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NONLEPTONIC DECAYS OF CHARMED MESONS $D \to 0^-0^-$ AND MIXING ANGLES IN SU(4)

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The experimental fact /1/ that for the Cabibbo-suppressed decays $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ we have $\Gamma(D^0 \to K^+K^-)/\Gamma(D^0 \to \pi^+\pi^-) > 1$ is a subject of the investigation of several theoretical models. The matter is that the standard theory $SU_3 \times SU_2 \times U_1$, in the spectator approximation, gives for the ratio $\lesssim 1$ /2/. To explain the observed pattern, certain phenomenological approaches have been proposed in which one includes such effects as the SU_3 breaking /3/, penguin diagrams /4/, right-handed currents /4/, final-state interactions /6,7/, soft gluons /8/.

At the same time, there is also another traditional approach to the Cabibbo-suppressed decays well describing the $\Delta I = 3/2$ transitions for the kaons, the phenomenological chiral Lagrangian method (PCLM) /9,11/. In this method, one takes the weak interaction Lagrangian in the Sakurai form /12/ with the chiral hadronic currents and the violation of the $\Delta I = 1/2$ rule is realized by the Oakes scheme /13/. Remind, the idea of Oakes is the rotation of both the currents and the SU_3 -chiral-symmetry breaking term around the $7th$ axis in SU_3 -space about the same Cabibbo angle, $\sin \theta_c \approx m_\pi / m_K$.

The aim of the present paper is to extend this method to the Cabibbo-suppressed decays of charmed hadrons. An application of the method to the Cabibbo-favored decays is considered in refs. /13,15/. Specifically, to see how the approach does work in the case of Cabibbo-suppressed decays, we consider only the $D \to 0^- 0^-$ decays neglecting the final-state interaction effects (the form factors).

In the charmed case, besides the above rotation in SU_3 -subspace, an additional rotation around the $10th$ axis in SU_4 -space is possible*). It is natural to expect that the new angle is smaller than θ_c but additional rotation may appreciably affect the Cabibbo-suppressed decays, particularly, $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$.

Let us start with the weak nonleptonic interaction Lagrangian (when $\theta_c = 0$)

*) Other rotations are suppressed by the charge conservation law.

$$L_W = L_W\,(\Delta I=0) + L_W^{\Delta S}\,(\Delta I=\tfrac{1}{2}) + L_W^{\Delta C}\,(\Delta I=1). \quad (1)$$

The first term

$$L_W\,(\Delta I=0) = \frac{G_F}{\sqrt{2}}\, J_\mu^{1+i2}\, J_\mu^{1-i2}.$$

Describes the $\Delta I = \Delta S = \Delta C = 0$ -transitions. Here G_F is the Fermi constant, $J_\mu^{a+ib} \equiv J_\mu^a + i J_\mu^b$ is the hadronic current associated with the chiral symmetry /10,11/

$$i\lambda_a J_\mu^a = exp\,(i\xi)\,\partial_\mu\, exp\,(-i\xi),$$

where $\xi = \lambda_i \varphi_i / F$, $F \approx$ 94 MeV, $\lambda_i \varphi_i$ is the 15-plet of pseudoscalar mesons. The second term describing the $\Delta I = 1/2$, $\Delta S = 1$, $\Delta C = 0$ -transitions is the Sakurai Lagrangian /12/,

$$L_W^{\Delta S}\,(\Delta I=\tfrac{1}{2}) = \sqrt{2}\, G_F\, d_{6ab}\, J_\mu^a J_\mu^b. \quad (2)$$

For the Lagrangian of $\Delta I = \Delta S = \Delta C = 1$ -transitions we suppose the explicit 20-plet (or sextet) dominance /14-16/

$$L_W^{\Delta C}\,(\Delta I=1) = \frac{G_F}{\sqrt{2}} \left(J_\mu^{1-i2} J_\mu^{13-i14} - J_\mu^{6+i7} J_\mu^{9-i10} + h.c. \right). \quad (3)$$

As the first step, we rotate the currents. The rotation of the currents around the 7th axis in SU_3- subspace about the angle Θ_7 is defined by

$$\lambda_a J_\mu^a(\Theta_7) = exp\,(i\lambda_7\Theta_7)(\lambda_a J_\mu^a)\, exp\,(-i\lambda_7\Theta_7).$$

Then for the $\Delta I = 3/2$, $\Delta S = 1$, $\Delta C = 0$ -transition from (1) we arrive at

$$L_W^{\Delta S}\,(\Delta I=\tfrac{3}{2}) = \frac{G_F}{\sqrt{2}}\, \epsilon \left(J_\mu^{1+i2} J_\mu^{4-i5} + h.c. \right)$$

satisfactorily describing the data on the Cabibbo-suppressed decays of kaons when $\Theta_7 \approx \Theta_c$, $\epsilon \equiv \cos\Theta_c \sin\Theta_c - 2\sin^2\Theta_c$ = 0.113 is close to its experimental value /17/ 0.111±0.007.

The charmed part of the rotated Lagragian is given by

$$L_W^{\Delta C}(\Theta_7) = \frac{G_F}{\sqrt{2}} \Big\{ C^2 \left[J_\mu^{1-i2} J_\mu^{13-i14} - J_\mu^{6+i7} J_\mu^{9-i10} \right]$$
$$+ CS \left[J_\mu^{1-i2} J_\mu^{11-i12} - J_\mu^{4-i5} J_\mu^{13-i14} + (J_\mu^3 - \sqrt{3} J_\mu^8) J_\mu^{9-i10} \right]$$
$$- S^2 \left[J_\mu^{4-i5} J_\mu^{11-i12} - J_\mu^{6-i7} J_\mu^{9-i10} \right] + h.c. \Big\},$$

where $C \equiv \cos\Theta_7$, $S \equiv \sin\Theta_7$. Notice, this charmed part has the same structure as that of the effective unnormalized Lagrangian of the standard theory (for example, see ref. /7/, when $|a_1| = |a_2| = 1$). However, the Lagrangian $L_W^{\Delta C}(\Theta_7)$ for the interesting ratio $\Gamma(D^0 \to K^+K^-) / \Gamma(D^0 \to \pi^+\pi^-)$ gives *) 0.75 far from the experimental value, ~ 3.7 /1/.

Let us now turn to the additional rotation acting on the current as

$$\lambda_a J_\mu^a(\Theta_7, \Theta_{10}) = exp\,(i\lambda_{10}\Theta_{10})(\lambda_a J_\mu^a(\Theta_7))\, exp\,(-i\lambda_{10}\Theta_{10}).$$

Then we have

$$L_W^{\Delta C}(\Theta_7, \Theta_{10}) = \frac{G_F}{\sqrt{2}} \Big[\xi_1 J_\mu^{1-i2} J_\mu^{13-i14} - \xi_2 J_\mu^{6+i7} J_\mu^{9-i10}$$
$$+ \xi_3 J_\mu^{1-i2} J_\mu^{11-i12} - \xi_4 J_\mu^{4-i5} J_\mu^{13-i14} + \xi_5 (J_\mu^3 - \sqrt{3} J_\mu^8) J_\mu^{9-i10}$$
$$+ \xi_6 J_\mu^{6-i7} J_\mu^{9-i10} - \xi_7 J_\mu^{4-i5} J_\mu^{11-i12} + h.c. \Big].$$

Here

$$\begin{aligned}
\xi_1 &\equiv \tilde{C}^2 C^2 + \tilde{C}\tilde{S}\,(CS - C^2 + S^2) + \tilde{S}^2 S^2 = 0.895\ (0.927) \\
\xi_2 &\equiv \tilde{C}^2 C^2 + \tilde{C}\tilde{S}\,(C^2 - S^2) + \tilde{S}^2 S^2 = 0.968\ (0.927) \\
\xi_3 &\equiv \tilde{C}^2 CS - \tilde{C}\tilde{S}\,(C^2 + 2CS) - \tilde{S}^2 CS = 0.186\ (0.256) \\
\xi_4 &\equiv \tilde{C}^2 CS + \tilde{C}\tilde{S}\,(S^2 - 2CS) - \tilde{S}^2 CS = 0.236\ (0.256)
\end{aligned} \quad (4)$$

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*) That result would be expected from the factorization approximation, but in ref. /7/ for some reason that ratio equals 1.4.

$$\xi_5 \equiv \tilde{c}^2 cs + 2\tilde{c}\tilde{s} cs - \tilde{s}^2 cs = 0.285 \quad (0.256)$$

$$\xi_6 \equiv \tilde{c}^2 s^2 - \tilde{c}\tilde{s}\,(c^2 - s^2) + \tilde{s}^2 c^2 = 0.032 \quad (0.073)$$

$$\xi_7 \equiv \tilde{c}^2 s^2 + \tilde{c}\tilde{s}\,(-cs + c^2 - s^2) + \tilde{s}^2 c^2 = 0.105 \quad (0.073)$$

$$\tilde{c} \equiv \cos\theta_{10}\,, \quad \tilde{s} \equiv \sin\theta_{10}\,,$$

in the parentheses ξ_i is indicated when $\theta_{10} = 0$.

As for the Cabibbo angle, the requirement that the rotated currents must describe the semileptonic decays of hadrons leads to

$$\theta_c = \theta_7 - \theta_{10}. \tag{5}$$

As the second step, we define the angles through the mass ratios. The symmetry breaking mass term in the $(4,4^*) + (4^*, 4)$ – model has the $SU_3 \times SU_3$ –symmetry form [18]

$$L_{SB} = F^2 \left(c_0 S_0 + c_8 S_8 + c_{15} S_{15} \right). \tag{6}$$

This is the generalized GMOR model [19]. Here S_i are defined by

$$\sum_{i=0}^{15} \lambda_i S_i = Re\, exp\left(i \lambda_K \frac{\varphi^K}{F} \right)$$

whereas the constants c_i are fixed from the physical masses of hadrons (see ref. [11]). In ref. [11] the further violation of the remaining symmetry was realized by the rotation of (6) around the 7th axis in SU_3-subspace, the same scheme as that of Oakes.

Let us rotate (6) around the 10th axis too. The additional mass relations thus obtained lead to the definitions:

$$\sin\theta_7 = \left(m^2_{D^+} - m^2_{D^0} + m^2_{\pi^+} \right)^{\frac{1}{2}} / \sqrt{2}\, m_{K^0} = 0.27 \tag{7}$$

$$\sin\theta_{10} = \left(m^2_{K^+} - m^2_{K^0} + m^2_{\pi^+} \right)^{\frac{1}{2}} / \sqrt{2}\, m_{D^0} = 0.05.$$

Then

$$\sin\theta_c = \sin(\theta_7 - \theta_{10}) = 0.22$$

is slightly different from the earlier value $\sin\theta_7 = m_\pi / m_K = 0.28$.

With these angles in the Lagrangian (4) we calculated the partial width ratios for the decays $D \to 0^- 0^-$ which are listed in the

table *) I. As one sees from the table, the rotation around the 10th axis about the angle θ_{10} indeed increases the ratio $\Gamma(D^0 \to K^+K^-)/\Gamma(D^0 \to \pi^+\pi^-)$ from 0.75 ($\theta_{10} = 0$) to 1.2, due to $\xi_4^2/\xi_3^2 = 1.6$. For other available D^0 decay data [1], $\Gamma(D^0 \to K^+K^-)/\Gamma(D^0 \to K^-\pi^+) = (11.3 \pm 3)\%$ and $\Gamma(D^0 \to \pi^+\pi^-)/\Gamma(D^0 \to K^-\pi^+) = (3.3 \pm 1.5)\%$, we can see that our results, 5.5 and 4.6 respectively, agree with the data up to $\sim 50\%$. It is interesting that for the recently observed [20] wrong-signed decay $D^0 \to K^+\pi^-$ for that one has $\Gamma(D^0 \to K^+\pi^-)/\Gamma(D^0 \to K^-\pi^+) < 4\%$, we predict 1.06%. Today there are few experimental data for $D^+ \to 0^-0^-$ decays and no ones for $D_s^+ \to 0^-0^-$ decays. For the data **)

$\Gamma(D^+ \to K^+\bar{K}^0)/\Gamma(D^+ \to \bar{K}^0\pi^+) = (31.7 \pm 10)\%$ and $\Gamma(D^+ \to \pi^+\pi^0)/\Gamma(D^+ \to \bar{K}^0\pi^+) < 21\%$ from the table one has $\Gamma(D^+ \to K^+K_S)/\Gamma(D^+ \to K_S\pi^+) = 77\%$ and $\Gamma(D^+ \to \pi^+\pi^0)/\Gamma(D^+ \to K_S\pi^+) = 18\%$, respectively. Here there is an agreement again on a level, $\lesssim 50\%$. As to the dominant decay modes of D^+ and D^+, they fastly decay to $\pi^+\eta$. So, the Cabibbo-suppressed decay $D^+ \to \pi^+\eta$ can in our scheme dominate even over the Cabibbo-favored $D^+ \to K^+\pi^0$ decay. Future experimental as well as theoretical tests are needed.

To summarize, in the Oakes scheme, when extended to the charmed case, the additional rotation around the 10th axis in SU_4– space is possible. This rotation slightly changes the Cabibbo angle-hadron mass relation, but can considerably affect the Cabibbo-suppresed decay rates. Agreement between the theoretical and experimental partial width ratios, in general, is reasonable within the experimental and theoretical errors. The remaining discrepancies ($\lesssim 50\%$) are probably due to the symmetry breaking [3] or/and final-state interactions effects [6,7] (i.e. form factors). For an explicit test of our approach future theoretical and experimental investigations are needed.

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*) For completeness, both the Cabibbo-favoured and the Cabibbo-suppresed decays are presented.

**) Experimentally, K^0 is identified through $K_S \to \pi^+\pi^-$ decay.

Table I.

The $D \to 0^- 0^-$ decay amplitudes, $M(D \to 0^- 0^-)$, and the partial width ratios, where $\sin\theta_7 = 0.27$, $\sin\theta_{10} = 0.05$. From (4) one has $M(D^0 \to K^- \pi^+) = G_F/\sqrt{2}\, F\,(4.572\,\xi_1 + 0.317\,\xi_2)$, $\Gamma(D^0 \to K^- \pi^+) = 17 \cdot 10^{10}\, s^{-1}$ (or $2.\ 10^{10}\, s^{-1}$ when $\theta_{10} = 0$); $M(D^+ \to K_S \pi^+) = G_F/\sqrt{2}\, F\,[-3.246\,\xi_1 + 3.475\,(\xi_2 + \xi_6) - 0.228\,\xi_7]$, $\Gamma(D^+ \to K_S \pi^+) = 0.61 \cdot 10^{10}\, s^{-1}$ (or $0.33 \cdot 10^{10}\, s^{-1}$ when $\theta_{10} = 0$); $M(D_s^+ \to K_S K^+) = G_F/\sqrt{2}\, F \cdot 3.639\,(\xi_2 + \xi_6 - \xi_7)$, $\Gamma(D_s^+ \to K_S K^+) = 9.88\ 10^{10}\, s^{-1}$ (or $9.35\ 10^{10}\, s^{-1}$ when $\theta_{10} = 0$).

$D^0 \to 0^- 0^-$	$\frac{\Gamma(D^0 \to 0^- 0^-)}{\Gamma(D^0 \to K^- \pi^+)}$ %	Amplitudes: $G_F/\sqrt{2}\, F \times$
$D^0 \to \bar{K}^0 \pi^0$	65.7	$-3.688\,\xi_2$
$D^0 \to \bar{K}^0 \eta$	16	$-1.926\,\xi_2$
$D^0 \to K^+ K^-$	5.5	$-4.572\,\xi_4$
$D^0 \to \pi^+ \pi^-$	4.6	$4.889\,\xi_3$
$D^0 \to \pi^0 \pi^0$	2.7	$2.446\,\xi_5$
$D^0 \to \pi^0 \eta$	3.6	$-2.940\,\xi_5$
$D^0 \to \eta \eta$	1.9	$-2.246\,\xi_5$
$D^0 \to K^0 \pi^0$	0.06	$3.229\,\xi_6$
$D^0 \to K^0 \eta$	0.18	$2.263\,\xi_7 - 0.522\,\xi_6$
$D^0 \to K^+ \pi^-$	1.06	$-4.255\,\xi_7 - 0.317\,\xi_6$
$D^+ \to 0^- 0^-$	$\frac{\Gamma(D^+ \to 0^- 0^-)}{\Gamma(D^+ \to K_S \pi^+)}$ %	
$D^+ \to \pi^+ \pi^0$	18	$3.514\,\xi_5 - 3.551\,\xi_3$
$D^+ \to \pi^+ \eta$	267	$-6.018\,\xi_5 + 1.843\,\xi_3$
$D^+ \to K^+ K_S$	77	$-3.246\,\xi_4$
$D^+ \to K^+ K_L$	77	$-3.246\,\xi_4$
$D^+ \to K^+ \eta$	5	$-1.744\,\xi_7$
$D^+ \to K^+ \pi^0$	16.4	$3.250\,\xi_7$
$D^+ \to K_L \pi^+$	0.4	$3.246\,\xi_1 + 3.476\,(\xi_6 - \xi_2) - 0.228\,\xi_7$
$D_s^+ \to 0^- 0^-$	$\frac{\Gamma(D_s^+ \to 0^- 0^-)}{\Gamma(D_s^+ \to K_S K^+)}$ %	
$D_s^+ \to \pi^+ \pi^0$	~ 0	$0.003\,\xi_1$
$D_s^+ \to \pi^+ \eta$	114	$-4.136\,\xi_1$
$D_s^+ \to K^+ \pi^0$	13.1	$-4.145\,\xi_5 - 0.244\,\xi_4$
$D_s^+ \to K^+ \eta$	5	$4.235\,\xi_4 - 6.303\,\xi_5$
$D_s^+ \to K_S \pi^+$	32.5	$3.635\,\xi_3 - 0.228\,\xi_4$
$D_s^+ \to K_L \pi^+$	32.5	$3.635\,\xi_3 - 0.228\,\xi_4$
$D_s^+ \to K_L K^+$	94.7	$3.639\,(\xi_2 - \xi_6 - \xi_7)$

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