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ON THE THEORY OF THE CLASSICAL RELATIVISTIC POINT-LIKE CHARGE

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1. INTRODUCTION

The problem of infinite self-energy of the point charge is normally regarded as resolved by the mass renormalization. But exact solutions of the renormalized equation of motion of the radiating point charge, i.e., the Lorentz-Dirac equation [1], seem inevitably to show either of the two pathological properties, run-away (self-acceleration) or preacceleration. The former represents that if the charge had once been given an acceleration its velocity continues to grow even if the force has ceased and it approaches asymptotically the one of light. Such unphysical solutions can be eliminated by imposing appropriate asymptotic conditions. Then a solution which satisfies the conditions is inevitably burdened by preacceleration, i.e., the charge starts to accelerate before the force begins to act, in contradiction with the causality. A detailed analysis of the situation with self- and preacceleration is given by Röhrlich [2]. For more recent papers on the problem we refer to introductory sections of refs. [3,4].

It is rather a common view that this problem of classical electrodynamics does not concern the reality, since it becomes essential for such intensities (forces) and wave lengths of external fields, that should be considered in the framework of quantum mechanics. But in the classical region it is regarded as sufficient (and even as necessary) to treat the radiation reaction as a perturbation (see, e.g., ref. [6], §73). Then there is no problem of the run-away and the preacceleration for the problem is related to exact solutions of the Lorentz-Dirac equation and not to its iterations by radiation reaction.

However, this is just the reason to investigate the origin of the contradiction between physical expectations and mathematical results. Indeed, in view of its relatively simple mathematics the theory of point charge may serve as a dynamical model for studying the consistency of a perturbative approach and of the exact theory, i.e., the problem which is both important and almost inaccessible in the quantum field theory. The problem of self-consistency of the classical electrodynamics does not seem also to be purely academic.

In the present paper we attempt to trace and to surmount the afore-mentioned difficulties of the classical charge dynamics by giving a more strict mathematical definition to divergent terms that arise in the course of solution of the Maxwell-Lorentz system for a point-like charge. In Sec.2 the essentials of the problem are represented as we understand them. In Sec.3 we introduce a regularization of the retarded Green functions of the electromagnetic field which leads to a relativistic-invariant integro-differential equation, which is apparently causal. We show that the Lorentz-Dirac equation is the limit of our equation for a vanishing regularization parameter. In Sec.4 we argue that if non-run-away solutions of our equation exist, they do not pre-accelerate.

2. MOTION OF A POINT CHARGE IN CLASSICAL ELECTRODYNAMICS

We start with the action, see, e.g., ref. [5]

$$S = m_0 c^2 \int d\lambda \sqrt{\frac{dz_\gamma}{d\lambda}\frac{dz^\gamma}{d\lambda}} + \int (dx)^4 \left\{ \frac{1}{16\pi} F_{\alpha\beta}(x) F^{\alpha\beta}(x) - \frac{1}{c} A_\alpha(x) j^\alpha(x) \right\};$$

m_0 being the (bare) mass of the charge, $F_{\alpha\beta} \equiv \partial_\alpha A_\beta - \partial_\beta A_\alpha$ is the electromagnetic field tensor, $z^\alpha(\lambda)$ being a world line of the charge and

$$j^\alpha(x) = ec \int_{-\infty}^{\infty} \delta^4(x - z(\lambda)) \frac{dz^\alpha}{d\lambda} d\lambda. \tag{1}$$

Variation of A and z^α and choice of the proper time τ as a parameter λ, i.e.,

$$\dot z^\alpha \dot z_\alpha = -c^2, \quad \dot z^\alpha \equiv \frac{d}{d\tau} z^\alpha(\tau), \tag{2}$$

give the system of Maxwell-Lorentz equations (in the Lorentz gauge):

$$\Box A_\alpha = -\frac{4\pi}{c} j_\alpha(x), \tag{3a}$$

$$m_0 \ddot z^\alpha(\tau) = \frac{e}{c} F^{\alpha\beta}(z) \dot z_\beta. \tag{3b}$$

If one considers the problem of charge motion in the (free) external field $A^{in}_\alpha(x)$, then it is natural to represent the solution of Eq. (3a) as a superposition of $A^{in}_\alpha(x)$ and of the retarded field

$$A_\alpha(x) = A^{in}_\alpha(x) + A^{ret}_\alpha(x) = A^{in}_\alpha(x) + \frac{4\pi}{c}\int (d\tilde x)^4 G^{ret}_{\alpha\beta}(x,\tilde x) j^\beta(\tilde x), \tag{4}$$

where

$$G^{\text{ret}}_{\alpha\beta}(x,\tilde{x}) = \frac{1}{4\pi}\theta(\Sigma(x),\tilde{x})\,\delta[\sigma(x,\tilde{x})]\eta_{\alpha\beta} \tag{5}$$

is the retarded Green function of the operator $\Box$, see, e.g., ref.[5]:

$$\Box G^{\text{ret}}_{\alpha\beta}(x,\tilde{x}) = -\delta^{(4)}(x-\tilde{x})\eta_{\alpha\beta},$$

$\eta_{\alpha\beta} \equiv \text{diag}(-1,1,1,1)$ is the Minkowsky metric tensor, $\theta(\Sigma(x),x')$ is the function that "takes values" 1 if x' is in the past and 0 if x' is in the future of a space-like hypersurface Σ, and

$$\sigma(x,\tilde{x}) = \frac{1}{2}(x^{\alpha}-\tilde{x}^{\alpha})(x_{\alpha}-\tilde{x}_{\alpha}). \tag{6}$$

Subsequent substitutions of Eq. (1) into Eq.(4) and of Eq. (4) into Eq. (3b) lead to a divergent term in the right-hand side of the latter. There are several approaches to subtraction of the infinite term of the form $\lim\limits_{\epsilon\to 0}\frac{\text{const}}{\epsilon}\ddot{z}_{\alpha}(\tau)$. Having been transposed into the left-hand side this term in the sum with an (infinite) bare mass m_0 is considered as the finite observable mass m. The renormalized equation thus obtained is called the Lorentz-Dirac equation (the LD equation) and has the form

$$m\ddot{z}_{\alpha}(\tau) = \frac{e}{c}F^{\text{in}}_{\alpha\beta}(z)\dot{z}^{\beta} + \frac{2}{3}\frac{e^2}{c^3}\left(\dddot{z}_{\alpha} - \frac{1}{c^3}\ddot{z}_{\gamma}\ddot{z}^{\gamma}\dot{z}_{\alpha}\right). \tag{7}$$

The force in the right-hand side proportional to e^2 is the radiation reaction (radiation damping). As was noted in Sec.1, exact solutions of Eq.(7) have pathological properties of run-away or preacceleration except the trivial solution $\ddot{z}_{\alpha}(\tau)\equiv 0$ for the trivial case of $F^{\text{in}}_{\alpha\beta}(z)\equiv 0$. One might suggest that incompatibility of causality and an acceptable asymptotic behaviour for $\tau\to\infty$ is the cost for the charge being struture-less and that the theory of finite-size charge would not meet difficulties of this kind. At present, however, a sufficiently strict relativistic-invariant formulation of the theory of the finite-sized charge is available only in the form of a complicated descriptive algorithm which does not seem to be accessible to any calculation or study (see[2], Sec.7).

These considerations motivate a more careful investigation of possible origins of the difficulties with the point charge theory. It is known[7] that the divergences in the quantum field theory originate actually from the mathematical fact that products of singular distributions involved in the theory are not in general distributions themselves. A strict removal of the divergences consists in attaching a strict meaning to such products via an intermediate regularization of factor distributions.

The situation in classical electrodynamics seems to be similar: according to Eqs. (4), (1) the expression for A^{ret}_{α} contains a product of five distributions $\theta(\Sigma(x),\tilde{x})$, $\delta[\sigma(x,\tilde{x})]$, $\delta^{(4)}(\tilde{x}-z(\tau))$ and has only the meaning of singular distribution at best. To derive the LD equation from the system of Eqs.(a,b) one should regularize the distributions and to remove the regularization after mass renormalization. However, if this is the case, it seems to be more consistent to define an explicit relativistic-invariant regularization of the retarded vector potential and to remove this regularization only in solutions of regularized motion equations. We will follow just this way.

3. REGULARIZED EQUATION OF MOTION FOR A RADIATING POINT CHARGE

It is well-known that distributions may be treated as improper limits of sequences in appropriate functional spaces. The most attractive way to regularize $A^{\text{ret}}_{\alpha}(x)$ in Eq. (4) is to substitute a sequence from a space of test functions on which the distribution $G^{\text{ret}}_{\alpha\beta}(x,\tilde{x})$ is well-defined instead of $j_{\alpha}(x)$. But if this were possible in a relativistic-invariant way we would have got a theory of a "smeared out" charge. The lack of such a theory was mentioned above.

An alternative possibility is to regularize the Green function $G^{\text{ret}}_{\alpha\beta}(x,\tilde{x})$. This end can be achieved by means of the following procedure:

1) replace $\delta(\sigma)$ by a sequence in the Schwartz space S of test functions:

$$\delta_R(\sigma) = \frac{1}{\sqrt{\pi}\,R^2}\exp(-\sigma^2/4R^4) \xrightarrow[R\to 0]{} \delta(\sigma). \tag{8}$$

2) Define a family of space-like hypersurfaces $\Sigma(x) = \text{const}$ orthogonal to the charge world-line $z^{\alpha}(\tau)$, i.e., $\partial_{\alpha}\Sigma(z)\sim\dot{z}_{\alpha}$ so that a point x, which is sufficiently close to the line belongs to one and only one hypersurface Σ; then one may write

$$\theta(\Sigma(x),\tilde{x}) = \theta(\Sigma(x),\Sigma(\tilde{x})).$$

It is worth noting that our restriction of arbitrariness in the choice of δ_R does not affect the generality of our results.

The above procedure leads to the following expression for the retarded potential

$$A^{\text{ret}}_{\alpha}(x;R) = \int_{-\infty}^{\infty} d\tilde{\tau}\,\theta[\Sigma(x)-\Sigma(\tilde{z})]\,\delta_R[\sigma(x,\tilde{z})]\,\dot{\tilde{z}}_{\alpha}, \qquad \tilde{z}_{\alpha}\equiv z_{\alpha}(\tilde{\tau}). \tag{9}$$

Of course, $A_\alpha^{ret}(x;R)$ is not a solution of Eq.(3a) unless $R=0$ but it is remarkable that the regularization conserves the Lorentz gauge

$$\partial^\alpha A_\alpha^{ret}(x;R) = 0 .$$

Finally, we obtain

$$F_{\alpha\beta}^{ret}(x;R) = 2e \int_{-\infty}^{\infty} d\tilde{\tau}\, \theta[\Sigma(x) - \Sigma(\tilde{x})]\delta_R'[\sigma(x,\tilde{z})]\, \dot{\tilde{z}}_{[\beta}\partial_{\alpha]}\, \sigma(x,\tilde{z}) ,$$

$$C_{[\alpha\beta]} \equiv \frac{1}{2}(C_{\alpha\beta} - C_{\beta\alpha}),\quad \delta_R'(\sigma) \equiv \frac{d}{d\sigma}\delta_R(\sigma), \tag{10}$$

and substitution of this expression into Eq.(35) leads to the following integro-differential equation

$$m_0 \ddot{z}_\alpha(\tau) = \frac{e}{c} F_{\alpha\beta}^{in}(z)\dot{z}^\beta + \frac{e^2}{c}\Phi_\alpha(\tau, R) \tag{11}$$

$$\Phi_\alpha(\tau, R) \equiv 2\dot{z}^\beta \int_{-\infty}^{\tau} d\tilde{\tau}\, \dot{\tilde{z}}_{[\beta}\partial_{\alpha]}\, \sigma(z,z)\delta_R'[\sigma(z,\tilde{z})]. \tag{11a}$$

This equation is relativistic-invariant and has no divergences if R is not zero. It is causal in the sense that the acceleration at a proper time τ is determined only by positions and velocities at this and preceding times. However, we have to know the prehistory of the charge because effectively we have "smeared out" the region of interaction of the charge. It is obvious that an essential contribution into the integral defining the nonlocal radiation reaction is given by the closest past of an order of $2R/c$.

We show now that, after renormalization, Eq.(10) is asymptotically the LD equation for $R\to 0$. For this let us make N integrations by parts in Φ_α:

$$\Phi_\alpha = \sum_{n=2}^{N}(-1)^n v_{n+1}(\tau,\tau)\frac{\partial^n u_\alpha(\tau,\tilde{\tau})}{\partial\tilde{\tau}^n}\Big|_{\tilde{\tau}=\tau} +$$

$$+ (-1)^{N+1}\int_{-\infty}^{\tau} d\tilde{\tau}\, v_{N+1}(\tau,\tilde{\tau})\frac{\partial^N u_\alpha(\tau,\tilde{\tau})}{\partial\tilde{\tau}^N}, \tag{12}$$

where

$$u_\alpha(\tau,\tilde{\tau}) = 2\dot{z}^\beta(\tau)\, \dot{z}_{[\beta}(\tilde{\tau})\partial_{\alpha]}\, \sigma[z(\tau), z(\tilde{\tau})],$$

$$v_n(\tau,\tilde{\tau}) = \int_{-\infty}^{\tilde{\tau}} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \dots \int_{-\infty}^{\tau_{k-1}} d\tau_k\, \delta_R'(\tau,\tau_k) = \tag{12a}$$

$$= \frac{1}{(k-1)!}\int_{-\infty}^{\tilde{\tau}} d\tau_1 (\tilde{\tau}-\tau_1)^{k-1}\delta_R'(\tau,\tau_1).$$

It is obvious that

$$0 < v_k(\tau,\tilde{\tau}) \le v_k(\tau,\tau) . \tag{13}$$

if $\tilde{\tau} < \tau$. Define $s \equiv |\sigma|^{1/2}/\sqrt{2}R$ and substitute here the expansion

$$z_\alpha(\tilde{\tau}) = \sum_{k=0}^{N}\frac{(-1)^k}{k!}\overset{(k)}{z}_\alpha(\tau)\, t^k + \frac{(-1)^{N+1}}{(N+1)!}\overset{(N+1)}{z}_\alpha(\omega)\, t^{N+1} , \tag{14}$$

where

$$\overset{(n)}{z}_\alpha \equiv d^n z_\alpha / d\tau^n , \quad t = \tau - \tilde{\tau},\quad \tilde{\tau} < \omega < \tau .$$

Reversion of this series gives

$$0 \le t(s) = \frac{2R}{c}s - \frac{1}{12c^2}\ddot{z}_\gamma \ddot{z}^\gamma \left(\frac{2R}{c}s\right)^3 + O\left[\left(\frac{2R}{c}s\right)^4\right] \le \frac{2R}{c}s , \tag{15a}$$

$$0 \le t'(s) = \frac{2R}{c}\left\{1 - \frac{1}{4c^2}\ddot{z}_\gamma \ddot{z}^\gamma \left(\frac{2R}{c}s\right)^2 + O\left[\left(\frac{2R}{c}s\right)^3\right]\right\} \le \frac{2R}{c} . \tag{15b}$$

The upper restrictions arise: in Eq.(15a), from the fact that the straight-line distance ($=2Rs$) between time-like situated points $z(\tau)$, $z(\tilde{\tau})$ is the longest one (the time-like extremal) and in Eq.(15b), as a property of triangle with three time-like sides.

Thus, we have

$$v_k(\tau,\tau) = \frac{1}{\sqrt{\pi}\, R^4 (k-1)!}\int_0^{\infty} ds\, t'(s)\, t^{k-1}(s)\, s^2 e^{-s^2} =$$

$$= \frac{4}{\sqrt{\pi}\, R^4 (k-1)!}\left(\frac{2R}{c}\right)^{k-4}\Gamma\left(\frac{k+2}{4}\right)\left\{1 + O\left[\left(\frac{2R}{c}\right)^2\right]\right\} \le \tag{16}$$

$$\le \frac{4}{\sqrt{\pi}\, R^4 (k-1)!}\left(\frac{2R}{c}\right)^{k-4}\Gamma\left(\frac{k+2}{4}\right) .$$

Using Eqs.(12a), (15a,b) we obtain, instead of Eq. (11),

$$m_0 z_\alpha = \frac{e}{c}F_{\alpha\beta}^{in}(z)\dot{z}^\beta - \frac{e^2}{c^2\sqrt{\pi}\, R}\Gamma\left(\frac{5}{4}\right)\left\{1 + O\left[\left(\frac{2R}{c}\right)^2\right]\right\}\ddot{z}_\alpha +$$

$$+ \frac{2}{3}\frac{e^2}{c^3}\left\{1 + O\left[\left(\frac{2R}{c}\right)^2\right]\right\}\left(\dddot{z}_\alpha - \frac{1}{c^2}\ddot{z}_\gamma \ddot{z}^\gamma \dot{z}_\alpha\right) + O\left(\frac{2R}{c}\right)$$

and after renormalization

$$m = m_0 + \delta m = m_0 + \frac{e^2\Gamma(5/4)}{c^2\sqrt{\pi}R} \qquad (17)$$

we come to the LD equation, Eq.(17), in the limit $R \to 0$.

At the same time it is apparent that for validity of the transition to the limit it is necessary for the function $\partial^4 u_\alpha(\tau,\tilde{\tau})/\partial\tilde{\tau}^4$ to have no nonintegrable singularities i.e., a certain smoothness of the world-like. The most important matter is, however, that the limit of a solution of Eq.(11) could not generally be a solution of the limit equation (i.e., the LD equation) at all. Considerations of the following section show that this seems actually to be the case.

4. EXISTENCE OF CAUSAL NON-RUN-AWAY SOLUTION

Suppose that the Lorentz force $(e/c)F^{in}_{\alpha\beta}\dot{z}^\beta$ acts a finite time and the world line is of class C^∞, i.e.,

$$|\overset{(n)}{z}_\alpha(\tau)| \le c_\alpha(n) < \infty \quad (n = 0,1,2,\ldots),\quad c_\alpha(n) = \text{const} \qquad (18)$$

and there is no self-acceleration

$$\lim_{\tau\to\infty} \overset{(k)}{z}_\alpha(\tau) = 0 \quad k = 2,3,\ldots\,. \qquad (19)$$

Renormalize Eq.(11) by the replacements

$$m_0 \to m,\quad \Phi_\alpha \to \Phi^{ren}_\alpha = \Phi_\alpha + \frac{c}{e^2}\delta m\,\ddot{z}_\alpha\,.$$

Φ^{ren}_α has apparently the same form of Eq.(12) up to the replacement of $v_3(\tau,\tau)$ by $v_3^{ren}(\tau,\tau) = v_3(\tau,\tau) - \delta m(c/e^2)$. Under conditions (18), (19) the remainder (integral) term in Eq.(12) vanishes for $N\to\infty$ sufficiently fast. To see this, it is sufficient to use Eq.(14) and the Taylor series for $z_\alpha(\tilde{\tau})$ with the remainder for $k=2$ and the following inequality

$$\exp\{-\sigma^2(\tau,\tau')/4R^4\} \le \exp\{[\sigma^2(\tau,\tilde{\tau}) + \sigma^2(\tilde{\tau},\tau')]/4R^4\} \le$$

$$\le \exp\{-[(\tau-\tilde{\tau})^4 + (\tilde{\tau}-\tau')^4](c/2R)^4\},$$

that arises for the same reasons as the upper restrictions in Eq.(15a,b). Consider now a sequence of truncated equations converging to Eq.(11)

$$m\ddot{z}_\alpha(\tau) = \frac{e}{c}F^{in}_{\alpha\beta}(z)\dot{z}^\beta + \frac{e^2}{c}\Phi^{ren}_{\alpha N}(\tau,N)\,, \qquad (20)$$

where

$$\Phi^{ren}_{\alpha N}(\tau,R) = -v_3^{ren}(\tau,\tau)\ddot{z}_\alpha(\tau) + \sum_{n=3}^{N}(-1)^n v_{n+1}(\tau,\tau)\left.\frac{\partial^n u_\alpha(\tau,\tilde{\tau})}{\partial\tilde{\tau}^n}\right|_{\tau=\tilde{\tau}}\,.$$

Simple transformations give

$$\left.\frac{\partial^n u_\alpha(\tau,\tilde{\tau})}{\partial\tilde{\tau}}\right|_{\tilde{\tau}=\tau} = (n-1)\{-c^2\overset{(n)}{z}_\alpha + \dot{z}^\beta\sum_{k=1}^{n-1}\frac{1}{k+1}\binom{n}{k}\overset{(k+1)}{z}_\beta\overset{(n-k)}{z}_\alpha\}. \qquad (21)$$

The coefficients v_3^{ren}, v_k, $k = 4,5\ldots$ are restricted functionals on C^∞-world lines. A further simplification of the system under consideration comes from the replacement of $v_{N+1}(\tau,\tau)$, the coefficient of the major derivative $\overset{(N)}{z}_\alpha(\tau)$ by the leading term of its expansion in powers of $(2R/c)$, Eq.(16). An integro-differential equation so obtained for Eq.(20) is equivalent to Eq.(11) up to terms of the order $O[(2R/c)^{N-2}]$ and may be represented in the form

$$m[(-t_N)^{N-2}\overset{(N)}{z}_\alpha + \ddot{z}_\alpha] = \frac{e}{c}F^{in}_{\alpha\beta}(z)\dot{z}^\beta -$$

$$-e^2c\{v_3^{ren}(\tau,\tau)\ddot{z}_\alpha - \frac{1}{c^2}\sum_{n=3}^{N-1}(-1)^n v_{n+1}(\tau,\tau)\left.\frac{\partial^n u_\alpha(\tau,\tilde{\tau})}{\partial\tilde{\tau}^n}\right|_{\tilde{\tau}=\tau} - \qquad (22)$$

$$-\frac{(-1)^N}{c^2}v_{N+1}(\tau,\tau)\dot{z}^\beta\sum_{k=1}^{N-1}\frac{1}{k+1}\binom{N}{k}\overset{(k+1)}{z}_\beta(\tau)\overset{(N-k)}{z}_\alpha(\tau)\} \equiv \Psi_\alpha[\tau;z(\cdot)]\,,$$

where

$$(t_N)^{N-2} = \frac{4e^2(N-1)}{m\sqrt{\pi}c^3N!}\Gamma\left(\frac{N+3}{4}\right)\left(\frac{2R}{c}\right)^{N-3}\,. \qquad (23)$$

Note that in view of Eq.(2) the right-hand side contains no derivatives higher than $\overset{(N-1)}{z}(\tau)$. Now we will take into account the asymptotical conditions (19) explicitly by the method proposed by A.A.Sokolov[8] for the nonrelativistic problem of the "luminous electron"; the relativistic version is due to Röhrlich[2], Sec.6-6.

Introducing an integrating multiplier one transforms Eq.(22) to the form

$$m\frac{d^{N-2}}{d\tau^{N-2}}\{(-t_N)^{N-2}e^{-\tau/t_N}\ddot{z}_\alpha(\tau)\} =$$

$$= \{\Psi_\alpha[\tau;z(\cdot)] + \sum_{k=1}^{N-3}\binom{N-2}{k}(-t_N)^k\overset{(k+2)}{z}_\alpha\}e^{-\tau/t_N} \equiv \tilde{\Psi}_\alpha\,.$$

After (N-2) -fold integration with allowing for the conditions (18), (19), using Eq.(12b), and introducing a new integration variable we obtain

$$m\ddot{z}_{\alpha}(\tau) = \frac{1}{(N-3)!}\int_{0}^{\infty} d\xi\, \xi^{N-3} e^{-\xi}\, \tilde{\Psi}[\tau + t_N \xi,\ z(\cdot)].$$

The shift of the argument by a constant t_N is just an evidence of preacceleration, see [2] sections 6-6, 6-7. However,

$$t_N \to 0 \quad \text{for} \quad N \to \infty,$$

i.e., the preacceleration ceases as Eq.(22) approaches the exact regularized equation (11). This is the case for arbitrarily small but nonzero values of R. If one sets $R=0$, then the sequence of equations (22) is truncated by $N=3$ and according to Eq.(23), $t_3 \neq 0$, i.e., nonvanishing preacceleration occurs.

These considerations give a reason to think that Eq.(11) has solutions with no pre- and self-acceleration. It is very significant that our arguments are valid only under keeping a nonlocal character of Eq.(22) via the coefficients $v_{n+1}(\tau,\tau)$. If they are expanded in powers of $2R/c$ to the same order of $O[(2R/c)^{N-2}]$ then one obtains a system of ordinary differential equations of order $N' > N$, and the higher derivatives which arise from the expansion of v_3^{ren} and v_4 enter into the system nonlinearly.

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К теории классического релятивистского точечного заряда

Устранение расходимости собственной массы точечного заряда в классической электродинамике рассматривается как доопределение произведения обобщенных функций путем регуляризации запаздывающей функции Грина. Получающееся при этом релятивистское инвариантное регуляризованное уравнение движения излучающего точечного заряда при снятии регуляризации переходит в известное уравнение Лоренца-Дирака. Приводятся аргументы в пользу того, что существует решение регуляризованного уравнения без тех патологических свойств /непричинность или самоускорение/, которые неизбежны для решения уравнения Лоренца-Дирака.

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On the Theory of the Classical Relativistic Point-Like Charge

Removal of the self-mass divergence of the point in the classical electrodynamics is treated as a more strict definition of a product of singular distributions via regularization of the retarded Green function. The regularized relativistic invariant equation of the radiating point charge motion thus obtained under the removal of regularization changes into the Lorentz-Dirac equation. It is known, that solutions of the latter necessarily prove at least either of two unphysical properties - self-acceleration or preacceleration. We show that condition of simultaneous absence of these properties is not contradictory for the regularized equation.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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