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ELASTIC AND INELASTIC SCATTERING OF 180 MeV $\pi^{\pm}$ ON ^{24}Mg

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Recently, we have seen excellent experimental data on the elastic and inelastic scattering of 180 MeV $\pi^{\pm}$-mesons on ^{24}Mg [1]. In ref. [1] also a fit of the differential cross sections was performed within the DWBA and using a rough collective model of ^{24}Mg. The values of the deformation parameters β_2 dictated by the fit of the individual levels differ considerably from the corresponding values extracted from the scattering of 40 MeV protons [2]. Most unexpected is indeed the value $\beta_2 = 0.82$ obtained [1] for the 2^+_1 (1.37 MeV) level. Among the possible explanations of this inconsistency, two seem to be most straightforward. One can expect that a coupled-channel (CC) calculation rather than DWBA should be appropriate for the mentioned fit. The other possible source of troubles is indeed a very crude collective model of ^{24}Mg used in ref. [1].

To check those points we have performed a calculation within the CC formalism using a parametrization of the ^{24}Mg nuclear densities as obtained from the (e, e') experiment [3] and processed in the spirit of the axial-symmetric rotor-model of ^{24}Mg. Starting from the Watson multiple scattering theory, we have obtained [4] the following system of integral equations

$$\langle \vec{Q}'n | F(E) | 0\vec{Q}_0 \rangle = \langle \vec{Q}'n | v(E) | 0\vec{Q}_0 \rangle - \frac{1}{(2\pi)^2} \times \sum_{m=0}^{\infty} \int \frac{\langle \vec{Q}'n | v(E) | m\vec{Q}'' \rangle \langle \vec{Q}''m | F(E) | 0\vec{Q}_0 \rangle}{E - E_m(Q'') + i\epsilon} \frac{d^3Q''}{M(Q'')} \qquad (1)$$

$n = 0, 1, 2, \ldots$ for the pion-nuclear amplitudes $F(E)$. Here, $M(Q'')$ is the pion-nucleus reduced mass [5]. To shorten the notation we use everywhere $E \equiv E_0(Q_0)$. The potential

$$\langle \vec{Q}'n | v(E) | m\vec{Q} \rangle = A \langle \vec{Q}'n | W f(\omega) | m\vec{Q} \rangle \qquad (2)$$

may be simply expressed in terms of the π-N amplitude $f(\omega)$

$$\langle \vec{k}' | f(\omega) | \vec{k} \rangle = A_0 + (\vec{t}\cdot\vec{\tau})A_T + i\frac{\vec{\sigma}\cdot[\vec{k}\times\vec{k}']}{kk'}(A_S + (\vec{t}\cdot\vec{\tau})A_{ST}), \qquad (3)$$

which depends on the pion initial $\vec{k}$ and final $\vec{k}'$ momenta in the π-N centre-of-mass system.

The functions $A_i \equiv A_i(\vec{k}', \vec{k}, \omega)$, $i = 0, S, T, ST$ are constructed from the experimental πN amplitudes $f^J_{\ell, t}(\omega)$ (we consider $\ell = 0,1,2$ waves and the phase-shift parametrization due to Salomon [6]) and πN form factors $g_\ell(k)$, which define the off-energy-shell extrapolation of the πN amplitude. We have chosen a simple form

$$g_\ell(k) = \frac{k^\ell}{(1+ak^2)^2} \tag{4}$$

which yields, using the value $a = 0.224\ fm^2$, almost the same off-shell extrapolation of πN amplitude in the resonating P_{33} wave as the model of separable πN potentials [7]. Unfortunately, the off-energy-shell extrapolation of the πN amplitude is a rather arbitrary part of the present-day pion-nucleus calculations.

When constructing the potential (2) we follow the prescription [8] for the relativistic transformation between the πN and π-nucleus centre-of-mass systems. In this way we end up (in π-nucleus c.m.s.) with the quantities $W_f(\omega)$, $\vec{Q}'$ and $\vec{Q}$ for the amplitude and the pion final and initial momentum, respectively. Further, the amplitudes $f^j_{\ell,t}(\omega)$ were averaged over (nucleon) Fermi momenta and the remaining slowly varying parts of the potential matrix (2) were evaluted for effective momenta [9]. As a result, we retain certain computational simplicity since the complete nuclear-structure input is still factorized in the form of matrix elements

$$\langle J_n T_n n | \exp(i\vec{q}\vec{r}_i)\,\Omega_i \,| J_m T_m m\rangle, \tag{5}$$

where $\vec{q} = \vec{Q}' - \vec{Q}$ and $\Omega_i = 1, \tau, \sigma, \sigma\tau$. In this way any microscopic or phenomenological model for the nuclear states $|J_n T_n n\rangle$ can be simply incorporated into our formalism.

To solve the system (1) we perform the angular decomposition. In the Green functions

$$\frac{1}{E(Q_0) - E_m(Q'') + i\epsilon} = \frac{P}{E(Q_0) - E_m(Q'')} - i\pi\delta(E(Q_0) - E_m(Q'')) \tag{6}$$

the principal-value integral is regularized [10] and represented by a Gaussian quadrature formula. The system (1) is then solved in the model space $m = 0,1,\dots N$ using the matrix inversion method. No phenomenological corrections for the true pion absorption were included in (2). The Coulomb interaction effects were included [11] in the $\langle 0 | v(E) | 0\rangle$ term only.

Now we proceed to describe the nuclear densities to be used in place of expressions (4). In ^{24}Mg eight "valence" nucleons occupy the 2s1d shell, the nucleus is therefore believed to be deformed. We postpone to our next communication the details of the collective axial-symmetric rotor model [12] we

use for this nucleus. Here we consider for the g.s. band ($K=0$, $J=0^+$, 2^+, 4^+, ...) and "γ-band" ($K=2^+$, $J=2^+$, 3^+, 4^+,...) the rough formula (a small term is omitted for $L \geq 4$) for the reduced matrix elements of the tensor operator M_{LK} of rank L

$$\langle J_f K_f \| M_L \| J_i K_i \rangle = [(2-\delta_{K_i K_f})(2J_i+1)]^{1/2} \begin{bmatrix} J_i & L & J_f \\ K_i & K_f - K_i & K_f \end{bmatrix} \langle K_f | M_{L K_f - K_i} | K_i \rangle, \tag{7}$$

where $[\ \cdot\ \cdot\ \cdot\ ]$ stands for the Clebsch-Gordan coefficients. The intrinsic state $|K\rangle$ of ^{24}Mg is the quasiparticle vacuum, $|K\rangle = |0\rangle$, for the g.s. band, or a two-quasiparticle state, $|K\rangle = a_1^+ a_2^+ |0\rangle$, for the γ-band. Using eq. (7) and disregarding small "valence" terms in the expression for $\langle K=2 | M_{L0} | K=2\rangle$ we were able to construct from the five measured [3] form factors the complete input for the CC meson-scattering program. This consists of 29 form factors if the 0^+ (g.s.), $2_1^+(1.37)$, $4_1^+(4.12)$, $2_2^+(4.24)$, and $4_2^+(6.0\ MeV)$ levels of ^{24}Mg are to be included explicitly in eq. (1).

The results of this coupled-channel calculation are shown in Fig. 1 for π^+ and π^- elastic scattering on ^{24}Mg. The comparison is made here with the experimental data and with the one-channel (optical model) calculations. It may be concluded that the calculated results reproduce qualitatively well the experimental data in the region of small scattering angles ($\theta < 90°$). There are definite discrepancies in the region of the first minimum and the diffractive patterns of the calculated curves are shifted somewhat to smaller scattering angles in comparison with the experiment. The channel coupling tends to improve the agreement between the experiment and the calculated results, however, the role of virtual excitations into the low-lying nuclear states turns out to be rather small. It is interesting to note that the difference between π^+ and π^- differential cross sections due to the Coulomb force is appreciable in the large scattering angle region. The difference is nicely reproduced by the CC calculation.

The coupled channel results obtained for inelastic scattering are displayed in Fig. 2 for final nuclear 2_1^+, 2_2^+ and 4_2^+ states and compared with the experiment and with the calculations. We do not show the results for 4_1^+ (4.12 MeV) level, which is extremely weakly excited and actually has not been resolved from the strong 2_2^+(4.24 MeV) level in the (π,π') experiment. The calculated curves agree well with the experiment in the small-scattering-angle region, and it can be concluded that the nuclear form factors deduced from the electron scattering experiments provide a good over-all fit to the inelastic pion scattering without the necessity of adjusting parameters.

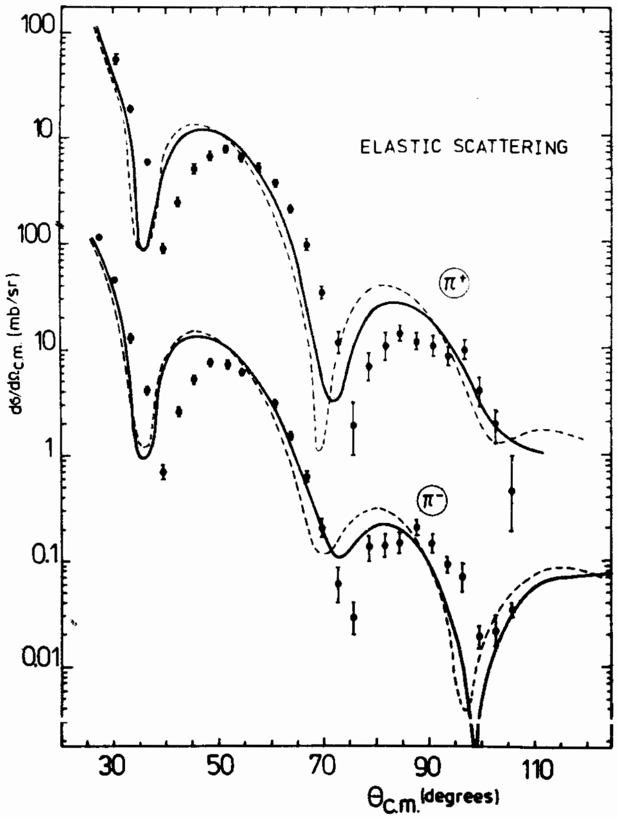

Fig.1. Coupled-channel (full line) and optical model (dashed line) calculations of the π elastic scattering are compared with the data of ref./1/.

If anything, the calculated cross sections overestimate slightly (e.g., in the first maximum of 2^+_1 level) the data; the conclusion, just opposite to that of ref./1/. The role of channel coupling is very small in the case of 2^+_1 level, however, for the weakly excited 4^+_2 level, the coupled channel result agrees better with the experiment than the standard DWIA calculation. We have also calculated within the same assumptions the cross sections for the inelastic scattering of π^- mesons on ^{24}Mg. The results are similar to those of fig.2. and point to the same conclusions.

Similarly to the elastic scattering there are discrepancies at large scattering angles, which can be caused by our ignorance of the detailed behaviour of nuclear form factors in a large momentum transfer region. Note that the electron scattering experiments were performed only for $q \lesssim 2$ fm^{-1}, which corresponds to the scattering angles $\theta \le 90°$ in pion scattering on ^{24}Mg at 180 MeV.

Also in the inelastic scattering we observe a shift of diffractive patterns towards smaller scattering angles of a similar magnitude as in the elastic scattering. A possible origin of this discrepancy is the renormalization of the elementary amplitude in nuclear medium (local field correction) which is not taken into account here, but the evaluation of such corrections is in progress.

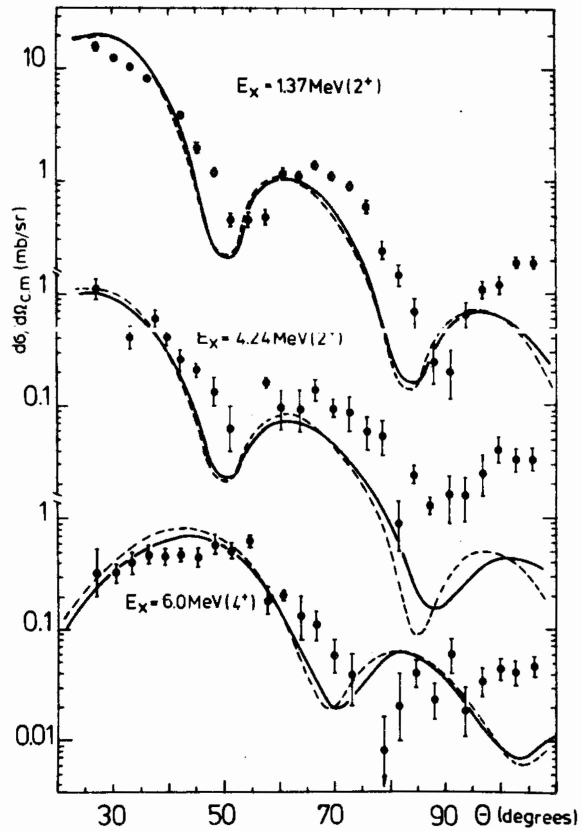

Fig.2. Coupled-channel (full line) and DWIA (dashed line) calculations for the three inelastic transitions in the π^+–^{24}Mg scattering. Data are from ref./1/.

In conclusion, we have shown that the multiple scattering formalism is indeed capable to explain semiquantitatively the experimental data for the 180 MeV pion elastic and inelastic scattering if realistic nuclear density ρ_0 and transition densities ρ_{tr} are supplied as an input. In testing runs we have seen that even small inaccuracy in the fit of ρ_{tr} is blown up in the calculation of (π,π') cross sections and deteriorates considerably the agreement with data.

Besides incorporating the local field corrections, as mentioned above, we also plan to improve upon the nuclear structure part that should allow us to avoid the fitting procedure for ρ_{tr}. An interesting problem left is then a simultaneous calculation of (π,π') and (p,p') cross sections with the same microscopic set of the nuclear densities.

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Гмитро М., Квасил Я., Мах Р. Упругое и неупругое рассеяние пионов с энергией 180 МэВ на ^{24}Mg E2-82-34

Уравнения метода связанных каналов, записанные в импульсном пространстве, были решены для рассеяния $\pi^{\pm}$-мезонов $/E_{\pi}=180$ МэВ/ на ^{24}Mg. Вычисляются упругие и неупругие дифференциальные сечения, причем были использованы плотность основного состояния ^{24}Mg и переходные ядерные плотности, которые правильно описывают данные по (e,e') рассеянию. Результаты хорошо согласуются с экспериментальными данными.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1982

Gmitro M., Kvasil J., Mach R. Elastic and Inelastic Scattering of 180 MeV $\pi^{\pm}$ on ^{24}Mg E2-82-34

Equations of the coupled channel method written in the momentum space are solved for the scattering of 180 MeV pions on ^{24}Mg. The elastic and inelastic differential cross sections are calculated by using the nuclear ground-state- and transition-densities, which describe correctly the (e,e') data for the same nuclear states. The results agree well with the recent (π,π') data.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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