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"MINIMAL" QUANTIZATION AND CONFINEMENT

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1. INTRODUCTION

There is a number of problems in gauge field theory, whose solution depends on the gauge choice. For example: computation of the fermion Green functions [1], quantization of theories with chiral anomalies [2], interpretation of the gauge ambiguities [3]. So, a physical principle restricting the arbitrariness in the gauge choice would be useful for their consideration.

In the present paper, we would like to draw attention to the "minimal" version of an operator quantization of gauge theories in which such a physical principle is just the gauge invariance. This version is based on the construction of the physical variables by using an explicit solution of the Gauss equation. For simplicity we shall illustrate the method by an example of QED and restrict ourselves to a short discussion of the non-Abelian theory.

2. COULOMB GAUGE AND "MINIMAL" QUANTIZATION

i) The Coulomb gauge is the most convenient and simple tool for describing bound states in the gauge theories, and it is more often than the others used for a path-integral construction by the canonical quantization [4]. Recall that one of the recent practical achievements of QED, the Lamb shift corrections can be calculated only in the Coulomb gauge [5].

A fault of this gauge for standard quantization is the relativistic noncovariance. For example, there is the problem of defining a moving electron wave function, i.e., the Green function residue

$$R = \lim_{\hat{p} \to m_R} (\hat{p} - m_R) G_R(p). \tag{1}$$

As it has been pointed out [5], the residue (1) is equal to unity ($R = 1$) in the rest system, while in a moving reference frame the value of R depends on the velocity and, generally speaking, looses its meaning due to the infrared divergence.

This problem in QED has been considered still in earlier papers for example [6]. (The same problems are pertinent to relativistic gauges, where the Green function has a cut instead of a pole, and the value of R can be equal to zero or infinity depending on the choice of gauge). The task of restoration of the Coulomb-gauge relativistic covariance is practically important for QCD, where the "Coulomb" version of potential confinement is used for proving spontaneous chiral symmetry breaking [7].

ii) On the other hand, it is well known [8] that there is a general proof of the Coulomb-gauge relativistic covariance in the framework of the canonical operator quantization. Most completely and consistently this method is formulated in the paper by Schwinger [9]. The Schwinger operator quantization is based on the choice of the gauge-invariant energy-momentum tensor (the Belifante tensor)

$$T_{\mu\nu} = F_\mu^\lambda F_{\lambda\nu} + \bar\Psi \gamma_\mu (i\partial - eA)_\nu \Psi - g_{\mu\nu} \mathcal{L} + \frac{i}{2}\partial^\lambda (\bar\Psi \Gamma_{\lambda\mu\nu} \Psi),$$
$$\Gamma_{\lambda\mu\nu} = \frac{1}{2}[\gamma_\lambda, \gamma_\mu]\gamma_\nu + g_{\mu\nu}\gamma_\lambda - g_{\nu\lambda}\gamma_\mu, \tag{2}$$

and on the nonlocal commutation relations

$$i[F_{oi}(\vec x, t), A_j^T(\vec y, t)] = (\delta_{ij} - \partial_i \frac{1}{\vec\partial^2}\partial_j)\delta^3(\vec x - \vec y). \tag{3}$$

Tensor (2) differs from the canonical one (used in the conventional Dirac quantization) by a total derivative. Together with the nonlocal term in commutator (3), it plays a very important role: they both restore the correct transformation properties of the transverse field operators

$$\delta A_\mu^T = i\epsilon_k [M_{ok}, A_\mu^T] = \delta_L^o A_\mu^T + \partial_\mu \Lambda,$$
$$\delta\Psi^T = i\epsilon_k [M_{ok}, \Psi^T] = \delta_L^o \Psi^T - ie\Lambda\Psi^T, \tag{4}$$
$$A_o^T = -\frac{1}{\vec\partial^2} j_o^T = -\frac{e}{\vec\partial^2}\bar\Psi^T \gamma_0 \Psi^T,$$

where M_{ok} is the infinitesimal operator of a boost transformation

$$M_{ok} = \int d^3x [t T_{ok}(A^T, \Psi^T) - x_k T_{oo}(A^T, \Psi^T)], \tag{5}$$

δ_L^o is the ordinary Lorentz transformation with parameters and

$$\Lambda = \epsilon_k \frac{1}{\vec\partial^2}(\partial_o A_k^T + \partial_k A_o^T) \tag{6}$$

is an additional gauge transformation by which a field A_i^T again becomes transversal in the new reference frame $\ell_\mu = \ell_\mu^o + \delta_L^o \ell_\mu^o$, $\ell_\mu^o = (1, 0, 0, 0)$

$$\partial^\ell \cdot A^\ell = 0, \quad (B\cdot A = B_\mu A_\mu)$$
$$A_\mu^\ell = A_\mu - \ell_\mu (A\cdot\ell), \quad A_\mu = A_\mu^T + \delta A_\mu^T, \quad \partial_\mu^\ell = \partial_\mu - \ell_\mu(\partial\cdot\ell). \tag{7}$$

Thus, the gauge of transverse fields A_i^T in the operator quantization is changed together with the time axis of quantization ℓ_μ and coincides with the Coulomb one only in the rest frame.

iii) Just the same transformation properties (4) in classical electrodynamics are inherent to the nonlocal classical variables

$$ieA_j^T = v(ieA_j + \partial_j)v^{-1} = ie(\delta_{jk} - \partial_j \frac{1}{\vec\partial^2}\partial_k)A_k,$$
$$\Psi^T = v\Psi, \quad (v = \exp\{ie\frac{1}{\vec\partial^2}\partial_j A_j\}), \tag{8}$$

that can be constructed by an explicit solution of the Gauss equation

$$\vec\partial^2 A_o = \partial_i \partial_o A_i - j_o. \tag{9}$$

It is easy to check that the substitution of the solution of eq. (9) into the Lagrangian

$$\mathcal{L}(x) = -\frac{1}{4}F_{\mu\nu}^2 + \bar\Psi[i\gamma_\mu(\partial_\mu + ieA_\mu) - m]\Psi, \tag{10}$$

leads to the expression

$$\mathcal{L}(x) = \frac{1}{2}(\partial_o A_k^T - \partial_k \frac{1}{\vec\partial^2} j_o)^2 - \frac{1}{4}F_{ij}^2 + A_i^T j_i + \bar\Psi\{i\gamma_\mu[\partial_\mu +$$
$$+ ie\partial_\mu \frac{1}{\vec\partial^2}(\partial_i A_i)]\}\Psi = \frac{1}{2}(\partial_o A_k^T - \partial_k \frac{1}{\vec\partial^2} j_o)^2 - \frac{1}{4}F_{ij} + A_i^T j_i + \bar\Psi^T[i\gamma_\mu\partial_\mu - m]\Psi^T$$

that depends only on variables (8). Therefore solving one gauge-invariant equation (9) we remove out two fields. The nonlocal variables are invariant under gauge transformations of the initial fields A_μ, Ψ: $ieA^g_\mu = g(ieA_\mu + \partial_\mu)g^{-1}$, $\Psi^g = g\Psi$ $A^T(A^g) = A^T(A)$, $\Psi^{Tg} = \Psi^T$ like the initial Lagrangian (10). The quantization of variables (8) is completely reproduced by the Schwinger scheme (2)-(4); and moreover, the very procedure of this variable construction by an explicit solution of the Gauss equation under the condition of gauge invariance justifies the application of the Belifante tensor (in terms of which the dependence on the longitudinal field $\partial_i A_i$ disappears) and the projectile operator in the commutation relation (3) (which arises in the explicit solution of the constraint equation (9)).

Thus, we have discussed three versions of quantization of fields in the "Coulomb gauge": i), ii), iii). In the first one the gauge is fixed and we are faced with the Lorents noncovariance. In the second version we fix the gauge but it automatically changes with the time-axis transformation. In such a way the Poincare algebra of the observables can be proved on an operator level. In the third version we do not need the constraint fixing the gauge, in such a "minimal" scheme (It is sufficient to solve eq. (9) explicitly and to use its part that transforms in a nonhomogeneous way $[(\vec{\partial}^2)^{-1}\partial_i A_i]$ for construction of nonlocal variables demanding their gauge invariance).

The only difference between the second and third versions is in the explicit construction of the physical variables. This construction leads to some additional physical consequences which we are going to discuss in the present paper.

3. COMPUTATION OF THE FERMION GREEN FUNCTIONS

Let us consider at first the computation of the one-particle fermion Green function

$$i(2\pi)^4\delta^4(p-q)G(p) = \int d^4x\, d^4y\, e^{ipx-iqy} < 0 \,|\, T(\Psi^T(x)\bar{\Psi}^T(y))\,|\, 0 > , \quad (11)$$

(where $\Psi^T, \bar{\Psi}^T$ are operators in the Heisenberg representation) in the relativistic version of Coulomb gauge. In the one-loop approximation G(p) has the form

$$G(p) = G_0(p) + G_0(p)\,\Sigma(p)G_0(p) + O(e^4), \; (G_0(p) = (\hat{p} - m)^{-1}),$$

where $\Sigma(p)$ is the electron self-energy of order e^2 which contains contributions from the transverse fields and the Coulomb interaction [5]

$$\Sigma(p) = \int \frac{(dq)}{q^2_\mu}[(\delta_{ij} - \frac{q_i q_j}{\vec{q}^2})\gamma_i \tilde{G}_0 \gamma_j + \gamma_0 \tilde{G}_0 \gamma_0 \frac{q^2_\mu}{\vec{q}^2}],$$

$$(dq) = \frac{e^2}{(2\pi)^4} id^4q, \; q^2_\mu = q^2_0 - \vec{q}^2 = q^2, \; \tilde{G}_0 = G_0(p-q). \quad (12)$$

It is known (8) that $\Sigma(p)$ can be represented as a sum of the invariant $\Sigma(p)$ and noninvariant $\Delta\Sigma(p)$ under Lorentz transformations terms *

$$\Sigma(p) = \Sigma_F(p) + \Delta\Sigma(p); \; (\delta^0_L \Sigma_F(p) = 0; \; \delta^0_L \Delta\Sigma(p) \neq 0)$$

$$\Sigma_F(p) = -\int (dq)\frac{1}{q^2}\gamma_\mu G_0 \gamma_\mu , \quad (13)$$

$$\Delta\Sigma(p) = \int \frac{(dq)}{q^2\vec{q}^2}[\hat{q}\tilde{G}_0\hat{q} + \underset{\sim}{q}\tilde{G}_0\hat{q} + \hat{q}\tilde{G}_0\underset{\sim}{q}]; \; (\underset{\sim}{q} = \gamma_i q_i).$$

The response of $\Delta\Sigma(p)$ to the Lorentz transformation can be got by changing the integration variables in eq.(13)

$$\delta^0_L q_0 = \epsilon_k q_k, \quad \delta^0_L q_k = \epsilon_k q_0$$

$$\delta^0_L \Delta\Sigma(p) = \epsilon_k \int \frac{(dq)}{q^2\vec{q}^2}[B_k \tilde{G}_0 \hat{q} + \hat{q}\tilde{G}_0 B_k], \quad (14)$$

where $B_k = q_k\gamma_0 + \gamma_k q_0 - \frac{2q_0 q_k}{\vec{q}^2} q_i\gamma_i - \frac{q_0 q_k}{\vec{q}^2}\hat{q}$. However, the total Lorentz transformation for the Green function contains also the gauge transformation (4):

$$(2\pi)^4 i\delta^4(p-q)\delta_\Lambda G(p) = ie\int d^4x\, d^4y\, e^{ipx-iqy}\{<0|\, T[\Psi^T(x)\bar{\Psi}^T(y)\times$$

$$\times \Lambda(y) - \Lambda(x)\Psi^T(x)\bar{\Psi}^T(y)]\,|\,0>\}.$$

Using the explicit form for Λ (6) we get the following expression

$$\delta_\Lambda \Sigma(p) = -\epsilon_k \int \frac{(dq)}{q^2\vec{q}^2}[B_k \tilde{G}_0(\hat{p} - m) + (\hat{p} - m)\tilde{G}_0 B_k],$$

* By "invariance" we shall understand the equality $G_0(p') = S_{p'p} G_0(p) S^{-1}_{p'p}$.

where B_k is given by the formula (14). As

$$G_o(p-q)(\hat{p}-m) = 1 + G_o(p-q)\hat{q}\ , \quad \int (dq) \frac{B_k}{q^2 \vec{q}^{\,2}} = 0.$$

We get that the total response of $\Sigma(p)$ to the Lorentz transformation is equal to zero

$$\delta_{L\,tot}\Sigma(p) = (\delta_L^o + \delta_\Lambda)\Sigma(p) = \theta. \qquad (15)$$

It is important to emphasize that the relativistic invariance (15) is broken for the canonical energy-momentum tensors since it differs from the Belinfante one by a total derivative, which is essential for δ_Λ.

Thus, the transition to another Lorentz frame will be accompanied by additional diagrams induced by a transformation of type (4) that is equivalent to the choice of the gauge (7).

If we choose the time axis (ℓ) parallel to the particle momentum (so that the fermion Coulomb field moves together with it) we get for the self-energy an expression without infrared divergences on the mass shell

$$\Sigma(p) = \int \frac{(dq)}{q^2} \frac{2}{\hat{p}-\hat{q}+m} - \int \frac{(dq)}{\vec{q}^{\,2}} \gamma_o \frac{1}{\hat{q}+m}\gamma_o = \frac{\alpha}{4\pi}\{m(3D+4) -$$

$$- D(\hat{p}-m) + \frac{1}{2}(\hat{p}-m)^2 [\frac{\hat{p}+m}{p^2}(\mathrm{Log}\frac{m^2-p^2}{m^2})(1 + \frac{\hat{p}(\hat{p}-m)}{2p^2}) - \frac{\hat{p}}{2p^2}]\}, \qquad (16)$$

here $D = \frac{1}{\epsilon} - \gamma_E + \mathrm{Log}4\pi$, ϵ is a dimensional regularization parameter. Due to eq.(15) the renormalized Green function (11) has the correct analytical properties

$$R = \lim_{\hat{p}\to m_R} (\hat{p} - m_R) G_R(p) = 1$$

and this result is relativistic-invariant.

4. THE PATH INTEGRAL

There are two approaches to formulating the path integral in gauge theories: heuristic and constructive ones. An example of the heuristic approach is the Faddeev - Popov ansatz, and an example of the constructive one is the proof of the Faddeev -

Popov ansatz by the operator canonical quantization [11,42]. The path integral construction is usually accomplished with the help of such modifications of operator quantization (for instance: a local commutation relation [11] or a canonical energy-momentum tensor [4]) which break relativistic covariance. No wonder, the path integral thus obtained leads to relativistically noncovariant Green functions for fermions [5].

Then a question arises whether it is possible to reproduce the relativistic version of the Coulomb-gauge operator quantization by the path-integral method. Since the "flowing" character of the gauge condition has to be reflected, our "constructive" path integral should explicitly depend on the time-axis of quantization ℓ_μ :

$$Z^\ell_{Const}[\bar{\eta},\eta] = \int d\bar{\Psi}\, d\Psi\, d^4A_\mu\, \delta[\partial^\ell \cdot A^\ell]\, \det[(\partial^\ell)^2] \times$$

$$\times \exp\{iS + i\int d^4x(\bar{\Psi}\eta + \bar{\eta}\Psi)\}, \qquad (17)$$

where $B^\ell_\mu = B_\mu - \ell_\mu(B\cdot\ell)$, $(B_\mu = A_\mu, \partial_\mu)$ (It should be noted that even the path integral (17) does not reflect all specific properties of a more fundamental operator method of quantization).

It is clear that all scattering amplitudes on the mass shell got by the path integral (17) do not depend on the time-like vector ℓ_μ except for the one-particle Green function residues (1). For their computation according to the operator quantization we have to choose the vector ℓ_μ parallel to the momentum characterizing Green function.

By changing the variables one can pass to the path integral in any gauge $f(A) = 0$

$$Z^\ell_{Const.}[\eta,\bar{\eta}] = \int d\bar{\Psi}\, d\Psi\, d^4A_\mu\, \delta[f(A)] \exp\{iS + i\int d^4x \times$$

$$\times [\bar{\Psi} v^\ell_f \eta + \bar{\eta}(v^\ell_f)^{-1}\Psi]\}, \qquad (18)$$

where

$$v^\ell_f(A_\mu) = \exp\{ie \frac{1}{(\partial^\ell)^2}\partial^\ell \cdot A^\ell\} \qquad (19)$$

is the transition matrix that restores the correct analytic properties of the Green one-particle functions in any gauge. On the level of "constructive" path integral (18) these one-particle functions do not depend on the gauge choice and have correct properties, while the Faddeev - Popov "heuristic" path integral, without v^ℓ_f , leads to their dependence on the gauge choice. Therefore, one can consider expression (18) as a more exact definition of the Faddeev - Popov path integral that follows from the relativistic covariant and gauge-invariant operator quantization.

One can show that the gauge-invariant construction of physical variables in the non-Abelian theory [12] up to some details leads to the Schwinger operator quantization [9] with the "running" gauge. This quantization corresponds to the path integral depending manifestly on the time-axis ℓ_μ (unlike the naive Coulomb-gauge path integral [4,11,13]) and to the one-particle Green functions with correct analytic properties (16), in any case, in the one-loop approximation.

On the level of a "constructive" integral of the type (18) the choice of gauge is only the one of integration variables. Therefore the gauge ambiguities that appear on a more fundamental level of the opeartor quantization should be presented also for a path integral for any gauge. It is important to emphasize that the inverse differential operator in the nonlocal commutation relation of the Schwinger quantization coincides with the operator in the Faddeev - Popov determinant. Thus the problem of determinant zeroes on the level of the operator quantization reduces to the problem of unique constructing of the nonlocal commutation relation of the type (3).

Let us consider the solution of this problem by the simplest example of Abelian theory. We have to separate all fields into two orthogonal classes with zero and nonzero eigenvalues of the operator ∂_i^2 and to quantize the fields from each class separately, then to construct the corresponding path integral that should not depend on the choice of variables or gauge. In the conventional commutation relation of the type of (3) the class of infrared fields

$$\partial_i^2 A_j(\vec{x},t) = 0\,, \quad A_j(\vec{x},t) = b_j(t) \tag{20}$$

drops out and only the fields

$$\partial_i^2 A_j(\vec{x},t) \neq 0, \quad \left(\int d^3x\, A_j(\vec{x},t) = 0\right) \tag{21}$$

are included. At first we consider the Abelian theory in the function class (20) in a finite volume of space

$$L = \int d^3x\left[-\frac{1}{4}F_{\mu\nu}^2(b) - \frac{\mu^2}{2}b_i^2 + b_i j_i(\vec{x},t)\right] =$$

$$= \frac{1}{2}V[\dot{b}_i^2 = \mu^2 b_i^2] + b_i\int d^3x\, j_i(\vec{x},t).$$

We introduce here the mass μ for the infrared regularization of the propagator of the quantum field

$$p_i = \frac{\partial L}{\partial \dot{b}_i} = V\dot{b}_i\,, \quad i[p_i, b_j] = \delta_{ij}\,, \quad \left(i[\dot{b}_i, b_j] = \frac{\delta_{ij}}{V}\right),$$

$$D_{ij}(t) = \frac{1}{2}\langle T(b_i(t), b_j(0))\rangle_o = \frac{\delta_{ij}}{2\pi V}\int dq_o \frac{e^{iq_o t}}{q_o^2 - \mu^2 + i\epsilon} = i\delta_{ij}\frac{e^{i\mu|t|}}{2\mu V}\,. \tag{22}$$

We see that there is such a limit of the infinite volume and zero mass

$$V \to \infty\,, \quad \mu \to 0, \quad 2\mu V = M^{-2} \neq 0, \tag{23}$$

where the propagator (22) is not zero

$$\lim_{\substack{V\to\infty \\ \mu\to o}} D_{ij}(t) = i\delta_{ij}\, M^2 \neq 0 \tag{24}$$

and we have the nontrivial evolution operator

$$\lim_{\substack{T,V\to\infty \\ \mu\to o}} \langle e^{iTH}\rangle = \exp\left\{\frac{M^2}{2}\left(\frac{\partial}{\partial b_i}\right)^2\right\} e^{ib_j J_j}\Big|_{b_i = o} \tag{25}$$

It is clear that in limit (23) the propagator of the total field $A_i = A_i^T + b_i$

$$i[F_{oi}(\vec{x},t), A_j(\vec{y},t)] = \left(\delta_{ij} - \partial_i\frac{1}{\vec{\partial}^2}\partial_j\right)\delta^3(\vec{x}-\vec{y}) + \frac{\delta_{ij}}{V}\,, \tag{26}$$

has the form of a sum of the usual transverse propagator and expression (24)

$$D_{ij}(x) = \frac{1}{i}\langle T(A_i(x)A_j(0))\rangle_o = D_{ij}^T(x) + i\delta_{ij}M^2,$$

or in the momentum representation

$$D_{ij}^T(q) = \left(\delta_{ij} - q_i\frac{1}{\vec{q}^2}q_j\right)\frac{1}{q^2 + i\epsilon} + i(2\pi)^4 M^2\delta^4(q)\delta_{ij}\,. \tag{27}$$

So, we have got one version of the "confinement" propagator [14] that reflects the collective excitation of infrared fields (20) in the whole space fields occupy. In view of this fact the attempts to get the confinement propagator by analytical calculations in the framework of the convention perturbation theory given only in the function class (21) look very doubtful.

For the generation functional of the Green functions for the Abelian theory with the commutation relations like (26) in limit (23) we get the following expression

$$Z[\eta,\bar{\eta}] = \exp\{\frac{M^2}{2}(\frac{\partial}{\partial b_i})^2\} Z(b|\eta,\bar{\eta})|_{b=0}, \quad (28)$$

where $Z(b|\eta,\bar{\eta})$ is the usual functional integral

$$Z(b|\eta,\bar{\eta}) = \int d\bar{\Psi} d\Psi d^4 A_\mu \delta(\partial_i A_i) \exp\{iS[A_0, A_i + b_i] + i\int d^4x [\bar{\Psi}\eta + \bar{\eta}\Psi]\}. \quad (29)$$

We can generalize the relativization procedure got above to expression (28)

$$b_i \to b^\ell_\mu = b_\mu - \ell_\mu (b\cdot\ell). \quad (30)$$

Thus the solution of the problem of zeroes of the determinant in the path integral on a more fundamental level of the operator canonical quantization leads to the stochastization of the usual path integral (29) over the zero eigenfunctions. In QED we omit the infrared fields (20) ($M = 0$). In QCD the inclusion of infrared gluon fields may be justified by the nonlinearity of theory and strong coupling of fields on the infrared limit accompanied by long-wave correlations and collective excitations.

5. PHYSICAL APPLICATION: CONFINEMENT IN THE SCHWINGER MODEL

The "minimal" quantization (see Section 2.iii) that does not use the gauge fixing as an initial assumption differs from the relativistic version of the Coulomb gauge only by the explicit construction of the nonlocal physical variables (8). Let us consider here some physical consequences of this construction by an example of the Schwinger model (see eq.(10) where $\mu = 0,1$, $m = 0$).

As is known, this model gave rise to the popular Wilson criterion - confinement based on a linearly rising quark potential [15]. Formally the choice of the variables (8) leads really to the Coulomb gauge with the linearly rising potential and to the following exact results for the current correlators and for the one-particle fermion Green function

$$\frac{1}{i} < T(j_\mu(x), j_\nu(0)) >_0 = \epsilon_{\mu\alpha}\epsilon_{\nu\beta}\, \partial_\alpha \partial_\beta\, \Delta_M(x-y), \quad (31)$$

$$\frac{1}{i} < T(\Psi(x)\bar{\Psi}(x)) >_0 = \exp\{-i\pi[\Delta_M(x-y) - \Delta_0(x-y)]\} G_0(x-y). \quad (32)$$

where Δ_M, Δ_0, G_0 are the Green functions of free scalar fields with masses $M = \sqrt{\frac{e^2}{\pi}}$, $M = 0$ and of a free fermion massless field, respectively.

Calculation of the fermion Green function in the model leads to the following asymptotics in the momentum space

$$G^T_{p\to\infty}(p) \sim \frac{\hat{p}}{p^2}, \qquad G_{p\to 0}(p) \simeq \frac{\hat{p}\sqrt{M}}{(p^2 + i\epsilon)^{5/4}}.$$

So, the probability of finding a particle with quark quantum numbers is not equal to zero

$$\lim_{\hat{p}\to 0} \hat{p}\, G(p) \neq 0, \quad (33)$$

that means, the Wilson criterion is not a criterion for confinement. Note, that this result is gauge-invariant in the sense, we discussed in Section 4.

The "minimal" constructive way of quantization (8) leads to another mechanism of confinement in the Schwinger model related to topological properties of the gauge field.

The explicit solution of the Gauss equation defines the nonlocal variables (8) up to the factor $g(x)$ which corresponds to a solution of the homogeneous Gauss equation

$$v = \exp\{i(\lambda(x) + \frac{1}{\vec{\partial}^2}\partial_i A_i)\} \equiv g(x)\exp\{i\frac{1}{\vec{\partial}^2}\partial_i A_i\}, \quad (34)$$

$$g(x) = \exp\{i\lambda(x)\}, \quad (35)$$

where $\partial_i^2 \lambda(x) = 0$. The g-factor has to be a smooth function without singularities as we solve eq.(35) in an empty space $R(1)$.

For a finite space $|x_1| < R$ there are nontrivial solutions of eq.(35) that represent a map of the space $R(1)$ onto the group $U(1)$-manifold with an integer degree of mapping n

$$g^{(n)}(x) = \exp\{i\frac{x}{R}\pi n\},\quad n = 0, \pm 1, \pm 2, \ldots$$

$$\lim_{|x|\to R} g^{(n)}(x) = \pm 1. \qquad (36)$$

So, our dynamical variables Ψ^T are topologically degenerated. This degeneration concerns all observables in the theory through the Green-function generating functional where the fermion sources acquire additional topological phases

$$S_{sour} = \int dx \{\Psi^T g^{(n)} \eta + \bar{\eta} g^{(-n)} \Psi^T\}.$$

This degeneration has to be removed by taking an average over its parameter (n). So, instead of the Green function (32) we get the following result

$$G(x-y) = \lim_{R\to\infty} \lim_{N\to\infty} \frac{1}{2N} \sum_{n=-N}^{N} g^{(n)}(x) g^{(-n)}(y) G^T(x-y) = \begin{cases} G^T(x-y), & x = y \\ 0, & x \neq y \end{cases} \qquad (37)$$

hence $G(p) = \int d^2x\, d^2y\, e^{ip(x-y)} G(x-y) = 0$. At the same time, for the two-current correlator we obtain the old result (31); it retains its pole at $p^2 = \frac{e^2}{\pi}$.

Such a picture is caused by destructive interference of the infinite number of phase factors $g^{(n)}(x)$. We would like to emphasize the noncommutativity of the limit procedures in (37) determined as in quantum statistics[16]; an opposite ordering leads to the old result (32).

The existence of nontrivial solutions (36) is enshured by the following relation which takes place in the theory under consideration

$$\pi_1(U(1)) = \mathbf{Z},$$

π_1 being the first homology group. This relation may be generalized for theories with a gauge group G in D-dimensional space-time as

$$\pi_{D-1}(G) = \mathbf{Z}. \qquad (38)$$

So, the same topological confinement mechanism should take place if such a criterion is satisfied.

For example, in QED_{3+1} this relation is not valid ($\pi_3(U(1)) = 0$), which is in agreement with the observability of the electron. In QCD_{3+1} criterion (38) is satisfied ($\pi_3(SU(3)) = Z$), which means vanishing of the amplitudes for coloured-particle creation[10]

$$\begin{pmatrix} S_{cc} & S_{hc} \\ S_{ch} & S_{hh} \end{pmatrix} \to \begin{pmatrix} 0 & 0 \\ 0 & S_{hh} \end{pmatrix}$$

where S_{ch} is an S-matrix element of a transition between a colour state (c) and a hadron one (h).

However, the usual free propagators of quarks and gluons are used in calculations of the colourless amplitudes. So, in correspondence with the unitarity principle

$$SS^+ = (1 + iT)(1 - iT^*) = 1$$

or

$$\sum_h \langle i|T|h\rangle\langle h|T^*|f\rangle = 2\,\mathrm{Im}\langle i|T|f\rangle, \quad (i, f \in h)$$

the inclusive processes provide a possibility of measuring the coloured particle quantum numbers. In this way, the "destructive interference - confinement" lays a foundation of the quark-hadron duality principle[17].

SUMMARY

In the present paper a gauge-invariant and relativistic-covariant operator construction of the physical variables and path integral in gauge theories has been proposed. We have shown that such a "minimal" quantization method solves the old QED problems as the correct definition of the electron wave function[5,6] or the residual of its Green function and leads to a new picture of the colour confinement. The latter is based on the destructive interference of the phase factors that appear in topologically degenerated theories ($\pi_{D-1}(G) = \mathbf{Z}$). In this picture the coloured-particle creation amplitudes are equal to zero due to this quantum phase interference. At the same time the free quark and gluon propagators are used

in calculation of the colourless observable amplitudes. So, the unitary principle provides a possibility for measurement of the coloured-particle quantum numbers in the inclusive processes. Note, that just these assumptions are implicitly used in the parton model and now-a-days, in QCD phenomenology at high and low energies.

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