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ON FEASIBILITY OF QUANTUM INTERFERENCE TRANSISTORS

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The rapid development of fabrication technologies has made it possible to prepare objects whose conductivity properties are influenced substantially by quantum interference : lithographically drawn metallic rings or more complicated planar graphs /1-3/, or semiconductor heterostructures produced by epitaxial-growth techniques /4/. Recently a new technique has been reported /5/ by which semiconductor graph structures could be produced. In this letter, we are going to discuss interference effects on a loop of metallic or semiconductor "wires" *).

A necessary condition for occurrence of the interference is that the mean free path of electrons is much longer than the "interferometer" size. At present, the wires can be made as thin as 200 Å and the size of the device is typically one order of magnitude larger. Hence we are close to the situation when one may neglect both elastic and inelastic scattering and suppose the electron motion in the device to be purely ballistic**).

From the viewpoint of applications, it is important how the interference effect can be controlled from outside. One possibility is to place the device into a homogeneous magnetic field perpendicular to the plane of the loop. Then an Aharonov-Bohm-type effect is observed : the conductivity exhibits oscillations with respect to the field intensity. A lot of experimental work in this direction has been done recently***). Unfortunately, a direct use of this effect in microelectronics

*) A theoretical analysis of interference in the heterostructures is more complicated, since the channel thickness cannot be neglected in this case. Under additional assumptions, however, it can be reduced to the situation treated here. A detailed discussion of this problem will be given elsewhere.

**) The mean free path in semiconductors can be made as long as $\lesssim 1\mu m$ so the assumption about the ballistic regime is justified in this case - cf.Refs.4 and 6 . The situation is not so good in metals, but even here experiments with ballistic electrons could be expected in the nearest future - see Ref.7 .

***) The magnetoresistance oscillations has been observed also in the semiconductor microstructures - Ref.10 . It was suggested that this AB-effect or its (unobserved up to now) electric counterpart might be used in switching devices - cf.Ref.11 . The heterostructures, however, are not the most suitable candidate for this purpose.

is hardly possible since it is difficult to generate the required magnetic field locally and with the desired intensity.

Another way is to use an electrostatic field whose intensity may be varied. It is our aim here to analyze this situation. The method we use starts from the splitting process that occurs when an electron passes through a branching point of the graph[12]. It will be described in detail in a subsequent paper ; in this letter we concentrate our attention on the results and their implications for construction of quantum interference transistors (QIT´s).

We consider a planar loop with two external leads and the electric field whose intensity is paralel to the graph plane and perpendicular to the leads (Fig.1). In order to derive the transmission coefficient for such a structure, one has to sew in a proper way four functions, $u_1(x) = e^{-ikx} + a\,e^{ikx}$ on the left lead, the transmitted wave $u_4(x) = b\,e^{-ikx}$ on the right lead*), and the functions u_2, u_3 that are solutions to the Schrödinger equations

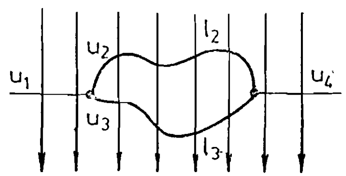

Fig.1. A planar loop with two leads in an electric field.

$$-\frac{\hbar^2}{2m^*}u_j''(x) + V_j(x)u_j(x) = Eu_j(x) \quad , \quad j = 2,3 \quad , \qquad (1)$$

where $E = \hbar^2k^2/2m^*$ and the potentials V_j are given by the electric intensity $\mathcal{E}$ and the shape of the loop ; the effective mass m^* of the electron is assumed to be the same on each branch of the graph. Solution to the problem depends substantially on the boundary conditions which must be fulfilled by the wavefunctions at the junctions. These conditions have been discussed extensively in Ref.12 . If we assume that the three wires connected at the left junction are physically equivalent, there is a two-parameter family of them**) compatible with the principles of quantum mechanics, namely

*) The orientation of axes is chosen so that the conditions (2) could be written in the same form as in Ref.12 .

**) Under this assumption, there are also two "exceptional" one-parameter families that will not be considered here.

$$\begin{aligned} u_1(0) &= A_1u_1'(0) + B_1u_2'(0) + B_1u_3'(0) \quad , \\ u_2(0) &= B_1u_1'(0) + A_1u_2'(0) + B_1u_3'(0) \quad , \qquad (2) \\ u_3(0) &= B_1u_1'(0) + B_1u_2'(0) + A_1u_3'(0) \quad . \end{aligned}$$

In a similar way, the boundary conditions for the second junction are expressed by means of two real parameters A_2, B_2 . After a series of algebraic manipulations that will be described in a subsequent paper, we arrive at the expression for the transmission coefficient $T(E)$,

$$T(E) = |b|^2 = \frac{B_2^2}{1 + k^2B_2^2}\,|c_2 + d_2|^2 \quad , \qquad (3)$$

where c_2, d_2 are given by the relations

$$c_2 = C_2(-k)_{21}d_1 + C_2(-k)_{22}d_2 \quad , \qquad (4a)$$

$$\begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = \left[\Pi_2^{-1}C_2(-k) - C_1(k)\Pi_3^{-1}\right]^{-1}\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \quad . \qquad (4b)$$

Here

$$z_1 = (A_1 - B_1)z_2 \quad , \quad z_2 = \frac{2ik}{1 + ik(B_1 - A_1)} \quad , \qquad (5a)$$

$$C_1(k) = B_1^{-1}(1 + ik(B_1 - A_1))^{-1} \times \qquad (5b)$$
$$\times \begin{pmatrix} A_1 + ik(B_1^2 - A_1^2) & (B_1 - A_1)\left[(A_1 + B_1)(1 - ikA_1) + 2ikB_1^2\right] \\ 1 - ikA_1 & -A_1 - ik(B_1^2 - A_1^2) \end{pmatrix}$$

and $C_2(k)$ is the analogous matrix built of A_2 and B_2 . Finally Π_j are transfer matrices

$$\begin{pmatrix} u_j(l_j) \\ u_j'(l_j) \end{pmatrix} = \Pi_j \begin{pmatrix} u_j(0) \\ u_j'(0) \end{pmatrix} \quad . \qquad (6)$$

The conductivity is now expressed by means of the Landauer formula[13]

$$G = \frac{e^2}{\pi\hbar}\,\frac{T(E)}{1 - T(E)} \quad , \qquad (7)$$

where E is assumed to be the Fermi energy for metals and the lowest transversal-mode energy for semiconductors*) ; recall that $\pi\hbar/e^2 \approx$ $\approx 12906\,\Omega$.

In order to calculate G , one has to find the matrices (6). We limit ourselves to the weak-field case, $F \equiv e\mathcal{E} \ll E/a$, where a is a characteristic size of the loop. If we take $a \approx 5 \cdot 10^{-5}$ cm and a typical Fermi energy for metals, it yields $\mathcal{E} \ll 10^5$ V/cm . For semiconductors this bound is correspondingly lower**). Then Π_j can be calculated in the WKB-approximation, which is applicable for $m\hbar F \ll (2mE)^{3/2}$, and substituted to the expression (4b).

Typical results are shown on Fig.2 . In both cases, the junctions are supposed to be the same and such that $A_1 = B_1 = -A_2 = -B_2 = 1$ ***). Variating the parameters that characterize the junctions, the loop shape and the energy E , we obtain other curves. The qualitative behaviour does not change, however. The conductivity always exhibits large oscillations with well-distiguished minima at reasonably low field intensities. Hence we can conclude that the prospect of constructing the QIT's based on the described effect is fully realistic. Moreover, they could operate at very low switching voltages, even from the milivolt region, because the electrodes can be placed close to the interference "circuit".

In conclusion, let us mention an additional and very appealing feature of the structures considered in this letter, namely the possibility of tailoring the conductance plot by choosing a proper loop shape. We regard this fact as a challenge to the experimentalists - a challenge that can be matched using the technological novelty quoted in the introduction.

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*) In metals the rhs of (7) should be averaged with the weight $f(E_F+eU) - f(E_F)$, where f denotes the Fermi-Dirac distribution. Since $kT \ll E_F$ and the typical applied voltage is such that $eU \ll E_F$, we see that (7) represents a good approximation to real situation. On the other hand, the decisive role in semiconductors is played by the transversal-mode energy. In the numerical calculations, we use $E = 0.05$ eV for the lowest mode in GaAs in agreement with the values obtained recently in Ref.14.

**) The weak-field assumption is needed only to avoid the discussion of the cases when the electron in the "upper" branch is tunneling through the barrier, or when E is near the top of the barrier and the exact solution to eq.(2) is required ; it is just a matter of convenience. Notice also that the simplest WKB-argument works for stronger fields too if the "upper" branch is flat enough.

***) We have taken $m^* = m_e$ for the metal and $m^* = 0.067\, m_e$ for GaAs, m_e being the electron mass.

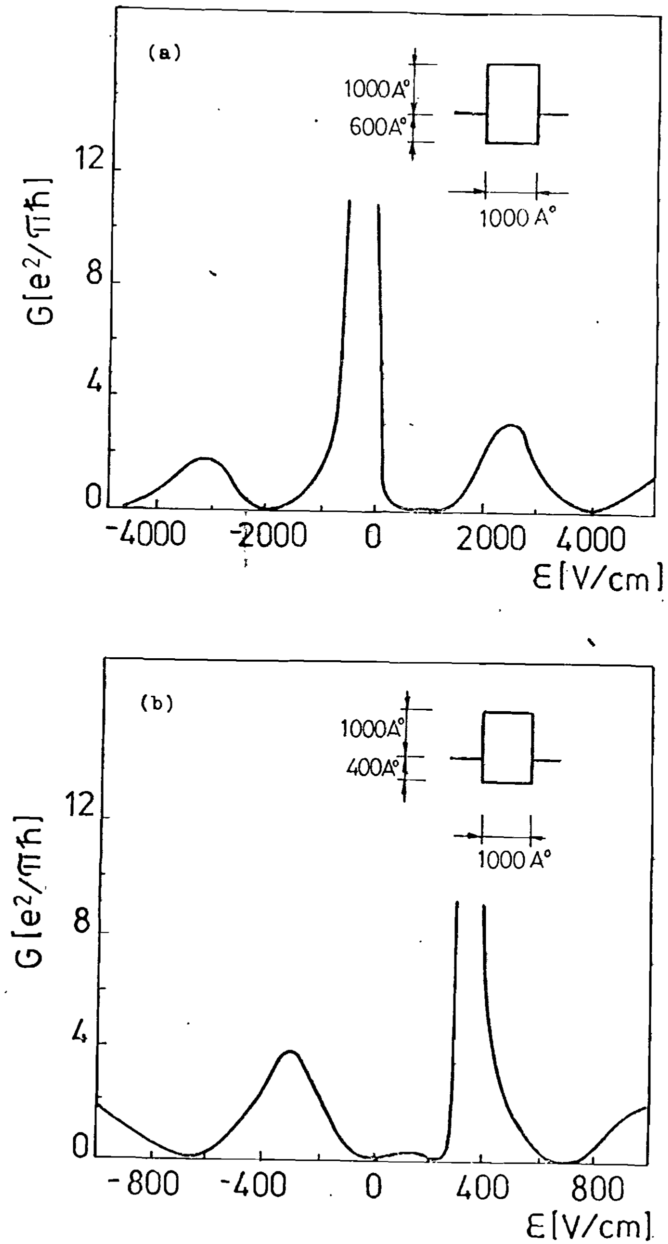

Fig.2. Conductivity vs field intensity for the sketched loops
(a) E = 11.7 eV (Fermi energy for aluminium)
(b) E = 0.05 eV (a 200 Å GaAs wire).

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