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CRITICAL COMMENT ON QUARK MASS PREDICTIONS FROM INFRA-RED FIXED POINTS

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Attractive features and success of grand unified theories at high energies do not exclude that the values of the low energy parameters are independent of the prediction of the GUT. It may well be that all observed symmetries aries as a result of infrared fixed points of the "low energy" theory rather than as a result of original "grand unified" symmetry[1]. If the renormalization group equations describing the evolution of the various couplings in the theory possess stable infrared fixed points, then the couplings will tend towards these points irrespective of their initial values. Their resulting values will then be determined by the low energy gauge group.

In a recent paper by Pendleton and Ross[2] this possibility was examined in a standard $SU_c(3)\times SU_L(2)\times U(1)$ model with six quarks. Their investigation was continued in paper [3]. The model has one Higgs doublet and the quark masses are given, via spontaneous symmetry breaking, through the Yukawa couplings: $m_q = g_q \cdot v$, where g_q is the Yukawa coupling and $v = \langle\phi\rangle_0$ is the vacuum expectation value of the Higgs field. The renormalization group equations written to one loop are:

$$16\pi^2 \frac{dg_U}{d\ell} = g_U\Big[\frac{3}{2}g_U^2 - \frac{3}{2}\sum_D g_D^2|\lambda_{UD}|^2 + \sum_n g_n^2 - \frac{3}{4}(3g^2+g'^2) - \frac{2}{3}g'^2 - 8g_{QCD}^2\Big], \tag{1}$$

$$16\pi^2 \frac{dg_D}{d\ell} = g_D\Big[\frac{3}{2}g_D^2 - \frac{3}{2}\sum_U g_U^2|\lambda_{UD}|^2 + \sum_n g_n^2 - \frac{3}{4}(3g^2+g'^2) + \frac{1}{3}g'^2 - 8g_{QCD}^2\Big], \tag{2}$$

$$16\pi^2 \frac{dg_L}{d\ell} = g_L\Big[\frac{3}{2}g_L^2 + \sum_n g_n^2 - \frac{3}{4}(3g^2+g'^2) - 3g'^2\Big], \tag{3}$$

$$16\pi^2 \frac{dg_{QCD}}{d\ell} = -7g_{QCD}^3 . \tag{4}$$

Here $\ell = \frac{1}{2}\ln\frac{\mu^2}{\mu_0^2}$, $U = u, c, t$ and $D = d, s, b$ refer to $Q=2/3$ and $Q=-1/3$ quarks, $L = e, \mu, \tau$ refers to leptons, the sum runs over all quarks and leptons including colors, and λ_{UD} are the Kobayashi-Maskawa mixing-matrix elements. Equations for g and g' couplings and the vacuum expectation value v are not significant for the present analysis.

Now our task is to find the IR fixed points of eqs. (1-4). Let us omit for the time being g and g' terms. It can be done because they are much smaller than g_{QCD} in the region of interest and what is important, the evolution equations for all three gauge couplings are independent to one-loop order. Then from eq. (3) it can be easily seen that there are no IR fixed points for g_L as far as the r.h.s. is positive. The lepton couplings vanish in the IR region irrespective of the behaviour of quark couplings. Hence, they can be dropped from eqs. (1-2).

For the investigation of the remaining equations it is useful to introduce the following variables

$$u=\frac{g_u^2}{g_{QCD}^2},\quad d=\frac{g_d^2}{g_{QCD}^2},\quad c=\frac{g_c^2}{g_{QCD}^2},\quad s=\frac{g_s^2}{g_{QCD}^2},\quad t=\frac{g_t^2}{g_{QCD}^2},\quad b=\frac{g_b^2}{g_{QCD}^2}.$$

Then, combining eqs. (1-2) with eq. (4) we have

$$\dot{u}=-u[\frac{9}{2}u+(3-\frac{3}{2}\lambda_{ud}^2)d+3c+(3-\frac{3}{2}\lambda_{us}^2)s+3t+(3-\frac{3}{2}\lambda_{ub}^2)b-1],$$

$$\dot{d}=-d[(3-\frac{3}{2}\lambda_{ud}^2)u+\frac{9}{2}d+(3-\frac{3}{2}\lambda_{cd}^2)c+3s+(3-\frac{3}{2}\lambda_{td}^2)t+3b-1],$$

$$\dot{c}=-c[3u+(3-\frac{3}{2}\lambda_{cd}^2)d+\frac{9}{2}c+(3-\frac{3}{2}\lambda_{cs}^2)s+3t+(3-\frac{3}{2}\lambda_{cb}^2)b-1],$$

$$\dot{s}=-s[(3-\frac{3}{2}\lambda_{us}^2)u+3d+(3-\frac{3}{2}\lambda_{cs}^2)c+\frac{9}{2}s+(3-\frac{3}{2}\lambda_{ts}^2)t+3b-1],$$

$$\dot{t}=-t[3u+(3-\frac{3}{2}\lambda_{td}^2)d+3c+(3-\frac{3}{2}\lambda_{ts}^2)s+\frac{9}{2}t+(3-\frac{3}{2}\lambda_{tb}^2)b-1],$$

$$\dot{b}=-b[(3-\frac{3}{2}\lambda_{ub}^2)u+3d+(3-\frac{3}{2}\lambda_{cd}^2)c+3s+(3-\frac{3}{2}\lambda_{tb}^2)t+\frac{9}{2}b-1], \qquad (5)$$

where the dot means $7g_{QCD}^2\frac{d}{dg_{QCD}^2}$. To find the fixed points, we have to equal the r.h.s. to zero. This gives us seven types of fixed points (as far as all quarks are on equal footing, we do not consider permutations):

I. $u=d=c=s=t=b=0.$

II. $u=d=c=s=b=0;\ t=2/9.$

III. $u=d=c=s=0;\ t,b\neq 0.$

IV. $u=d=s=0;\ c,t,b\neq 0.$

V. $u=d=0;\ c,s,t,b\neq 0.$

VI. $u=0;\ d,c,s,t,b\neq 0.$

VII. $u=d=c=s=t=b=\frac{1}{18}.$

The numerical values depend on the mixing-matrix parameters.

In papers [2,3] the II fixed point was chosen. The authors argued this choice by the smallness of other couplings which can be dropped from the equations. However, we will show that it is not possible because the equations for different couplings are not independent. In fact, the II fixed point is not IR stable and is a saddle-point as will be shown below.

In order to establish whether the fixed point is UR stable or not, we use the standard method. Despite the nonlinearity of eqs. (5) they can be reduced to a linear form in a vicinity of the fixed point. We introduce the infinitesimal deviations from a fixed point $u=\bar{u}+\delta u$, $d=\bar{d}+\delta d$, $c=\bar{c}+\delta c$, $s=\bar{s}+\delta s$, $t=\bar{t}+\delta t$, $b=\bar{b}+\delta b$, where $\bar{u}$, etc., denote a fixed point of some type. Then we arrive at a linear homogeneous system of equations for small deviations from the fixed point:

$$\delta\dot{x}_i=-A_{ij}(\bar{x})\delta x_j\ , \qquad (6)$$

where δx_i means δu, etc., and matrix A depends on a given type of a fixed point.

The IR stability criterium now is the negative definiteness of the solutions to the characteristic equation

$$\mathrm{Det}(A-\Lambda E)=0,$$

where E is a unit matrix.

IR stability: $\Lambda_i<0.$

If we apply this criterium to different types of fixed points listed above, we find

I. $\Lambda_i>0,\quad i=1,2,3,4,5,6.$

II. $\Lambda_1<0;\ \Lambda_i>0,\quad i=2,3,4,5,6.$

III. $\Lambda_j<0,\ j=1,2;\ \Lambda_i>0,\ i=3,4,5,6.$

IV. $\Lambda_j<0,\ j=1,2,3;\ \Lambda_i>0,\ i=4,5,6.$

V. $\Lambda_j<0,\ j=1,2,3,4;\ \Lambda_i>0,\ i=5,6.$

VI. $\Lambda_j<0,\ j=1,2,3,4,5;\ \Lambda_6>0.$

VII. $\Lambda_j<0,\ j=1,2,3,4,5,6.$

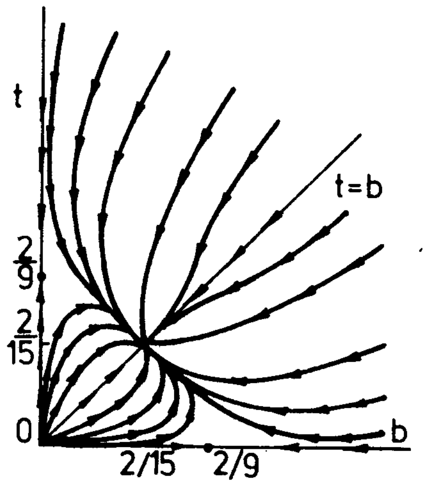
This means that the type I fixed point is totally IR unstable, while the type VII one is totally IR stable. All the other fixed-points are stable only in some sectors in a six-dimensional phase space and are unstable elsewhere. For instance, the PR fixed point, type II in our notation, is IR stable only if all the couplings but t are exactly zero, that is not the case. Also unstable is the III fixed point exploited in paper [3] for the prediction of the quark masses of the possible fourth generation. Both these fixed points are saddle-points.

We illustrate the situation by a simple example of two couplings. It is a projection of a six-dimensional phase space on a plane of two variables. In this case we have

$$\dot{t} = -t\left[\frac{9}{2}t + 3b - 1\right], \qquad \dot{b} = -b\left[\frac{9}{2}b + 3t - 1\right], \tag{7}$$

where we ignore mixing. The IR stable point is $t = b = \frac{2}{15}$ and the saddle-points are $t = 0,\ b = \frac{2}{9}$ and $b = 0,\ t = 2/9$. The solutions of eqs. (7) are shown in the phase diagram. The arrows show the decrease of $\ell = \frac{1}{2}\ln\frac{\mu^2}{\mu_0^2}$.

Hence, the only IR stable fixed point is the totally symmetric one $u = d = c = s = t = b = 1/18$ irrespective of the mixing-matrix parameters. During the evolution from the scale of M_X the solution of eqs. (5) will be attracted by various saddle-points but eventually will be driven to the stable one.



Figure

However, it will be reached only in the IR limit where eqs. (1-4), strictly speaking, are not valid due to confinement phenomena at 1 GeV. So we can investigate the approach of quark couplings to a fixed point. Due to unequal initial conditions the couplings will sweep to it with different rate. If at the scale of M_X $u, d, c, s, b \ll t$ and t is of an order of the stable point, then at the scale of M_W t will be close to the IR stable point and the other couplings can be still much smaller. Thus, we can predict only the value of t which is of the order of 1/18. This means that

$$g_t^2 \sim \frac{1}{18} g_{QCD}^2 \tag{8}$$

and the top quark mass can be predicted once g_{QCD} and v are known.

With $g_{QCD}^2(4M_W^2) = 4\pi/7$ and $v = 175$ GeV it gives

$$m_t \simeq 55 \text{ GeV}. \tag{9}$$

If we include the effects of g and g', the situation becomes more complicated since g_{QCD}, g and g' evolve differently. We obtain some kind of the effective fixed point

$$u = c = t = \frac{1}{18} + \frac{1}{8}\frac{g^2}{g_{QCD}^2} + \frac{47}{216}\frac{g'^2}{g_{QCD}^2}, \qquad d = s = b = \frac{1}{18} + \frac{1}{8}\frac{g^2}{g_{QCD}^2} - \frac{25}{216}\frac{g'^2}{g_{QCD}^2}. \tag{10}$$

Thus, at the scale of M_W g_t will tend to

$$g_{t,eff}^2 = \frac{1}{18}\left[g_{QCD}^2 + \frac{9}{4}g^2 + \frac{47}{12}g'^2\right]. \tag{11}$$

This corresponds to the top quark mass

$$m_t \simeq 70 \text{ GeV}, \tag{12}$$

where we have assumed that $g' = \frac{4\pi}{128}$ and $\sin^2\theta_W \simeq 0{,}23$ at M_W.

The same analysis can be applied to the Higgs coupling λ. The renormalization group equation to one loop order is

$$16\pi^2 \frac{d\lambda}{d\ell} = 4\lambda^2 + 4\lambda\sum_n g_n^2 - 12\sum_n g_n^4 - 3\lambda(3g^2 + g'^2) + \frac{9}{4}\left[2g^4 + (g^2 + g'^2)^2\right]. \tag{13}$$

Again, if we ignore g and g', λ has a set of fixed points corresponding to the choice of the soluiton of eqs. (5). If we choose the VII type of the solution for quark couplings, we have the IR stable fixed point

$$\lambda \simeq 0{,}0367\, g^2_{QCD}\,. \tag{14}$$

This gives the Higgs mass

$$m_H = \left(\frac{2}{3}\lambda\right)^{1/2} v \simeq 37 \text{ GeV}\,. \tag{15}$$

Including the effects of g and g' moves the effective fixed point to

$$\lambda_{eff} \simeq 0{,}0720\, g^2_{QCD} \tag{16}$$

leading to

$$m_H \simeq 51 \text{ GeV}. \tag{17}$$

The problem which makes our predictions (12), (17) not so well defined is how far are the couplings from their IR values at the scale of M_W and how fast these points are reached. The only definite statement is that at the scale of M_W the couplings are closer than at M_X. As was already mentioned by PR [2], the fixed point is approached very slowly if the initial value is close to it and is approached quite rapidity otherwise. The numerical analysis shows that the values of t in the interval .15.-.30 are reached quite rapidly, when all other couplings are small. Similar estimates are valid also for the Higgs coupling. Hence we have approximately 200% uncertainty from above the predicted values.

Summarizing, in the standard six-quark model with one Higgs doublet we predict the values of the masses for the heaviest quark and Higgs meson at the scale of 0 (100 GeV) to be

$$m_{top} \simeq 70\text{–}160 \text{ GeV}, \tag{18}$$

$$m_{Higgs} \simeq 50\text{–}90 \text{ GeV}. \tag{19}$$

These values practically coincide with those of PR.

In the case when there are other generations beyond the first three ones, which acquire their masses due to the same Higgs doublet, we will have the following IR stable fixed point solution (ignoring g and g')

$$g_i^2 = \left(\frac{2}{9} - \frac{1}{n_f}\right) g^2_{QCD}\,, \quad i = 1, 2, \dots, n_f \tag{20}$$

for the Yukawa couplings and

$$\lambda = \left[\sqrt{\left(\frac{2}{3}n_f + 5\right)^2 + \frac{16}{n_f}\left(\frac{2}{3}n_f - 3\right)^2} - \left(\frac{2}{3}n_f + 5\right)\right]\frac{g^2_{QCD}}{4} \tag{21}$$

for the Higgs coupling. It differs from the values obtained in [3] for $n_f = 8$. As we have already mentioned, in paper [3] the saddlepoints are exploited.

From eqs. (20), (21) it follows that with n_f increasing, the IR values of the couplings, and hence masses, also increase. It will not be the case if heavier quarks obtained their masses due to other Higgs fields with larger masses. Then these heavy particles will decouple from lighter ones.

A further comment concerns the behaviour of the mass corresponding to a symmetric IR point. Combining eqs. (1-2) with the equation for the vacuum expectation value

$$16\pi^2 \frac{dv}{d\ell} = v\left[-\sum_n g_n^2 + \frac{3}{4}(3g^2 + g'^2)\right], \tag{22}$$

we get in the case when $u = d = c = s = t = b$ and ignoring g and g' terms

$$16\pi^2 \frac{dm_{SYMM}}{d\ell} = -8 g^2_{QCD}\, m_{SYMM}\,. \tag{23}$$

Together with eq. (4) this gives

$$m_{SYMM}(\ell)/m_{SYMM}(\ell_0) = [g_{QCD}(\ell)/g_{QCD}(\ell_0)]^{8/7}. \tag{24}$$

The solution of eqs. (1-4) will tend to this curve in the IR region.

The general conclusion out of all this analysis agrees with that of Pendleton and Ross. In any system of equations there is an IR stable fixed point solution. The difference is that we show that IR stable point corresponds to a symmetric point and all other fixed points are saddle-points. This does not allow us to predict the quark mass spectra but only gives a rough estimate on the mass of the heaviest quark. Another possibility arises if we start with some spectra at low energies and follow the evolution of masses to the region of grand unification. Then we will be interested in ultraviolet fixed points which will depend on the details of GUT group.

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Казаков Д.И. Критическое замечание о предсказаниях масс кварков на основе фиксированных точек уравнений ренормгруппы E2-82-4

Критически анализируются аргументы Пендльтона и Росса, связывающих массы кварков с инфракрасными фиксированными точками уравнений ренормгруппы. Найдены все фиксированные точки и показано, что устойчивая точка симметрична по всем кваркам и лежит ниже решения ПР. В 6-кварковой модели она приводит к предсказанию для массы t-кварка $m_t \gtrsim 70$ ГэВ и для массы хигговского бозона $m_H \gtrsim 51$ ГэВ.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Kazakov D.I. Critical Comment on Quark Mass Predictions from Infra-Red Fixed Points E2-82-4

The Pendleton and Ross arguments that quark masses may be related by the infrared fixed points of the underlying field theory are critically examined. All fixed points are found, and it is shown that the infrared stable point is symmetrical in quark masses and lies lower than the PR solution. For the 6-quark model we predict the top quark mass $m_t \gtrsim 70$ GeV and the Higgs mass $m_H \gtrsim 51$ GeV.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1982