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F 22
E17-87-269

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EXACT RESULTS FOR THE EMISSION
FROM ONE AND TWO ATOMS
IN AN IDEAL CAVITY
AT MULTIPHOTON RESONANCE

Submitted to "Journal of Physics A"

## 1. INTRODUCTION

The Jaynes-Cumimings model of a two-1evel atóm interacting with a single mode of the electromagnetic field in a lossless cavity is one of the few exactly soluble models in quantum optics. For many years, theoretists have analyzed various aspects of this model (for reviews see ${ }^{\prime 1,21}$ ). A number of interesting effects such as vacuum-field Rabi oscillations, collapse and revival have been predicted and then observed experimental$1 y^{\prime / 3}=6 / \cdot$ Recently, Sanchez-Mondragon et a1. ${ }^{\prime \prime}$ ' and Agarwal ${ }^{\prime 8}$ have calculated the emission spectrum for fluorescence photons in this model. The fluorescence and absorption spectra have also been obtained for the model of a single-atom one-mode system with cavity damping '9,10' and for the two-atom generalized models $/ 8,11$. It has been shown that the emission of an atom in a cavity is drastically modified as compared with its behaviour in free space. For example, Sanchez-Mondragon, Narozhny and Eberly ${ }^{17 /}$ have shown that the spontaneous emission spectrum of an atom excited in a lossless cavity has a doublet structure arising from vacuum-field Rabi oscillations. The coonerative and multiphoton'transition effects ${ }^{\prime} 8.12^{\prime}$ make the structure of the spectrum very complicated and thus might lead to difficulties in the study of such a spectrum.

In this paper we examine the emission from one and two twolevel atoms interacting through the multiphoton-transition mechanism with one resonant mode in an ideal cavity. Exact quantum electrodynamic results for the dipole correlation functions and the spectra of multiphoton-induced fluorescence are presented.

The paper is organized as follows. In Sect. 2 we study the single-atom system. Section 3 is devoted to the two-atom system. In Sect. 4 we give a summary.

## 2. A SINGLE ATOM

In the rotating wave approximation, the effective Hamiltonian for a single two-level atom interacting with a one-mode
radiation field in a lossless resonant cavity through the multiphoton transition mechanism /24, 25 ! is
$H=H_{o}+H_{A F}=\left(\hbar \omega_{o} R^{z}+\hbar \omega_{f} a^{+} a\right)+\hbar g\left(a^{+m} R^{-}+a^{m} R^{+}\right)$.
Here $R^{ \pm}$and $R^{2}$ are the spin $1 / 2$ operators of the atom, $a$ and $a^{+}$ $a^{+}$are the usual photon operators of the quantized cavity mode, $\omega_{\mathrm{f}}$ and $\omega_{\mathrm{o}}$ are the frequences of the cavity mode and the atom, m . is the photon multiple and g is the coupling constant. The exact multiphoton-resonance $\omega_{0}=m \omega_{f}$ is assumed to occur.

We assume that the atom was initially in the excited state $1+>$ and the cavity field was in an arbitrary state $\rho_{\mathrm{F}}=$
$=\Sigma_{n}, p_{n n}{ }^{\prime}|n\rangle\left\langle n^{\prime}\right|$, i.e, the initial density matrix was
$\rho(0) \stackrel{A}{=}(1+\rangle\langle+|) \otimes \rho_{\mathrm{F}}=\sum_{\mathrm{nn}}, \mathrm{p}_{\mathrm{nn}},|+; \mathrm{n}\rangle\langle+; \mathrm{n}|$.
By solving the corresponding equations of the motion the wave functions $\exp \left[-\mathrm{iH}_{\mathrm{AF}}^{\mathrm{t}} / \mathrm{h}^{\prime}\right] \pm ; \mathrm{n}>$ in the interaction picture are found to be
$\exp \left[-i H_{A F} t / \hbar\right]\left|+; n>=\cos \left(\Omega_{n+m} t\right)\right|+; n>-i \sin \left(\Omega_{n+m}^{t}\right) \mid-; n+m>$,
$\exp \left[-i H_{A F} t / \hbar\right]|-; n\rangle=\cos \left(\Omega_{n} t\right)\left|-; n>-i \sin \left(\Omega_{n} t\right)\right|+; n-m>$,
where
$\Omega_{n}=g \sqrt{n(n-1) \ldots(n-m+1)}=g \sqrt{\frac{n!}{(n-m)!}}$
Then, from"the definition of the Heisenberg operators we get for the two-time dipole correlation function $\mathrm{D}(\mathrm{t}, r)$ the expression
$\mathrm{D}(\mathrm{t}, \tau) \equiv\left\langle\mathrm{R}^{+}(\mathrm{t}+\tau) \mathrm{R}^{-}(\mathrm{t})\right\rangle=\sum_{\mathrm{n}} \mathrm{p}_{\mathrm{nn}} \mathrm{D}_{\mathrm{n}}(\mathrm{t}, \tau)$,
where
$\mathrm{D}_{\mathrm{n}}(\mathrm{t}, \tau)=\frac{1}{4} \mathrm{e}^{\mathrm{i} \omega_{\mathrm{o}} \tau}\left\{\cos \left[2 \Omega_{\mathrm{n}+\mathrm{m}}^{\mathrm{t}}+\left(\Omega_{\mathrm{n}+\mathrm{m}}+\Omega_{\mathrm{n}}\right)_{\tau}\right]+\cos \left[2 \Omega_{\mathrm{n}+\mathrm{m}^{\mathrm{t}}}+\right.\right.$

$$
\begin{equation*}
\left.\left.+\left(\Omega_{\mathrm{n}+\mathrm{m}}-\Omega_{\mathrm{n}}^{\prime}\right)_{\tau}\right]+\cos \left[\left(\Omega_{\mathrm{n}+\mathrm{m}^{2}}+\Omega_{\mathrm{n}}\right)_{\tau}\right]+\cos \left[\left(\Omega_{\mathrm{n}+\mathrm{m}}-\Omega_{\mathrm{n}}^{\prime}\right) \tau\right]\right\} \tag{6}
\end{equation*}
$$

We proceed to the study of the emission spectrum of the atom. No source of relaxation has been assumed in the model. Therefore, the only source of line width is the width of the detector. Following Eberly and Wodkiewicz '13:/ we define the time-dependent spectrum of the multiphoton-induced fluorescence radiated into other modes $/ 7,10$ :/ as
$\mathrm{S}(\nu, \mathrm{T})=\Gamma \beta \int_{0}^{\mathrm{T}} \mathrm{d} \mathrm{t}_{1} \int_{0}^{\mathrm{T}} \mathrm{dt}_{2}<\mathrm{R}^{+}\left(\mathrm{t}_{1}\right) \mathrm{R}^{-}\left(\mathrm{t}_{2}\right)>\dot{x} \exp \left[-(\Gamma-\mathrm{i} \nu)\left(\mathrm{T}-\mathrm{t}_{1}\right)-(\Gamma+\mathrm{i} \nu)\left(\mathrm{T}^{\prime}-\mathrm{t}_{2}\right)\right]$,
where $\Gamma$ is the bandwidth of the detecting mechanism, $T$ is the time at which the spectrum is evaluated, and $\beta$ is a measure of fluorescence into other modes. Expression (7) can be transformed so that we have only to evaluate the time ordered correlation function $\mathrm{D}(\mathrm{t}, r), r>0$. Then, taking into account Eqs.(5) and (6), one can show that
$\mathrm{S}(\nu, \mathrm{T})=\sum_{\mathrm{n}} \mathrm{P}_{\mathrm{nn}} \mathrm{S}_{\mathrm{n}}(\nu, \mathrm{T})$,
where
$S_{n}(\nu, T)=2 \Gamma \beta \operatorname{Re} \int_{0}^{T} \mathrm{~d} \tau \mathrm{e}^{(\Gamma-\mathrm{i} \nu) \tau} \int_{0}^{\mathrm{T}-\tau} \mathrm{dt} \mathrm{e}^{-2 \Gamma(\mathrm{~T}-\mathrm{t})} \quad \mathrm{D}_{\mathrm{n}}(\mathrm{t}, \tau \cdot)=$
$=\frac{1}{4} \Gamma \beta R \sum_{\mathrm{k}, \mathrm{j}}\left(\eta_{\mathrm{j}}+2 \mathrm{i} \Gamma\right)^{-1} \times\left[\left(\nu-\omega_{\mathrm{o}}+\lambda_{\mathrm{k}}-\eta_{\mathrm{j}}-\mathrm{i} \Gamma\right)^{-1}\left(\mathrm{e}^{-\mathrm{i} \eta_{\mathrm{j}} \mathrm{T}}-\mathrm{e}^{\mathrm{i}\left(\nu \omega_{\mathrm{o}}+\lambda_{\mathrm{k}}-\mathrm{i} \Gamma\right) \mathrm{T}}\right)+\right.$
$\left.+\left(1-\omega_{0}+\lambda_{k}+i \Gamma\right)^{-1}\left(e^{-i\left(\nu-\omega_{0}+\lambda_{k}-i \Gamma\right) T}-e^{-2 \Gamma^{\prime} T}\right)\right]+$

$$
+\left(\nu-\omega_{0}\right) \rightarrow-\left(\nu-\omega_{0}\right),
$$

and
$\eta_{1}=0, \eta_{2}=2 \Omega_{n+m}, \quad \lambda_{1}=\Omega_{n+m}-\Omega_{n}, \quad \lambda_{2}=\Omega_{n+m}+\Omega_{n}$.
If we consider the long-time limit of (9): $\Gamma \mathrm{T} \gg 1$, and if we ignore the small oscillating terms '10/ corresponding to $\exp \left(-\mathrm{i} \eta_{2} \mathrm{~T}\right)$, then we get
$\mathrm{S}_{\mathrm{n}}(\nu, \mathrm{T} \rightarrow \infty)=\frac{\Gamma \beta / 8}{\Gamma^{2}+\left(\nu-\omega_{\mathrm{o}}+\Omega_{\mathrm{n}+\mathrm{m}}-\Omega_{\mathrm{n}}\right)^{2}}+\frac{\Gamma \beta / 8}{\Gamma^{2}+\left(\nu-\omega_{\mathrm{o}}+\Omega_{\mathrm{n}+\mathrm{m}}+\Omega_{\mathrm{n}}\right)^{2}}+\left(\nu-\omega_{\mathrm{o}}\right) \rightarrow$
$\rightarrow-\left(\nu-\omega_{\mathrm{o}}\right)$.

It is seen from Eqs.(8), (9) and (10) that if initially the field is in a Fock state $|\mathrm{n}\rangle$, then the fluorescence spectrum $\mathrm{S}(\nu, \mathrm{T})=\mathrm{S}_{\mathrm{n}}(\nu, \mathrm{T})$ consists of several lines whose positions and widths are determined by $\omega_{o} \pm\left(\Omega_{\mathrm{n}+\mathrm{m}} \pm \Omega_{\mathrm{n}}\right)$ and $\Gamma$. Below we consider two cases of general interest in the narrow-band detection limit.

Case (1) $n \leq m-1$. It is seen from Eqs. (4) that $\Omega_{n}=0$ in this case. The spectrum $\mathrm{S}_{\mathrm{n}}(\nu, \mathrm{T})$ then exhibits a doublet with peaks at $\nu=\omega_{\mathrm{o}} \pm \Omega_{\mathrm{n}+\mathrm{m}}$ if $\Gamma \ll \Omega_{\mathrm{n}+\mathrm{m}}$. Note that the particular case $\mathrm{n}=0$ corresponds to the spontaneous emission and hence our result is in agreement with the result of Sanchez-Mondragon et al. ${ }^{17 /}$ and Agarwal $/ 8 . /$ showing the doublet structure of the spontaneous emission spectrum in the situation with one-photon-resonance $m=1$. In the general situation with multiphoton resonance the separation of the two peaks at $\nu=\omega_{0} \pm \mathrm{g} \sqrt{\mathrm{m}!}$ of the spontaneous emission spectrum is just equal to the frequency $2 \mathrm{~g} \sqrt{\mathrm{~m}!}$ of the vacuum-field Rabi oscillations and is called the multiphoton-resonance vacuum-field Rabi splitting ${ }^{8 /}$. This splitting is reminiscent of the fluorescence line splitting predicted in the presence of an intense driving laser field by Mollow ${ }^{14: /)^{\prime}}$ and observed experimentally ${ }^{\prime 15-17!}$, but it is not the same because it occurs here in the absence of photons in the cavity mode.

Case (2): $n \gg m^{2}$. From the asymptotic expression
$\Omega_{n+m}-\Omega_{n} \approx \frac{1}{2} g^{2} n^{m / 2-1}$,
$\Omega_{n+m}+\Omega_{n} \approx 2 g\left(n^{m / 2}+\frac{1}{4} m n^{m / 2-1}\right), \quad n \gg m^{2}$,
we find that the spectrum $\mathrm{S}_{\mathrm{n}}(\nu, \mathrm{T})$ with $\mathrm{n} \gg \mathrm{m}^{2}$ has (2a) four peaks at

$$
\begin{equation*}
\nu=\omega_{0} \pm \frac{1}{2} \mathrm{gm}^{2} \mathrm{n}^{\mathrm{m} / 2-1}, \quad \omega_{0} \pm 2 g\left(\mathrm{n}^{\mathrm{m} / 2}+\frac{1}{4} \mathrm{mn}^{\mathrm{m} / 2-1}\right), \tag{13}
\end{equation*}
$$

if $\Gamma \ll \mathrm{gm}^{2} \mathrm{n}^{\mathrm{m} / 2-1}$
(2b) three peaks at

$$
\begin{equation*}
\nu=\omega_{0}, \quad \omega_{0} \pm 2 \mathrm{~g} \sqrt{\mathrm{n}}, \tag{14}
\end{equation*}
$$

if $m=1$ and $\sqrt{n} \gg g / \Gamma, \Gamma / g^{\prime / 8 /}$. If we consider the long-time limit, then we find from Eq.(11) that the four peaks in the case (2a) are equally intense, whereas the central peak in the
case (2b) is twice as intense as side peaks. The triplet obtained in the case (2b) is in fact the exact analog of the Mollow triplet of intense-laser line splitting 14,18 !

## 3. TWO-ATOM SYSTEM

In this section, we examine the effect of cooperativity on the fluorescence spectrum, in particular, we consider what happens to the spectrum owing to the presence of two excited atoms with multiphoton-transitions in a lossless resonant cavity.The two-atom generalization of the Hamiltonian (1) is given by
$\mathrm{H}=\mathrm{H}_{\mathrm{o}}+\mathrm{H}_{\mathrm{AF}}$
$H_{0}=\hbar \omega_{f} \mathbf{a}^{+} \mathbf{a}+\sum_{j=1}^{2} \hbar \omega_{o} R_{j}^{z} \equiv \hbar \omega_{f} \mathbf{a}^{+} \mathbf{a}+\hbar \omega_{o} R^{z}$,
$H_{A F}=\sum_{j=1}^{2} \hbar g\left(a^{+m} R_{j}^{-}+a^{m} R_{j}^{+}\right) \equiv \hbar g\left(a^{+m} R^{-}+a^{m} R^{+}\right)$,
where $R^{z,+,-}=\sum_{j=1}^{2} R_{j}^{z,+,-}$ are now spin-1 collective operators. We calculate the correlation function $\mathrm{D}(\mathrm{t}, \tau)$ and the spectrum $\mathrm{S}(\nu, \mathrm{T})$ assuming that each atom initially is in the excited state and that the field is in an arbitrary state $\rho_{\cdot}=\sum_{n n}, p_{n n} \times x$ $x|n\rangle\left\langle n^{\prime}\right|$. Let us denote the atomic spin-1 eigenstates $\mid R=1$, $M_{R}=0, \pm 1 \mid$ by $\left|S_{0, \pm 1}\right\rangle$, i.e.,
$\left|\mathrm{S}_{\mathrm{o}}\right\rangle=(|+,-\rangle+\mid-,+>) / \sqrt{2},\left|\mathrm{~S}_{1}\right\rangle=\mid+,+>$,
$\left|S_{-1}\right\rangle=\mid-,->$.
It follows from the structure of the Hamiltonian (15) that the time-dependent wave functions $\exp (-i H t / \hbar)\left|S_{1} ; n>, \exp (-i H t / \hbar) \times\right|$ $\times \mid S_{0} ; n+m>$ and $\exp (-i H t / \hbar) \mid S_{-1} ; n+2 m>$ are linear superpositions of the three basic states $\left|S_{1} ; n\right\rangle,\left|S_{0} ; n+m\right\rangle$ and $\mid S_{-1} ;$ $\mathrm{n}+2 \mathrm{~m}>$. The coefficients that appear in these superpositions are to be obtained from the solutions of the Schrödinger equations. The dipole correlation function $D(t, \tau)=\left\langle R^{+}(t+\tau) R^{-}(t)\right\rangle$ is calculated in terms of the above coefficients to read
$\mathrm{D}(\mathrm{t}, \tau)=\sum_{\mathrm{n}} \mathrm{p}_{\mathrm{nn}} \mathrm{D}_{\mathrm{n}}(\mathrm{t}, \tau)$,
$\mathrm{D}_{\mathrm{n}}(\mathrm{t}, \tau)=2 \mathrm{e}^{\mathrm{i} \omega_{\mathrm{o}} \tau}\left\{\alpha_{\mathrm{n}}(\mathrm{t}+\tau)\left\lceil\alpha_{\mathrm{n}}(\mathrm{t}) \mu_{\mathrm{n}}(\tau)-\beta_{\mathrm{n}}(\mathrm{t}) \chi_{\mathrm{n}}(\tau)\right\}+\right.$

$$
\begin{equation*}
\left.+\beta_{\mathrm{n}}(\mathrm{t}+\tau)\left[a_{\mathrm{n}}(\mathrm{t}) \chi_{\mathrm{n}}(\tau)+B_{\mathrm{n}}(\mathrm{t}) \phi_{\mathrm{n}}(\tau)\right]\right\} \tag{17b}
\end{equation*}
$$

Here the notation
$\alpha_{n}(t)=1-\frac{g^{2} q_{n+m}}{\vec{\Omega}_{n+m}^{2}} \sin ^{2}\left\lceil\vec{\Omega}_{n+m^{t}}\right\rceil$,
$\beta_{n}(t)=\frac{g \sqrt{q} \frac{n+m}{} / 2}{\tilde{\Omega}_{n+m}} \sin \left[2 \widetilde{\Omega}_{n+m}^{t}\right]$,
$x_{\mathrm{n}}(\mathrm{t})=\frac{\mathrm{g}{\sqrt{\Phi_{n+m}^{\prime}}}^{\prime 2}}{\tilde{\Omega}_{\mathrm{n}}} \sin \left[2 \tilde{\Omega}_{\mathrm{n}} \mathrm{t}\right]$,
$\mu_{n}(t)=\cos \left[2 \tilde{\Omega}_{n} t \mid\right.$.
$\phi_{n}(t)=1-\frac{g^{2} q_{n+m}}{\tilde{\Omega}_{n}^{2}} \sin ^{2}\left[\tilde{\Omega}_{n} t\right]$,
and
$q_{n}=n!/(n-m)!, \quad \vec{a}_{n}=g \sqrt{\left(q_{n}+q_{n+m}\right) / 2}$,
has been introduced. Note that $D_{n}(t, r)$ has the structure

Therefore, the time-dependent fluorescence spectrum defined by Eq. (7) has the form ' 10 '
$\mathrm{S}(\nu, \mathrm{T})=\sum_{\mathrm{n}} \mathrm{p}_{\mathrm{nn}} \mathrm{S}_{\mathrm{n}}\left({ }^{\prime}, \mathrm{T}\right)$,
$\mathrm{S}_{\mathrm{n}}(\nu, \mathrm{T})=2 \Gamma \beta \mathrm{Re} \underset{\mathrm{kj}}{\Sigma} \mathrm{A}_{\mathrm{kj}}\left(\tilde{\eta}_{\mathrm{j}}+2 \mathrm{i} \Gamma\right)^{-1}$,
$\left[\left(\nu-\omega_{\mathrm{o}}+\vec{\lambda}_{\mathrm{k}}-\tilde{\eta}_{\mathrm{j}}-\mathrm{i} \Gamma\right)^{-1}\left(\mathrm{e}^{-\mathrm{i} \tilde{\eta}_{\mathrm{j}} \mathrm{T}}-\mathrm{e}^{\mathrm{i}\left(\nu-\omega_{\mathrm{o}}+\vec{\lambda}_{\mathrm{k}}-\mathrm{i} \Gamma\right) \mathrm{T}}\right)+\left(\nu-\omega_{\mathrm{o}}+\lambda_{\mathrm{k}}+\mathrm{i} \Gamma\right)^{-1} \times\right.$
$\left.\times\left(\mathrm{e}^{-\mathrm{i}\left(\nu-\omega_{\mathrm{o}}+\tilde{\lambda}_{\mathrm{k}}-\mathrm{i} \Gamma\right) \mathrm{T}}-\mathrm{e}^{-2 \Gamma \mathrm{~T}}\right)\right]+\left(\nu-\omega_{\mathrm{o}}\right) \rightarrow-\left(\nu-\omega_{\mathrm{o}}\right)$.

Here $\mathrm{S}_{\mathrm{n}}(\nu, \mathrm{T})$ is the collective fluorescence spectrum of the two-atom system in the situation when the cavity field is initially in the Fock state $|\mathrm{n}\rangle$. Using Eqs. (17b), (18), (20) and (21b) one can show that in the long time limit the spectrum $\mathrm{S}_{\mathrm{n}}(\nu, \mathrm{T} \rightarrow \infty)$ is explicitly given by

$$
\begin{align*}
& \mathrm{S}_{\mathrm{n}}(\nu, \mathrm{~T} \rightarrow \infty)=\frac{\mathrm{g}^{4} \mathrm{q}_{\mathrm{n}+2 \mathrm{~m}}^{2}}{4 \tilde{\Omega}_{\mathrm{n}+\mathrm{m}}^{4}} \frac{\Gamma \beta}{\Gamma^{2}+\left(\nu-\omega_{\mathrm{o}}-2 \tilde{\Omega}_{\mathrm{n}}\right)^{2}}+\frac{\mathrm{g}^{4} \mathrm{q}_{\mathrm{n}} \mathrm{q}_{\mathrm{n}+\mathrm{m}}}{8 \tilde{\Omega}_{\mathrm{n}}^{2} \tilde{\Omega}_{\mathrm{n}+\mathrm{m}}^{2}} \times \\
& \times \frac{\Gamma \beta}{\Gamma^{2}+\left(\nu-\omega_{\mathrm{o}}-2 \tilde{\Omega}_{\mathrm{n}+\mathrm{m}}\right)^{2}}+\frac{\mathrm{g}^{4} \mathrm{q}_{\mathrm{n}+\mathrm{m}}^{2}}{16 \tilde{\Omega}_{\mathrm{n}+\mathrm{m}}^{2}}\left(\frac{1}{\tilde{\Omega}_{\mathrm{n}}}-\frac{1}{\tilde{\Omega}_{\mathrm{n}+\mathrm{m}}}\right)^{2} \times  \tag{22}\\
& \times \frac{\Gamma \beta}{\Gamma^{2}+\left(\nu-\omega_{\mathrm{o}}-2 \tilde{\Omega}_{\mathrm{n}+\mathrm{m}}-2 \tilde{\Omega}_{\mathrm{n}}\right)^{2}}+\frac{\mathrm{g}^{4} \mathrm{q}_{\mathrm{n}+\mathrm{m}}^{2}}{16 \tilde{\Omega}_{\mathrm{n}+\mathrm{m}}^{2}}\left(\frac{1}{\tilde{\Omega}_{\mathrm{n}}}+\frac{1}{\tilde{\Omega}_{\mathrm{n}+\mathrm{m}}}\right)^{2} \times \\
& \times \frac{\Gamma \beta}{\Gamma^{2}+\left(\nu-\omega_{\mathrm{o}}-2 \tilde{\Omega}_{\mathrm{n}+\mathrm{m}}+2 \tilde{\Omega}_{\mathrm{n}}\right)^{2}}+\left(\nu-\omega_{\mathrm{o}}\right),-\left(v-\omega_{\mathrm{o}}\right) .
\end{align*}
$$

This spectrum will have as many as from 1 to 8 splittings depending on the resolution of the following frequencies:

$$
\begin{align*}
& v_{ \pm 1}-\omega_{0} \pm 2\left(\tilde{\Omega}_{n+m}-\tilde{\Omega}_{n}\right), \quad \nu_{ \pm 2}=\omega_{0} \pm 2 \tilde{\Omega}_{n}, \\
& \prime_{ \pm 3}=\omega_{0} \pm 2 \tilde{\Omega}_{n \cdot m}, \quad \nu_{ \pm 4}=\omega_{0} \pm 2\left(\tilde{\Omega}_{n}+m+\tilde{\Omega}_{n}\right) . \tag{23}
\end{align*}
$$

The line widths are defined by the width [' of the detector. Note that the expressions of $\nu_{ \pm 4}$ in Eqs. (23) describe the quantum electrodynamic analog of the so-called cooperative additional sidebands '19-23'. The existence of these higher harmonics in the vacuum and weak cavity fields has recently been shown by Agarwal ${ }^{\prime 8}$ ' and Cheltsov $/ 11^{\prime}$. Now, we consider in detail two cases of general interest in the narrow-band detection limit.

Case (1): $n \leq m-1$. In.this case, $q_{n}$, and therefore the second term in the right-hand side of Eq.(22), are equal to zero. The spectrum (22) then has six peaks at $\nu=\omega_{0} \pm 2 \Omega_{n}, \omega_{o} \pm$ $\pm 2\left(\tilde{\Omega}_{\mathrm{n}+\mathrm{m}} \pm \widetilde{\Omega}_{\mathrm{n}}\right)$ if $\Gamma \ll 2\left(\tilde{\Omega}_{\mathrm{n}+\mathrm{m}}-\widetilde{\Omega}_{\mathrm{n}}\right)$. In particular, we find that the spontaneous emission spectrum $\mathrm{S}_{\mathrm{n}=\mathrm{o}}(\nu, \mathrm{T} \rightarrow \infty)$ peaks at the frequencies
$\nu_{ \pm 1}=\omega_{0} \pm \mathrm{g}\{\sqrt{2[m!+(2 m)!/ m!]}-\sqrt{2(m!)}\}$,
$\nu_{ \pm 2}=\omega_{0} \pm \mathrm{g} \sqrt{2(\mathrm{~m}!)}$,
$\nu_{ \pm 4}=\omega_{0} \pm g\{\sqrt{2[m!+(2 m)!/ m!}+\sqrt{2(m!)}\}$.

The relations between the heights of the peaks (24a), (24b) and (24c) are
$I_{ \pm 1}: I_{ \pm 2}: I_{ \pm 4}=\left[\sqrt{m!+\frac{(2 m)!}{m!}}+\sqrt{m!}\right]^{2}: \frac{4}{m!}\left[\frac{(2 m)!}{m!}\right]^{2}:\left[\sqrt{m!+\frac{(2 m)!}{m!}}-\sqrt{m!}\right]^{2}$.

When $m=1$ these expressions reduce to those obtained by Agarwal ${ }^{\prime} 8: /$ for the vacuum-field Rabi splittings of the spectrum of a two-atom system with one-photon resonance.

Case (2): $n \gg m^{2}$. From Eqs.(19) we get the following asymptotic expressions:
$q_{n} \approx q_{n+m} \approx q_{n+2 m} \approx n^{m}, \quad \tilde{\Omega}_{n} \approx g\left(n^{m / 2}+\frac{1}{4} m^{m / 2-1}\right)$,
$\tilde{\Omega}_{\mathrm{n}+\mathrm{m}} \approx \mathrm{g}\left[\mathrm{n}^{\mathrm{m} / 2}+\frac{1}{4} \mathrm{~m}(2 \mathrm{~m}+1) \mathrm{n}^{\mathrm{m} / 2-1}\right]$,
for $\mathrm{n} \gg \mathrm{m}^{2}$. Therefore, the spectrum $\mathrm{S}_{\mathrm{n}}(\nu, \mathrm{T} \rightarrow \infty)$ with $\mathrm{n} \gg \mathrm{m}^{2}$ will have eight peaks at the firequencies
$\nu_{ \pm 1} \approx \omega_{0} \pm \mathrm{gm}^{2} \mathrm{n}^{\mathrm{m} / 2-1}$,
$\nu_{ \pm 2} \approx \omega_{0} \pm 2 \mathrm{~g}\left(\mathrm{n}^{\mathrm{m} / 2}+\frac{1}{4} \mathrm{~m} \mathrm{n}^{\mathrm{m} / 2-1}\right)$,
$\nu_{ \pm 3}=\omega_{0} \pm 2 \mathrm{~g}\left[\mathrm{n}^{\mathrm{m} / 2}+\frac{1}{4} \mathrm{~m}(2 \mathrm{~m}+1) \mathrm{n}^{\mathrm{m} / 2-1}\right]$,
$\nu_{ \pm 4} \approx \omega_{0} \pm 4 g\left[n^{m / 2}+\frac{1}{4} m(m+1) n^{m / 2-1}\right]$,
if
$\Gamma \ll \mathrm{gm}^{2} \mathrm{n}^{\mathrm{m} / 2-1}$.

The relations between the heights of these peaks are approximately found from Eqs. (22) and (26) to be
$I_{ \pm 1}: I_{ \pm 2}: I_{ \pm 3}: I_{ \pm 4} \approx 2: 2: 1:\left(\mathrm{m}^{4} / 8 \mathrm{n}^{2}\right)$.
Since $m^{4} / 8 n^{2} \ll 1$, the extreme side peaks $\nu_{ \pm 4}$ are very weak compared with the other peaks. For the case $\Gamma \sim \mathrm{gm}^{2} \mathrm{n}^{\mathrm{m} / 2-1}$, it is possible that instead of the two peaks $\nu_{ \pm 1}$ one gets the new central peak $\nu_{o}=\omega_{0}$. The number of peaks then is seven. Increasing the detection bandwidth $\Gamma$, we may decrease the number of spectrum peaks up to 1 . It should be noted here that the nar-row-band detection condition (28) implies, in the case $m \geq 3$, the requirement of a strong cavity field, but in the case $m=$ $=1$ it restricts the field intensity. In the case of one-photon resonance and strong cavity field: $m=1 ; \sqrt{n} \gg \mathrm{~g} / \Gamma, \Gamma / \mathrm{g}$, the spectrum $\mathrm{S}_{\mathrm{n}}(\nu, \mathrm{T} \rightarrow \infty)$ has only five peaks at the frequencies
$\nu_{\mathrm{o}}=\omega_{\mathrm{o}}, \quad \nu_{ \pm 2}=\omega_{\mathrm{o}} \pm 2 \mathrm{~g} \sqrt{\mathrm{n}}, \quad \nu_{ \pm 4}=\omega_{\mathrm{o}} \pm 4 \mathrm{~g} \sqrt{\mathrm{n}}$,
because of $\nu_{ \pm 1} \rightarrow \nu_{0}, \nu_{ \pm 3} \rightarrow, \nu_{ \pm 2}$. The relations between the heights of these peaks now become
$I_{o}: I_{ \pm 2}: I_{ \pm 4}=4: 3: \frac{1}{8 n^{2}}$.
If $n$ is very large so that the extreme sidebands $\nu_{ \pm 4}=\omega_{0} \pm 4 g \sqrt{n}$ can be neglected, then the spectrum $\mathrm{S}_{\mathrm{n}}(\nu, \mathrm{T} \rightarrow \infty)$ will have a three-peak structure. The results expressed by Eqs.(30) and (31), and in particular, the decrease of the heights of the cooperative sidebands $\nu_{ \pm 4}$ as $1 / \mathrm{n}^{2}$ as $\mathrm{n} \rightarrow \infty$ are in agreement with the results of Cheltsov :/11:/ having applied a different method.

Finally, we emphasize that for an arbitrary state of the cavity field characterized by the photon distribution $p_{n n}$, one has to average the spectra $\mathrm{S}_{\mathrm{n}}(\nu, \mathrm{T})$ in Eqs. (9) and (21b) with respect to the distribution $p_{n n}$. All the spectral characteristics will essentially depend on the statistical properties of $p_{n n}$. In particular, the line widths, and consequently the peak number will be determined not only by $\Gamma$ but also by the dispersion of this distribution.

## 4. SUMMARY

We have investigated the emission from one and two two-1evel atoms in an ideal cavity with one mode at multiphoton re-
sonance. The exact quantum electrodynamic results for the twotime dipole correlation functions and the time-dependent spectra of multiphoton-induced fluorescence have been obtained. Here, we have examined the general situation with an arbitrary state of the quantised cavity mode. The multiphoton-resonance vacuum-field Rabi splittings have been calculated. The structure of the spectra in the situation with a Fock state of the cavity field has also been studied. The side peaks at the higher harmonics $\nu_{ \pm 4}$ of the collective spectrum $\mathrm{S}_{\mathrm{n}}(\nu, \mathrm{T} \rightarrow \infty)$ have been shown to exist in vacuum and weak cavity fields, their intensities tending to zero as $\mathrm{m}^{4} / \mathrm{n}^{2}$ as $\mathrm{n} \rightarrow \infty$. Out results clearly show the complications of the spectra caused by the quantum discrete nature of the cavity field, the photon multiplicity of resonance and the atomic cooperativity.

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