bar{\nu}_{\mathrm{R}}$ were known. He assumed that existed also right-handed neutrino $\nu_{\mathrm{R}}$ and left-handed antineutrino $\bar{\nu}_{\mathrm{L}}$, particles which did not participate in the standard weak interaction (later he called such particles sterile neutrinos). In analogy with (37), Pontecorvo suggested that

$$|\bar{\nu}_{\mathrm{R}}\rangle = \frac{1}{\sqrt{2}}(|\nu_{1\mathrm{R}}\rangle + |\nu_{2\mathrm{R}}\rangle), \qquad |\nu_{\mathrm{R}}\rangle = \frac{1}{\sqrt{2}}(|\nu_{1\mathrm{R}}\rangle - |\nu_{2\mathrm{R}}\rangle), \qquad (38)$$

where $\nu_1$ and $\nu_2$ are Majorana neutrinos with masses $m_1$ and $m_2$.

If at $t = 0$ reactor $\bar{\nu}_{\mathrm{R}}$s were produced, at the time $t$, we had

$$|\bar{\nu}_{\mathrm{R}}\rangle_t = \frac{1}{\sqrt{2}}\left(e^{-iE_1 t}|\nu_{1\mathrm{R}}\rangle + e^{-iE_2 t}|\nu_{1\mathrm{R}}\rangle\right)$$

$$= \frac{1}{2}\left(e^{-iE_1 t} + e^{-iE_2 t}\right)|\bar{\nu}_{\mathrm{R}}\rangle + \frac{1}{2}\left(e^{-iE_1 t} - e^{-iE_2 t}\right)|\nu_{\mathrm{R}}\rangle, \qquad (39)$$

where $E_{1,2} = \sqrt{p^2 + m_{1,2}^2} \simeq E + \frac{m_{1,2}^2}{2E}$, $E = p$.

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[8] We neglected a small effect of the CP violation.

The probability of the reactor $\bar{\nu}_\text{R}$s to survive at the distance $L \simeq t$ was given by the expression

$$P(\bar{\nu}_\text{R} \to \bar{\nu}_\text{R}) = \frac{1}{2} \left( 1 + \cos \frac{\Delta m^2 L}{2E} \right) . \tag{40}$$

Here, $\Delta m^2 = m_2^2 - m_1^2$.

Pontecorvo proposed to search for neutrino oscillations by detecting reactor antineutrinos $\bar{\nu}_\text{R}$ at different distances from reactors. He also discussed a possibility to search for neutrino oscillations by measuring the flux of solar neutrinos $\nu_\text{L}$.

In 1962, Maki, Nakagawa and Sakata [23] on the basis of the Nagoya model, in which baryons were considered as bound states of neutrinos and a vector boson $B^+$, came to an idea of neutrino masses and mixing. They assumed "that there exists a representation which defines the true neutrinos $\nu_1$ and $\nu_2$ through orthogonal transformation"

$$\nu_1 = \cos \delta \nu_e - \sin \delta \nu_\mu \, , \qquad \nu_2 = \sin \delta \nu_e + \cos \delta \nu_\mu \, . \tag{41}$$

Neutrino oscillations were not considered in the MNS paper. From (41), they concluded that "weak neutrinos $\nu_e$ and $\nu_\mu$ are not stable due to the occurrence of virtual transition $\nu_e \rightleftarrows \nu_\mu$". From dimensional arguments, MNS estimated a transition time ($\tau \simeq \frac{1}{\Delta m}$) and in connection with the Brookhaven experiment, which was going on at that time, they noticed that "the absence of $e^-$ (in the Brookhaven experiment) will be able not only to verify the two-neutrino hypothesis but also to provide an upper limit of the mass difference $\Delta m$".

Neutrino oscillation hypothesis was actively worked out in Dubna in the seventies and eighties (see reviews [24, 25]). In accordance with spontaneously broken gauge theories, we assumed that the source of the neutrino masses and mixing is *a neutrino mass term* in the total Lagrangian. We considered all possible neutrino mass terms.

The neutrino mass term is a Lorenz-invariant product of left-handed and right-handed components of neutrino fields. Let us make the following remark. Any fermion field $\psi(x)$ can be presented in the form of

$$\psi = \psi_\text{L} + \psi_\text{R} \, , \tag{42}$$

where left-handed and right-handed components $\psi_\text{L}$ and $\psi_\text{R}$ are determined by the relations

$$\gamma_5 \psi_\text{L,R} = \mp \psi_\text{L,R} \, , \qquad \bar{\psi}_\text{L,R} \gamma_5 = \pm \bar{\psi}_\text{L,R} \, . \tag{43}$$

From the second relation, we find

$$C \gamma_5^T C^{-1} C \bar{\psi}_\text{L,R}^T = \gamma_5 C \bar{\psi}_\text{L,R}^T = \pm C \bar{\psi}_\text{L,R}^T \, , \tag{44}$$

where $C$ is the matrix of the charge conjugation, which satisfies the relations

$$C\gamma_\alpha^T C^{-1} = -\gamma_\alpha, \qquad C\gamma_5^T C^{-1} = \gamma_5, \qquad C^T = -C. \tag{45}$$

From (44), it follows that $(\psi_{\mathrm{L,R}})^c = C\bar{\psi}_{\mathrm{L,R}}^T$ is a right-handed (left-handed) component.

The standard leptonic charged current has the form

$$j_\alpha^{\mathrm{CC}}(x) = 2 \sum_{l=e,\mu,\tau} \nu_{l\mathrm{L}}(x)\gamma_\alpha l_{\mathrm{L}}(x). \tag{46}$$

Can the neutrino mass term include only left-handed flavor fields $\nu_{l\mathrm{L}}$?

For the simplest case of two flavor neutrinos, the positive answer to this question was given by Gribov and Pontecorvo [26]. In the case of three flavor neutrinos for the neutrino mass term, we have

$$\mathcal{L}^{\mathrm{L}}(x) = -\tfrac{1}{2}\sum_{l',l} \bar{\nu}_{l'\mathrm{L}}(x)\, M_{l'l}^{\mathrm{L}}(\nu_{l\mathrm{L}}(x))^c + \mathrm{h.c.} = -\tfrac{1}{2}\bar{\nu}_{\mathrm{L}}(x)\, M^{\mathrm{L}}\, (\nu_{\mathrm{L}}(x))^c + \mathrm{h.c.} \tag{47}$$

Here,

$$\nu_{\mathrm{L}}(x) = \begin{pmatrix} \nu_{e\mathrm{L}}(x) \\ \nu_{\mu\mathrm{L}}(x) \\ \nu_{\tau\mathrm{L}}(x) \end{pmatrix} \tag{48}$$

and $M^{\mathrm{L}}$ is a $3 \times 3$ matrix. Taking into account that neutrino fields satisfy the Fermi–Dirac statistics, we have

$$\bar{\nu}_{\mathrm{L}}\, M^{\mathrm{L}}\, (\nu_{\mathrm{L}})^c = \bar{\nu}_{\mathrm{L}}\, M^{\mathrm{L}}\, C\bar{\nu}_{\mathrm{L}}^T = -\bar{\nu}_{\mathrm{L}}\left(M^{\mathrm{L}}\right)^T C^T \bar{\nu}_{\mathrm{L}}^T = \bar{\nu}_{\mathrm{L}}\left(M^{\mathrm{L}}\right)^T (\nu_{\mathrm{L}})^c. \tag{49}$$

Thus, $M^{\mathrm{L}}$ is a symmetrical matrix

$$\left(M^{\mathrm{L}}\right)^T = M^{\mathrm{L}}. \tag{50}$$

The matrix $M^{\mathrm{L}}$ can be presented in the form of

$$M^{\mathrm{L}} = U\, m\, U^T, \tag{51}$$

where $U^\dagger U = 1$ and $m_{ik} = m_i\delta_{ik}$, $m_i > 0$.

Using (51), we can bring the mass term $\mathcal{L}^{\mathrm{L}}$ to the standard diagonal form

$$\begin{aligned} \mathcal{L}^{\mathrm{L}}(x) &= -\tfrac{1}{2}\overline{U^\dagger \nu_{\mathrm{L}}(x)}\, m\left(U^\dagger \nu_{\mathrm{L}}(x)\right)^c + \mathrm{h.c.} \\ &= -\tfrac{1}{2}\bar{\nu}^{\mathrm{M}}(x)\, m\, \nu^{\mathrm{M}}(x) = -\tfrac{1}{2}\sum_{i=1}^{3} m_i\bar{\nu}_i(x)\nu_i(x). \end{aligned} \tag{52}$$

Here,

$$\nu^{\mathrm{M}}(x) = U^{\dagger}\nu_{\mathrm{L}}(x) + \left(U^{\dagger}\nu_{\mathrm{L}}(x)\right)^c = \begin{pmatrix} \nu_1(x) \\ \nu_2(x) \\ \nu_3(x) \end{pmatrix}. \qquad (53)$$

From (48) and (53), it follows that $\nu_i(x)$ is the field of neutrinos with mass $m_i$. It satisfies the Majorana condition

$$\nu_i^c(x) = C\bar{\nu}_i^T(x) = \nu_i(x) \qquad (54)$$

which means that $\bar{\nu}_i \equiv \nu_i$.

It also follows from (53) that the flavor neutrino field $\nu_{l\mathrm{L}}(x)$ is given by the mixture of the fields of the Majorana neutrinos with definite masses

$$\nu_{l\mathrm{L}}(x) = \sum_{i=1}^{3} U_{li}\nu_{i\mathrm{L}}(x). \qquad (55)$$

The mass term $\mathcal{L}^{\mathrm{L}}$ is called *the Majorana mass term*.

If in the Lagrangian there are not only flavor left-handed neutrino fields $\nu_{l\mathrm{L}}(x)$ but also right-handed fields $\nu_{l\mathrm{R}}(x)$, it will be two additional possibilities for the neutrino mass term. The simplest possibility is the *Dirac mass term*

$$\mathcal{L}^{\mathrm{D}}(x) = -\sum_{l'l} \bar{\nu}_{l'\mathrm{L}}(x)\, M_{l'l}^{\mathrm{D}}\, \nu_{l\mathrm{R}}(x) + \mathrm{h.c.} = \bar{\nu}_{\mathrm{L}}(\mathrm{x})\, \mathrm{M}^{\mathrm{D}}\, \nu_{\mathrm{R}}(\mathrm{x}) + \mathrm{h.c.} \qquad (56)$$

Here, $M^{\mathrm{D}}$ is a complex, nondiagonal matrix. It can be presented in the form of

$$M^{\mathrm{D}} = U\, m\, V^{\dagger}, \qquad (57)$$

where $U^{\dagger}U = 1$, $V^{\dagger}V = 1$, $m_{ik} = m_i\delta_{ik}$, $m_i > 0$.

With the help of (57), we can present the mass term $\mathcal{L}^{\mathrm{D}}$ in the standard diagonal form

$$\mathcal{L}^{\mathrm{D}}(x) = -\sum_{i=1}^{3} m_i\bar{\nu}_i(x)\nu_i(x). \qquad (58)$$

Thus, $\nu_i(x)$ is the field of neutrino with the mass $m_i$. For the flavor neutrino field $\nu_{l\mathrm{L}}(x)$, we have the mixing relation

$$\nu_{l\mathrm{L}}(x) = \sum_{i=1}^{3} U_{li}\nu_{i\mathrm{L}}(x). \qquad (59)$$

The Lagrangian with mass term (58) is invariant under the following global gauge transformation

$$\nu_i(x) \to e^{i\Lambda}\nu_i(x)\,, \qquad l(x) \to e^{i\Lambda}l(x)\,, \qquad q(x) \to q(x)\,, \qquad \Lambda = \text{const.} \tag{60}$$

Invariance under transformation (60) means that the total lepton number $L$ is conserved and $\nu_i(x)$ is the field of the Dirac neutrino

$$L(\nu_i) = -L(\bar\nu_i) = 1\,.$$

Dirac mass term (56) can be generated by the standard Higgs mechanism.

   *The Dirac and Majorana mass term*

$$\mathcal{L}^{\text{D+M}}(x) = \mathcal{L}^{\text{L}}(x) + \mathcal{L}^{\text{D}}(x) + \mathcal{L}^{\text{R}}(x) \tag{61}$$

is the most general neutrino mass term [27]. Here, $\mathcal{L}^{\text{L}}(x)$ and $\mathcal{L}^{\text{D}}(x)$ are given by (47) and (56), and

$$\mathcal{L}^{\text{R}}(x) = -\tfrac{1}{2}\sum_{s',s}\overline{(\nu_{s'\text{R}}(x))^c}\,M^{\text{R}}_{s's}\,\nu_{s\text{R}}(x) + \text{h.c.} = -\tfrac{1}{2}\overline{(\nu_{\text{R}}(x))^c}\,M^{\text{R}}\,\nu_{\text{R}}(x) + \text{h.c.}\,, \tag{62}$$

where $M^{\text{R}}$ is a complex, symmetric matrix.

   Lagrangian (62) does not conserve the total lepton number $L$. This means that neutrinos with definite masses are Majorana particles. After the standard diagonalization of the mass term $\mathcal{L}^{\text{D+M}}(x)$, we find

$$\mathcal{L}^{\text{D+M}}(x) = -\tfrac{1}{2}\sum_{i=1}^{6}m_i\bar\nu_i(x)\nu_i(x)\,. \tag{63}$$

Here, $\nu_i^c(x) = \nu_i(x)$ is the field of the Majorana neutrino with the mass $m_i$. For the flavor fields $\nu_{l\text{L}}(x)$, we have

$$\nu_{l\text{L}}(x) = \sum_{i=1}^{6}U_{li}\,\nu_{i\text{L}}(x)\,, \qquad l = e, \mu, \tau\,, \tag{64}$$

where $U$ is an unitary $6 \times 6$ matrix. Thus, left-handed flavor fields $\nu_{l\text{L}}(x)$ are combinations of the left-handed components of six Majorana fields with definite masses. The left-handed components of sterile fields $(\nu_{s\text{R}}(x))^c$ are combinations of the left-handed components of the same Majorana fields

$$(\nu_{s\text{R}}(x))^c = \sum_{i=1}^{6}U_{si}\,\nu_{i\text{L}}(x)\,, \qquad s = s_1, s_2, s_3\,. \tag{65}$$

There are different possibilities in the case of the Dirac and Majorana mass term (61):

1. If masses of more than three Majorana particles $\nu_i$ are small, transitions of flavor neutrinos $\nu_l$ ($l = e, \mu, \tau$) into sterile states become possible. There exist indications in favor of such transitions obtained in different short-baseline neutrino experiments. We will discuss these indications later.

2. If $M^{\mathrm{L}} = 0$ and $M^{\mathrm{D}} \ll M^{\mathrm{R}}$, in this case, in the mass spectrum of the Majorana particles, there are three light neutrino masses $m_i$ ($i = 1, 2, 3$) and three heavy masses $M_a$ ($a = 1, 2, 3$). This is a basis for the famous seesaw mechanism of the neutrino mass generation [28] which explains smallness of masses $m_i$.

## 3. Neutrino oscillations

If there are neutrino mixing, a new quantum phenomenon, neutrino oscillations, become possible. We considered this phenomenon in some details in Dubna in the seventies and eighties [24, 25]. Here, we will briefly consider the present status of neutrino oscillations (see [29]).

All existing CC neutrino interaction data are described by the SM Lagrangian

$$\mathcal{L}_I^{\mathrm{CC}} = -\frac{g}{2\sqrt{2}} j_\alpha^{\mathrm{CC}} W^\alpha + \text{h.c.} , \qquad j_\alpha^{\mathrm{CC}} = \sum_l \bar{\nu}_{l\mathrm{L}} \gamma_\alpha l_{\mathrm{L}} , \qquad (66)$$

where $\nu_{l\mathrm{L}}$ is given by the relation

$$\nu_{l\mathrm{L}} = \sum_i U_{li} \, \nu_{i\mathrm{L}} . \qquad (67)$$

We call flavor muon neutrino $\nu_\mu$ a particle which is produced together with $\mu^+$, say, in the decay $\pi^+ \to \mu^+ + \nu_\mu$. Electron antineutrino $\bar{\nu}_e$ is a particle which is produced together with $e^-$ in the $\beta$-decay $n \to p + e^- + \bar{\nu}_e$, *etc.*

What are the states of flavor neutrinos in the case of the neutrino mixing (67)?

The standard theory of the neutrino oscillations is based on the assumption that state of the flavor neutrino $\nu_l$ with momentum $\vec{p}$ is given by *a coherent superposition of states of neutrinos with definite masses*

$$|\nu_l\rangle = \sum_i U_{li}^* \, |\nu_i\rangle \qquad (l = e, \mu, \tau) . \qquad (68)$$

Here, $|\nu_i\rangle$ is the state of neutrino with mass $m_i$, momentum $\vec{p}$ and energy $E_i = \sqrt{p^2 + m_i^2} \simeq E + \frac{m_i^2}{2E}$ ($m_i \ll p \equiv E$).

Relation (68) is based on the Heisenberg uncertainty relation from which it follows that it is impossible to resolve emission of neutrinos with small mass-squared differences in weak decays[9].

*Small neutrino mass-squared differences can be resolved in special experiments with large distances between neutrino sources and detectors.* A possibility to reveal $\Delta m_{ik}^2$ is based on the time–energy uncertainty relation

$$\Delta E \Delta t \geq 1 \,. \tag{69}$$

Taking into account that $\Delta E_{ki} = |E_i - E_k| \simeq \frac{|\Delta m_{ki}^2|}{2E}$ and for the ultrarelativistic neutrinos $\Delta t \simeq L$ ($L$ is the source-detector distance) from (69), we find the following condition to resolve $|\Delta m_{ki}^2|$:

$$\frac{\left|\Delta m_{ki}^2\right|}{2E} \, L \geq 1 \,. \tag{70}$$

Let us now briefly consider neutrino oscillations in vacuum. If at $t = 0$ the flavor neutrino $\nu_l$ is produced, the state of neutrino at the time $t$ will be the superposition of states with different energies (the nonstationary state)

$$|\nu_l\rangle_t = e^{-iH_0 t}|\nu_l\rangle = \sum_i e^{-iE_i t} U_{li}^* \, |\nu_i\rangle \,. \tag{71}$$

Flavor neutrinos (and antineutrinos) can be detected in a neutrino detector. From (68) and (71), we find

$$|\nu_l\rangle_t = \sum_{l'} |\nu_{l'}\rangle \, \sum_i U_{l'i} e^{-iE_i t} U_{li}^* \,. \tag{72}$$

Thus, the probability of the $\nu_l \to \nu_{l'}$ transition is given by the expression

$$P(\nu_l \to \nu_{l'}) = \Big| \sum_i U_{l'i} e^{-iE_i t} U_{li}^* \Big|^2 \,. \tag{73}$$

This expression will be simplified if we:

— extract in (73) a common phase factor $e^{-iE_p t}$, where the index $p$ is arbitrary,

— use the unitarity $\sum_i U_{l'i} U_{li}^* = \delta_{l'l}$.

---

[9] The difference of momenta of neutrinos $\nu_i$ and $\nu_k$ is given by the relation $|\Delta p_{ki}| \simeq \frac{|\Delta m_{ki}^2|}{2E} = \frac{1}{L_{ki}}$. On the other hand, for the quantum-mechanical uncertainty of the momentum, we have $(\Delta p)_{\mathrm{QM}} \simeq \frac{1}{d}$ ($d$ is a microscopic size of a source). In neutrino oscillation experiments, $L_{ki}$ is a large macroscopic distance ($\sim 10^3$ km in atmospheric and accelerator experiments, $\sim 10^2$ km in the reactor KamLAND experiment, *etc.*). Thus, $L_{ki} \gg d$ and $|\Delta p_{ki}| \ll (\Delta p)_{\mathrm{QM}}$.

We find

$$P(\nu_l \to \nu_{l'}) = \left| \delta_{l'l} - 2i \sum_i U_{l'i} U_{li}^* e^{-i\Delta_{pi}} \sin \Delta_{pi} \right|^2, \tag{74}$$

where

$$\Delta_{pi} = \frac{\Delta m_{pi}^2 L}{4E}, \qquad \Delta m_{pi}^2 = m_i^2 - m_p^2. \tag{75}$$

From (74), we obtain the following general expression for the neutrino transition probability in vacuum

$$P\left( \overset{(-)}{\nu_l} \to \overset{(-)}{\nu_{l'}} \right) = \delta_{l'l} - 4 \sum_i |U_{li}|^2 \left( \delta_{l'l} - |U_{l'i}|^2 \right) \sin^2 \Delta_{pi}$$

$$+ 8 \sum_{i>k} \text{Re} \left( U_{l'i} U_{li}^* U_{l'k}^* U_{lk} \right) \cos \left( \Delta_{pi} - \Delta_{pk} \right) \sin \Delta_{pi} \sin \Delta_{pk}$$

$$\pm 8 \sum_{i>k} \text{Im} \left( U_{l'i} U_{li}^* U_{l'k}^* U_{lk} \right) \sin \left( \Delta_{pi} - \Delta_{pk} \right) \sin \Delta_{pi} \sin \Delta_{pk}, \tag{76}$$

where sign $+$ $(-)$ refers to $\nu_l \to \nu_{l'}$ $(\bar{\nu}_l \to \bar{\nu}_{l'})$ transition and $i, k \neq p$.

In the simplest case of two-neutrino oscillations, we will choose $p = 1$. In this case, $i = 2$ and there are no $i > k$ terms. For the neutrino transition probability, we have

$$P\left( \overset{(-)}{\nu_l} \to \overset{(-)}{\nu_{l'}} \right) = \delta_{l'l} - 4|U_{l2}|^2 \left( \delta_{l'l} - |U_{l'2}|^2 \right) \sin^2 \Delta_{12}. \tag{77}$$

From the unitarity of the mixing matrix, we have: $|U_{l2}|^2 = \sin^2 \theta$, $|U_{l'2}|^2 = \cos^2 \theta$, $(l' \neq l)$ ($\theta$ is the mixing angle). From (77), we find the following classical expressions for neutrino disappearance and appearance probabilities:

$$P\left( \overset{(-)}{\nu_l} \to \overset{(-)}{\nu_l} \right) = 1 - \sin^2 2\theta \ \sin^2 \frac{\Delta m_{12}^2 L}{4E} \tag{78}$$

and

$$P\left( \overset{(-)}{\nu_l} \to \overset{(-)}{\nu_{l'}} \right) = \sin^2 2\theta \ \sin^2 \frac{\Delta m_{12}^2 L}{4E}, \qquad l' \neq l. \tag{79}$$

From these expressions, it follows that neutrino oscillations are characterized by the amplitude $\sin^2 2\theta$ and oscillation length

$$L_{\text{osc}} = 4\pi \frac{E}{\Delta m_{12}^2} \simeq 2.47 \frac{E \ [\text{MeV}]}{\Delta m_{12}^2 \ [\text{eV}^2]} m. \tag{80}$$

In the three-neutrino case, probabilities of the neutrino transition depend on six parameters: three mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$, CP phase $\delta$ and two neutrino mass-squared differences. Two neutrino mass spectra are possible in

this case. Usually, neutrino masses are labeled in such a way that $m_2 > m_1$. Possible neutrino mass spectra are determined by the mass $m_3$. There are two possibilities:

1. Normal ordering (NO) $m_3 > m_2 > m_1$.

2. Inverted ordering (IO) $m_2 > m_1 > m_3$.

We will determine (large) atmospheric mass-squared difference in the following way

$$\Delta m_{\mathrm{A}}^2 = \Delta m_{23}^2 \quad (\mathrm{NO}), \qquad \Delta m_{\mathrm{A}}^2 = |\Delta m_{13}^2| \quad (\mathrm{IO}). \tag{81}$$

The (small) solar mass-squared difference for both mass spectra is determined as follows

$$\Delta m_{\mathrm{S}}^2 = \Delta m_{12}^2. \tag{82}$$

From analysis of the neutrino oscillation data, it follows that two neutrino oscillation parameters are small

$$\frac{\Delta m_{\mathrm{S}}^2}{\Delta m_{\mathrm{A}}^2} \simeq 3 \times 10^{-2}, \qquad \sin^2 \theta_{13} \simeq 2.5 \times 10^{-2}. \tag{83}$$

If we neglect contribution of these parameters to the neutrino transition probabilities, we will obtain simple two-neutrino formulas which describe the basic feature of the three-neutrino oscillations (the leading approximation).

Let us consider $\overset{(-)}{\nu_\mu} \to \overset{(-)}{\nu_\mu}$ transitions in the atmospheric range of $\frac{L}{E}$ ($\Delta_{\mathrm{A}} \simeq 1$, $\Delta_{\mathrm{S}} \ll 1$). From (76), we find

$$P\left(\overset{(-)}{\nu_\mu} \to \overset{(-)}{\nu_\mu}\right) \simeq 1 - \sin^2 2\theta_{23} \; \sin^2 \frac{\Delta m_{\mathrm{A}}^2 L}{4E}. \tag{84}$$

For $\bar{\nu}_e \to \bar{\nu}_e$ transition probability in the solar range of $\frac{L}{E}$ (KamLAND experiment, $\Delta_{\mathrm{S}} \simeq 1$, $\Delta_{\mathrm{A}} \gg 1$), we obtain the following expression

$$P(\bar{\nu}_e \to \bar{\nu}_e) \simeq 1 - \sin^2 2\theta_{12} \; \sin^2 \frac{\Delta m_{\mathrm{S}}^2 L}{4E}. \tag{85}$$

During many years, these expressions were used for the analysis of atmospheric, accelerator and reactor neutrino oscillation data. In order to analyze data of modern neutrino oscillation experiments, the exact three-neutrino expressions for transition probabilities must be used. In Table I, we present the result of the global analysis of present-day data [30].

| Parameter | Normal ordering | Inverted ordering |
|---|---|---|
| $\sin^2\theta_{12}$ | $0.304^{+0.013}_{-0.012}$ | $0.304^{+0.013}_{-0.012}$ |
| $\sin^2\theta_{23}$ | $0.452^{+0.052}_{-0.028}$ | $0.579^{+0.025}_{-0.037}$ |
| $\sin^2\theta_{13}$ | $0.0218^{+0.0010}_{-0.0010}$ | $0.0219^{+0.0011}_{-0.0010}$ |
| $\delta$ (in °) | $(306^{+39}_{-70})$ | $(254^{+63}_{-62})$ |
| $\Delta m^2_{\rm S}$ | $(7.50^{+0.19}_{-0.17}) \times 10^{-5}$ eV$^2$ | $(7.50^{+0.19}_{-0.17}) \times 10^{-5}$ eV$^2$ |
| $\Delta m^2_{\rm A}$ | $(2.457^{+0.047}_{-0.047}) \times 10^{-3}$ eV$^2$ | $(2.449^{+0.048}_{-0.047}) \times 10^{-3}$ eV$^2$ |

We see from this table that:

— neutrino oscillations parameters are known with accuracies from $\sim 3\%$ ($\Delta m^2_{\rm S,A}$) to $\sim 10\%$ ($\sin^2\theta_{23}$),

— the existing data do not allow to distinguish normal and inverted neutrino mass ordering,

— the CP phase is practically unknown.

It is expected that in the future neutrino oscillation experiments:

1. Neutrino oscillation parameters will be measured with % accuracy.

2. Neutrino mass ordering will be determined.

3. The CP phase $\delta$ will be measured.

We can hope that future neutrino oscillation and other experiments will allow to answer the most fundamental question: what is the origin of small neutrino masses? What new physics was discovered with discovery of neutrino oscillations? In the last part of this paper, we will discuss a plausible mechanism of a generation of small neutrino masses.

## 4. The most economical mechanism of neutrino mass generation

After the discovery of the Higgs boson at the LHC, the Standard Model acquire a status of *the theory of elementary particles in the electroweak range (up to $\sim$ 300 GeV)*.

The Standard Model is based on the following principles:

— Local gauge invariance.

— Unification of the electromagnetic and weak interactions.

— Spontaneous breaking of the electroweak symmetry.

The Standard Model teaches us that in the framework of these principles, *the nature chooses the simplest, most economical possibilities* (see [31]).

In fact, the first step in the creation of the Standard Model was the theory of two-component massless left-handed neutrinos. $SU_L(2)$ is the simplest symmetry which allows to include also left-handed leptons (and quarks). Unification of the weak and electromagnetic interactions requires to include in the theory right-handed fields *of charged fields* (leptons and quarks). The simplest group which allow to unify the weak and electromagnetic interactions is $SU_L(2) \times U(1)$.

Neutrinos have no electromagnetic interaction. Unification of the weak and electromagnetic interactions does not require right-handed neutrino fields. The most economical possibility: *there are no right-handed neutrino fields in the Standard Model Lagrangian.*

Notice also that the standard electroweak interaction of fermions and vector gauge bosons is the minimal (compatible with the local gauge invariance) interaction and Higgs doublet is a minimal possibility which allows to generate masses of $W^{\pm}$ and $Z^0$ bosons. *After the spontaneous symmetry breaking, neutrinos remain two-component massless particles.* Thus, neutrino masses and mixing can be generated only by a beyond the Standard Model mechanism.

The method of the effective Lagrangian is a powerful, general method which allows to describe effects of a beyond the Standard Model physics. The effective Lagrangian is a nonrenormalizable Lagrangian invariant under the transformations of $SU_L(2) \times U_Y(1)$ group and built from the Standard Model fields. In general, the effective Lagrangian is a sum of operators of dimension five and more.

Let us consider the terms

$$\left( \bar{\psi}_{l\mathrm{L}}^{\mathrm{lep}} \ \tilde{\phi} \right), \qquad \left( \tilde{\phi}^{\dagger} \ \psi_{l\mathrm{L}}^{\mathrm{lep}} \right), \qquad l = e, \mu, \tau. \tag{86}$$

Here,

$$\psi_{l\mathrm{L}}^{\mathrm{lep}} = \left( \begin{array}{c} \nu'_{l\mathrm{L}} \\ e'_{\mathrm{L}} \end{array} \right), \quad \phi = \left( \begin{array}{c} \phi_+ \\ \phi_0 \end{array} \right) \tag{87}$$

are lepton and Higgs doublets and $\tilde{\phi} = i\tau_2 \phi^*$ is a conjugated doublet.

It is obvious that terms (86) are $SU_L(2) \times U_Y(1)$ invariant and have dimension $M^{5/2}$. After spontaneous symmetry breaking, we have

$$\left( \bar{\psi}_{l\mathrm{L}}^{\mathrm{lep}} \ \tilde{\phi} \right) \to \frac{v}{\sqrt{2}} \bar{\nu}'_{l\mathrm{L}}, \qquad \left( \tilde{\phi}^{\dagger} \ \psi_{l\mathrm{L}}^{\mathrm{lep}} \right) \to \frac{v}{\sqrt{2}} \nu'_{l\mathrm{L}}, \tag{88}$$

where $v = (\sqrt{2} G_{\mathrm{F}})^{-1/2} \simeq 246$ GeV is the vacuum expectation value of the Higgs field.

It follows from (88) that the possible effective Lagrangian which generates the neutrino mass term has the form [32]

$$\mathcal{L}_I^{\text{eff}} = -\frac{1}{\Lambda} \sum_{l_1,l_2} \left( \bar{\psi}_{l_1\text{L}}^{\text{lep}} \tilde{\phi} \right) Y'_{l_1 l_2} \left( \tilde{\phi}^T \left( \psi_{l_2\text{L}}^{\text{lep}} \right)^c \right) + \text{h.c.} , \qquad (89)$$

where the parameter $\Lambda$ (dimension $M$) characterizes a scale of a beyond the SM physics and $Y'$ is $3 \times 3$ dimensionless, symmetrical matrix. Let us stress that:

— $\Lambda \gg v$.

— The Lagrangian $\mathcal{L}_I^{\text{eff}}$ is *the only possible effective Lagrangian which generates the neutrino mass term.* It is a dimension five operator.

— The Lagrangian $\mathcal{L}_I^{\text{eff}}$ does not conserve the total lepton number.

After spontaneous symmetry breaking, from (89), we come to the Majorana mass term

$$\mathcal{L}^{\text{M}} = -\frac{1}{2} \frac{v^2}{\Lambda} \sum_{l_1,l_2} \bar{\nu}_{l_1\text{L}} Y_{l_1 l_2} (\nu_{l_2\text{L}})^c + \text{h.c.} \qquad (90)$$

The symmetrical matrix $Y$ can be presented in the form of

$$Y = U y U^T , \qquad (91)$$

where $U^\dagger U = 1$ and $y_{ik} = y_i \delta_{ik}$, $y_i > 0$. From (90) and (91), we find

$$\mathcal{L}^{\text{M}} = -\frac{1}{2} \sum_{i=1}^{3} m_i \bar{\nu}_i \nu_i . \qquad (92)$$

Here,

$$\nu_i = \nu_i^c \qquad (93)$$

is the field of the neutrino Majorana with the mass

$$m_i = \frac{v^2}{\Lambda} y_i = \frac{v}{\Lambda} (y_i v) . \qquad (94)$$

For the field of the flavor neutrino $\nu_{l\text{L}}$, we have the standard mixing relation

$$\nu_{l\text{L}} = \sum_{i=1}^{3} U_{li} \nu_{i\text{L}} . \qquad (95)$$

The quantity $(y_i v)$ is a "typical" Dirac fermion mass in the Standard Model. From (94), we conclude that Majorana neutrino masses generated by the

effective Lagrangian (89) are much less than the Dirac masses of leptons and quarks. The suppression factor is given by

$$\frac{v}{\Lambda} = \frac{\text{scale of SM}}{\text{scale of a new physics}} \ll 1. \tag{96}$$

In order to estimate the scale of a new physics $\Lambda$, we assume hierarchy of neutrino masses $m_1 \ll m_2 \ll m_3$. In this case, $m_3 \simeq \sqrt{\Delta m_{\text{A}}^2} \simeq 5 \times 10^{-2}$ eV. If we also assume that $y_3 \simeq 1$, we find $\Lambda \simeq 10^{15}$ GeV.

Thus, if neutrino masses are generated via beyond the Standard Model effective Lagrangian in this case:

— Neutrino with definite masses $\nu_i$ are Majorana particles.

— The number of neutrinos with definite masses is equal to the number of lepton–quark generations (three).

In conclusion, we will briefly discuss possibilities to test these predictions.

Observation of the neutrinoless double $\beta$-decay ($0\nu\beta\beta$-decay) of $^{76}$Ge, $^{136}$Xe and other even–even nuclei

$$(A, Z) \to (A, Z + 2) + e^- + e^- \tag{97}$$

would be a direct proof that $\nu_i$ are Majorana particles (see review [33]).

The $0\nu\beta\beta$-decay is the second order in $G_{\text{F}}$ process with virtual neutrino. For the neutrino propagator, we have

$$\sum_i U_{ei}^2 \left( \frac{1 - \gamma_5}{2} \right) \frac{\gamma\, p + m_i}{p^2 - m_i^2} \left( \frac{1 - \gamma_5}{2} \right) \simeq m_{\beta\beta} \frac{1}{p^2} \left( \frac{1 - \gamma_5}{2} \right). \tag{98}$$

Here,

$$m_{\beta\beta} = \sum_{i=1}^{3} U_{ei}^2 m_i \tag{99}$$

is the effective Majorana mass. From (98), it follows that the matrix element of the $0\nu\beta\beta$-decay is proportional to $m_{\beta\beta}$.

The probability of the process is given by the expression

$$\frac{1}{T_{1/2}^{0\nu}} = |m_{\beta\beta}|^2 \; |M^{0\nu}|^2 \; G^{0\nu}(Q, Z). \tag{100}$$

Here, $T_{1/2}^{0\nu}$ is the half-live of the $0\nu\beta\beta$-decay, $M^{0\nu}$ is a nuclear matrix element and $G^{0\nu}(Q, Z)$ is the known phase factor.

Many experiments on the search for $0\nu\beta\beta$-decay of different nuclei were performed. Up to now, the $0\nu\beta\beta$-decay was not observed. Very large lower bounds for half-lives of the decay were obtained in different experiments.

We will present here some recent data:

— EXO-200: $T_{1/2}^{0\nu}(^{136}\text{Xe}) > 1.1 \times 10^{25}$ y, $|m_{\beta\beta}| < (1.9 - 4.5) \times 10^{-1}$ eV.

— KamLAND-Zen: $T_{1/2}^{0\nu}(^{136}\text{Xe}) > 2.6 \times 10^{25}$ y, $|m_{\beta\beta}| < (1.4 - 2.8) \times 10^{-1}$ eV.

— GERDA: $T_{1/2}^{0\nu}(^{76}\text{Ge}) > 3.0 \times 10^{25}$ y, $|m_{\beta\beta}| < (2 - 4) \times 10^{-1}$ eV.

Half-life of $0\nu\beta\beta$-decay strongly depends on neutrino mass ordering. If there is the inverted ordering (and neutrinos are Majorana particles) from oscillation data, it follows that

$$|m_{\beta\beta}| \simeq \text{a few} \times 10^{-2} \text{ eV} . \tag{101}$$

If there is the normal ordering of neutrino masses, $|m_{\beta\beta}|$ will be smaller. Next experiments on the search for $0\nu\beta\beta$ are planned to reach region (101).

If the number of neutrinos with definite masses is equal to three, there are no transitions of flavor neutrinos into sterile states. Indications in favor of such transitions were obtained in the LSND $\bar{\nu}_\mu \to \bar{\nu}_e$ short baseline experiment, in the MiniBooNE experiment in which $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ transitions were searched for, in the reactor $\bar{\nu}_e \to \bar{\nu}_e$ experiments and in the source $\nu_e \to \nu_e$ experiments (see [34–36]).

Existing data can be explained by neutrino oscillations with $\Delta m^2 \simeq 1$ eV$^2$, much larger than $\Delta m_A^2$. There exists, however, a tension between data: probability of $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu$ disappearance predicted from analysis of the existing short baseline data is not compatible with direct experiments.

Many new short baseline accelerator, reactor and source experiments on the search for transitions of flavor neutrinos into sterile states are in preparation at the moment. There is no doubt that in a few years, the sterile neutrino problem will be solved.

In conclusion, let us summarize the main challenges which will be solved in future:

1. Are neutrinos $\nu_i$ Majorana or Dirac particles?

2. Is neutrino mass ordering normal or inverted?

3. What is the value of the CP phase $\delta$?

4. Are there transitions of flavor neutrinos $\nu_l$ into sterile states?

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