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A.A.Chumbalov*, S.S.Kamalov

OFF-SHELL EFFECTS IN THE COHERENT π^0-PHOTOPRODUCTION OFF NUCLEI

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* Kazakh State University, Alma-Ata

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I. INTRODUCTION

In the present paper we discuss the off-shell effects in the coherent π^{o} -photoproduction analysed in the framework of the distorted-wave impulse approximation (DWIA). In order to formulate the problem let us write T-matrix of the process in terms of DWIA in the momentum space[1]

$$\langle \underline{q}_0|T(E)|\underline{k}\lambda\rangle = \langle \underline{q}_0|V_{\pi\gamma}|\underline{k}\lambda\rangle + \frac{1}{(2\pi)^3}\int \frac{\langle \underline{q}_0|T'_{\pi\pi}(E)|\underline{q}\rangle\langle \underline{q}|V_{\pi\gamma}|\underline{k}\lambda\rangle}{E - E(\underline{q}) + i\varepsilon}\, d\underline{q}\,, \qquad (1)$$

where

$$\langle \underline{q}|V_{\pi\gamma}|\underline{k}\lambda\rangle = \int\int \rho(\underline{p}',\underline{p})\langle \underline{q},\underline{p}'|t_{\pi\gamma}(\omega)|\underline{p},\underline{k}\lambda\rangle\, d\underline{p}\, d\underline{p}', \qquad (2)$$

$\underline{q}$ and $\underline{p}(\underline{p}')$ are the pion and initial (final) nucleon momenta, respectively, $\underline{k} = E_\gamma \hat{\underline{k}}$ and $\lambda = \pm 1$ are the photon momentum and polarization,

$$\rho(\underline{p}',\underline{p}) = \langle 0|\sum_{j=1}^{A} \delta(\underline{p}'-\underline{p}_j')\,\delta(\underline{p}-\underline{p}_j)\,|0\rangle \qquad (3)$$

is the nuclear density, $t_{\pi\gamma}(\omega)$ is the pion photoproduction t -matrix on the free nucleon. Here, the quantity ω has the meaning of the full pion-nucleon (πN) energy in the πN -centre of-mass system when pion is on-shell (i.e. when $E_\pi = [m_\pi + q_0^2]^{1/2}$). $T'_{\pi\pi}(E) = T_{\pi\pi}(E)(A-1)/A$ is the matrix of elastic pion-nuclear scattering that depends on the full energy E for a pion-nuclear (π A) system (i.e. $E = E_\pi(q_0) + E_A(q_0)$). This matrix can be determined by solving the πA -scattering problem[2,3] in the KMT version of the Multiple Scattering Theory[4].

It follows from eqs.(1,2) that to take into account the strong πA -interaction in the final state it is necessary to perform the integration over q from 0 to ∞. As a consequence, the problem of determination of the $t_{\pi\gamma}(\omega)$ -matrix in the off-shell region (i.e. when $q \neq q_0$) arises. This problem is closely connected with the choice of the reaction energy ω since different off-shell extrapolation of the $t_{\pi\gamma}(\omega)$ -matrix can be realized by different determination of the parameter ω in the off-shell region. The main task of the present work is to investigate the sensitivity of the π^0 -photoproduction process to the different choice of ω.

Note that an analogous problem arises in the πA -scattering. In this case it has been shown[5,6] that various assumptions about ω and its relation with the energy E lead to different results in the Δ_{33} -region. We think that coherent π^0 -photoproduction must be more sensitive to different determination of the reaction energy ω since in this process the nonresonant pion S -wave contribution is absent and therefore the resonance (3,3)-multipole dominates. As a consequence, we have a very sharp energy dependence for the $t_{\pi\gamma}(\omega)$ matrix. In such a situation it is very important to know the correct determination of the reaction energy ω.

II. ON-SHELL PHOTOPRODUCTION AMPLITUDE

For the determination of the $t_{\pi\gamma}$ -matrix in eq.(2) the expression for the corresponding free nucleon amplitude $\tilde{f}_{\pi\gamma}$ in the πN c.m. frame is used. The on-shell relation for $t_{\pi\gamma}$ and $\tilde{f}_{\pi\gamma}$ is

$$\langle \underset{\sim}{q}, \underset{\sim}{p}' | t_{\pi\gamma}(\omega) | \underset{\sim}{p}, \underset{\sim}{K}\lambda \rangle = -2\pi\,\delta(\underset{\sim}{\mathcal{P}_f} - \underset{\sim}{\mathcal{P}_i}) \sqrt{\frac{W_i W_f}{E_\pi E_\gamma E_N E_N'}}\, \langle \tilde{\underset{\sim}{q}} | f^{(\lambda)}_{\pi\gamma}(\omega) | \tilde{\underset{\sim}{K}} \rangle . \quad (4)$$

where W_i and W_f are the invariants for γN and πN systems

$$W_i^2 = \left(E_\gamma + E_N(p)\right)^2 - \underset{\sim}{\mathcal{P}_i}^2 \quad ; \quad W_f^2 = \left(E_\pi(q) + E_N(p')\right)^2 - \underset{\sim}{\mathcal{P}_f}^2 \quad (5)$$

with $\underset{\sim}{\mathcal{P}_i} = \underset{\sim}{K} + \underset{\sim}{p}$, $\underset{\sim}{\mathcal{P}_f} = \underset{\sim}{q} + \underset{\sim}{p}'$ and $E_N(p) = (M_N^2 + p^2)^{1/2}$. The pion ($\tilde{\underset{\sim}{q}}$) and photon ($\tilde{\underset{\sim}{K}}$) momenta in the c.m. frame are connected with the corresponding momenta in an arbitrary frame by the Lorents transformation

$$\tilde{\underset{\sim}{q}} = \underset{\sim}{q} + \left[\frac{\underset{\sim}{\mathcal{P}_f}\cdot \underset{\sim}{q}}{E_\pi(q) + E_N(p') + W_f} - E_\pi(q)\right]\frac{\underset{\sim}{\mathcal{P}_f}}{W_f} \; ; \; \tilde{\underset{\sim}{K}} = \underset{\sim}{K} + \left[\frac{\underset{\sim}{\mathcal{P}_i}\cdot \underset{\sim}{K}}{E_\gamma + E_N(p) + W_i} - E_\gamma\right]\frac{\underset{\sim}{\mathcal{P}_i}}{W_i} \cdot \quad (6)$$

The general expression for $\tilde{f}_{\pi\gamma}$ and its partial decomposition is well-known[7]. Here, we use the spin-independent part of $\tilde{f}_{\pi\gamma}$ since only it contributes in the coherent π^0 - photoproduction on zero-spin nuclei. Keeping only p - and d -pion partial waves, one obtains

$$\langle \tilde{\underset{\sim}{q}} | \tilde{f}^{(\lambda)}_{\pi\gamma}(\omega) | \tilde{\underset{\sim}{K}} \rangle = \left[(2M_{1+} + M_{1-}) + 3\tilde{x}(3M_{2+} + 2M_{2-})\right][\tilde{\underset{\sim}{\eta}} \times \tilde{\underset{\sim}{\varkappa}}]\cdot \underset{\sim}{\epsilon}_\lambda , \quad (7)$$

where $\tilde{\underset{\sim}{\eta}}$ and $\tilde{\underset{\sim}{\varkappa}}$ are the unit pion and photon momenta in the πN c.m. frame, respectively, $\tilde{x} = \tilde{\underset{\sim}{\eta}} \cdot \tilde{\underset{\sim}{\varkappa}}$. The multipoles $M_{\ell\pm}(\omega)$ are taken from refs.[7,8].

For averaging over nucleon momentum distribution in the nucleus for the calculation of $V_{\pi\gamma}$ (see eq.(2)), it is convenient to perform the following substitution

$$\underset{\sim}{p} = -\frac{\underset{\sim}{K}}{A} - \frac{A-1}{2A}(\underset{\sim}{K} - \underset{\sim}{q}) + \underset{\sim}{v} \equiv \underset{\sim}{p}_i^{eff} + \underset{\sim}{v} , \quad (8a)$$

$$\underset{\sim}{p}' = -\frac{\underset{\sim}{q}}{A} + \frac{A-1}{2A}(\underset{\sim}{K} - \underset{\sim}{q}) + \underset{\sim}{v} \equiv \underset{\sim}{p}_f^{eff} + \underset{\sim}{v} . \quad (8b)$$

According to ref.[6], for zero-spin nuclei linear in v terms in $t_{\pi\gamma}$ give zero contributions and quadratic ones in v are of the order $(m_\pi/M)^2 \sim 0.02$. Consequently, for our purpose it is sufficient to neglect v in (7) and take $p \approx p_i^{eff}$, $p' \approx p_f^{eff}$

(the factorization approximation). Then, the plane wave part $V_{\pi\gamma}$ can be expressed as

$$\langle \underset{\sim}{q} | V_{\pi\gamma} | \underset{\sim}{k}\lambda \rangle = -2\pi A \left[\frac{W_i\, W_f}{E_\gamma E_\pi(q) E_N(p) E_N(p')} \right]^{1/2} \langle \underset{\sim}{\tilde{q}} | \tilde{f}^{(A)}_{\pi\gamma}(\omega) | \underset{\sim}{\tilde{k}} \rangle F^{ch}_A(Q) / f^{ch}_p(Q), \quad (9)$$

where $F_A^{ch}(Q)$ and $f_p^{ch}(Q)$ are the nuclear and proton charge form factors, respectively, $\underset{\sim}{Q} = \underset{\sim}{k} - \underset{\sim}{q}$ is the transferred momentum.

Note that the factorization approximation guarantees the momentum and energy conservation in the πA, πN and γN systems simultaneously when $q = q_o$. However, in the off-shell region (where $q \neq q_o$) the total energy for the πN -system disagrees with the total energy for the γN system (i.e. $W_i \neq W_f$ when $q \neq q_o$). In this region the physical meaning of the parameter ω is uncertain.

III. OFF-SHELL EFFECTS. RESULTS AND DISCUSSION

As it has been noted above, the connection (4) between the matrices $t_{\pi\gamma}(\omega)$ and $\tilde{f}_{\pi\gamma}(\omega)$ is correct only in the on-shell sense. While computing the principal value of integral in (1), one postulates that eq.(4) is relevant in the off-shell region as well. However, the uncertainty in the definition of the energy ω in the off-shell region remains. In what follows, to study the effects of different definition of ω, we shall use the following alternative expressions:

$$\omega_o = W_i \approx \left[(E_\pi(q_o) + E_N(p'))^2 - \underset{\sim}{P}_f^2 \right]^{1/2} \quad (10a)$$

$$\omega_1 = \left[m_\pi^2 + M_N^2 + 2E_\pi(q_o) E_N(p') - 2\underset{\sim}{q}\cdot\underset{\sim}{p}' \right]^{1/2} \quad (10b)$$

$$\omega_2 = W_f = \left[(E_\pi(q) + E_N(p'))^2 - \underset{\sim}{P}_f^2 \right]^{1/2}. \quad (10c)$$

According to ref.[6], the reaction energy ω may be determined also as $\omega_3 = (W_i + W_f)/2$ and $\omega_4 = \sqrt{W_i\, W_f}$.

All definitions for ω_i (i = 0÷4) are on-shell equivalent but they provide us with rather different values in off-shell-region. For the illustration of this fact the dependence of ω_o, ω_1 and ω_2 on pion momentum q is plotted in Fig.1. in the case of π^o -photoproduction off ^{12}C at E_γ^{LAB} = 290 MeV and θ_π = 25° (in the πA c.m. frame E_γ = 283 MeV, q_o = = 1.26 fm^{-1} and $\omega(q_o)$ = 1200 MeV).

The off-shell behaviour of the partial amplitudes $M_{\ell\pm}(\omega)$ at the fixed reaction energy ω_i (i = 0÷4) may be determined, e.g., from the separable model of πN -interaction[9] like it was done in the πA -scattering problem[2,3]

$$M_{\ell\pm}(\omega) = M_{\ell\pm}(\omega_i)\, g^{(\ell)}_{\pi N}(\tilde{q}) / g^{(\ell)}_{\pi N}(\tilde{q}_i), \quad (11)$$

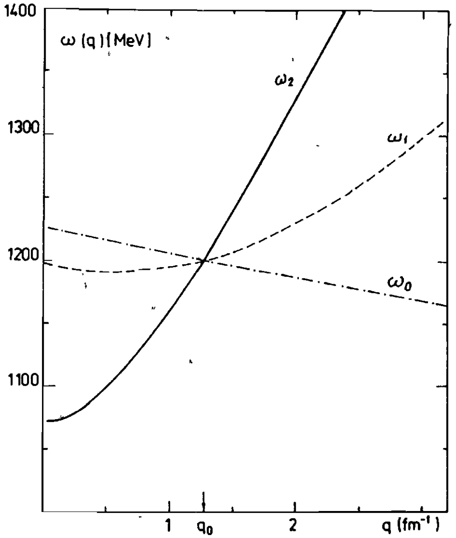
where

$$g^{(\ell)}_{\pi N}(\tilde{q}) = q^\ell / (1 + \alpha q^2)^2, \qquad \alpha = 0.224 \text{ fm}^2. \quad (12)$$

In eqs. (11,12) $\tilde{q}_i$ is the pion momentum in πN c.m. frame that corresponds to the energy ω_i for the πN -system. The appropriate threshold behaviour of the amplitude $M_{\ell\pm}(\omega)$ as $\tilde{q} \to 0$ is guaranteed by the term $\tilde{q}^\ell$ in eq.(12).

Notice that for $\omega_2 = W_f(q)$, one may not introduce the factor $g^{(\ell)}_{\pi N}(q)$ because in this case $\tilde{q} = q_2$. The right threshold behaviour of $\tilde{f}_{\pi\gamma}$ and its cutting off as $\tilde{q} \to \infty$ are provided by the energy dependence of $M_{\ell\pm}(\omega)$ multipoles for which one has[7]

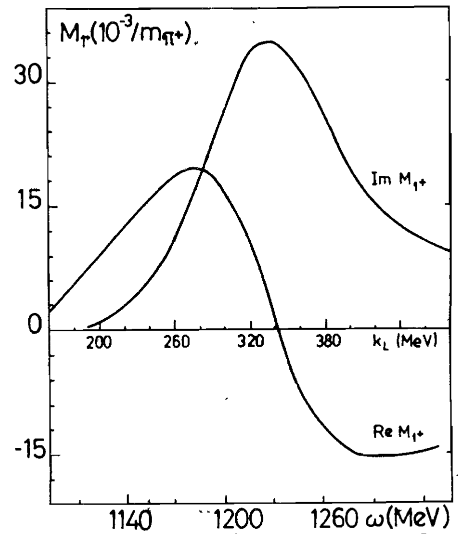
$$M_{\ell\pm} \xrightarrow[\tilde{q}\to 0]{} \tilde{q}^\ell \ ; \ M_{\ell+} \xrightarrow[\tilde{q}\to\infty]{} \frac{1}{\omega^{\ell+1}} \ ; \ M_{\ell-} \xrightarrow[\tilde{q}\to\infty]{} \frac{1}{\omega^\ell}. \quad (13)$$



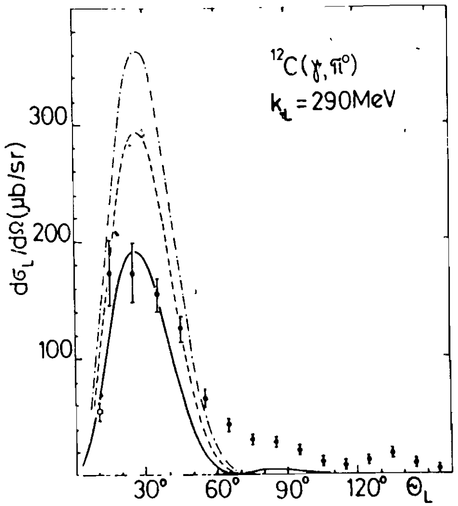
F i g. 1. q -dependence of different definition of the reaction energy ω in the off-shell region corresponding to the on-shell value q_o = 1.26 fm^{-1} ($k_L \equiv E_\gamma^{LAB}$ = = 290 MeV).

The energy dependence of the multipole $M_{1+}(\omega)$ dominant in (7) is plotted in Fig. 2. One can see that the region most sensitive to the choice of energy ω may be $E_\gamma^{LAB} \sim$ 290-340 MeV where the real and imaginary parts of this multipole have sharp energy dependence. Our DWIA results* corresponding to the different definition for the off-shell behaviour of the parameter ω (see Fig.2) are depicted in Figs.3 and 4. As it was expected, Δ_{33} -region is the most sensitive to the choice of ω . In this region results may differ as much as 1.5-2 times. This sensitivity decreases with E_γ^{LAB} .

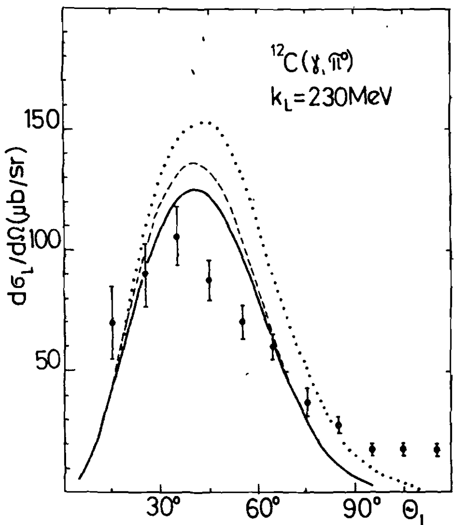
*The results of this paper differ from our previous results (see ref./14/). This is due to several reasons: 1) earlier, multipoles $M_{\ell\pm}(\omega)$ were calculated at fixed energy $\omega(q_o)$, now ω is the variable of integration; 2) in the Δ_{33} -resonance region we use the new BD-amplitude[8] instead of BDW [7] ; 3) we use the new πA -optical potential[3] describing not only the differential cross sections[15] but also σ_{TOT} and σ_{el} for the elastic πA -scattering.



F i g. 2. Energy dependence of the $M_{1+}(\omega) = [M_{1+}^{(1)} + 2M_{1+}^{(3)}]/3$ multipole [8] .



F i g. 3. Angular distribution for $^{12}C(\gamma,\pi^o)^{12}C$ at the photon energy k_L = 290 MeV obtained by using the reaction energy $\omega = \omega_2$ (dash-dot), $\omega = \omega_1$ (dashed) and $\omega = \omega_2$ (solid). The data are from ref. [10] .



F i g. 4. The same as in Fig. 3 at k_L = 230 MeV. The result corresponding to the choice $\omega = \omega_o$ is not shown because it only slightly differs from that corresponding to $\omega = \omega_2$. Dotted line is the result of the plane wave impulse approximation.

In our opinion $\omega = \omega_2$ is the most consistent choice. This is not only because this choice provides us with the best agreement with the experimental data from ref. [10]. Actually, such a conclusion may be considered as a consequence of the Relativistic Potential theory [11] with the help of which one can determine the off-shell relation between the $t_{\pi\gamma}$-matrix in an arbitrary frame and the corresponding amplitude $\tilde{f}_{\pi\gamma}$ in πN c.m. frame. Such an expression was obtained in [12] for the πN -scattering matrix. Generalizing this method for the two-potentials problem [13] one can see that in expression (4) it is necessary to replace the amplitude $\tilde{f}_{\pi\gamma}$ by the auxiliary matrix $f_{\pi\gamma}$ connected with each other as follows:

$$\langle \tilde{q} | f^{(\lambda)}_{\pi\gamma}(\omega) | \tilde{K} \rangle = \langle \tilde{q} | \tilde{f}_{\pi\gamma}(W_f) | \tilde{K} \rangle - \frac{1}{(2\pi)^2} \int \frac{dq'}{\mu(q)} \langle \tilde{q} | f_{\pi\pi}(W_f(q')) | q' \rangle \tag{14}$$
$$\times \langle q' | \tilde{f}^{(\lambda)}_{\pi\gamma}(W_f(q')) | \tilde{K} \rangle \left[\frac{1}{W_f - W_f(q') + i\varepsilon} - \frac{1}{\omega - W_f(q') + i\varepsilon} \right],$$

where $\mu(q)$ is the reduced mass of πN -system, $W_f = E_\pi(\tilde{q}) + E_N(\tilde{q}) = [(E_\pi(q) + E_N(p'))^2 - \mathcal{P}_f^2]^{1/2}$ has the meaning of the eigenvalue of the relativistic free Hamiltonian of the πN -system ($\hat{h}^o_{\pi N} = \hat{h}_\pi + \hat{h}_N$) in the c.m. frame, $f_{\pi\pi}$ is the πN -scattering amplitude. Note that we neglect in (14) the terms of $\mathcal{P}^2_{i(f)}/W_i W_f$ order and use the first Born approximation for the electromagnetic interaction.

It can easily be seen from eq.(14) that if we set up $\omega = W_f$ the contribution of the second term in eq.(14) will be zero. As a result, we obtain the simplest off-shell connection (4) between the $t_{\pi\gamma}$ -matrix in an arbitrary frame and the corresponding amplitude $\tilde{f}_{\pi\gamma}$ in the πN - c.m. frame (half off-shell connection [12]).

IV. SUMMARY

We have demonstrated the strong sensitivity of the coherent π^o -photoproduction off nuclei to the choice of the reaction energy ω for the elementary $t_{\pi\gamma}(\omega)$ -matrix in the off-shell region. The main reason for such a sensitivity is the resonant energy dependence of the $t_{\pi\gamma}$ -matrix. The best agreement with the experimental data was obtained when ω was chosen as the eigenvalue of the free relativistic Hamiltonian for the πN - system (i.e. $\omega^2 = (E_\pi(q) + E_N(p'))^2 - (q + p')^2$). This conclusion is consistent with the results of the Relativistic Potential theory [11,22].

Note that in our calculations performed for the charged pion photoproduction off ^{16}O, ^{12}C and ^{10}B we have observed only 10-20% difference between the results corresponding to various choices of the energy ω. This is mainly due to the fact that in the case of charged pion photoproduction of the nonresonant S -wave E_{0+} multipole dominates and consequently $t_{\pi\gamma}$ has smooth energy dependence.

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