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2278/82

17/II-82

E2-82-36

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THE STRUCTURE FUNCTIONS OF PSEUDO-SCALAR MESONS IN A COMPOSITE MODEL WITH QCD INTERACTION

Submitted to "ТМФ"

*IHEP, Serpukhov

1982

1. INTRODUCTION

The present paper is a sequel to our works [1-3]; it is devoted to the investigation of lepton-hadron inelastic scattering processes on the basis of relativistic wave functions (WF) of quark-antiquark bound states. As WF we will employ approximate solutions found here to the covariant three-dimensional equation obtained in the framework of the single-time description of a two-particle system [4-8] and coincident in form with the equation that had been deduced within the Hamilton formulation of quantum field theory [9-11]. The kernel of the equation is chosen in the relativistic configurational representation in a form of the Coulomb quasipotential that corresponds, as it is shown in refs. [12,13], to the one-gluon exchange in quantum chromodynamics (QCD). For comparison the case of the quasipotential corresponding in the momentum space to the electromagnetic interaction of quarks in quantum electrodynamic (QED) is also considered.

Our paper is organized as follows: The next section will be devoted to the analysis of scaling properties of structure functions of the deep-inelastic scattering of the electron on a hadron calculated in ref. [3] in the framework of the single-time formulation of quantum field theory. In the third section the approximate solutions will be found for the quasipotential equation for the WF of the two-particle system with the interaction quasipotentials taken in the form of the one-photon and one-gluon exchange amplitudes. These solutions in the fourth section are applied to obtain an explicit form of the mesons structure functions and for a numerical analysis of the scaling violation in a preasymptotic region.

2. THE SCALING PROPERTIES OF THE STRUCTURE FUNCTIONS IN THE SINGLE-TIME FORMULATION OF QUANTUM FIELD THEORY

In ref. [3] the following expression was obtained for the structure functions of the electron-hadron scattering in the deep-inelastic region:

$$F_2(\zeta, W^2) = (1-\zeta)\cdot F_1(\zeta, W^2) = 8M(Q_1^2+Q_2^2)(1-\zeta)\cdot \int_{\left|\ln\frac{M\zeta}{m}\right|}^{\ln\frac{W^2}{mM\zeta}} d\chi_k \times$$

$$\times \frac{\cosh\chi_k - \zeta M/2m}{\sinh\chi_k}|\phi(\chi_k)|^2 - \frac{16mM^2(Q_1^2+Q_2^2)\zeta(1-\zeta)}{W^2}\int_{\frac{M\zeta}{m}}^{\ln\frac{W^2}{mM\zeta}} d\chi_k \times \tag{1}$$

$$\times \sinh\chi_k\cdot|\phi(\chi_k)|^2 ,$$

where the scaling variable

$$\zeta = \frac{\nu + M - \sqrt{\nu^2+Q^2}}{M} \tag{2}$$

was introduced, W^2 is the invariant squared mass of a system of hadrons in the final state, Q_1 and Q_2 are charges of the quark and antiquark, $\phi(\chi_k)$ is the WF of the quark-antiquark bound system that depends on the quark rapidity $\chi_k = \ln(|\vec{k}| + k_0)m^{-1}$ and is related to the meson WF in the relativistic configurational representation (introduced in ref. [10]) by the Fourier sine-transform over the rapidity (for details see ref. [11]):

$$\phi(\chi_k) = 4\pi\int_0^\infty dr\cdot r\phi(r)\sin mr\chi_k . \tag{3}$$

The scaling variable ζ, namely

$$1-\zeta = \frac{\sqrt{\nu^2+Q^2}-\nu}{M} \tag{4}$$

was firstly introduced in ref. [14] but still did not get further application because the upper limit of its change at fixed Q^2 is not equal to 1 and depends on Q^2 in contrast to the usual Bjorken variable $x = Q^2/2M\nu$. In the framework of the single-time formalism, as is shown in ref. [15], it turns out to be more natural to investigate the behaviour of structure functions in terms of variables ζ and W^2 (instead of x and Q^2) even in the case of many-particle intermediate states. In this case the difficulty mentioned in ref. [14] with the range of changing the variable ζ is eliminated because at fixed W^2 the variable ζ can take any value from the interval $[0,1]$. Moreover the variable W^2 is a more natural characteristic of inelasticity of the hadron electroproduction process.

It is not difficult to show that the Bjorken variable x is expressed through ζ as follows

$$x = (1-\zeta)\frac{W^2 - \zeta M^2}{W^2 - \zeta(2-\zeta)M^2}. \tag{5}$$

From (5) it is clear that the appearance of the variable ζ is connected with taking into account of the mass of the hadron-target because at $M=0$ these variables are in fact reduced to each other: $x = 1-\zeta$.

In ref. [16,17] the scaling variable ξ has been proposed that is expressed through the variable ζ in the following way

$$\xi = \frac{1}{2}(1-\zeta)(1+\sqrt{1+4m^2/Q^2}). \tag{6}$$

This variable was introduced in the framework of the parton model to take into account nonzero quarks masses and to explain the violation of Bjorken scaling. At first sight it seems that the structure functions are scale-invariant in ξ up to kinematical factors in the whole region of changing the inelastic electroproduction variables and a violation of ξ-scaling appears because of the mentioned factors. However, it is to be noted that the upper limit of changing the variable ξ depends on Q^2.

And finally, let us note that in our paper [2] one more scaling variable ξ_M was introduced. It is connected with the variable ζ as follows:

$$\xi_M = -\frac{1}{2}\zeta(1+\sqrt{1-4m^2/W^2}) \tag{7}$$

and changes at fixed W^2 in the limits: $-(1+\sqrt{1-4m^2/W^2})/2 \le \xi_M \le 0$.

Let us investigate now the asymptotics of the structure functions (1) in the deep inelastic region $W^2 \gg M^2$. The asymptotics of the second integral can be represented in the form

$$2\int_{|\ln\frac{M\zeta}{m}|}^{z} d\chi_k \cdot \sinh\chi_k \cdot |\phi(\chi_k)|^2 \simeq e^z \sum_{n=0}^{\infty}(-1)^n \frac{d^n}{dz^n}|\phi(z)|^2, \tag{8}$$

where $z = \ln(W^2/mM\zeta) \to \infty$, therefore instead of (1) we have

$$F_2(\zeta, W^2) = (1-\zeta)\cdot F_1(\zeta, W^2) = 8M(Q_1^2 + Q_2^2)\cdot(1-\zeta) \times \\ \times\Big[\int_{|\ln\frac{M\zeta}{m}|}^{z} d\chi_k \cdot \frac{\cosh\chi_k - \zeta M/2m}{\sinh\chi_k}|\phi(\chi_k)|^2 - \sum_{n=0}^{\infty}(-1)^n \frac{d^n}{dz^n}|\phi(z)|^2\Big]. \tag{9}$$

The remaining integral is convenient to be divided into two parts (retaining only the leading terms at $z \to \infty$):

$$\int_{|\ln\frac{M\zeta}{m}|}^{z} d\chi_k \frac{\cosh\chi_k - \zeta M/2m}{\sinh\chi_k}|\phi(\chi_k)|^2 = \int_{|\ln\frac{M\zeta}{m}|}^{\infty} d\chi_k \times \\ \times \frac{\cosh\chi_k - \zeta M/2m}{\sinh\chi_k}|\phi(\chi_k)|^2 - \int_z^{\infty} d\chi_k \cdot |\phi(\chi_k)|^2. \tag{10}$$

According to (9), (10) the structure functions can also be divided into two parts

$$F_i(\zeta, W^2) = F_i^S(\zeta) - F_i^P(\zeta, W^2), \tag{11}$$

where $F_i^S(\zeta)$ is the scaling part of the structure function (i.e., that depends on ζ only)

$$F_2^S(\zeta) = (1-\zeta)\cdot F_1^S(\zeta) = 8M(Q_1^2+Q_2^2)(1-\zeta)\cdot\int_{|\ln\frac{M\zeta}{m}|}^{\infty} d\chi_k \times \\ \times \frac{\cosh\chi_k - \zeta M/2m}{\sinh\chi_k}|\phi(\chi_k)|^2, \tag{12}$$

and $F_i^P(\zeta, W^2)$ describes an approach to scaling at $W^2 \to \infty$ and according to above calculations it has the following leading asymptotic term

$$F_2^P(\zeta, W^2) = (1-\zeta)\cdot F_1^P(\zeta, W^2) = \\ = 8M(Q_1^2 + Q_2^2)(1-\zeta)\cdot\sum_{n=0}^{\infty}(-1)^n \frac{d^n}{dz^n}\int_z^{\infty} d\chi_k \cdot |\phi(\chi_k)|^2. \tag{13}$$

Thus, we come to the conclusion that in the deep inelastic region ($W^2 \gg M^2$) the structure functions of a composite hadron become scale-invariant, i.e., they depend on the variable ζ only. Preasymptotic terms are completely determined by the asymptotic of the WF $\phi(\chi_k)$ at $\chi_k \to \infty$, and in case when $\phi(\chi_k)$ decreases in a power way in χ_k, the leading correction violating scaling will contain logarithmic terms (leading log's) only.

In the next section we will investigate the asymptotics of the WF on the basis of an approximate solution of the quasipotential equation and thereby we will determine a character of the behaviour of preasymptotic corrections.

To conclude this section let us note that the difference of the results of our approach and refs. [16,17] results from the different interpretation of composed systems in the single-time formalism and parton model. In the latter all quarks composing a hadron are off-mass-shell, but while calculating the structure functions in a hard collision it is assumed that the quark after collision belongs to the mass shell. In the quasipotential approach all quarks certainly have fixed masses (the way of continuation from the whole system energy-momentum shell is only defined) and the considered in our works scaling properties of the structure functions are obtained without any extra assumptions.

3. APPROXIMATE SOLUTIONS OF THE RELATIVISTIC EQUATION FOR THE WAVE FUNCTION

Quasipotential equations for the WF of the system of two spinor particles were found in refs.[6, 11, 18, 19] in the helicity representation. In refs.[13, 20] the quasipotential equation was projected onto the spin states of the bound system, and in notation of ref.[3] (formula (2.19)) it has following form:

$$2\Delta^{\circ}_{p,m\lambda_p}\cdot(2m)^{-1}\,(M-2\Delta^{\circ}_{p,m\lambda_p})\cdot\phi_{s,\sigma}(\vec{\Delta}_{p,m\lambda_p}) = \frac{1}{(2\pi)^3}\int\frac{d\vec{\Delta}_{k,m\lambda_p}}{2\Delta^{\circ}_{k,m\lambda_p}}V^{s',\sigma'}_{s,\sigma}(\vec{\Delta}_{p,m\lambda_p};\vec{\Delta}_{k,m\lambda_p})\cdot\phi_{s',\sigma'}(\Delta_{k,m\lambda_p}). \qquad (14)$$

The quasipotential of quark-antiquark interaction in the helicity basis is natural to be defined in the form

$$V(\vec{\Delta}_{p,m\lambda_p};\vec{\Delta}_{k,m\lambda_p}) = \bar{u}(\vec{\Delta}_{p,m\lambda_p})\gamma^{\mu}u(\vec{\Delta}_{k,m\lambda_p})\times v(-\vec{\Delta}_{k,m\lambda_p})\gamma_{\mu}\bar{v}(-\vec{\Delta}_{p,m\lambda_p})\cdot V_0(\vec{\Delta}_{p,m\lambda_p}-\vec{\Delta}_{k,m\lambda_p}), \qquad (15)$$

where V_0 is a quasipotential local part corresponding to the gluon exchange in QCD and the photon exchange in QED.

After simple calculations for the WF of a pseudoscalar meson introduced in ref.[3] by relation (2.22) we arrive at the following equation

$$2\Delta^{\circ}_{p,m\lambda_p}\cdot(2m)^{-1}\,(M-2\Delta^{\circ}_{p,m\lambda_p})\cdot\phi_{0,0}(\vec{\Delta}_{p,m\lambda_p}) = \frac{4}{(2\pi)^3}\int\frac{d\vec{\Delta}_{k,m\lambda_p}}{2\Delta^{\circ}_{k,m\lambda_p}}(2\Delta^{\circ}_{p,m\lambda_p}\cdot\Delta^{\circ}_{k,m\lambda_p}-m^2)\times V_0(\vec{\Delta}_{p,m\lambda_p}-\vec{\Delta}_{k,m\lambda_p})\cdot\phi_{0,0}(\vec{\Delta}_{k,m\lambda_p}). \qquad (16)$$

Let us note that unlike the case of spinless quarks the kernel of the obtained quasipotential equation is no longer local, as the factors $\Delta^{\circ}_{p,m\lambda_p}$ and $\Delta^{\circ}_{k,m\lambda_p}$ appear there.

If we restrict ourselves to considering the s-state, then owing to the spherical symmetry it is convenient to pass in eq. (16) to the quark rapidities putting $\Delta^{\circ}_{p,m\lambda_p}=m\cosh\chi_p$ and $\Delta^{\circ}_{k,m\lambda_p}=m\cosh\chi_K$ and integrating over the polar-angle:

$$\cosh\chi_p(\cos\chi_0-\cosh\chi_p)\cdot\phi(\chi_p) = \frac{(2m)^3}{2(2\pi)^2}\int_0^{\infty}d\chi_k\times[\cosh(\chi_p-\chi_k)+\cosh(\chi_p+\chi_k)-1]\times\int_{|\chi_p-\chi_k|}^{\chi_p+\chi_k}dy\cdot\sinh y\cdot V_0(2m\cdot\sinh y/2)\cdot\phi(\chi_k), \qquad (17)$$

where $\cos\chi_0=M/2m$. Here we have introduced the WF $\phi(\chi_p)$, depending on the rapidity, by the relation

$$\phi_{0,0}(|\vec{\Delta}_{p,m\lambda_p}|)\equiv\phi_{0,0}(m\sinh\chi_p) = \frac{4\pi}{m\sinh\chi_p}\phi(\chi_p), \qquad (18)$$

and entering into the expressions for the structure functions in the second section of this paper. Moreover, here the following parametrization of the transfer momentum in the quasipotential is introduced:

$$Q^2=-(p-k)^2=-(\Delta_{p,m\lambda_p}-\Delta_{k,m\lambda_p})^2=(2m\cdot\sinh y/2)^2. \qquad (19)$$

With the help of the transformation (3) it is easy to pass into the relativistic coordinate space where eq. (17) will be of the form

$$\frac{\hat{H}_0}{2m}(\cos\chi_0-\frac{\hat{H}_0}{2m})\cdot\phi_{0,0}(\vec{r}) = [2\frac{\hat{H}_0}{2m}V_0(\vec{r})\frac{\hat{H}_0}{2m}-V_0(\vec{r})]\cdot\phi_{0,0}(\vec{r}). \qquad (20)$$

There, as usual,

$$\hat{H}_0 = 2m\cdot\cosh\frac{i}{m}\frac{\partial}{\partial r}+\frac{2i}{r}\cdot\sinh\frac{i}{m}\frac{\partial}{\partial r}, \qquad (21)$$

and

$$V_0(2m\cdot\sinh y/2) = \frac{4\pi}{m\cdot\sinh y}\int_0^{\infty}dr\cdot r\,V_0(r)\sin mry. \qquad (22)$$

In refs.[12, 13] it is shown that the Coulomb quasipotential in the relativistic coordinate space

$$V_0(r) = -g^2/r \qquad (23)$$

can be identified with the QCD one-gluon exchange contribution because in the momentum space this quasipotential has the same asymptotics at large Q^2 as the QCD one-gluon exchange amplitude:

$$V_0^{\text{eff.QCD}}(2m\cdot\sinh y/2) = -\frac{4\pi g^2}{m^2 y \sinh y} \underset{Q^2\to\infty}{\simeq} -\frac{8\pi g^2}{Q^2\cdot\ln Q^2/m^2}, \tag{24}$$

where as the parameter Λ characteristic for QCD the quark mass m stands.

In this case (QCD) eq. (17) has obviously the following form:

$$\cosh\chi_p(\cosh\chi_p - \cos\chi_0)\cdot\phi(\chi_p) =$$

$$= \frac{g^2\cdot 2m}{2\pi}\int_0^\infty d\chi_k\cdot[\cosh(\chi_p-\chi_k) + \cosh(\chi_p+\chi_k) - 1]\times \tag{25}$$

$$\times\ln\frac{\chi_p+\chi_k}{|\chi_p-\chi_k|}\cdot\phi(\chi_k).$$

For comparison, let us consider also the quasipotential of the electromagnetic interaction that in the momentum space according to QED equals

$$V_0^{\text{QED}}(2m\cdot\sinh y/2) = \frac{4\pi g^2}{-Q^2} = -\frac{\pi g^2}{m^2\cdot\sinh^2 y/2}\times. \tag{26}$$

After substituting this expression into eq. (17) we find

$$\cosh\chi_p(\cosh\chi_p - \cos\chi_0)\cdot\phi(\chi_p) =$$

$$= \frac{g^2\cdot 2m}{4\pi}\int_0^\infty d\chi_k\cdot[\cosh(\chi_p-\chi_k) + \cosh(\chi_p+\chi_k) - 1]\times \tag{27}$$

$$\times\ln\frac{\cosh(\chi_p+\chi_k)-1}{\cosh(\chi_p-\chi_k)-1}\cdot\phi(\chi_k).$$

Finally, let us consider as the quasipotential directly the one-gluon exchange contribution represented usually in QCD as follows

$$V_0^{\text{QCD}}(2m\cdot\sinh y/2) = \frac{4\pi\alpha_s(Q^2)}{Q^2}. \tag{28a}$$

Restricting ourselves for $\alpha_s(Q^2)$ to the one-loop approximation

$$\alpha_s(Q^2)\Big|_{Q^2\gg\Lambda^2} = \frac{4\pi}{\beta_0\cdot\ln Q^2/\Lambda^2}, \tag{28b}$$

where $\beta_0 = 11 - 2/3\cdot n_f$ (n_f is a number of flavors), we represent (28a) in the form

$$V_0^{\text{QCD}}(2m\cdot\sinh y/2) = \frac{2\pi^2}{\beta_0\, m^2\cdot\sinh^2 y/2\cdot\ln(2m/\Lambda\cdot\sinh y/2)}, \tag{28c}$$

Signs of equality in (28a)-(28c) have a conventional character because the quantity $\alpha_s(Q^2)$ calculated in QCD within perturbation theory is determined, strictly speaking, in the range of applicability of perturbation theory only, i.e., at $Q^2\gg\Lambda^2$. Therefore, the expressions (28a) and (28c) are applicable in the region of large transfer momenta only.

Eq. (17) with the potential (28c) takes the form:

$$\cosh\chi_p(\cos\chi_0 - \cosh\chi_p)\cdot\phi(\chi_p) =$$

$$= \frac{2m}{\beta_0}\int_0^\infty d\chi_k\cdot[\cosh(\chi_p-\chi_k) + \cosh(\chi_p+\chi_k) - 1]\times \tag{29}$$

$$\times\ln\frac{\ln\frac{m^2}{\Lambda^2}[\cosh(\chi_p+\chi_k)-1]}{\ln\frac{m^2}{\Lambda^2}[\cosh(\chi_p-\chi_k)-1]}\cdot\phi(\chi_k).$$

In the case of spinless quarks in refs.[21-23] exact solutions were found to the quasipotential equation in the coordinate space for the potential (23) and

$$V_0(r) = -\frac{g^2}{r}\,\mathrm{cth}\,\pi m r, \tag{30}$$

and what is more, the latter is the QED potential in the coordinate space. However, one has not yet succeeded in solving eq. (20) even for these simplest quasipotentials and also eqs. (25) or (27) in the momentum space. Nevertheless, one may try to find the asymptotics of the WF in the momentum space using eq. (17) because just this asymptotics determines the prescaling part of the structure functions (3) or to construct approximate solutions of the quasipotential equation.

For this purpose we will pass to the limit $\chi_p\to\infty$ directly in the integrand in the right-hand side of eq. (17) and suppose that $\chi_p\gg\chi_k$. This assumption may be justified because with rapidly decreasing the WF it is just this region that gives the main contribution to the integral. As a result eqs. (25), (26) and (28) are significantly simplified and take the form

$$(\cosh\chi_p - \cos\chi_0)\cdot\phi(\chi_p) = \int_0^\infty d\chi_k\cdot L(\chi_p,\chi_k)\cdot\phi(\chi_k), \tag{31}$$

where in the case of the effective QCD potential (23), (24) the kernel has the form

$$L_{\text{QCD}}^{\text{eff.}}(\chi_p,\chi_k) = \frac{g^2}{\pi}\ln\left|\frac{\chi_p+\chi_k}{\chi_p-\chi_k}\right|. \tag{32}$$

Analogously, we find for the QCD one-gluon exchange amplitude (28)

$$L_{QCD}(\chi_p, \chi_k) = \frac{1}{\beta_0} \ln \left| \frac{\ln \left| \frac{m^2}{\Lambda^2} \sinh \frac{\chi_p + \chi_k}{2} \right|}{\ln \left| \frac{m^2}{\Lambda^2} \sinh \frac{\chi_p - \chi_k}{2} \right|} \right| \qquad (33)$$

and for the QED amplitude (26)

$$L_{QED}(\chi_p, \chi_k) = \frac{g^2}{2\pi} \ln \left| \sinh \frac{\chi_p + \chi_k}{2} \Big/ \sinh \frac{\chi_p - \chi_k}{2} \right| . \qquad (34)$$

Let us consider first the case of QED. Upon integrating in the right-hand side of eq. (31) with $L_{QED}(\chi_p, \chi_k)$ (34) by parts we obtain

$$(\cosh \chi_p - \cos \chi_0) \cdot \phi_{QED}(\chi_p) = \qquad (35)$$
$$= \frac{g^2 \cdot 2m}{2\pi} \int_0^\infty \frac{d\chi_k \cdot \sinh \chi_p}{\cosh \chi_p - \cosh \chi_k} \cdot \int_{\chi_k}^\infty d\chi_k' \cdot \phi_{QED}(\chi_k')$$

hence it follows at large values of χ_p

$$\phi_{QED}(\chi_p) \underset{\chi_p \to \infty}{\simeq} \frac{\mathrm{const} \cdot \sinh \chi_p}{\cosh \chi_p \cdot (\cosh \chi_p - \cos \chi_0)} . \qquad (36)$$

Thus for $\ell = 0$ we have determined the asymptotics of the WF that is a solution of eq. (14) with the potential (15), (26). Since eq. (14) with the potentials (15), (25) in the nonrelativistic limit turns into the Schrödinger equation with the Coulomb potential, then, as it is easy to see, the interpolating function

$$\phi_{0,0}^{QED}(m \sinh \chi_p) \equiv \frac{4\pi}{m \sinh \chi_p} \phi_{QED}(\chi_p) = \frac{\mathrm{const}}{(\cosh \chi_p - \cos \chi_0)^2} \qquad (37)$$

will have the right nonrelativistic limit as well as the asymptotics following from (36)

$$\phi_{0,0}^{QED}(m \sinh \chi_p) \underset{\chi_p \to \infty}{\simeq} \frac{1}{\cosh \chi_p \cdot (\cosh \chi_p - \cos \chi_0)} \simeq \frac{1}{\cosh^2 \chi_p} .$$

The asymptotics of the kernel (33) has the form

$$L_{QCD}(\chi_p, \chi_k) \underset{\chi_p \to \infty}{\simeq} \frac{\chi_k}{\chi_p} . \qquad (38)$$

After passing to the limit $\chi_p \to \infty$ directly in the integrand of (31) in the case of the kernel (33) we find

$$\phi_{0,0}^{QCD}(\chi_p) \underset{\chi_p \to \infty}{\simeq} \frac{\mathrm{const}}{\chi_p \cdot \sinh \chi_p \cdot (\cosh \chi_p - \cos \chi_0)} \simeq \frac{1}{\chi_p \cdot \cosh^2 \chi} \cdot (39)$$

It is interesting to note that in the asymptotic limit (38) the kernel (33) has lost the dependence on the QCD scale para-

meter Λ. As a result, it does not enter into the asymptotics of the WF (39) also. This result agrees with the fact known in the QCD perturbation theory /24/ that the magnitude of the scale parameter Λ is not of great importance when the one-loop approximation (28b) is chosen for the "running" coupling constant $\alpha_s(Q^2)$ and calculations are restricted to the first order in $(\ln Q^2/\Lambda^2)^{-1}$. Therefore, to determine Λ, it is necessary to apply to formulae containing the two-loop approximation for $\alpha_s(Q^2)$. The question of approximating the one-gluon exchange amplitude with the two- and three-loop formulae chosen for $\alpha_s(Q^2)$ by the effective potential $V_0^{eff.QCD}$ (24) is analysed in ref. /25/. It is easy to check that the kernel $L_{QCD}^{eff.}(\chi_p, \chi_k)$ has limit analogous to (38)

$$L_{QCD}^{eff.}(\chi_p, \chi_k)\Big|_{\chi_p \to \infty} \simeq \frac{\chi_k}{\chi_p} , \qquad (40)$$

and, therefore, the asymptotics of the WF $\phi_{0,0}^{eff.QCD}$ and $\phi_{0,0}^{QCD}$ coincide also.

However, the effective QCD potential (23), (24) being regular as $Q^2 \to \infty$ allows, in contrast to the singular as $Q^2 \to \Lambda^2$ potential (28a), (28c), one to find the WF in a region wider than the limit of large χ_p. Indeed, the expressions (23) and (30), i.e., the transforms of the potentials (24) and (26) in the relativistic configurational representation, coincide at large distances $r > (m\pi)^{-1}$, where $\coth \pi r m \to 1$. According to the transformations (3) and (21), (22) to large values of r there correspond the regions of small values of rapidities which in these regions coincide with the corresponding momenta and transfer momenta

$$\chi_p = \ln \frac{p_0 + |\vec{p}|}{m} \underset{|\vec{p}| \to 0}{\to} \frac{|\vec{p}|}{m} , \quad y \underset{|\vec{q}| \to 0}{\to} \frac{|\vec{q}|}{m} \equiv \frac{|\vec{p} - \vec{k}|}{m} . \qquad (41)$$

Hence, in the region of small momenta and transfer momenta the potentials (24) and (26) coincide

$$V_0^{eff.QCD}(2m \cdot \sinh y/2)\Big|_{y \to 0} = V_0^{QED}(2m \cdot \sinh y/2)\Big|_{y \to 0} = \qquad (42)$$
$$= -\frac{4\pi g^2}{(my)^2} \simeq -\frac{4\pi g^2}{|\vec{q}|^2} ,$$

and, therefore, the WF's coincide also

$$\phi_{0,0}^{eff.QCD}(m \sinh \chi_p)\Big|_{\chi_p \to 0} = \phi_{0,0}^{QED}(m \sinh \chi_p)\Big|_{\chi_p \to 0} = \qquad (43)$$
$$= \frac{\mathrm{const}}{(\cosh \chi_p - \cos \chi_0)^2}\Big|_{\chi_p \to 0} = \frac{4 \cdot \mathrm{const}}{(\chi_p^2 + \chi_M^2)^2} , \quad \chi_M^2 = 2(1 - \cos \chi_0) .$$

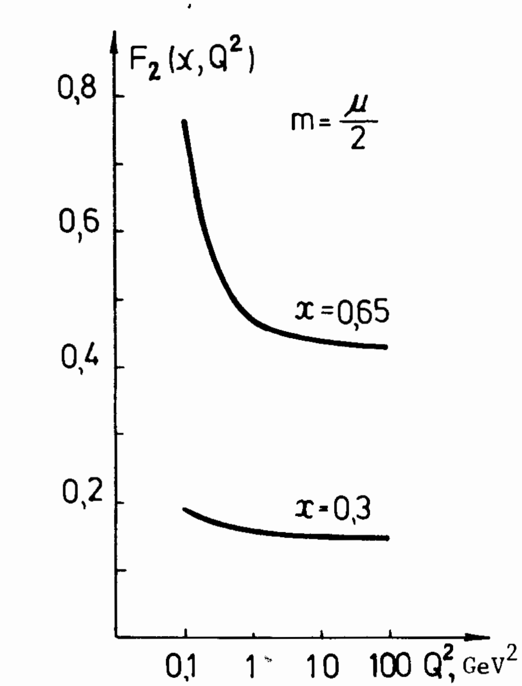
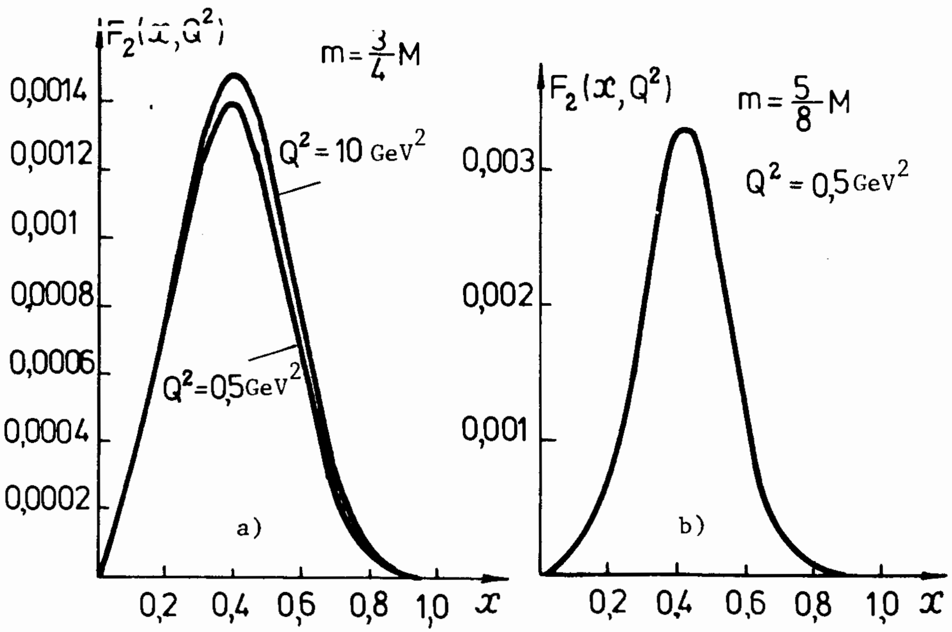


Fig.1. The behaviour of the structure function F_2 versus Q^2 at fixed x.

Fig.2. The behaviour of the structure function F_2 as a function of x at fixed Q^2. The curves corresponding to $Q^2 = 0.5$ GeV^2 and $Q^2 = 10$ GeV^2 practically coincide with each other at $m=5/8M$.



Comparing the asymptotics of WF $\phi^{\mathrm{eff.QCD}}_{0,0}(m \sinh\chi_p)$ of the form (39) with the form of this WF for small $\chi_p \to 0$ we can conclude that the function

$$\phi^{\mathrm{eff.QCD}}_{0,0}(m \sinh\chi_p) = \frac{\mathrm{const}\cdot\chi_p}{\sinh\chi_p \cdot (\cosh\chi_p - \cos\chi_0)(\chi_p^2+\chi_0^2)} \qquad (44)$$

satisfies both boundary for $\chi_p \to 0$ and $\chi_p \to \infty$ conditions (39) and (43), is defined in the whole interval of changing $0 \le |p| < \infty$ and, therefore, it can be considered as an interpolating function. We will use a function of just this form for the numerical calculation of the prescaling part of the structure function $F_2(\zeta, W^2)$ (13).

In Figs.1-3 the behaviour is depicted of the structure function parametrized by the variables ζ and W^2 as well as by the standard Bjorken variables x and Q^2. One may note the following properties. The structure function F_2 comes to the scaling regime at increasing Q^2 or W^2, and what is more, the ζ-scaling sets in before the x-scaling. The structure function decreases or increases with increasing Q^2 depending on the value of the parameter, quark mass m.

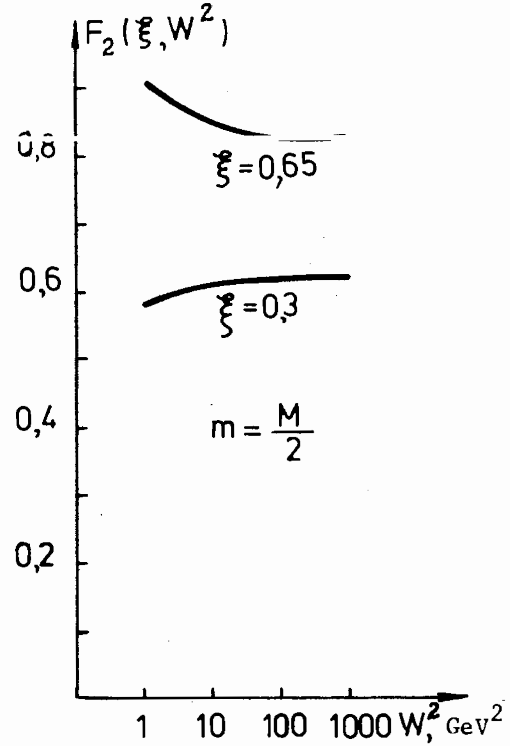


Fig.3. The behaviour of the structure function F_2 as a function of W^2 at fixed ζ.

4. CONCLUSIONS

In the present work approximate analytic solutions have been found for the quasipotential equation for the wave function with potentials corresponding to the one-photon and one-gluon exchange and also with the QCD effective potential. These solutions allow us to obtain the explicit form of structure functions and other chatacteristics of two-particle systems. The behaviour of the meson structure function is established in the case of the QCD effective potential.

The authors are thankful to V.G.Kadyshevsky, S.P.Kuleshov and A.V.Sidorov for useful discussions and also to V.V.Sanadze for the help in numerical computations at computer.

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Received by Publishing Department
on January 19 1982.

Капшай В.Н. и др. Структурные функции псевдоскалярных мезонов в составной модели с хромодинамическим взаимодействием E2-82-36

Решение уравнения для волновой функции кварк-антикварковой системы с ядром взаимодействия, взятым в виде амплитуды одноглюонного обмена, используется для вычисления асимптотического поведения структурных функций мезонов. Полученные выражения содержат члены, приводящие как к степенному, так и к логарифмическому нарушению скейлинга.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1982

Kapshay V.N. et al. The Structure Functions of Pseudo-Scalar Mesons in a Composite Model with QCD Interaction E2-82-36

Solutions of the equation for the wave function of a quark-antiquark system with the kernel in a form of the one-gluon exchange amplitude are used for calculating the asymptotic behaviour of the mesons structure functions. The obtained expressions contain terms giving a power violation of scaling as well as a logarithmic one.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Prepri[illegible] Dubna 1982