объединенный
институт
ядерных
исследований
дубна

E2-87-370

A.V.Kotikov

THE CALCULATION OF MOMENTS OF STRUCTURE FUNCTION OF DEEP INELASTIC SCATTERING IN QCD

Submitted to "Теоретическая и математическая физика"

1987

I. INTRODUCTION

In paper[1] the method of multiloop calculations, the method of uniqueness[2-5], has been generalized to the diagrams containing arbitrary number of momenta on lines. These diagrams appear when the method of "gluing"[6] is used to calculate the moments of structure functions of deep-unelastic lepton-hadron scattering (DIS) in QCD. There is another method to obtain moments of structure functions. It is called the method of "projectors"[7,8]. In this paper we demonstrate that the combination of the method of "projectors" and that of uniqueness is more prospective for calculating complicated diagrams. We show examples of the calculation of typical diagrams arising in DIS. The developed method is applied to the calculation of the two-loop correction to the longitudinal singlet structure function in DIS.

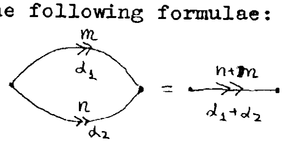
2. BASIC FORMULAE

Let us briefly consider the calculation rules. (A more full review can be found in paper[1]). All the calculations are performed in the coordinate representation. The lines of graphs are associated with powers of the type $1/(x^2)^\alpha$, α being called the index of the line, the arrow with subscript μ corresponds to a vector x^μ , the black arrow corresponds to derivative $\frac{\partial}{\partial x_\mu} \equiv \partial_\mu$, two arrows (black arrows) with subscript n correspond to the product of vectors $x^{\mu_1} \dots x^{\mu_n}$ (derivatives $\partial_{\mu_1} \dots \partial_{\mu_n}$)

$$\underset{0\quad\ \alpha\quad\ x}{\overset{\mu}{\longrightarrow}} = \frac{x^\mu}{(x^2)^\alpha} , \quad \underset{0\quad\ \alpha\quad\ x}{\overset{\mu}{\longrightarrow}} = \frac{d}{dx_\mu} \frac{1}{(x^2)^\alpha} , \quad \underset{0\quad\ \alpha\quad\ x}{\overset{n}{\twoheadrightarrow}} = \frac{\prod_{i=1}^{n} x^{\mu_i}}{(x^2)^\alpha} . \tag{1}$$

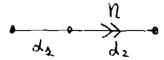
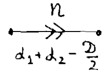
These are the following formulae:

A simple loop



(2)

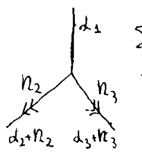
A chain

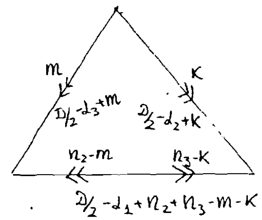
 $= A^{0,n}(\alpha_1,\alpha_2)$  $+ \dots ,$ (3)

where $A^{n,m}(\alpha_1,\alpha_2) = \dfrac{\pi^{D}\, a_n(\alpha_1)\, a_m(\alpha_2)}{a_{n+m}(\alpha_1+\alpha_2-\frac{D}{2})}$, $a_n(\alpha) = \dfrac{\Gamma(n+\frac{D}{2}-\alpha)}{\Gamma(\alpha)}$

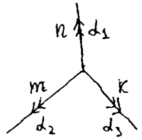
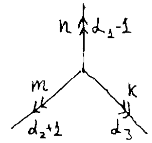
(Hereafter we will neglect the terms $\sim g^{\mu_i \mu_j}$).

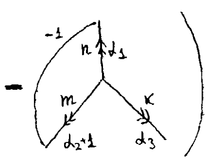
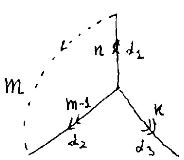
A vertex (Uniqueness relation)

 $\overset{\sum \alpha_i = D}{=\!=\!=} \pi \sum_{m=0}^{n_2} \sum_{k=0}^{n_3} C^{m}_{n_2} C^{k}_{n_3}\, a_{n_2+k}(\alpha_2+n_2) \cdot$

$\cdot\, a_{n_3+m}(\alpha_3+n_3)\, a_{n_2+n_3-k-m}(\alpha_1) \cdot$  . (4)

A vertex (integration by parts)

 $(D-2\alpha_1-\alpha_2-\alpha_3+n+m+k) = \alpha_2 \Big($  $-$

$-$  $\Big) + m$  $+ (\alpha_2 \leftrightarrow \alpha_3 , m \leftrightarrow k)$. (5)

Equation (5) allows us to change indices of the lines of diagrams by an integer. We can also change indices of the lines with the help of the point group of transformations[3]. Elements of the group are as follows:

a) transition to the momentum representation,

b) conformal inversion $x \to x' = x/x^2$,

c) special series of the transformations which allow to do one vertex unique. Then the relation (4) is applied to this vertex.

Consider the action of the group of transformation on the specific two-loop diagram presented in Fig. 1

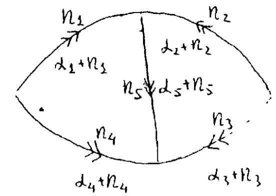
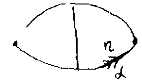
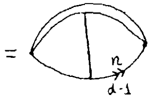
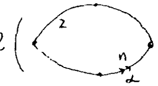
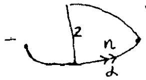
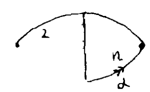
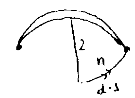


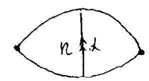
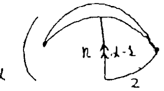
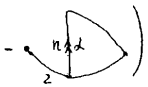
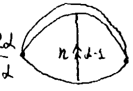
Fig. 1.

The table of transformations is given in Appendix. The notation used is given in paper[3]. Note that with the help of this group of transformations (see Appendix) we cannot only change the index of lines, but sometimes we can reduce the sum of products of vectors to a unique product.

To calculate complicated diagrams, it is often convenient to use functional relations analogous to those of ref.[2,5]. This sometimes simplifies the calculations. For example, we receive[1] for the diagram presented in Fig. 1, when $\alpha_3+n_3=\alpha$, $n_3=n$, $\alpha_j(j\neq3)=1$, $n_j(j\neq3)=0$ and $\alpha_5+n_5=\alpha$, $n_5=n$, $\alpha_j(j\neq5)=1$, $n_j(j\neq5)=0$:

 $=$  $- \frac{1}{2\varepsilon}\Big[2\Big($  $-$  $\Big) -$ (6a)

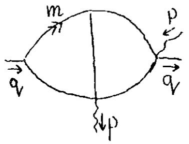
$-$  $+$  $\Big]$

 $= \dfrac{2}{n+D-2-2\alpha}\Big($  $-$  $\Big) - \dfrac{n+2D-4-2\alpha}{n+D-2-2\alpha}$  (6b)

Hereafter, we neglect the index equal to unity.

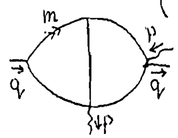
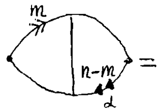
3. METHOD OF "PROJECTORS"

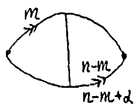
In this paper, we apply a special case of the method of "projectors", the method of "differentiation" which allows us to obtain coefficients of degrees of $\frac{pq}{q^2}$ to the diagrams depending on two momenta p and q, when $p^2 = 0$ and $Q^2 = -q^2 > 0$ is large. These coefficients will be called the moments of the diagram. For example, we obtain an n-moment of the diagram:

$$I = \prod_{i=1}^{m} p_{\lambda_i}$$  $$\underset{Q^2\to\infty}{\overset{p^2=0}{=}} \sum_K I_K \frac{p_{\nu_1}\cdots p_{\nu_K}\, q^{\nu_1}\cdots q^{\nu_K}}{q^{2(K+\alpha+2\varepsilon)}}, \qquad (7)$$

where $\xrightarrow[\vec{K}]{m} = \dfrac{\prod_{i=1}^{m} K_{\lambda_i}}{K^2}$.

Differentiating (7) with respect to $\left(\frac{d}{dp_{\mu_i}}\right)^n$ and supposing $p = 0$, we get to the left:

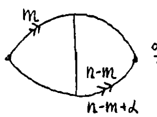
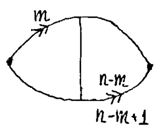
$$\frac{d}{dp_{\mu_1}}\cdots\frac{d}{dp_{\mu_n}}\Big\{ p_{\lambda_1}\cdots p_{\lambda_m}$$  $$\Big\}\Big|_{p=0} = \hat{S}\,\frac{n!}{(n-m)!}$$  $$=$$

$$= \hat{S}\, n!\, \frac{2^{n-m}\,\Gamma(n-m+\alpha)}{\Gamma(\alpha)\,(n-m)!}$$  ,

where $\hat{S}$ is a symmetrizator over indeces λ_i, μ_j, $i = 1,\dots,m$; $j = 1,\dots,n$, and to the right:

$$\sum_K I_K \frac{q^{\nu_1}\cdots q^{\nu_K}}{q^{2(K+\alpha+2\varepsilon)}} \frac{d}{dp_{\mu_1}}\cdots\frac{d}{dp_{\mu_n}}\left(p^{\nu_1}\cdots p^{\nu_K}\right)\Big|_{p=0} = \hat{S}\, n!\, I_n \frac{q^{\nu_1}\cdots q^{\nu_n}}{q^{2(n+\alpha+2\varepsilon)}}.$$

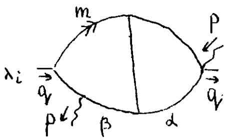
Hence, we have the following expression for the moment of the diagram:

$$I_n \frac{q^{\nu_1}\cdots q^{\nu_n}}{q^{2(n+\alpha+2\varepsilon)}} = \hat{S}\, \frac{2^{n-m}\,\Gamma(n-m+\alpha)}{(n-m)!\,\Gamma(\alpha)}$$  $$\overset{\alpha=1}{=} \hat{S}\cdot 2^{n-m}$$  .

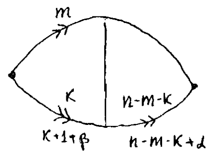
Further, the symmetrizator $\hat{S}$ will be neglected.

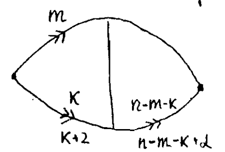
Note that this transformation from the diagram to its moment will be right at any indices of the lines of the diagram.

A similar conclusion can be drawn for the diagram

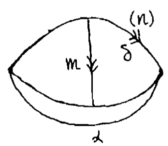
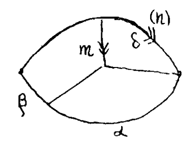
$$I = \prod_{i=1}^{m} p_{\lambda_i}$$  . (8)

Its moment is as follows:

$$I_n = \sum_{K=0}^{n-m} \frac{2^{n-m}\,\Gamma(K+\beta)\,\Gamma(n-m-K+\alpha)}{\Gamma(\beta)\,\Gamma(\alpha)\,K!\,(n-m-K)!}$$  $$=$$

$$\overset{\alpha=\beta=1}{=} 2^{n-m} \sum_{K=0}^{n-m}$$  .

Note that expressions for the moments of diagrams (7),(8) are transformed by the method of "gluing"/8/ to integrals of the form

 ,  ,

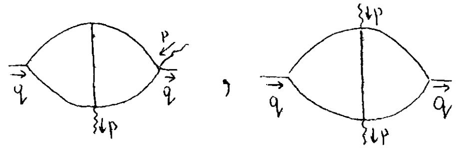
where index (n) means a traceless product of n vectors and index δ is defined in ref./6/. The first of the diagrams is nearly equivalent to the diagram (7) because of one trivial integration, the second is more complicated than the diagram (8).

Hence, the method of "differentiation" gives for the moment of the diagram a more simple expression than the method of "gluing".

4. EXAMPLES OF THE DIAGRAM EVALUATION

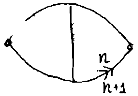
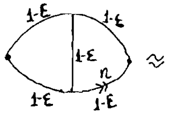
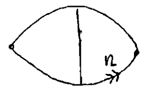
We demonstrate the effeciency of the proposed method calculating a number of typical diagrams taken from practical calculations. The calculations are made in the x-space for the dual diagram. The dual diagram arises from the initial one by replacing all p_i by x_i with the diagram-integral correspondence as in the x-space. The transition to the dual diagram will be denoted by $\underset{=}{\mathfrak{D}}$.

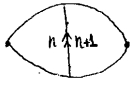
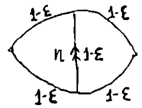
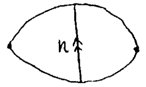
At first consider the two simplest diagrams (Fig.2).



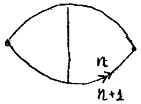
F i g. 2.

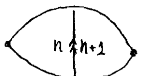
Their moments coincide up to 2^n with the diagram represented in Fig. 1, when $\alpha_i = 1$; $n_3 = n$, $n_j (j \neq 3) = 0$ and $n_5 = n$, $n_j (j \neq 5) = 0$, respectively. Note that dual diagrams equal initial diagrams. Using the momentum transformation from the table in Appendix we get


$$\overset{MR}{=} \frac{\Gamma(n+1+2\varepsilon)}{n!\,\Gamma(1+2\varepsilon)}$$

$$\approx$$



$$\overset{MR}{=} \frac{\Gamma(n+1+2\varepsilon)}{n!\,\Gamma(1+2\varepsilon)}$$

$$\approx$$

,

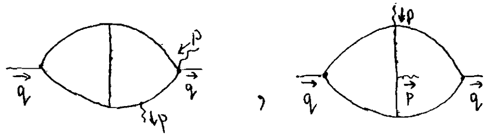
where $\approx$ means the equality up to $O(\varepsilon^0)$. The latter diagrams are calculated in paper[1]. Thus, we have


$$= \frac{1}{n+1} \left[S_3(n) + S_2(n) S_1(n) - T(n) + 6\,\zeta(3) \right] \qquad (9a)$$


$$= \left(1 + (-1)^n\right) \left[\frac{(1-\delta_n^0)\, 2 K_2(n)}{n(n+1)} + \delta_n^0 \cdot 3\, \zeta(3) \right], \qquad (9b)$$

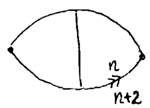
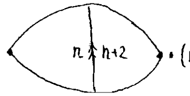
where $S_i(n) = \sum_{K=1}^{n} \frac{1}{K^i}$, $K_i(n) = \sum_{K=1}^{n} \frac{(-1)^{K+1}}{K^i}$, $T(n) = \sum_{K=1}^{n} \frac{S_1(K)}{K^2}$, $\zeta(n) = S_n(\infty)$ is the Riemannian ζ-function.

Consider more complicated diagrams (Fig. 3).

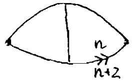
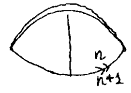
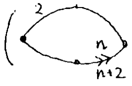
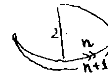
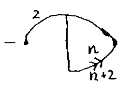
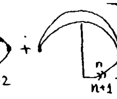


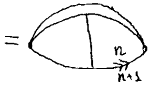
F i g. 3.

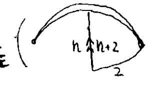
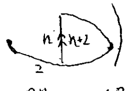
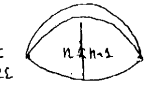
Their moments up to 2^n are given by (the dual diagrams also equal initial diagrams):

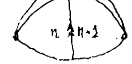

$$\cdot (n+1) \quad ,$$

$$\cdot (n+1) \; .$$

Using the equation (6), we get


$$=$$

$$- \frac{1}{2\varepsilon} \Big[2 \Big($$

$$-$$

$$\Big) -$$

$$+$$

$$\Big] =$$

$$=$$

$$- \frac{1}{2\varepsilon} \left[2 A^{0,0}_{(1,2)} \left(A^{0,n}_{(1,n+2)} - A^{0,n}_{(1,n+2+\varepsilon)} \right) - A^{0,n}_{(1,n+2)} A^{0,n}_{(2,n+2+\varepsilon)} + A^{0,n}_{(2,n+1)} A^{0,n}_{(1,n+2+\varepsilon)} \right]$$


$$= - \frac{2}{n+2+2\varepsilon} \Big($$

$$-$$

$$\Big) - \frac{n+4\varepsilon}{n+2+2\varepsilon}$$

$$=$$

$$= - \frac{2}{n+2+2\varepsilon} \left(A^{0,n}_{(2,n+1)} A^{0,n}_{(1,n+2+\varepsilon)} - A^{0,n}_{(1,n+2)} A^{0,n}_{(2,n+2+\varepsilon)} \right) - \frac{n+4\varepsilon}{n+2+2\varepsilon}$$

.

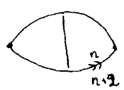
Now using the expansion of Γ-functions:

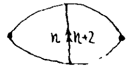
$$\frac{\Gamma(n+1+a\varepsilon)}{n!\,\Gamma(1+a\varepsilon)} = 1 + S_1(n) a\varepsilon + \left[S_1^2(n) - S_2(n) \right] \frac{a^2 \varepsilon^2}{2} + \left[S_1^3(n) - 3 S_2(n) S_1(n) + 2 S_3(n) \right] \frac{a^3 \varepsilon^3}{3!} + \ldots,$$

$$\Gamma(1+a\varepsilon) = \exp\left[-\gamma a\varepsilon + \sum_{n=2}^{\infty} \frac{\zeta(n)}{n} (-a)^n \varepsilon^n \right].$$

where γ is the Euler constant.

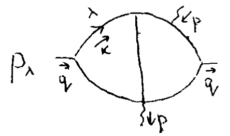
We finally get


$$\cdot (n+1) = \frac{S_1(n+1)}{2\varepsilon^2} + \frac{1}{4\varepsilon} \left[3 S_1^2(n+1) - 5 S_2(n+1) \right] + \frac{7}{12} S_3(n+1) - \frac{5}{4} S_2(n+1) S_1(n+1) + \frac{19}{6} S_3(n+1) - 2\,T(n+1) \qquad (10a)$$


$$\cdot (n+1) = \frac{1+(-1)^n}{n+2} \left[\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon} \left(2 S_1(n+1) - \frac{2}{n+2} \right) + 2 S_1(n+1) - 2 S_2(n+1) - 2 K_2(n+1) - \frac{4 S_1(n+1)}{n+2} + \frac{4}{(n+2)^2} \right]. \qquad (10b)$$

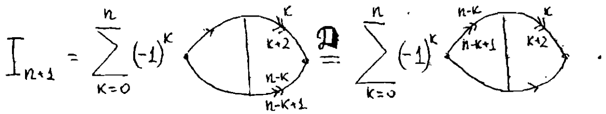
Note that the infrared divergences present in the expansion (in ε) for moments do not contribute to the coefficient functions of the Wilson expansion since they cancel out with the divergences which appear in the renormalization of the corresponding operators[7,8].

As a third example, consider the diagram (Fig.4).

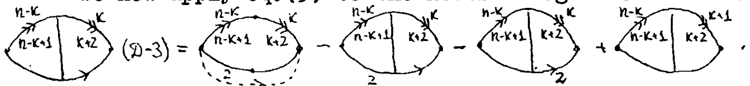


F i g. 4.

The presence of the momentum K^λ is of no importance but simplifies a final answer. Up to 2^n $n+1$-th moment is introduced as follows

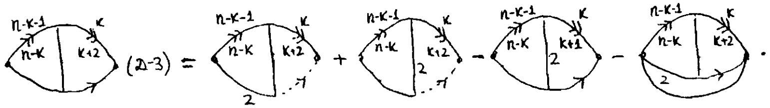


We now apply eq.(5) to the lower integration vertex. We get



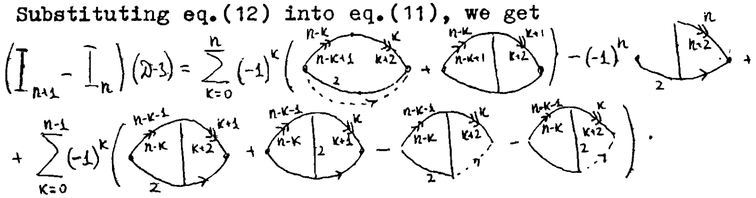
(11)

Appling the same equation, but with another isolated line we come to



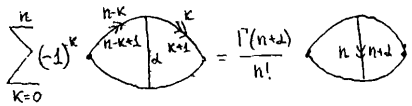
(12)

Substituting eq.(12) into eq.(11), we get

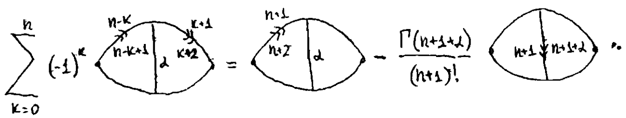


(13)

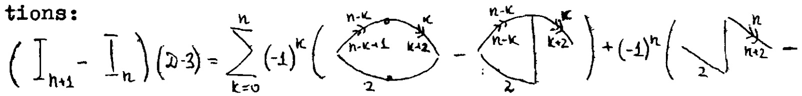
Note that these are the equalities

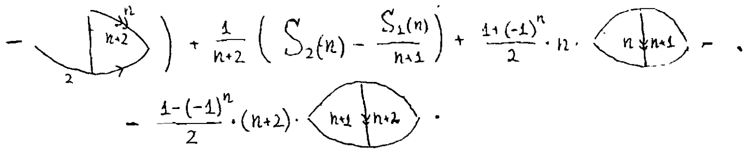


(14)



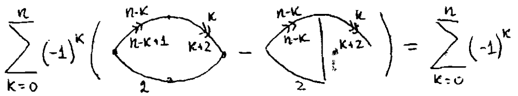
Substituting eq.(14) into eq.(13) we have, after simple transformations:





(15)

It is a wonderful fact: in all the complicated diagrams of expression (15) we could get rid of sums. The obtained diagrams were considered earlier (see 9b). The remaining sum is calculated in all orders in ε after calculating the integral:



$$= \sum_{k=0}^{n} (-1)^k A^{0,0}(1,1)\Big(A^{n-k,k}(n-k+1,k+2) - A^{n-k,k}(n-k+1+\varepsilon,k+2)\Big) = \frac{1}{\varepsilon^2}\frac{\Gamma(n+1+\varepsilon)}{(n+1)!} - \frac{1}{\varepsilon^2}\frac{\Gamma(n+1+2\varepsilon)}{\Gamma(n+2+\varepsilon)} - \frac{(-1)^n}{\varepsilon}\frac{\Gamma(n+1+2\varepsilon)}{(n+1)!\,(n+1+\varepsilon)} \quad (16)$$

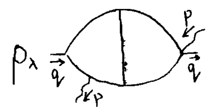
Appling formula (9a), (13) to the integrals in the right-hand side of equation (15), we get

$$I_{n+1} - I_n = \frac{1}{(1-2\varepsilon)(n+1)}\left(\frac{1-2\cdot(-1)^n}{\varepsilon(n+1)} + 2S_2(n) - 4K_2(n) - \frac{4\cdot(-1)^n S_1(n)}{n+1} + \frac{1}{(n+1)^2}\right).$$

Solving the obtained recurrence relation, we have

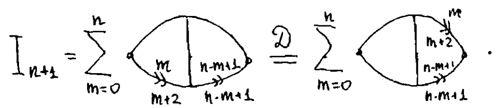
$$I_n\cdot(1-2\varepsilon) = \frac{1}{\varepsilon}\big(S_2(n) - 2K_2(n)\big) + S_3(n) + 2S_2(n)S_1(n) - 2T(n) - 4K_2(n)S_1(n) + 4K_3(n)\big). \quad (17)$$

As the last example, consider the diagram (Fig. 5).

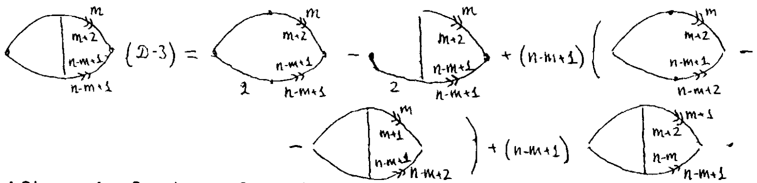


F i g. 5.

The $n+1$-th moment is introduced up to 2^n by:

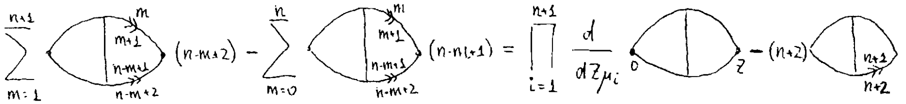


We apply eq.(5) to the lower integration vertex. We get



(18)

After simple transformations of complicated diagrams of equation (18) we have



Like in eq.(15), the sum is calculated before integration. In simple diagrams the sum is calculated in all orders in ε .

A final result can be represented in the form:

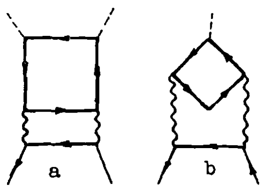
$$I_n \cdot (1-2\varepsilon) = -\frac{1}{\varepsilon}\cdot S_2(n) + S_3(n) - 2\, S_2(n)\, S_1(n) + 2\, T(n) .$$

Finally note that the methods of "differentiation" and "uniqueness" allow us to separate almost all complicated integrals and complicated sums. Moreover, after integrating simple diagrams the remaining sums are calculated in all orders in ε .

5. DEEP-INELASTIC SCATTERING. α_s-CORRECTION TO THE LONGITUDIAL SINGLET STRUCTURE FUNCTION

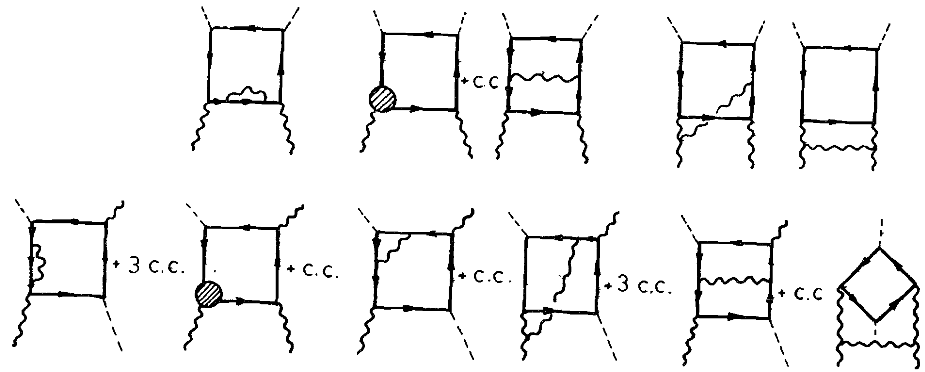
In this section we present the diagrams contributing to the two-loop correction to the longitudial singlet structure function in DIS. The contribution consists of two parts: scattering on a quark and scattering on a gluon.

One part contains diagrams contributing to the longitudial non-singlet structure function /1,9/ and additional diagrams presented in Fig. 6:



F i g. 6.

The second part contains diagrams presented in Fig. 7.



F i g. 7.

The number of all diagrams (Fig. 6,7) is doubled due to reverse motion of the fermion loop. The diagram (a) (Fig. 6) and diagrams (a-e) (Fig. 7) are also doubled because of the contribution of crossing diagrams.

In the figures the solid, wavy and dotted lines stand for the quark, gluon, and photon, respectively.

To get the longitudinal part, we multiply the diagram by a projector $p_\mu p_\nu / q^2$, where p and q are quark and photon momenta. We are interested in the coefficient function of an n-th moment. To find it, we use the technique of "differentiation" (see 3). The obtained integrals will be expressed through the diagrams analogous to those presented in 4.

The final result for quark and gluon structure functions can be represented in the form

$$C^{\psi}_{n,L}(1,\alpha_s) = \frac{\alpha_s}{4\pi}\cdot\frac{4C_F}{n+1}\left(1+\frac{\alpha_s}{4\pi}R^{\psi}_{n,L}(\overline{MS})\right)$$

$$C^{G}_{n,L}(1,\alpha_s) = \frac{\alpha_s}{4\pi}\cdot\frac{16\,T_F}{(n+1)(n+2)}\left(1+\frac{\alpha_s}{4\pi}R^{G}_{n,L}(\overline{MS})\right),$$

where

$$R^{\psi}_{n,L}(\overline{MS}) = R^{NS}_{n,L}(\overline{MS}) + \frac{2^3 T_F}{(n+1)(n+2)}\Big[S_1(n)\Big(-1-\frac{4}{n-1}+\frac{2}{n}\Big)-1+\frac{5}{3}\cdot\frac{1}{n-1}-\frac{3}{n}+\frac{1}{n+1}+\frac{4}{3}\cdot\frac{1}{n+2}-\frac{2}{n^2}\Big] \qquad n=2m,$$

$$R^{G}_{n,L}(\overline{MS}) = C_A\Big[2S_1^2(n)-2S_2(n)+4K_2(n)+2S_1(n)\Big(4-\frac{2}{n-1}+\frac{2}{n}-\frac{4}{n+1}\Big)+3+\frac{17}{3}\cdot\frac{1}{n-1}-\frac{6}{n}+\frac{1}{n+1}+\frac{11}{12}\cdot\frac{1}{n+2}-\frac{4}{n^2}+\frac{4}{(n+1)^2}-\frac{8}{(n+2)^2}\Big]-$$

$$-2C_F\Big[\frac{2}{5}\Big\{\frac{1-\delta_n^2}{n-2}\big(4K_2(n)-3\big)+\delta_n^2\big(6\zeta(3)-7\big)\Big\}-2K_2(n)\Big(1+\frac{4}{5}\cdot\frac{1}{n+3}\Big)+$$

$$+S_1(n)\Big(3/2+\frac{1}{n}-\frac{1}{n+1}\Big)+\frac{14}{5}-\frac{3}{2}\cdot\frac{1}{n}+\frac{1}{2}\cdot\frac{1}{n+1}+\frac{6}{5}\cdot\frac{1}{n+3}-\frac{1}{n^2}+\frac{1}{(n+1)^2}\Big] \qquad n=2m,$$

where $R^{NS}_{n,L}(\overline{MS})$ is found in ref./1/.

6. CONCLUSION

We propose the method of calculating of structure functions of DIS. Unlike papers/10,11/, all the calculations are performed algorithmically. The method allows us to calculate two-loop (in principle, also higher) corrections to the structure functions of DIS, and it can be applied to other problems having diagrams which depend on two moments (when the square of one of them equals zero) or propagator-type diagrams with a large number of momenta on the lines. We present the results of calculation of the two-loop correction to the longitudial structure function of DIS. In future this result will be applied to determine the α_S -correction to the ratio $R=\sigma_L/\sigma_T$, where σ_L and σ_T are cross sections of the longitudinal and transverse photon scattering on a nucleon.

The author is grateful to Kazakov D.I. for permanent interest and help in the work; and also to Gorishny S.G. for useful remarks.

APPENDIX

The table of transformations of the diagram presented in Fig. 1.

	MR	Conformal inversion		Point insertions							
		L	R	↗	↖	↘	↙	↑	↓	→	←
α_1'	$\frac{D}{2}-\alpha_2-n_2$	$D-S_1-n_{S1}$	α_1+n_1	$S_1-\frac{D}{2}$	$\frac{D}{2}-\alpha_5$	α_1	$t_1-\frac{D}{2}$	$\frac{D}{2}-\alpha_2$	α_1	$\frac{D}{2}-\alpha_4$	α_1
n_1'	n_2	n_1+l+K	0	n_1	m	n_1	$n_{t1}-K-m$	m	n_1	m	n_1
α_2'	$\frac{D}{2}-\alpha_3-n_3$	α_2+n_2	$D-S_1-n_{S1}$	$\frac{D}{2}-\alpha_5$	$S_2-\frac{D}{2}$	$t_2-\frac{D}{2}$	α_2	$\frac{D}{2}-\alpha_1$	α_2	α_2	$\frac{D}{2}-\alpha_3$
n_2'	n_3	0	n_2+n_5+m-l	m	n_2	$n_{t2}-K-m$	n_2	K	n_2	n_2	m
α_3'	$\frac{D}{2}-\alpha_4-n_4$	α_3+n_3	$D-S_2-n_{S2}$	$t_2-\frac{D}{2}$	α_3	$\frac{D}{2}-\alpha_5$	$S_2-\frac{D}{2}$	α_3	$\frac{D}{2}-\alpha_4$	α_3	$\frac{D}{2}-\alpha_4$
n_3'	n_4	0	n_3+l+K	$n_{t2}-K-m$	n_3	m	n_3	n_3	m	n_3	K
α_4'	$\frac{D}{2}-\alpha_1-n_1$	$D-S_2-n_{S2}$	α_4+n_4	α_4	$t_1-\frac{D}{2}$	$S_2-\frac{D}{2}$	$\frac{D}{2}-\alpha_5$	α_4	$\frac{D}{2}-\alpha_3$	$\frac{D}{2}-\alpha_1$	α_2
n_4'	n_1	n_4+n_5+m-l	0	n_4	$n_{t1}-K-m$	n_2	m	n_4	K	K	n_2
α_5'	$\frac{D}{2}-\alpha_5-n_5$	α_5+n_5	α_5+n_5	$\frac{D}{2}-\alpha_2$	$\frac{D}{2}-\alpha_1$	$\frac{D}{2}-\alpha_4$	$\frac{D}{2}-\alpha_3$	$S_1-\frac{D}{2}$	$S_2-\frac{D}{2}$	$t_1-\frac{D}{2}$	$t_2-\frac{D}{2}$
n_5'	n_5	0	0	K	K	K	K	n_5	n_5	$n_{t1}-m-K$	$n_{t2}-m-K$
S_1'	$\frac{3D}{2}-t_2-n_{t2}$	$D-\alpha_1-n_1$	$D-\alpha_1-n_1$	$\frac{D}{2}+\alpha_1$	$\frac{D}{2}+\alpha_2$	S_1	S_1	$\frac{D}{2}+\alpha_5$	$d-D/2$	S_1	S_1
n_{S1}'	n_{t2}	n_1+l+K	n_2+n_5+m-l	n_1+K+m	n_2+K+m	$n_{S1}+n_3-m$	$n_{S1}+n_4-m$	n_5+K+m	n_{S2}	$n_{S1}+n_4-K$	$n_{S1}+n_3-K$
S_2'	$\frac{3D}{2}-t_1-n_{t1}$	$D-\alpha_4-n_4$	$D-\alpha_4-n_4$	S_2	S_2	$\frac{D}{2}+\alpha_4$	$\frac{D}{2}+\alpha_3$	$d-\frac{D}{2}$	$\frac{D}{2}+\alpha_5$	S_2	S_2
n_{S2}'	n_{t1}	n_4+n_5+m-l	n_3+l+K	$n_{S2}+n_2-m$	$n_{S2}+n_1-m$	n_4+K+m	n_3+K+m	n_{S2}	n_5+K+m	$n_{S2}+n_1-K$	$n_{S2}+n_2-K$
t_1'	$\frac{3D}{2}-S_1-n_{S1}$	$2D-d-n_d$	t_1+n_{t1}	t_1	$\frac{D}{2}+\alpha_4$	t_1	$\frac{D}{2}+\alpha_1$	t_1	t_1	$\frac{D}{2}+\alpha_5$	$d-\frac{D}{2}$
n_{t1}'	n_{S1}	$n_{t1}+K+m$	0	$n_{t1}-n_5+K$	n_{t1}	$n_{t1}-n_5+K$	n_{t1}	$n_{t1}-n_1+m$	$n_{t1}-n_4+m$	n_{t1}	n_d-m-K
t_2'	$\frac{3D}{2}-S_2-n_{S2}$	t_2-n_{t2}	$2D-d-n_d$	$\frac{D}{2}+\alpha_3$	t_2	$\frac{D}{2}+\alpha_2$	t_2	t_2	t_2	$d-\frac{D}{2}$	$\frac{D}{2}+\alpha_5$
n_{t2}'	n_{S2}	0	$n_{t2}+K+m$	n_{t2}	$n_{t2}-n_5+K$	n_{t2}	$n_{t2}-n_5+K$	$n_{t2}-n_1+K$	$n_{t2}-n_3+K$	n_d-m-K	n_{t2}
d'	$\frac{5D}{2}-d-n_d$	$2D-t_1-n_{t1}$	$2D-t_2-n_{t2}$	d	d	d	d	$\frac{D}{2}+S_2$	$\frac{D}{2}+S_1$	$\frac{D}{2}+t_2$	$\frac{D}{2}+t_1$
n_d'	n_d	$n_{t1}+K+m$	$n_{t2}+K+m$	n_d	n_d	n_d	n_d	$n_{S2}+m+K$	$n_{S1}+m+K$	n_d	n_d
C'	C_1	$C_2(n_2,n_3;n_5)$	$C_2(n_1,n_4;n_5)$	$C_3(S_1;\alpha_1)$	$C_3(S_1;\alpha_2)$	$C_3(S_2;\alpha_4)$	$C_3(S_2;\alpha_3)$	$C_3(S_1;\alpha_5)$	$C_3(S_2;\alpha_5)$	$C_4(t_2;\alpha_1,\alpha_4)$	$C_4(t_1;\alpha_2,\alpha_3)$

Here the index (the momentum) of the vertex, triangle and of the diagram is a sum of indiced (momenta) of constituent lines:

$$S_1 \equiv \alpha_1+\alpha_2+\alpha_5\,,\quad S_2 \equiv \alpha_3+\alpha_4+\alpha_5\,;\quad t_1 \equiv \alpha_1+\alpha_4+\alpha_5\,;\quad t_2 \equiv \alpha_2+\alpha_3+\alpha_5\,,\quad \alpha = \sum_{i=1}^{5}\alpha_i$$

$$n_{S1} \equiv n_1+n_2+n_5\,,\quad n_{S2} \equiv n_3+n_4+n_5\,;\quad n_{t1} \equiv n_1+n_4+n_5\,,\quad n_{t2} \equiv n_2+n_3+n_5\,;\quad n_\alpha = \sum_{i=1}^{5} n_i\,.$$

Nonprimed indices (momenta) correspond to the initial diagram; primed indices, to the diagram obtained with the help of one of the following transformations:

transition to the momentum representation (MR),

two conformal inversions (with the left-handed basis (L) and with the right-handed basis (R)),

and eight operations of point insertion.

In the lowest line we give the transformation coefficients which can be represented in the form

$$C_1 = \frac{\prod_{i=1}^{5} a_{n_i}(\alpha_i+n_i)\,(-)^{n_\alpha+n_5}}{a_{n_\alpha}(\alpha-D+n_\alpha)}\,,$$

$$C_3(S_1;\alpha_1) = \sum_{m=0}^{n_2}\sum_{k=0}^{n_5} C_{n_2}^{m} C_{n_5}^{k}\, \frac{a_{n_1}(\alpha_1+n_1)\,a_{n_2+k}(\alpha_2+n_2)\,a_{n_5+m}(\alpha_5+n_5)}{a_{n_1}(S_1-\frac{D}{2}+n_1)} \cdot \frac{\Gamma(\alpha_2+\alpha_5-\frac{D}{2}+n_2+n_5-m-k)}{\Gamma(\alpha_2+\alpha_5-\frac{D}{2})}$$

analogously for other $C_3(S_j;\alpha_i)$ (with factor $(-)^k$ for $C_3(S_2,\alpha_i),\ i=1,2$)

$$C_4(t_2,\alpha_1,\alpha_4) = \sum_{m=0}^{n_1}\sum_{k=0}^{n_4} C_{n_1}^{m} C_{n_4}^{k}\,(-1)^{n_1-m}\, \frac{a_{n_\alpha}(\alpha_2-\frac{D}{2}+n_\alpha)\,a_{n_1+k}(\alpha_1+n_1)\,a_{n_4+m}(\alpha_4+n_4)}{a_{n_\alpha}(\alpha-D+n_\alpha)} \cdot \frac{\Gamma(\alpha_1+\alpha_4-\frac{D}{2}+m+k)}{\Gamma(\alpha_1+\alpha_4-\frac{D}{2})}$$

and similarly for $C_4(t_1,\alpha_2,\alpha_3)$

$$C_2(n_2,n_3,n_5) = \sum_{m=0}^{n_3}\sum_{k=0}^{n_2}\sum_{\ell=0}^{n_5} (-1)^{m+\ell+k+n_2+n_3}$$

and for $C_2(n_1,n_4,n_5)$.

REFERENCES

1. Kazakov D.I., Kotikov A.V. JINR Preprint, E2-86-204, Dubna, 1986.
2. Kazakov D.I. Phys.Lett., 1983, B38, p. 406.
3. Vassiliev A.N., Pis'mak Yu.M., Honkonen Yu.R. TMF, 1981, 47, p.291.
4. Kazakov D.I. TMF, 1984, 58, p. 357.
5. Kazakov D.I. TMF, 1985, 62, p. 127.
6. Chetyrkin K.G., Gorishny S.G., Tkachov F.V. Phys.Lett., 1982, 118B, p. 407.
7. Gorishny S.G., Larin S.A., Tkachov F.V. Phys.Lett., 1983, 124B, p. 217.
8. Gorishny S.G., Larin S.A. Nucl.Phys.,1987,B283,p.452.
9. Kazakov D.I., Kotikov A.V. JINR Preprint, E2-86-715, Dubna, 1986.
10. Duke D.W., Kimel I.D., Sowell G.A. Phys.Rev., 1982, D25, p. 71.
11. Devoto A., Duke D.W., Kimel I.D., Sowell G.A. Phys.Rev., 1985, D30, p. 541.

Received by Publishing Department
on May 28, 1987.