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HADRON STATIC PROPERTIES IN THE MODEL CONSIDERING THE STRUCTURE OF QCD VACUUM

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The model based on a dominating role of the interaction of quarks with QCD vacuum fields was proposed in works[1,2]. It was shown that the model well reproduces the hadron mass spectrum and allows one to explain such features of hadron spectroscopy as hadron electromagnetic mass differences[3] and mixing of unitary multiplets[2].

In the present work, within the model[1,2] we calculate static characteristics of hadrons: mean squared charge radii (MCR), magnetic moments and axial-vector constants. We shall take into account only the one-particle corrections to these quantities neglecting effects of quark-quark interaction.

In the proposed model, such an approximation is justified because quarks interact with the vacuum basically in an additive manner[1]. This property may be violated mainly due to the interaction induced by instantons. To what extent this violation depends on the static properties of hadrons will be shorting discussed in the conclusion.

The one-particle contribution to mean-square charge radius is defined by the expression

$$\langle \vec{r}^{\,2} \rangle_{ch} = \sum_i e_i \int d\vec{r}\, q_i^+(r)\, r^2 q_i(r), \tag{1}$$

where e_i is the charge of an i-th quark, $q_i(r)$ is the one-particle wave function.

As the solution of zero approximation in the interaction of quarks the wave functions of the bag model[4,5] were chosen. For the 1S state they are

$$q(\vec{r}) = \frac{\bar{N}}{\sqrt{4\pi}} \begin{pmatrix} \left(\dfrac{\omega + m}{\omega}\right)^{1/2} i\, j_0(\kappa r/R) U \\ \left(\dfrac{\omega - m}{\omega}\right)^{1/2} (\vec{\sigma}\cdot\hat{\vec{r}})\, j_1(\kappa r/R) U \end{pmatrix}, \tag{2}$$

where R is the bag radius, m is the quark mass, $\omega = (\kappa^2/R^2 + m^2)^{1/2}$ is the one-particle energy, κ is the solution of the equation

$$\operatorname{tg}\kappa = \kappa/(1 - mR - \omega R),$$

$$\bar{N}^{-2} = R^3 j_0^2(\kappa)[2\omega(\omega - 1/R) + m/R]/[\omega(\omega - m)], \qquad (3)$$

U is Dirac spinor.

Substituting the wave functions (2) into (1) we obtain

$$\langle \vec{r}^2 \rangle_{ch} = R^2 \sum_i e_i \{ \alpha_i [2\kappa_i^2(\alpha_i - 1) + 4\alpha_i + 2\lambda_i - 3] -$$

$$- \frac{3}{2}\lambda_i(4\alpha_i + 2\lambda_i - 2\kappa_i^2 - 3)\}/(3\kappa_i^2[2\alpha_i(\alpha_i - 1) + \lambda_i]), \qquad (4)$$

where $\alpha_i = R\omega_i$, $\lambda_i = m_i R$, $\alpha_i^2 = \lambda_i^2 + \kappa_i^2$, κ_i is the solution of (3) for $m = m_i$ ($\kappa_0 = 2.043$ for $m = 0$).

In the case of massless quarks we find

$$\langle \vec{r}^2 \rangle_{ch} = R^2 \frac{2\kappa_0^2(\kappa_0 - 1) + 4\kappa_0 - 3}{6\kappa_0^2(\kappa_0 - 1)} \sum_i e_i .$$

Neglecting masses of u- and d-quarks [3] we have for MCR of the proton, neutron and π-meson

$$\langle \vec{r}^2 \rangle_p = 0.69 \text{ fm}^2, \quad \langle \vec{r}^2 \rangle_n = 0, \quad \langle \vec{r}^2 \rangle_\pi = 0.61 \text{ fm}^2. \qquad (5)$$

Here we use equilibrium bag radii from ref. [2] :

$R_p = R_n = 5.80$ GeV^{-1} , $R_\pi = 5.44$ GeV^{-1} .

The values (5) should be compared with experimental ones

$$\langle r^2 \rangle_p^{exp} = 0.68 \text{ fm}^2, \quad \langle r^2 \rangle_n^{exp} = -0.116 \text{ fm}^2, \quad \langle r^2 \rangle_\pi^{exp} = 0.44 \text{ fm}^2.$$

As we see, the model is in favourable agreement with the MCR experimental data. The standard model gives the following values [5]

$$\langle r^2 \rangle_p^{MIT} = 0.53 \text{ fm}^2, \quad \langle r^2 \rangle_n^{MIT} = 0, \quad \langle r^2 \rangle_\pi^{MIT} = 0.24 \text{ fm}^2,$$

which are in a worse agreement with the data than (5).

In our model, the magnetic moment of particle is defined by the electromagnetic current of quarks in the bag. A one-particle contribution to the magnetic moment is expressed as

$$\vec{\mu} = \frac{1}{2} \sum_i \int (\vec{r} \times \vec{j}_i)\, d\vec{r}, \qquad (6)$$

where $\vec{j}_i = e_i \bar{q}_i(r) \vec{\gamma} q_i(r)$ is the current of an i-th quark. Taking matrix elements of the operator (6) between spin-unitary wave functions of components of the baryon octet [6] , for the magnetic moments we obtain the expressions

$$\mu_p = \mu_0, \quad \mu_n = -\frac{2}{3}\mu_0, \quad \mu_\Lambda = \frac{1}{3}\mu_s, \quad \mu_{\Sigma^+} = \frac{8}{9}\mu_0 + \frac{1}{9}\mu_s, \quad \mu_{\Sigma^-} =$$

$$= -\frac{4}{9}\mu_0 + \frac{1}{9}\mu_s, \quad \mu_{\Xi^0} = -\frac{2}{9}\mu_0 - \frac{4}{9}\mu_s, \quad \mu_{\Xi^-} = \frac{1}{9}\mu_0 - \frac{4}{9}\mu_s, \quad \mu_{\Lambda\Sigma^0} = -\frac{1}{\sqrt{3}}\mu_0 . \qquad (7)$$

In formula (7) the magnetic moment of an i-th quark is equal to

$$\mu_i = \frac{R(4\alpha_i + 2\lambda_i - 3)}{6(4\alpha_i(\alpha_i - 1) - \lambda_i)} .$$

Table 1.

The magnetic moments of the members of baryonic octet (in $1/2M_N$ units). (The values used in fitting the model parameters are underlined).

Hadron	R(GeV^{-1})	μ_{th}	μ_{MIT}	μ_{NR}	μ_{exp}
p	5.80	2.20	1.90	<u>2.79</u>	2.7928
n	5.80	-1.47	-1.27	<u>-1.86</u>	-1.9130
Λ	5.56	-0.60	-0.49	<u>-0.61</u>	-0.613±0.004
Σ^+	5.56	2.08	1.84	<u>2.68</u>	2.379±0.020
Σ^-	5.56	-0.74	-0.68	-1.04	-1.14 ±0.05
Ξ^0	5.32	-1.21	-1.06	-1.44	-1.250±0.014
Ξ^-	5.32	-0.54	-0.44	-0.51	-0.69 ±0.04
$\Lambda\Sigma^0$	5.56	-1.23	-1.10	-1.61	-1.60 ±0.07

The results of calculations with $m_u = m_d = 0$, $m_s = 220$ MeV and the values of radii R obtained in ref. [2] are given in Table 1, together with the MIT results [5] (μ_{MIT}) and the nonrelativistic quark model. It should be noted that in the nonrelativistic calculations a better agreement with the experiment is reached by using two additional parameters: the magnetic moments of nonstrange and strange quarks, while our and MIT models are parameterless ones. At the same time the proposed model describes the magnetic moments of the members of baryonic octet much better than the MIT model.

Let us consider now lepton decays of baryons: $B \to B' e^- \nu$. The ratio of decay constants is defined by the operator average:

$$\frac{g_A}{g_V} = \frac{\langle B'\uparrow | \int d\vec{x}\, \bar{q}(\vec{x}) \lambda^+ S_z q(\vec{x}) | B\uparrow \rangle}{\langle B'\uparrow | \int d\vec{x}\, \bar{q}(\vec{x}) \lambda^+ q(\vec{x}) | B\uparrow \rangle}, \tag{8}$$

where $\lambda^+ = \lambda_1 + i\lambda_2$ and $\lambda^+ = \lambda_4 + i\lambda_5$ for decays without and with changing the strangeness, respectively, λ_i are $SU_F(3)$ generators in the flavour space. The right-hand side of eq. (8) is easily calculated by using SU(6)-symmetric nonrelativistic wave functions /6/ . However, as first was shown by P.N.Bogolubov /4/, relativistic effects of quark motion play an important role in calculating axial-vector constants.

For instance, let us consider the decay $n \to p e^- \nu$. The nonrelativistic model gives $g_A/g_V = 5/3$ whereas the relativistic formalism leads to the well-known formula /4/

$$\frac{g_A}{g_V} = \frac{5}{3} \langle \sigma_z \rangle, \tag{9}$$

where $\langle \sigma_z \rangle$ is the average of z-component of the quark spin operator. In the case of massless quarks (2) we have

$$\langle \sigma_z \rangle = \frac{\int_0^{\kappa_0} dx\, x^2 (j_0^2(x) - \frac{1}{3} j_1^2(x))}{\int_0^{\kappa_0} dx\, x^2 (j_0^2(x) + j_1^2(x))} = 0.653 .$$

So, the correction gets equal to 30 percent for ultrarelativistic quarks. In the general case ($m_q \neq 0$) we obtain

$$\langle \sigma_z \rangle_{ij} = \int d\vec{x}\, q_i^+(\vec{x}) \sigma_z q_j(\vec{x}) =$$

$$= \frac{3\kappa_i \kappa_j (2(\alpha_i - \alpha_j) + \lambda_i - \lambda_j)}{3(\kappa_i^2 - \kappa_j^2)[2\alpha_i(\alpha_i - 1) + \lambda_i]^{1/2}[2\alpha_j(\alpha_j - 1) + \lambda_j]^{1/2}} . \tag{10}$$

Making use of (10), the relativistic corrections for the transitions with and without changing strangeness were calculated.

The results of our calculations of g_A/g_V ratio are reported in Table 2. There are also given the results obtained by using MIT /5/ and the nonrelativistic quark models.

Table 2.

Axial-vector decay constants of the members of baryonic octet

Decay	$(\frac{g_A}{g_V})_{th}$	$(\frac{g_A}{g_V})_{MIT}$	$(\frac{g_A}{g_V})_{NR}$	$(\frac{g_A}{g_V})_{exp}$
$n \to p e^- \nu$	1.088	1.088	5/3	1.254±0.006
$\Lambda \to p e^- \nu$	0.713	0.709	1	0.694±0.025
$\Sigma^- \to n e^- \nu$	-0.238	-0.236	-1/3	-0.362±0.043
$\Xi^- \to \Lambda e^- \nu$	0.238	0.236	1/3	0.25 ±0.043
$\Xi^- \to \Sigma^0 e^- \nu$	1.188	1.181	5/3	
$\Xi^0 \to \Sigma^+ e^- \nu$	1.188	1.181	5/3	

As we can see, the model gives practically the same values as the MIT version. This correspondence is due to the fact that the ratio g_A/g_V depends on the dimensionless parameter $m_s R$ which is almost the same in both models. It should be also noted that these models agree much better with experiment than the nonrelativistic quark model.

So, the model proposed in /1,2/ in the one-particle approximation describes static characteristic of hadrons with an accuracy not worse than 20%. The discrepancies with experiment are probably due to two basic reasons. First, a correct consideration of the c.m. motion is needed in the bag model; the corrections caused by this effect may turn out to be significant /8,9/ . Second, the interaction of quarks induced by instantons /1,2/ violates additivity. This violation reaches 100% for π-meson and 20% for a baryonic octet. In particular, it leads to a strong collapse of the π-meson wave function for a net quark-quark state in 1S-mode and the formation of scalar diquarks in baryons /2,10/ .

In our opinion, the visuable mean-square raduis of π-meson is caused by a large mixing of its wave function with radial excitations and quark-antiquark sea arising through instanton. On the other hand, the diquark mechanism may explain the differences of data from the predictions of the SU(6)-symmetric quark model for baryonic sector and obtain, for example, nonzero mean-square charge radius of the neutron [11-13]. A detailed consideration of these effects will be published elsewhere.

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