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дубна

E4-85-953

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CROSS SECTIONS FOR SCATTERING OF DEUTERIUM MESIC ATOMS ON DEUTERIUM NUCLEI

Submitted to "Physics Letters A"

1985

1. To describe kinetics of muon catalyzed fusion which has attracted much interest recently one must know cross sections and rates of various processes involving mesic atoms (cf., e.g., ref. 1).

In this paper cross sections σ^S_{ij} for reaction

$$\begin{array}{ccc} d\mu(F=3/2)+d & \xrightarrow{\sigma^S_{22}} & d\mu(F=3/2)+d \\ & \sigma^S_{21} \nearrow \quad \searrow \sigma^S_{12} & \\ d\mu(F=1/2)+d & \xrightarrow{\sigma^S_{11}} & d\mu(F=1/2)+d \end{array} \qquad (1)$$

are calculated. Here σ^S_{11} and σ^S_{22} are elastic-scattering cross sections of $d\mu$-atoms in para and ortho states (F=1/2 and F=3/2, respectively) on deuterium nuclei; σ^S_{12} and σ^S_{21} are cross sections for transitions between those states; S is the total spin of the $d\mu + d$ system; F is the spin of the $d\mu$-atom. Considered energy region is $\mathcal{E} \leq 50$ eV*). The ground states of $d\mu$-atoms are only considered, i.e., two lower reaction channels can be open: channel 1 with the $d\mu$-atom spin F=1/2 and channel 2 with F=3/2, the gap between their thresholds being equal to the hyperfine splitting energy of the $d\mu$-atom $\Delta\mathcal{E}^{hfs} = 0.049$ eV.

When describing reaction (1) we took into account the exchange interaction where the total spin $\vec{S} = \vec{F} + \vec{S}_d$ was conserved. The spin may take values S=1/2, 3/2 for the transitions $F=3/2 \rightleftarrows F=1/2$. Spin-spin and spin-orbital interactions which change S and whose values are several orders of magnitude lower than that of the exchange interaction /2/ are not considered here. Therefore the elastic scattering ($F=3/2 \to F=3/2$) is only possible for the S=5/2 state.

Earlier /2-4/ reaction (1) was studied within the limits of low collision energy $\mathcal{E} \lesssim \Delta\mathcal{E}^{hfs}$.

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*) The initial energy of deuterium mesic atoms produced in the ground state does not exceed ~ 2 eV for D_2 and D_2+T_2 mixture and ~ 50 eV for D_2+H_2.

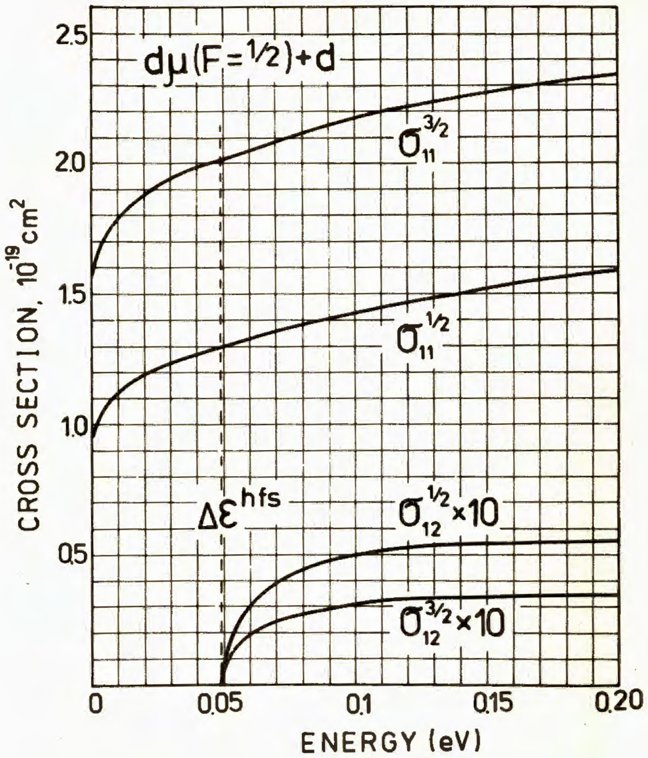


Fig. 1. Cross sections σ^S_{11} for elastic scattering of $d\mu$-atoms in the para state (F=1/2) on deuterium nuclei and for para-ortho transition σ^S_{12} . Results for low collision energies (S-wave only) are shown.

2. The cross sections have been calculated in the adiabatic representation /5/ where the problem of slow collisions in a three-body system is reduced to the multichannel scattering problem /6,7/ with a great number of closed channels, N- $\nu \sim 100$:

$$\frac{d^2}{dR^2} y_i^{(\gamma)}(R) + \left(k_i^2 - \frac{J(J+1)}{R^2}\right) y_i^{(\gamma)}(R) - \sum_{j\neq i}^{N} U^S_{ij}(R)\, y_j^{(\gamma)}(R) = 0 \qquad (2)$$

$i = 1,\dots,N$; $\gamma = 1,\dots,\nu$; $\nu = 1,2$ is the number of open channels. Here k_i is the momentum in the channel ($k_i^2 \geq 0$ at $i \leq \nu$ and $k_i^2 < 0$ at $i > \nu$); J is the angular momentum of the system; the wave

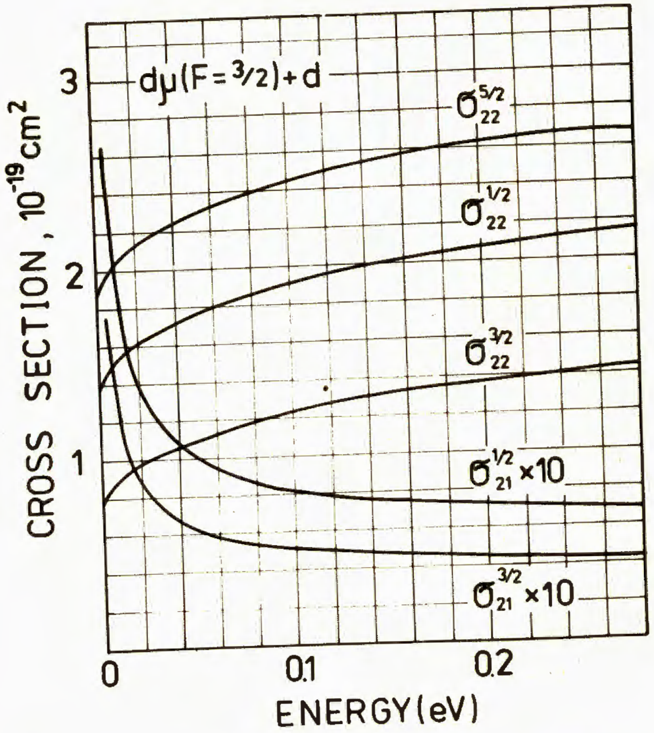


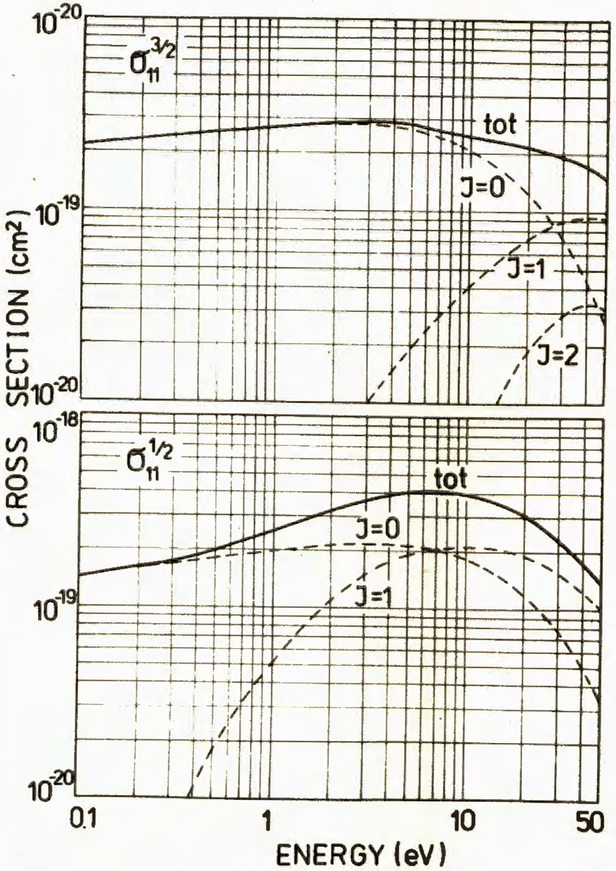
Fig. 2. Elastic scattering s-wave cross sections σ^S_{22} of ortho-$d\mu$-atoms (F=3/2) and for ortho-para transitions σ^S_{21} in the low energy region.

functions $y_1^{(\gamma)}(R) \equiv y^{(\gamma)}_{1s\sigma, F=1/2}(R)$, $y_2^{(\gamma)}(R) \equiv y^{(\gamma)}_{1s\sigma, 3/2}(R)$, ... of the relative nuclear motion are regular at R=0 and correspond to subsystems $d\mu_{1s}$ (F=1/2)+d and $d\mu_{1s}$ (F=3/2)+d in the asymptotic region $R \to \infty$ where they have the form:

$$\left\{ y_i^{(\gamma)}(R) \right\} = \begin{Bmatrix} y_1^{(1)} & \dots & y_1^{(\nu)} \\ \vdots & & \vdots \\ y_N^{(1)} & \dots & y_N^{(\nu)} \end{Bmatrix} = \begin{cases} \hat{j}_J(R) - \hat{n}_J(R)\hat{T} & \text{for } i \leq \nu \\ \exp(-|k_i|R) & \text{for } i > \nu . \end{cases} \qquad (2a)$$

Here $\hat{j}_J(R) = j_J(k_i R)\delta_{ij}$, $\hat{n}_J(R) = n_J(k_i R)\delta_{ij}$ and $j_J(k_i R), n_J(k_i R)$ are Bessel's spherical functions; $\hat{T}$ is the reaction matrix to be found. Potentials $U^S_{ij}(R)$ are formed of the effective potentials of

Fig. 3. Total and partial (s,p,d-waves) elastic scattering cross sections σ_{11}^{S} of para-$d\mu$-atoms for $0.1 \le \varepsilon \le 50$ eV.



the three-body problem $U_{ij}(R)$ /5/. The spin is introduced according to the rules /2/ by means of the transformations A(S,J):

$$U^S = A(S,J)\,U\,A^{-1}(S,J),\ \text{where}\ U = \begin{pmatrix} U_{igjg}, U_{igju} \\ U_{iujg}, U_{iuju} \end{pmatrix}$$

$$A(1/2; J=0,2) = \frac{1}{\sqrt{3}}\begin{pmatrix} -1 & \sqrt{2} \\ \sqrt{2} & 1 \end{pmatrix};\quad A(3/2; J=0,2) = \frac{1}{\sqrt{6}}\begin{pmatrix} -\sqrt{5} & 1 \\ 1 & \sqrt{5} \end{pmatrix}$$

$$A(1/2; J=1) = \frac{1}{\sqrt{3}}\begin{pmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{pmatrix};\quad A(3/2; J=1) = \frac{1}{\sqrt{6}}\begin{pmatrix} 1 & \sqrt{5} \\ -\sqrt{5} & 1 \end{pmatrix} \qquad (3)$$

$$U^{5/2} = \begin{cases} U_{igjg} & \text{for } J=0,2 \\ U_{iuju} & \text{for } J=1. \end{cases}$$

Here we take into account the wave-functions symmetry of three spin particles $d\mu + d$ and their spin-spin contact interaction /2/.

The method from ref. 7 is used to solve the problem (2)-(3). The matrix cross section

$$\sigma_{ij}^{S} = \frac{\pi}{k_i^2}\left|\delta_{ij} - \{(1+i\hat{T})(1-i\hat{T})^{-1}\}_{ij}\right|^2 \qquad (4)$$

is found by the calculated reaction matrix T. Preliminary results of calculations were given in ref. 8.

3. In Figs 1-5 one can see the results of calculations of cross sections σ_{ij}^{S} versus collision energy $\varepsilon = k^2/2M$ (M is the reduced mass of $d\mu + d$ system). Cross sections for elastic scattering of $d\mu$-atoms in para states σ_{11}^{S} and for transitions to ortho states σ_{12}^{S} are shown in Fig. 1 for $\varepsilon \le$ 0.2 eV. Analogous quantities σ_{22}^{S}, σ_{21}^{S} for ortho state are presented in Fig. 2 for $\varepsilon \le 0.28$ eV. The S-wave cross sections are only shown for possible values of total spin S, as the contribution of higher partial waves can be neglected for the considered energy region (except for σ_{21}^{S} case shown more completely in Fig. 5 for $\varepsilon \ge 0.1$ eV). Noteworthy is the threshold behaviour at $\varepsilon = \Delta\varepsilon^{hfs} = 0.049$ eV (Fig. 1).

In the limit $\varepsilon \to 0$ the T-matrix for J=0 can be presented in the form $T_{ij} = -a_{ij} \cdot \sqrt{k_i k_j}$ via introducing the scattering lengths a_{ij}. In the Table we present our best values of scattering lengths obtained at N=92 and the results at N=2 for comparison with the results of previous calculations.

Table. Scattering lengths for $d\mu + d \to d\mu + d$ processes (in units $a_\mu = 2.56 \cdot 10^{-11}$ cm)

S	N	a_{11}			$a_{12} = a_{21}$			a_{22}		
		this work	ref.3 a)	ref.4 b)	this work	ref.3 a)	ref.4 b)	this work	ref.3 a)	ref.4 b)
1/2	92	3.40			-0.95			4.07		
	2	3.76	3.5	3.61	-1.09	-1.1	-0.92	4.53	4.7	4.26
3/2	92	4.40			-0.75			3.06		
	2	4.92	5.1	4.58	-0.86	-0.86	-0.73	3.38	3.5	3.27
5/2	92							4.74		

a) two-level approximation of adiabatic approach by the method of phase functions

b) improved two-level approximation.

The known formula $k_i/T_{ii} = -1/a_{ii} + 3\pi M/(2a_{ii}^2)\cdot k_i^2 + 3M/a_{ii}\,\ln(9Mk_i^2/32)$

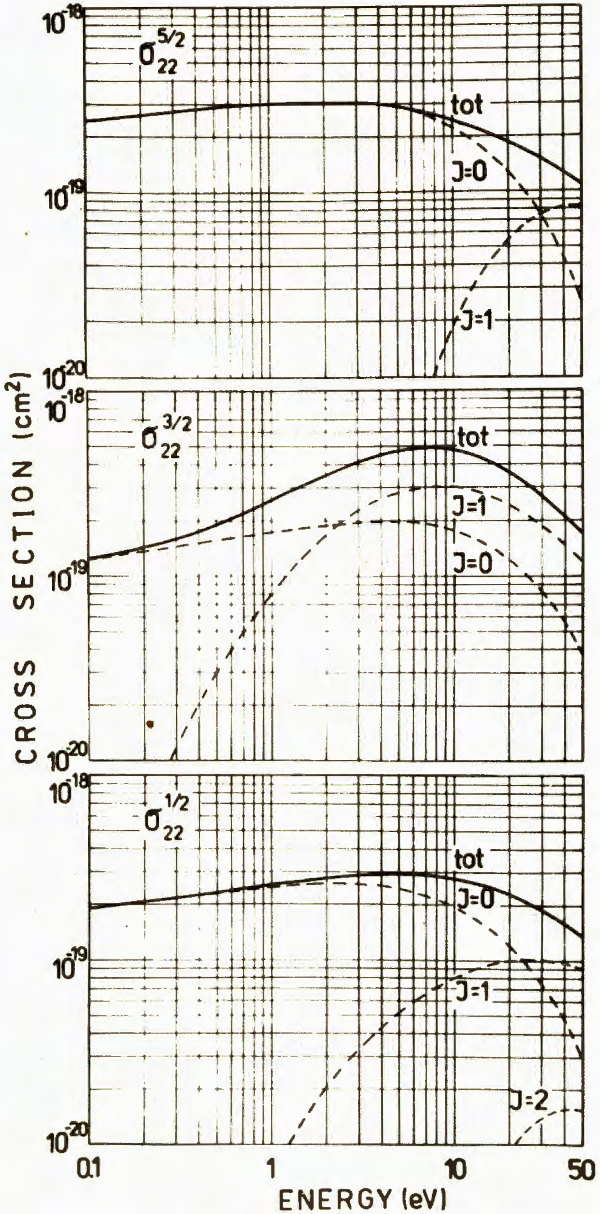


Fig. 4. Total and partial (s,p,d-waves) elastic scattering cross sections $\sigma^{s}_{22}(\ )$ of ortho- $d\mu$ -atoms for $0.1 \leq \varepsilon \leq 50$ eV.

can be used to obtain , with an accuracy $\leq$ 2%, the diagonal elements of T-matrix at $\varepsilon \lesssim 0.04$ eV.

Figs 3-5 show the total as well as partial (s, p and d-wave) cross sections for collision energies $0.1 \leq \varepsilon \leq 50$ eV. For the energies 25 eV the cross sections reach a unitary limit for J=0, since phase-shifts $\delta_{11} = \arctan T_{11}$ jump as much as π there. A similar behaviour of σ^{s}_{ij} (J=1) is observed at $\sim$ 35 eV.

Now one can use the results obtained for σ^{s}_{ij} to determine total cross sections of $d\mu$ -atoms for any known population of hf-states.

Accuracy of the calculations of σ^{s}_{ij} is $\lesssim$ 5% for the energy region $\varepsilon \lesssim 1$ eV and is a little worse ($\sim$10%) for $\varepsilon \gtrsim 2$ eV. It is determined by the errors of numerical integration of the system of equations (2) and by convergence of solution in the number N of functions used in expanding the wave function of the three-body system in the basis of solutions of a two-centre problem /7/. The numerical study has shown that the prescribed accuracy is achieved at N=92 (J=0) or N=136 (J=1,2) and for the set of nodes over R: 0.1(0.1)20(1)R_{max} used for numerical integration. The quadrature formulae of O(h^2) order have been used, and R_{max} has been chosen to achieve the assumed accuracy.

4. Using the cross sections σ^{s}_{12} and σ^{s}_{21} we have calculated the temperature dependence of the rates of ortho-para transitions of $d\mu$-atoms:

$$\langle \lambda_{21} \rangle = \frac{1}{6} \int_0^\infty (\lambda^{1/2}_{21} + 2\lambda^{3/2}_{21})\, \rho(\varepsilon, T)\, d\varepsilon$$

$$\langle \lambda_{12} \rangle = \frac{1}{3} \int_0^\infty (\lambda^{1/2}_{12} + 2\lambda_{12})\, \rho(\varepsilon, T)\, d\varepsilon$$

$$\lambda_{hf} = \langle \lambda_{21} \rangle - \tfrac{1}{2} \langle \lambda_{12} \rangle ,$$

where $\lambda^{s}_{ij} = N_0 v \sigma^{s}_{ij}$, $N_0 = 4.25 \cdot 10^{22} cm^{-3}$ is the liquid hydrogen density, $\rho(\varepsilon, T)$ is the Maxwellian distribution of $d\mu$ -atoms at temperature T. The calculations have been made under the assumption of the statistical distribution of F-states ($n_{F=3/2} : n_{F=1/2} = 2$). The results are shown in Fig. 6 together with the experimental value at T = 34 K /9/. An interesting behaviour of ortho-to-para transition in a deuterium target is the decrease of effective rate λ_{hf}(T) with growing temperature. This is due to the fact that $\Delta\varepsilon^{hfs}$ is of the same order of magnitude as the Maxwellian distribution width.

5. The results obtained can be employed for the calculation of kinetics of μ-atomic and μ-molecular processes in hydrogen isotope mixture.

However, when comparing the calculated elastic cross sections of mesic atoms with the experimental ones /10/, one should take into

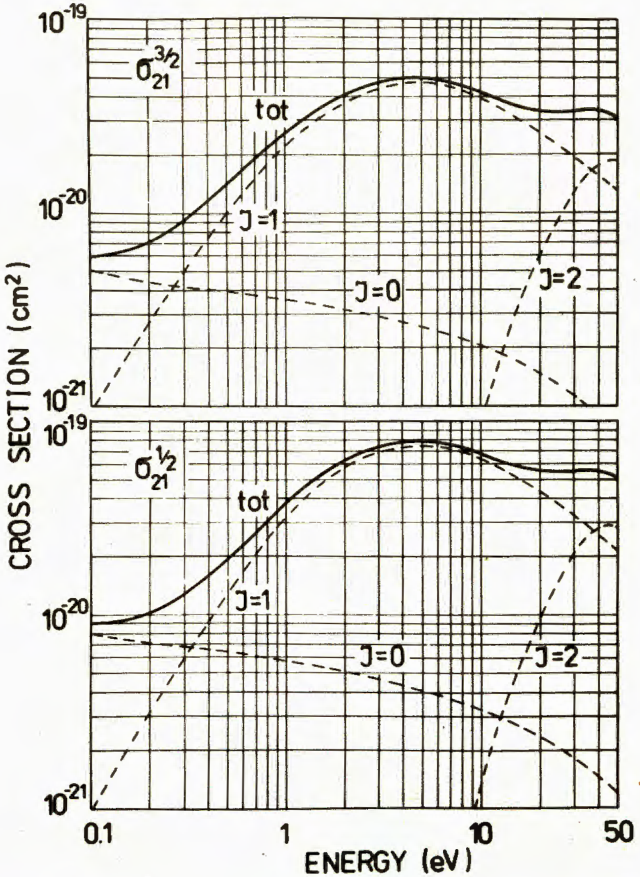


Fig. 5. Total and partial (s, p, d-waves) cross sections $\sigma^{s}_{21}(\varepsilon)$ for ortho-para transitions. Cross sections $\sigma^{s}_{12}(\varepsilon)$ for para-ortho transition can be obtained by using relations: $\sigma_{12}(\varepsilon) = (1 - \Delta\varepsilon^{hfs}/\varepsilon)\,\sigma_{21}(\varepsilon')$; $\varepsilon = \varepsilon' + \Delta\varepsilon^{hfs}$.

account the electron screening /11/ and the molecular structure effects. They can significantly affect the cross sections at low collision energies $\varepsilon \lesssim 0.1$ eV. On the other hand, their influence on the ortho-para transitions is insignificant, as such transition processes occur in the region of mesic-atom interactions with the nuclei, i.e. at distances $R \sim 2\text{-}3 \cdot a_{\mu}$, where the electron screening effects are weak. The obtained temperature dependence of the spin-

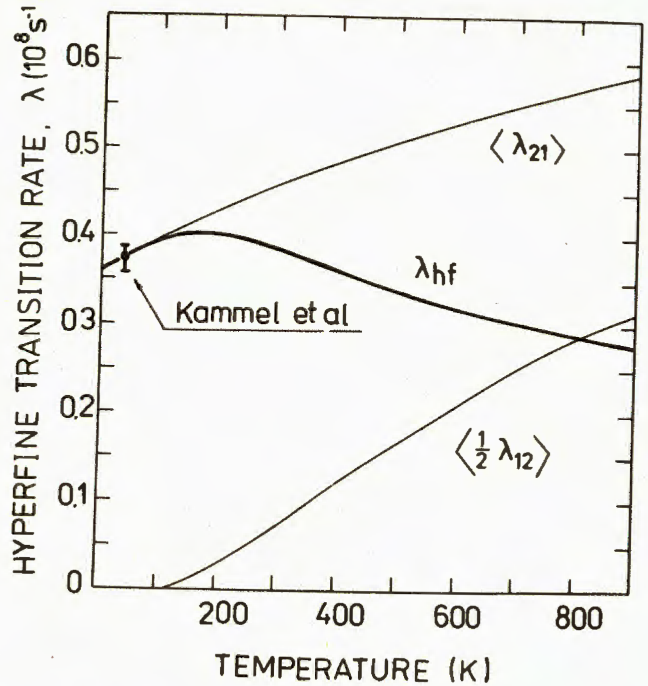


Fig. 6. Temperature dependence of the Maxwell averaged ortho-para and para-ortho transition rates (λ_{21} and λ_{12} , respectively) in a deuterium target. Effective spin flip-rate $\lambda_{hf}(T)$ (assuming the population ratio of hf-states $n_{F=3/2} : n_{F=1/2} = 2$) as well as the experimental value at T=34K /10/ are also shown. Results are normalized to the liquid hydrogen density.

flip rate for $d\mu$-atoms can be measured since it appears in the experimentally accessible region.

The authors express their gratitude to Professor L.I.Ponomarev for his interest in the work and useful discussions; to Drs. M.P. Faifman, S.I.Vinitsky and A.Adamczak for help and discussions; to Professor V.P.Dzhelepov and Professor V.G.Zinov for support.

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Received by Publishing Department
on December 27, 1985.

Вниманию организаций и лиц, заинтересованных в получении публикаций Объединенного института ядерных исследований

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Сечения рассеяния мезоатомов дейтерия на ядрах дейтерия

Вычислены сечения упругого рассеяния мезоатомов дейтерия в орто- и парасостояниях на ядрах дейтерия в интервале энергий 0-50 эВ, а также сечения переворота спина мезоатома в таких столкновениях. Расчеты выполнены в адиабатическом представлении, в котором исходная задача о медленных столкновениях в системе трех частиц сведена к многоканальной задаче рассеяния. Получена температурная зависимость скорости орто-пара переходов $\lambda_{hf}(T)$ мезоатомов в дейтериевой мишени. Вычисленная скорость $37{,}2\cdot 10^{6}$ $с^{-1}$ при 34 К хорошо согласуется с экспериментальным значением λ_{exp} (34 К)=37,0±0,74·10 $с^{-1}$ (группа Вена-SIN, 1983г.).

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1985

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Cross Sections for Scattering of Deuterium Mesic Atoms on Deuterium Nuclei

Cross sections have been calculated for elastic scattering of deuterium mesic atoms in ortho and para states on deuterium nuclei in the 0÷50 eV energy range; cross sections for spin-flip have also been calculated. The calculations have been performed in the adiabatic representation where the initial problem of slow collisions in a three-body system is reduced to the multichannel scattering problem. The temperature dependence of the ortho-para transition rate $\lambda_{hf}(T)$ for $d\mu$-atoms is obtained for a deuterium target. The calculated rate $\lambda_{hf}(34K) = 37.2\cdot 10^{6}\ s^{-1}$ is in a good agreement with the recent experimental value, λ_{exp} (34K) = $(37.0 \pm 0.74)\cdot 10^{6} s^{-1}$ (Vienna-SIN group, 1983).

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1985